Medical Physics

A2 Physics ▪ Topic 24

A-Level Physics

What we will cover

1. Ultrasound 24.1
2. Diagnostic information from echoes 24.1.1
3. X-rays 24.2
4. Controls, layers, sharpness, CT 24.2.1
5. PET scanning 24.3
6. Recap

1 Ultrasound

Sheet 24.1 – what you need to know

You must be able to:

- explain the piezo-electric effect both ways
- use $Z = \rho c$ and $\alpha = (Z_2 - Z_1)^2/(Z_2 + Z_1)^2$
- use $I = I_0 e^{-\mu x}$, and solve it for μ

What the exam asks

"Explain how ultrasound waves are detected by a piezoelectric crystal." **[2]**

"Calculate the linear attenuation coefficient μ ... Give a unit with your answer." **[2]**

"Explain, without calculation, what is likely to happen when ultrasound is incident on a tissue–water boundary." **[2]**

Practise on exercise sheet 24.1, then try the real exam questions on past-paper sheet 24.1.

Piezo-electric – two linked effects

A **piezo-electric** 压电 crystal (quartz, PZT) changes shape when a p.d. is put across it, and makes an e.m.f. when its shape is changed.

apply a p.d.

the crystal vibrates –
used to make ultrasound

squeeze it

an e.m.f. appears – used
to detect ultrasound

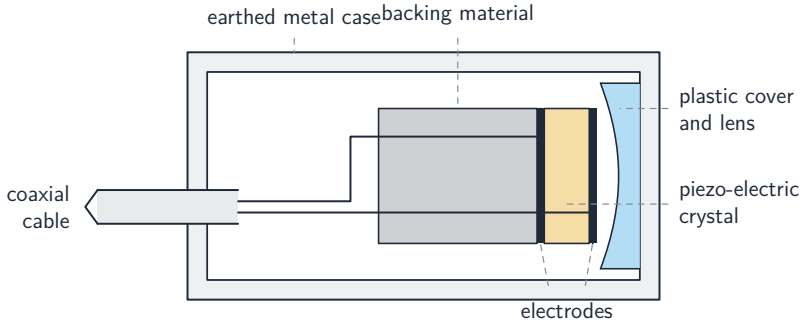
One transducer both sends and listens

An alternating p.d. (a few MHz) makes the crystal vibrate, sending **longitudinal** 纵波 waves above $20\,\mathrm{kHz}$ (1–$10\,\mathrm{MHz}$ for imaging). Returning ultrasound makes it vibrate and produce an e.m.f.

> One transducer switches between sending and listening. **Coupling gel** 耦合剂 pushes out the air – without it almost all the ultrasound would reflect at the skin.

Inside the transducer

The crystal sits between a damping block (to kill ringing so pulses stay short) and a matching layer (to cut the reflection at the skin).



Specific acoustic impedance

$$Z = \rho c$$

unit $\mathrm{kg\,m^{-2}\,s^{-1}}$

At a boundary the fraction reflected is

$$\frac{I_\mathrm{R}}{I_0} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2$$

material	Z / $10^6\,\mathrm{kg\,m^{-2}\,s^{-1}}$
air	0.0004
soft tissue	1.63
bone	7.8

boundary	**reflected**
tissue / air	99.9%
tissue / bone	43%

Gel removes the air gap, so the pulse gets in.

Impedance, reflection and attenuation

acoustic impedance	$Z = \rho c$ – **density** 密度 times the speed of sound. Soft tissue: $1060 \times 1540 \approx 1.6 \times 10^6$ kg m^{-2}s^{-1}
reflection at a boundary	the fraction reflected depends on the difference in Z between the two media
best for imaging	different enough to give an echo, but not so different that nothing passes on
why gel is used	air's Z is so low that almost everything would reflect at the skin
attenuation	$I = I_0 e^{-\mu x}$; higher-energy X-rays penetrate further, so their μ is smaller
contrast	bone has a much larger μ than soft tissue, which is why it shows white

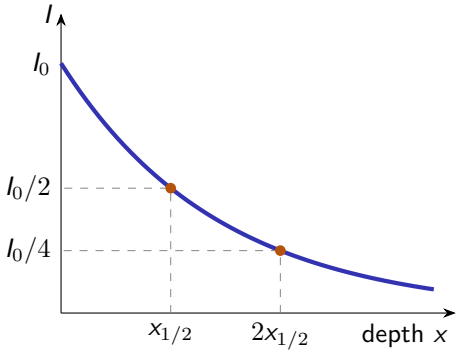
To find a thickness, rearrange to $x = \dfrac{1}{\mu} \ln \dfrac{I_0}{I}$. Half-value thickness is the special case $I = I_0/2$.

Attenuation – intensity falls exponentially

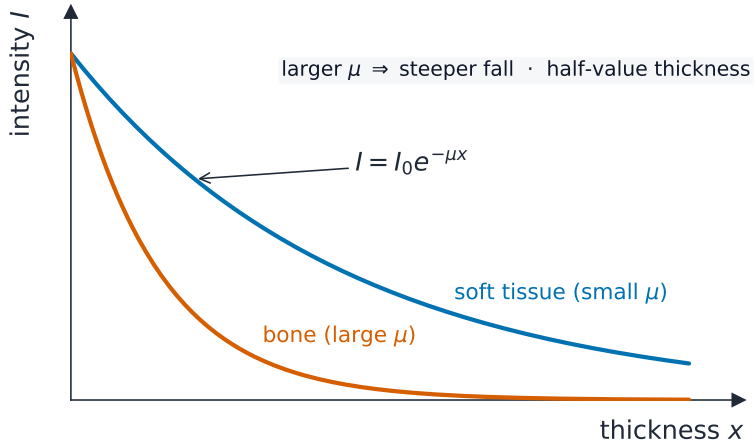
$$I = I_0 e^{-\mu x}$$

μ is the **attenuation coefficient** 衰减系数 (unit m^{-1}). The same form applies to X-rays.

Each half-value thickness $x_{1/2} = \ln 2/\mu$ halves the intensity again.



Attenuation – intensity falls exponentially, in detail



Intensity falls exponentially with thickness; a larger attenuation coefficient (bone) falls off faster.

Impedance and attenuation: the answers that score

"Define specific acoustic impedance." [2]

The product of the density of a medium and the speed of sound in it: $Z = \rho c$. (So $\rho = Z/c$ in a table.)

"Explain, without calculation, what happens at a tissue–water boundary." [2]

Their Z values are similar, so very little is reflected: almost all is transmitted.

Check: $\alpha = \left(\frac{1.63-1.48}{1.63+1.48}\right)^2 = 0.002$, only 0.2%.

Ultrasound falls to 35% of its intensity through $4.0\,\mathrm{cm}$ of soft tissue. Find μ, with a unit.

$$\frac{I}{I_0} = e^{-\mu x} \;\Rightarrow\; \mu = \frac{1}{x}\ln\frac{I_0}{I}$$

$$\mu = \frac{1}{0.040\,\mathrm{m}}\ln\frac{1}{0.35} = 26\,\mathrm{m}^{-1}$$

2 Echoes

Sheet 24.1.1 – what you need to know

You must be able to:

- describe how a crystal sends and receives ultrasound
- find a depth from an echo time, and what fraction of the pulse returns
- choose a frequency; explain the gel, and why lungs cannot be imaged

What the exam asks

"Describe how the piezoelectric crystal in a transducer generates ultrasound waves for use in medical diagnosis." **[3]**

"Describe how α depends on the relative values of Z_1 and Z_2." **[2]**

"Explain why ultrasound cannot be used to produce an image inside an air-filled cavity such as the lungs." **[2]**

Practise on exercise sheet 24.1.1, then try the real exam questions on past-paper sheet 24.1.

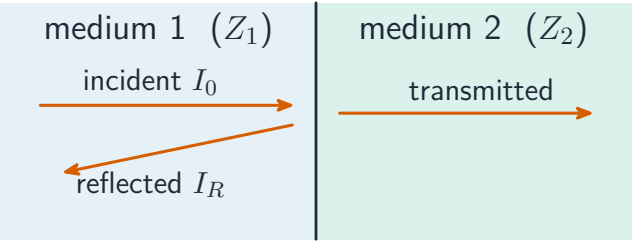
Pulse-echo: depth from the time delay

1. send a short pulse
2. at each tissue boundary, part reflects and part goes on
3. detect each echo
4. depth $d = ct/2$ (there and back); amplitude $\Rightarrow$ strength of the reflection

Echo after $60\,\mu$s; $c = 1500\,\mathrm{m\,s^{-1}}$

$$d = \frac{ct}{2} = \frac{1500 \times 60 \times 10^{-6}}{2} = 0.045\,\mathrm{m}\ (4.5\,\mathrm{cm})$$

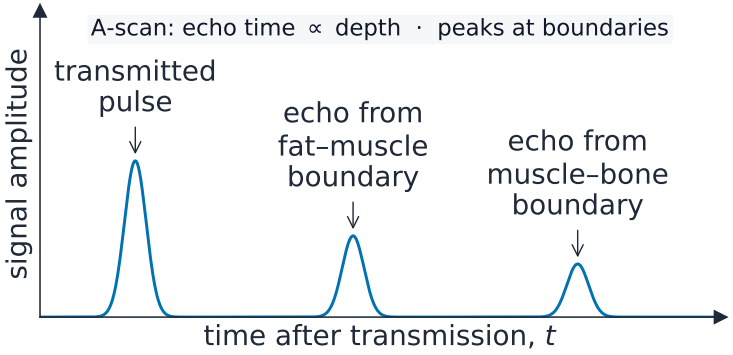
Pulse-echo: depth from the time delay, in detail



$$\frac{I_R}{I_0} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2$$ — a bigger impedance difference gives a bigger echo

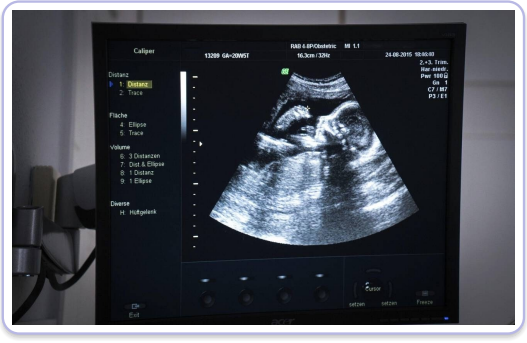
At a boundary, part of the pulse reflects and part transmits – a bigger impedance difference gives a bigger echo.

Pulse-echo imaging



An A-scan shows the transmitted pulse and the echoes from each tissue boundary

What the operator sees



A B-scan builds a 2-D image from many pulse-echo lines swept across the body.

Making and receiving ultrasound

"Describe how the crystal generates ultrasound." [3]

An alternating p.d. is applied across the crystal;
it expands and contracts, so it vibrates;
at its resonant frequency the vibrations are largest, sending out ultrasound.

"Explain how ultrasound is detected." [2]

The returning wave makes the crystal vibrate, which produces an alternating p.d. across it.

- pulses: the same crystal listens between them, and the delay gives the depth
- lungs: air's Z is tiny, so nearly all is reflected at the first air boundary and nothing returns from inside
- higher f: shorter λ, finer detail, but more attenuation, so less depth
- non-ionising, so safe for a fetus

Worked example – how much of the echo comes back

A boundary lies $5.0\,\mathrm{cm}$ deep in tissue with $\mu = 23\,\mathrm{m}^{-1}$; it reflects a fraction $\alpha = 0.010$. What fraction of the sent intensity returns to the probe?

The wave travels there and back: $2d = 0.10\,\mathrm{m}$.

$$\frac{I}{I_0} = \alpha\, e^{-\mu(2d)} = 0.010 \times e^{-23\times 0.10}$$
$$= 0.010 \times 0.100 = 1.0 \times 10^{-3}$$

Only a thousandth comes back, so the receiver must be very sensitive.

Using d instead of $2d$ is the usual lost mark.

Gel removes the air between probe and skin. Without it, almost all the ultrasound reflects at the skin.

3

X-rays

Sheet 24.2 – what you need to know

You must be able to:

- describe the X-ray tube, with its polarity
- use $\lambda_{\min} = hc/eV$ and photon momentum $p = E/c$
- explain contrast; find the percentage attenuated

What the exam asks

"Use + and − signs to label terminals X and Y to indicate the polarity of the high voltage." **[1]**

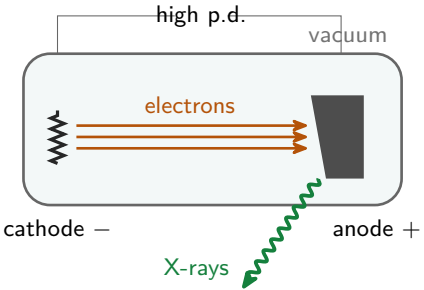
"Explain why X-rays can be used to produce images of internal body structures that have good contrast." **[3]**

"Calculate the percentage of the initial intensity of the wave that is attenuated by the tissue." **[2]**

Practise on exercise sheet 24.2, then try the real exam questions on past-paper sheet 24.2.

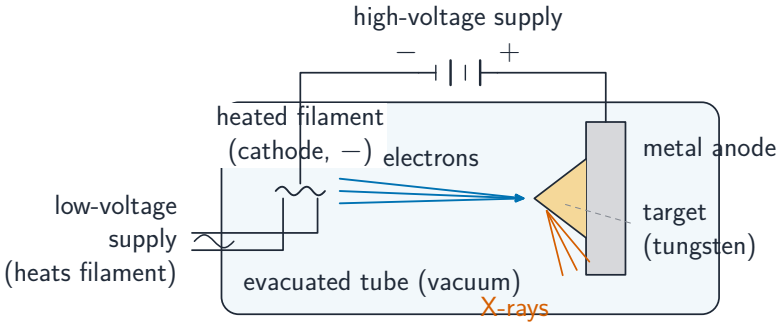
How an X-ray tube works

1. heated **cathode** 阴极 emits electrons by **thermionic emission** 热电子发射
2. a high p.d. accelerates them across a vacuum to a metal **target** 靶 anode (often tungsten)
3. they slow sharply: most KE becomes heat; a small part is X-ray photons (**Bremsstrahlung** 韧致辐射). Knocking out inner electrons gives **characteristic** 特征 lines.



The X-ray tube

Most of the electrons' kinetic energy becomes heat; a small part leaves as X-ray photons, so the anode is angled and cooled.



Minimum wavelength and imaging

The most energy one photon can have is eV:

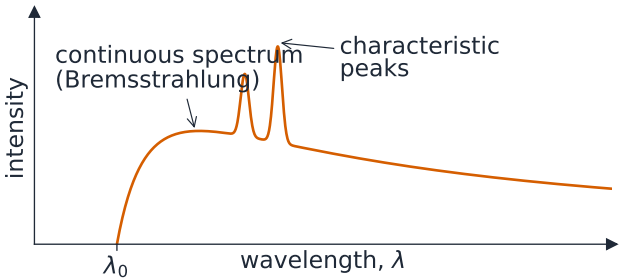
$$hf_{\max} = eV, \qquad \lambda_{\min} = \frac{hc}{eV}$$

Bone (higher density and proton number) attenuates more and looks lighter; soft tissue looks darker. A **contrast medium** 造影剂 (barium meal) makes soft tissues stand out. Half-value thickness $x_{1/2} = \ln 2/\mu$ halves the intensity.

$x = 3.0\,\mathrm{cm}$, $\mu = 40\,\mathrm{m}^{-1}$

$I/I_0 = e^{-40\times 0.030} = e^{-1.2} \approx 0.30$

Minimum wavelength

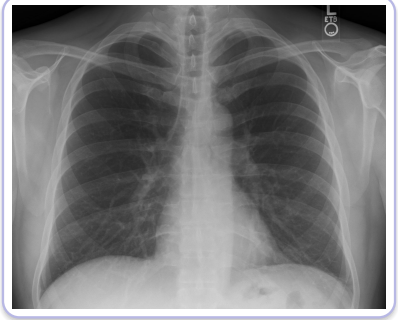


X-ray: continuous bremsstrahlung + characteristic peaks

A typical X-ray spectrum – a continuous Bremsstrahlung curve cut off at λ_0, with sharp characteristic peaks

Why bone shows white

Dense bone (high proton number) absorbs strongly and leaves the film unexposed; the air-filled lungs let X-rays through and darken it.



X-ray contrast and percentage attenuated

"Explain why X-rays can produce images with good contrast." [3]

Different tissues have different attenuation coefficients μ;
so very different fractions are transmitted;
so the intensity reaching the detector differs sharply between regions.

A $100\,\mathrm{keV}$ photon: $E = 1.6 \times 10^{-14}\,\mathrm{J}$,
$p = E/c = 5.3 \times 10^{-23}\,\mathrm{N\,s}$,
$\lambda_{\min} = 1.2 \times 10^{-11}\,\mathrm{m}$.

$3.0\,\mathrm{cm}$ of tissue transmits 72%. What percentage is attenuated by $5.0\,\mathrm{cm}$?

$$\mu = \frac{1}{0.030} \ln \frac{1}{0.72} = 11.0\,\mathrm{m}^{-1}$$

$$\frac{I}{I_0} = e^{-11.0 \times 0.050} = 0.58$$

$$\text{attenuated} = 1 - 0.58 = 42\%$$

The trap: the question asks for the part removed, $1 - I/I_0$, not the part transmitted.

Ultrasound and X-rays side by side

	ultrasound	**X-rays**
produced by	a vibrating piezo-electric crystal	electrons hitting a metal target
image made from	waves reflected at boundaries	waves transmitted through the body
ionising?	no	yes: keep the dose low
best for	soft tissue, a fetus, moving organs	bone; CT for 3-D slices

Ultrasound fails at air and bone boundaries (almost total reflection); X-rays show little difference between soft tissues unless a contrast medium is used.

4 Controls and CT

Sheet 24.2.1 – what you need to know

You must be able to:

- say what the filament current and the p.d. each control
- attenuate through layers; use the half-value thickness
- tell sharpness from contrast; explain CT

What the exam asks

"Calculate the percentage of the X-ray energy that is absorbed by a $2.8\,\mathrm{cm}$ thickness of this medium." **[3]**

"Explain how X-rays are produced at the metal target." **[2]**

"State the name of the charged particles that are accelerated by the applied p.d." **[1]**

Practise on exercise sheet 24.2.1, then try the real exam questions on past-paper sheet 24.2.

The tube's two controls, layers and sharpness

- filament current: more electrons, so more photons – the intensity
- accelerating p.d.: the largest photon energy – the hardness
- an aluminium filter removes soft, low-energy X-rays that only add to the skin dose
- sharpness: a small focal spot, a still patient, the detector close to the patient

"Explain how X-rays are produced at the target." [2]

Electrons decelerate rapidly in the target; the kinetic energy they lose leaves as X-ray photons.

Find the percentage of the X-ray energy absorbed by $2.8\,\mathrm{cm}$ of a medium with $\mu = 0.20\,\mathrm{cm}^{-1}$.

$$\frac{I}{I_0} = e^{-0.20\times 2.8} = 0.57 \;\Rightarrow\; 43\% \text{ absorbed}$$

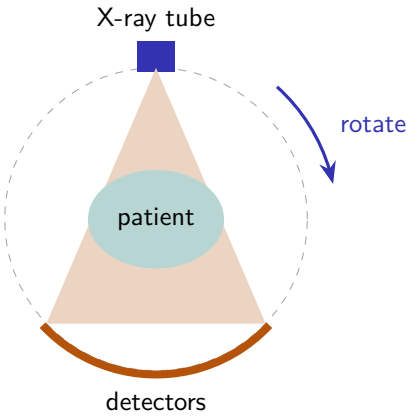
Through layers the exponents add: $I = I_0 e^{-(\mu_1 x_1 + \mu_2 x_2)}$.

CT – many angles, then a computer

Computed tomography 计算机断层扫描:

1. tube and detectors rotate, imaging one thin slice from many angles
2. a computer combines them into a 2-D cross-section
3. the patient moves on; the next slice is imaged
4. slices stack into a 3-D image

> Tissues that overlap on one X-ray are separated.



5

PET

Sheet 24.3 – what you need to know

You must be able to:

- say what a tracer is and write the β^+ decay
- explain annihilation into two $511\,\mathrm{keV}$ photons
- locate the event from a coincidence and a time difference

What the exam asks

"State what is meant by a tracer." **[2]**

"State the name of the particles that are detected." **[1]**

"Explain how these particles are formed inside the body." **[2]**

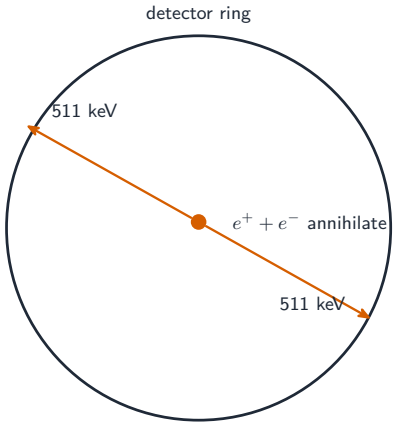
Practise on exercise sheet 24.3, then try the real exam questions on past-paper sheet 24.3.

A β^+ tracer, then annihilation

A **tracer** 示踪剂 (often FDG, fluorine-18 on a glucose analogue) is taken up more by active tissue. In PET it is a β^+ emitter – it gives out a **positron** 正电子.

The positron meets an electron and they **annihilate** 湮灭: two photons, opposite directions, each $511\,\mathrm{keV}$ ($= m_e c^2$).

$$2hf = 2m_e c^2$$



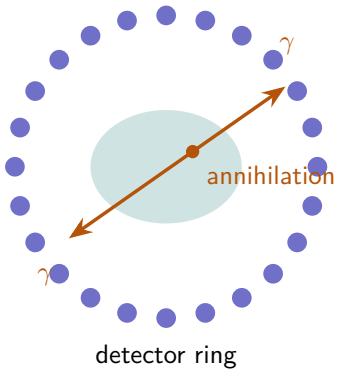
two photons, opposite directions — the coincidence fixes the line

A coincidence fixes the line

The two photons fly off back to back and hit the detector ring. Two simultaneous arrivals (a coincidence) fix the line the annihilation lay on.

Many lines from many angles build a 3-D map of the tracer: tissues with high metabolic activity.

> Arrival-time differences refine the position along the line (time-of-flight PET).

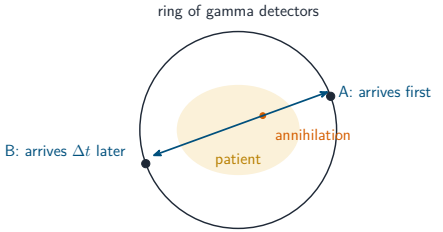


Tracers and annihilation

a tracer 示踪剂	a substance containing radioactive nuclei, introduced into the body
how it works	it is taken up more by the tissue being studied, so that tissue emits more strongly
the choice	a γ or β^+ emitter with a short half-life, so the dose falls quickly
an antiparticle 反粒子	the same mass, opposite charge – the positron is the electron's
annihilation 湮灭	a particle meets its antiparticle and their mass turns into electromagnetic energy
the signature	two γ photons of 0.51 MeV leave in opposite directions, and detecting both fixes the line the event lay on

PET works because momentum conservation forces the two photons back to back. That geometry is what locates the tracer.

PET: the answers that score, and locating the event



two photons detected together: the event lies on the line AB
the time difference gives where on the line: $d = c\,\Delta t/2$ from the midpoint

Photon B arrives $0.40\,\mathrm{ns}$ after A. How far from the midpoint was the annihilation?

$$d = \frac{c\,\Delta t}{2} = \frac{3.00 \times 10^{8} \times 0.40 \times 10^{-9}}{2} = 6.0\,\mathrm{cm}$$

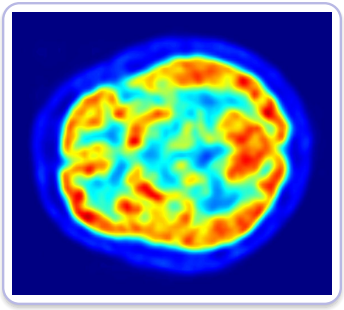
"State what is meant by a tracer." [2]

A substance containing a radioactive nuclide that is put into the body and absorbed by the tissue being studied.

"Name the particles detected, and explain how they form." [3]

Gamma-ray photons. A positron from β^{+} decay meets an electron and they annihilate: two $511\,\mathrm{keV}$ photons go off in opposite directions (momentum is conserved).

A finished PET image



Positron emission tomography 正电子发射断层成像 (PET) maps function, not shape: warm colours mark where the tracer collected – the tissue with the fastest **metabolism** 代谢.

6 Recap

Exam tips

- **Ultrasound** reflects at boundaries (acoustic impedance); a coupling gel reduces reflection at the skin.
- **X-rays** are attenuated as $I = I_0 e^{-\mu x}$; contrast media (barium, iodine) absorb strongly.
- **PET** uses a positron emitter; annihilation gives two γ-rays in opposite directions, located by the detector ring.

Topic 24 – key takeaways

1 Ultrasound

piezo; $d = ct/2$; gel; $Z = \rho c$

2 X-rays

$\lambda_{\min} = hc/eV$; $I = I_0 e^{-\mu x}$

3 CT

many angles of one slice, then stack

4 PET

two $511\,\mathrm{keV}$ photons; coincidence line

Check yourself

1. Why $d = ct/2$, not ct?
2. Why put gel between the transducer and the skin?
3. What sets $\lambda_{\min}$ of an X-ray spectrum?
4. Why do PET photons fly in opposite directions?

Answers **1.** there and back **2.** air/skin reflects almost everything **3.** $hf_{\max} = eV$ **4.** momentum conservation (pair at rest)