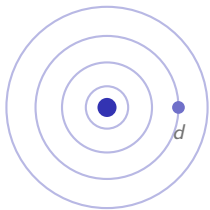
Astronomy and Cosmology

A2 Physics ▪ Topic 25

A-Level Physics



What we will cover

1. Luminosity, flux and standard candles 25.1
2. Stellar temperatures and radii 25.2
3. Redshift, Hubble, the Big Bang 25.3
4. Recap

1 Standard candles

Sheet 25.1 – what you need to know

You must be able to:

- define luminosity and use $F = L/4\pi d^2$
- explain how a standard candle gives a distance
- use the Doppler shift, including from a rotating star

What the exam asks

"Explain how standard candles are used to determine the distance to a galaxy." **[3]**

"... Show that the radius of the Sun is $6.93 \times 10^8\,\mathrm{m}$." **[3]**

"State and explain how the wavelength observed from Z compares with the emitted wavelength." **[2]**

Practise on exercise sheet 25.1, then try the real exam questions on past-paper sheet 25.1.

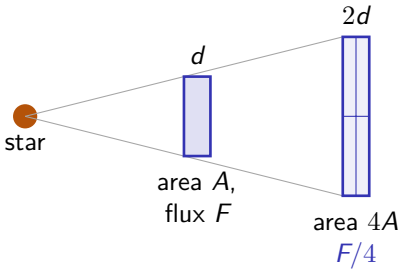
Total power, then power per unit area

Luminosity 光度 L: the **total power** a star radiates (W). At distance d it has spread over a sphere $4\pi d^2$:

$$F = \frac{L}{4\pi d^2}$$

radiant flux intensity
辐射通量密度, $\mathrm{W\,m^{-2}}$

Measure F, know $L \Rightarrow d = \sqrt{\frac{L}{4\pi F}}$.



inverse-square law
平方反比定律: double d, and F falls to a quarter.

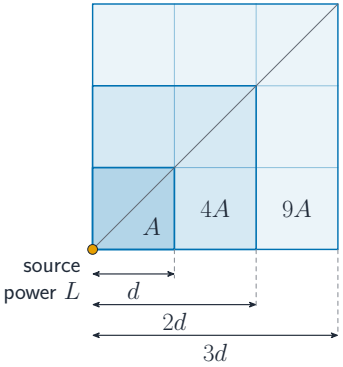
Worked example – flux at the Earth

Sun: $L = 3.8 \times 10^{26}\,\mathrm{W}$, $d = 1.5 \times 10^{11}\,\mathrm{m}$

$$F = \frac{L}{4\pi d^2} = \frac{3.8 \times 10^{26}}{4\pi (1.5 \times 10^{11})^2}$$

$$\approx 1.4 \times 10^{3}\,\mathrm{W\,m^{-2}}$$

Worked example – flux at the Earth, in detail



$$F = \frac{L}{4\pi d^2}$$

flux: F, $\frac{F}{4}$, $\frac{F}{9}$

The same power spreads over a larger area as distance grows, so flux falls as $1/d^2$

An object whose luminosity is known from its type

Find one in a distant galaxy, measure F, get $d = \sqrt{L/(4\pi F)}$. Reaches far beyond **parallax** 视差.

Cepheids

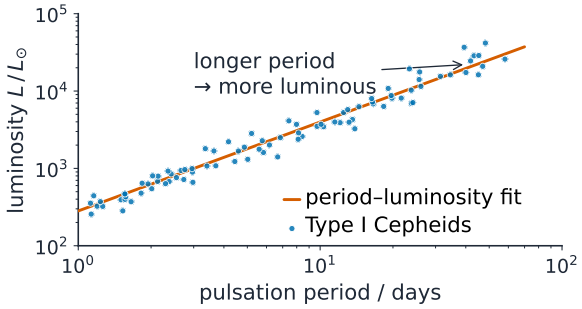
造父变星

pulsation period is tightly linked to luminosity – the period gives L

Type Ia supernovae

a white dwarf at critical mass always has about the same peak luminosity

Standard candles

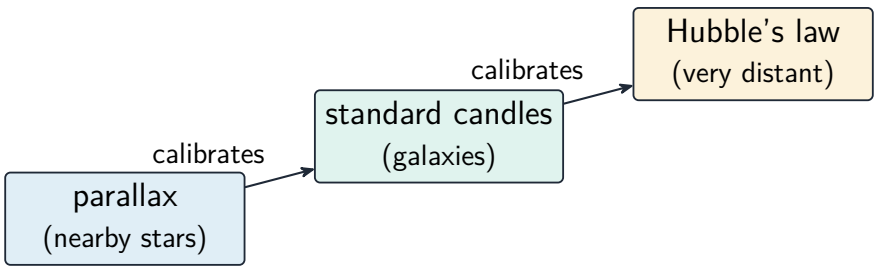


Cepheid: longer period ⇒ brighter · standard candles

For Cepheid variables the pulsation period sets the luminosity, making them standard candles

Each rung calibrates the next

Parallax reaches the nearest stars; Cepheids reach nearby galaxies; supernovae reach the far ones. Every rung is calibrated by the one below it.



each rung is calibrated by the one below, reaching ever further

Standard candles: the answers that score

"Explain how standard candles are used to determine the distance to a galaxy." [3]

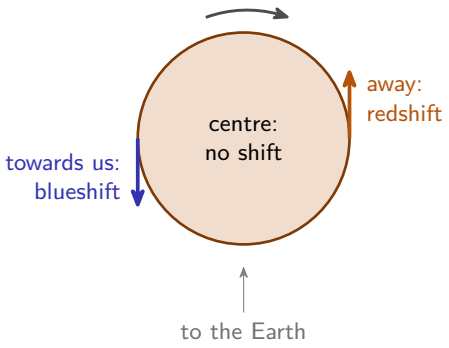
A standard candle is an object of known luminosity L (a Cepheid, a type Ia supernova);
measure its radiant flux intensity F at the Earth;
then $d = \sqrt{\frac{L}{4\pi F}}$.

"Suggest two reasons why stars differ in apparent brightness." [2]

Different luminosities; different distances.

Flux lower than predicted (dust absorbs some light)? Then d comes out too large.

Measuring the Sun's radius from its rotation



At one limb the $656.30\,\mathrm{nm}$ line is shifted by $0.00460\,\mathrm{nm}$. The period is $2.07 \times 10^6\,\mathrm{s}$. Show that $r = 6.93 \times 10^8\,\mathrm{m}$.

$$v = c\frac{\Delta\lambda}{\lambda} = 3.00 \times 10^8 \times \frac{0.00460}{656.30}$$
$$= 2.10 \times 10^3\,\mathrm{m\,s^{-1}}$$
$$r = \frac{vT}{2\pi} = \frac{2.10 \times 10^3 \times 2.07 \times 10^6}{2\pi}$$
$$= 6.93 \times 10^8\,\mathrm{m}$$

The centre of the disc moves across the line of sight, so its light is not shifted.

2 Stellar radii

Sheet 25.2 – what you need to know

You must be able to:

- use Wien's law $\lambda_{\max} T = \text{constant}$
- use $L = 4\pi r^2 \sigma T^4$, including by ratios
- find a star's radius or distance from L, F and T

What the exam asks

"Explain why the observed wavelength and the emitted wavelength have different values." **[2]**

"Explain how the temperature determined using the observed wavelength compares with the true value." **[2]**

"Determine the distance of the star from the Earth." **[2]**

Practise on exercise sheet 25.2, then try the real exam questions on past-paper sheet 25.2.

Wien – hotter means bluer

A blackbody spectrum peaks at $\lambda_{\max}$ set by temperature:

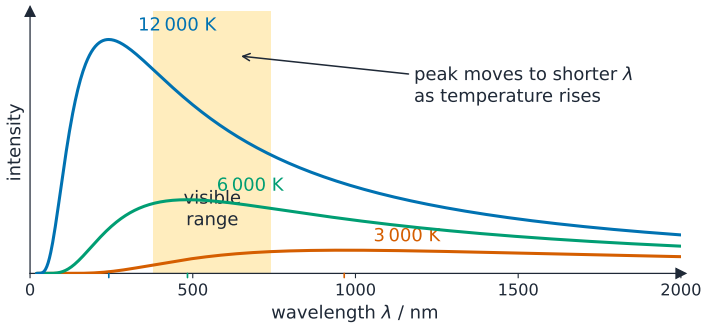
$$\lambda_{\max} T = b \approx 2.90 \times 10^{-3}\,\mathrm{m\,K}$$

Cool red star $\sim 3000\,\mathrm{K}$ peaks in the infrared; the Sun $\sim 5800\,\mathrm{K}$ near $500\,\mathrm{nm}$; a hot blue-white star $\sim 20\,000\,\mathrm{K}$ in the ultraviolet.

$\lambda_{\max} = 500\,\mathrm{nm}$

$T = b/\lambda_{\max} = 2.90 \times 10^{-3}/(500 \times 10^{-9}) \approx 5800\,\mathrm{K}$

Wien – hotter means bluer, in detail



Wien: $\lambda_{\max} T = \text{const} \cdot \text{hotter} \Rightarrow \text{peak shifts left}$

A hotter black body radiates more, and its peak wavelength shifts towards the blue (Wien's law)

Stefan–Boltzmann – $L \propto r^2 T^4$

A star as a blackbody sphere: The **Stefan–Boltzmann law** 斯特藩-玻尔兹曼定律, with the **Stefan–Boltzmann constant** 斯特藩-玻尔兹曼常数 σ:

$$L = 4\pi\sigma r^2 T^4, \qquad \sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$$

Twice the radius $\Rightarrow$ four times L. Twice the temperature $\Rightarrow$ sixteen times L. To estimate a radius: $\lambda_{\max} \to T$ (Wien); F and $d \to L$; then

$$r = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

Estimating a star's radius

Wien, then Stefan–Boltzmann

1. $\lambda_{\max} \Rightarrow T$
   $(\lambda_{\max} T = 2.9 \times 10^{-3}\,\mathrm{m\,K})$
2. $L = 4\pi d^2 F$ from flux and distance
3. $L = 4\pi r^2 \sigma T^4 \Rightarrow r = \sqrt{\dfrac{L}{4\pi\sigma T^4}}$

Two laws and a distance are enough to size a star nobody can resolve.

Luminosity, and the law behind it

Luminosity	The **luminosity** 光度 L of a star is its **total power** 总功率 of radiation – the **energy** 能量 radiated per second in all directions.
Stefan–Boltzmann law	The **Stefan–Boltzmann law**: $L = 4\pi r^2 \sigma T^4$, where $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4} is the **Stefan–Boltzmann constant**.
Why it gives a radius	Wien's law gives T from the peak wavelength, the flux and distance give L – so the only unknown left is r.

$L \propto T^4$: doubling the surface temperature multiplies the power by sixteen, which is why hot stars are so much brighter than their size suggests.

From flux to a star's radius, and the distance ladder

Stefan–Boltzmann law 斯特藩-玻尔兹曼定律	$L = 4\pi r^2 \sigma T^4$; σ is the **Stefan–Boltzmann constant** 斯特藩-玻尔兹曼常量
the route	Wien's law gives T; flux and distance give L; rearrange for r
why it matters	radii of stars that cannot be resolved as a disc
the ladder	each method is calibrated by the one below it
the rungs	radar, parallax, standard candles, then Hubble's law
Big Bang evidence	galactic redshift, the **cosmic microwave background** 宇宙微波背景, the H/He abundance

Each rung reaches further and is less precise. An error low on the ladder carries all the way up.

Ratio reasoning with $L = 4\pi r^2 \sigma T^4$

Star B has the same luminosity as the Sun but twice its surface temperature. Compare its radius.

$r^2 \propto \dfrac{L}{T^4}$, so $r \propto \dfrac{1}{T^2}$: $r_B = \frac{1}{4}\, r_\odot$.

Hot and small: a white dwarf.

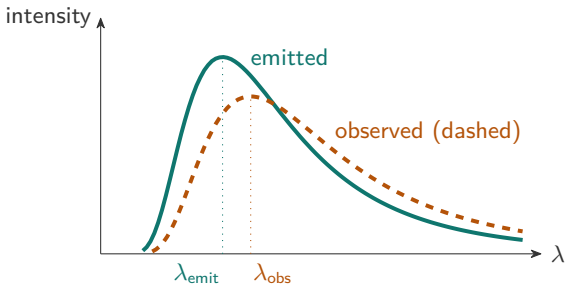
Same radius, half the temperature?

$L \propto T^4$: L is $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$ as large.

A star of luminosity $3.8 \times 10^{28}\,\mathrm{W}$ gives a flux of $2.0 \times 10^{-9}\,\mathrm{W\,m^{-2}}$ at the Earth. How far away is it?

$$d = \sqrt{\frac{L}{4\pi F}} = \sqrt{\frac{3.8 \times 10^{28}}{4\pi \times 2.0 \times 10^{-9}}}$$
$$= 1.2 \times 10^{18}\,\mathrm{m}$$

A redshifted spectrum gives too low a temperature



"Explain why the observed and emitted wavelengths differ." [2]

The star (galaxy) is moving away, so every wavelength is redshifted: $\lambda_{\text{obs}} > \lambda_{\text{emit}}$.

"How does the temperature from the observed wavelength compare with the true value?" [2]

$T = b/\lambda_{\max}$ and the observed $\lambda_{\max}$ is longer, so the temperature found is too low.

The Pillars of Creation

The Pillars of Creation in the Eagle Nebula – clouds of gas and dust lit by hot, newly formed stars

3 Hubble's law

Sheet 25.3 – what you need to know

You must be able to:

- use $\Delta\lambda/\lambda \approx \Delta f/f \approx v/c$
- use Hubble's law $v = H_0 d$ and estimate the age of the Universe
- explain how redshift supports the Big Bang

What the exam asks

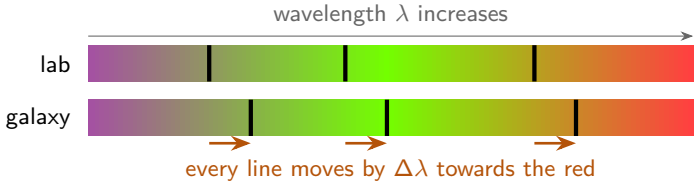
"Explain how redshift leads to the idea that the Universe is expanding." **[3]**

"Sketch the variation of v with d." **[2]**

"State the name of the quantity represented by the gradient of the line." **[1]**

Practise on exercise sheet 25.3, then try the real exam questions on past-paper sheet 25.3.

Spectral lines shifted to the red



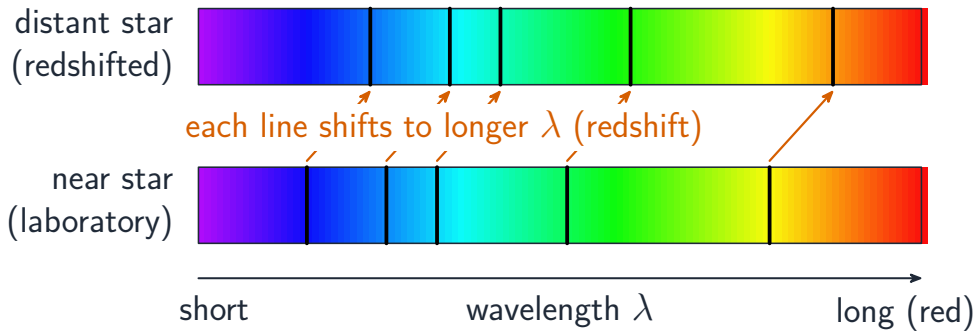
Lines from distant galaxies sit at longer wavelengths: **redshift** 红移 – the galaxy is **receding**. For $v \ll c$:

$$\frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$$

Almost every distant galaxy is redshifted (a few near ones are **blueshifted** 蓝移): the Universe is expanding.

The same lines, moved

A galaxy's lines keep their pattern but sit at longer wavelengths – which is how the shift is recognised as a shift, and not as different elements.



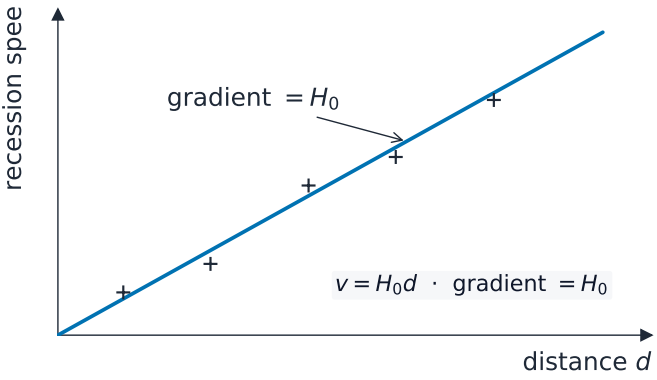
Hubble's law

$$v \approx H_0 \cdot d$$

where H_0 is the **Hubble constant** 哈勃常数, $\approx 2.3 \times 10^{-18}\,\mathrm{s}^{-1}$. Always SI units. Running the expansion backwards, distances shrink to zero at $t = -1/H_0$ – the **Big Bang** 大爆炸. For steady expansion the age is

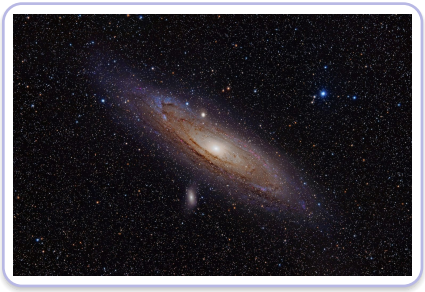
$$T_{\mathrm{age}} \approx \frac{1}{H_0} \approx 14 \text{ billion years}$$

Hubble's law, in detail



Hubble's law: a galaxy's recession speed is proportional to its distance, $v = H_0 d$

How far the ladder reaches



Andromeda – Cepheids put it well outside our own galaxy.

Almost every smudge here is a galaxy, most of them receding.

Each rung calibrates the next: parallax, then **Cepheid** 造父变星 variables, then redshift.

Worked example – a receding galaxy

Emitted $4.62 \times 10^{-7}\,\mathrm{m}$, seen at $4.91 \times 10^{-7}\,\mathrm{m}$. $H_0 = 2.3 \times 10^{-18}\,\mathrm{s}^{-1}$.

$\Delta\lambda = 0.29 \times 10^{-7}\,\mathrm{m}$, so $v \approx (\Delta\lambda/\lambda)c \approx 1.9 \times 10^{7}\,\mathrm{m\,s}^{-1}$.

Distance $d = v/H_0 \approx 8.3 \times 10^{24}\,\mathrm{m}$.

Redshift and the expanding Universe

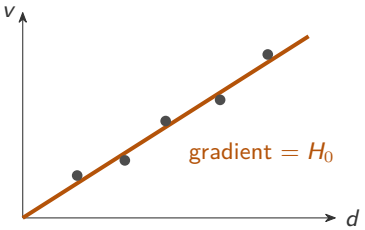
"Explain how redshift leads to the idea that the Universe is expanding." [3]

Light from distant galaxies is redshifted, so they are moving away;

the further the galaxy, the greater its redshift (speed);

so all of space is expanding, as from a single point.

An approaching source is blueshifted: $\Delta\lambda$ is negative.



Sketch: a straight line through the origin. The gradient is the Hubble constant.

Worked example – two recipes for one galaxy

A Cepheid puts a galaxy at $d = 1.0 \times 10^{24}\,\mathrm{m}$. A line emitted at $656.3\,\mathrm{nm}$ is observed at $661.1\,\mathrm{nm}$. Find H_0 and estimate the age of the Universe.

$$v = c\frac{\Delta\lambda}{\lambda} = 3.00 \times 10^8 \times \frac{4.8}{656.3}$$
$$= 2.2 \times 10^6\,\mathrm{m\,s^{-1}}$$

$$H_0 = \frac{v}{d} = \frac{2.2 \times 10^6}{1.0 \times 10^{24}} = 2.2 \times 10^{-18}\,\mathrm{s^{-1}}$$

$$t \approx \frac{1}{H_0} = 4.5 \times 10^{17}\,\mathrm{s}$$
$$\approx 1.4 \times 10^{10}\,\mathrm{years}$$

assuming the galaxies have always moved apart at the same speed.

Distance from a standard candle; speed from the redshift. The two together give H_0.

4 Recap

Exam tips

- $F = L/4\pi d^2$ (inverse-square): a standard candle has known luminosity L, so the distance follows from the measured flux F.
- **Wien's law** ($\lambda_{max} \propto 1/T$) gives surface temperature; **Stefan's law** $L = 4\pi r^2 \sigma T^4$.
- **Redshift** $z = \mathbf{\Delta}\lambda/\lambda \approx v/c$; Hubble's law $v = H_0 d$ leads to the age of the universe (evidence for the Big Bang).

Topic 25 – key takeaways

1 Flux

$F = L/(4\pi d^2)$; inverse-square

2 Candles

Cepheids, Type Ia; d from known L

3 Star

Wien $\lambda_{\max} T = b$; $L = 4\pi\sigma r^2 T^4$

4 Hubble

$v = H_0 d$; age $\approx 1/H_0$

Check yourself

1. Double the distance: what happens to flux?
2. What makes a Cepheid a standard candle?
3. A hotter star: longer or shorter $\lambda_{\max}$?
4. Hubble's law points to what origin of the Universe?

Answers **1.** falls to a quarter **2.** period gives luminosity **3.** shorter **4.** the Big Bang