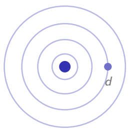
Astronomy and Cosmology

A2 Physics ▪ Topic 25

A-Level Physics



What we will cover

1 Luminosity and flux

Total power, then power per unit area

Luminosity 光度 L is the total power a star radiates in all directions. Unit: W. At distance d that power has spread over a sphere $4\pi d^2$, so the **radiant flux intensity** 辐射通量密度 is

$$F = \frac{L}{4\pi d^2}$$

Inverse-square: doubling d cuts the flux to a quarter. A telescope measures F; if L is known,

$$d = \sqrt{\frac{L}{4\pi F}}$$

Worked example – flux at the Earth

Sun: $L = 3.8 \times 10^{26}\,\mathrm{W}$, $d = 1.5 \times 10^{11}\,\mathrm{m}$

$$F = \frac{L}{4\pi d^2} = \frac{3.8 \times 10^{26}}{4\pi(1.5 \times 10^{11})^2}$$

$$\approx 1.4 \times 10^{3}\,\mathrm{W\,m^{-2}}$$

2 Standard candles

An object whose luminosity is known from its type

Find one in a distant galaxy, measure F, get $d = \sqrt{L/(4\pi F)}$. Reaches far beyond **parallax** 视差.

Cepheids
造父变星
pulsation period is tightly linked to luminosity – the period gives L

Type Ia supernovae
a white dwarf at critical mass always has about the same peak luminosity

Each rung calibrates the next

Parallax reaches the nearest stars; Cepheids reach nearby galaxies; supernovae reach the far ones. Every rung is calibrated by the one below it.

parallax (nearby stars) → calibrates → standard candles (galaxies) → calibrates → Hubble's law (very distant)

each rung is calibrated by the one below, reaching ever further

3 Stellar temperature

Wien – hotter means bluer

A blackbody spectrum peaks at $\lambda_{\max}$ set by temperature:

$$\lambda_{\max} T = b \approx 2.90 \times 10^{-3}\,\mathrm{m\,K}$$

Cool red star $\sim 3000\,\mathrm{K}$ peaks in the infrared; the Sun $\sim 5800\,\mathrm{K}$ near $500\,\mathrm{nm}$; a hot blue-white star $\sim 20\,000\,\mathrm{K}$ in the ultraviolet.

$\lambda_{\max} = 500\,\mathrm{nm}$

$T = b/\lambda_{\max} = 2.90 \times 10^{-3}/(500 \times 10^{-9}) \approx 5800\,\mathrm{K}$

Stefan–Boltzmann – $L \propto r^2 T^4$

A star as a blackbody sphere:

$$L = 4\pi\sigma r^2 T^4, \qquad \sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$$

Twice the radius $\Rightarrow$ four times L. Twice the temperature $\Rightarrow$ sixteen times L. To estimate a radius: $\lambda_{\max} \to T$ (Wien); F and $d \to L$; then

$$r = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

Estimating a star's radius

Wien, then Stefan–Boltzmann

1. $\lambda_{\max} \Rightarrow T$ $(\lambda_{\max} T = 2.9 \times 10^{-3}\,\mathrm{m\,K})$
2. $L = 4\pi d^2 F$ from flux and distance
3. $L = 4\pi r^2 \sigma T^4 \Rightarrow r = \sqrt{\dfrac{L}{4\pi\sigma T^4}}$

Two laws and a distance are enough to size a star nobody can resolve.

4 Redshift and Hubble

Spectral lines shifted to the red

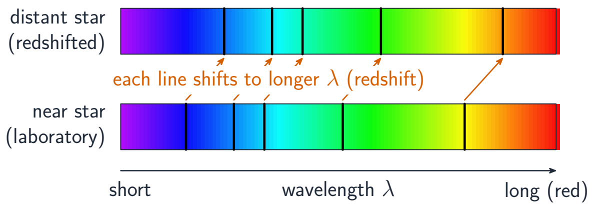
Lines from distant galaxies sit at longer wavelengths than in the lab – **redshift** 红移. Read as a Doppler shift, the galaxy is receding. For $v \ll c$:

$$\frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$$

Almost every distant galaxy is redshifted (a few near ones are **blueshifted** 蓝移 by local motion). Galaxies are, on average, moving apart – the Universe is expanding.

The same lines, moved

A galaxy's lines keep their pattern but sit at longer wavelengths — which is how the shift is recognised as a shift, and not as different elements.



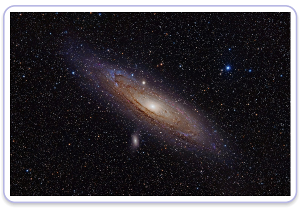
Hubble's law

$$v \approx H_0 \cdot d$$

$H_0 \approx 2.3 \times 10^{-18}\,\mathrm{s}^{-1}$. Always SI units. Running the expansion backwards, distances shrink to zero at $t = -1/H_0$ – the **Big Bang** 大爆炸. For steady expansion the age is

$$T_{\mathrm{age}} \approx \frac{1}{H_0} \approx 14 \text{ billion years}$$

How far the ladder reaches



Andromeda — Cepheids put it well outside our own galaxy.

Almost every smudge here is a galaxy, most of them receding.

Worked example – a receding galaxy

Emitted $4.62 \times 10^{-7}\,\mathrm{m}$, seen at $4.91 \times 10^{-7}\,\mathrm{m}$. $H_0 = 2.3 \times 10^{-18}\,\mathrm{s}^{-1}$.

$\Delta\lambda = 0.29 \times 10^{-7}\,\mathrm{m}$, so $v \approx (\Delta\lambda/\lambda)c \approx 1.9 \times 10^{7}\,\mathrm{m\,s^{-1}}$.
Distance $d = v/H_0 \approx 8.3 \times 10^{24}\,\mathrm{m}$.

5 Recap

Topic 25 – key takeaways

1 Flux

$F = L/(4\pi d^2)$; inverse-square

2 Candles

Cepheids, Type Ia; d from known L

3 Star

Wien $\lambda_{\max} T = b$; $L = 4\pi\sigma r^2 T^4$

4 Hubble

$v = H_0 d$; age $\approx 1/H_0$

Check yourself

1. Double the distance: what happens to flux?
2. What makes a Cepheid a standard candle?
3. A hotter star: longer or shorter $\lambda_{\max}$?
4. Hubble's law points to what origin of the Universe?

Answers **1.** falls to a quarter **2.** period gives luminosity **3.** shorter **4.** the Big Bang