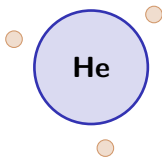


Nuclear Physics

A2 Physics ■ Topic 23

A-Level Physics



What we will cover

- 1 Mass defect and binding energy 23.1
- 2 The binding-energy curve 23.1.1
- 3 Radioactive decay 23.2
- 4 Count rate and log graphs 23.2.1
- 5 Recap

1

Mass defect

Sheet 23.1 – what you need to know

You must be able to:

- use $E = mc^2$ in kg and J, and $1 \text{ u} = 931.5 \text{ MeV}$
- find energy released from masses, from binding energies, and per mole
- balance nuclear equations

What the exam asks

“Calculate the energy released in the formation of 1.00 mol of helium-4.” [4]

“Determine the energy, in eV, released when a nucleus of polonium-193 decays into a nucleus of lead-189.” [2]

“Determine a value for the luminosity of Sirius. Give a unit with your answer.” [2]

Practise on exercise sheet 23.1, then try the real exam questions on past-paper sheet 23.1.

Mass and energy are the same thing

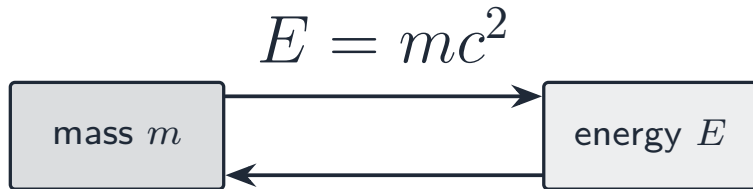
Mass–energy equivalence 质能等价, from special relativity:

$$E = mc^2, \quad \Delta E = c^2 \Delta m$$

$c = 3.00 \times 10^8 \text{ m s}^{-1}$. Nuclear masses are tiny but c^2 is huge, so a small mass change is a large energy. The conversion you will use again and again:

$$1 \text{ u} \longleftrightarrow 931 \text{ MeV}$$

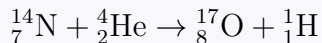
The exchange rate



A mass of one atomic mass unit is worth 931.5 MeV – which is why nuclear energies are quoted in MeV and chemical ones in eV.

1 u $\leftrightarrow$ 931 MeV. Nuclear masses are tiny and c^2 is enormous, so a mass change too small to weigh releases an energy you can measure.

Balancing a nuclear reaction



Nucleon number 核子数 is conserved (top: $14 + 4 = 17 + 1$) and charge is conserved (bottom: $7 + 2 = 8 + 1$).

To fill in an unknown: identify the species, then balance the top and bottom numbers.

A nucleus weighs less than its parts

The mass of a **nucleus** 原子核 is less than its Z **protons** 质子 plus N neutrons, $Zm_p + Nm_n$. The difference is the **mass defect** 质量亏损:

$$\Delta m = (Zm_p + Nm_n) - m_{\text{nucleus}}$$

That missing mass was released as **binding energy** 结合能 when the nucleus formed:

$$B = \Delta m \cdot c^2$$

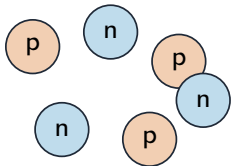
Helium-4, $\Delta m = 0.0304 \text{ u}$

$$B = 0.0304 \times 931 \approx 28 \text{ MeV}$$

The missing mass

Separated nucleons weigh **more** than the nucleus they make. That difference is the **mass defect** 质量亏损, and $E = mc^2$ turns it into the binding energy.

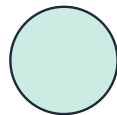
separate nucleons



$$\text{mass} = Zm_p + Nm_n$$

assemble
→

nucleus



mass is *less*
(mass defect Δm)

binding
energy B
→

Calculating the energy released

- 1 total mass of the reactants
- 2 total mass of the products
- 3 $\Delta m = m_{\text{reactants}} - m_{\text{products}}$ (positive when energy is released)
- 4 $\Delta E = c^2 \Delta m$, or in u: $\Delta E \text{ (MeV)} = \Delta m \text{ (u)} \times 931$

Mass decreases by 0.020 u

$$\Delta E = 0.020 \times 931 \approx 19 \text{ MeV}$$

Worked example – energy in kilograms and joules

Find the energy released when 1.00 mol of helium-4 forms from free protons and neutrons. Masses: $m_p = 1.007276 \text{ u}$, $m_n = 1.008665 \text{ u}$, $m_{\text{He}} = 4.001506 \text{ u}$.

$$\begin{aligned}
 \Delta m &= 2(1.007276) + 2(1.008665) - 4.001506 & E &= \Delta m c^2 = 5.04 \times 10^{-29} \times (3.00 \times 10^8)^2 \\
 &= 0.030376 \text{ u} & &= 4.54 \times 10^{-12} \text{ J per nucleus} \\
 &= 0.030376 \times 1.66 \times 10^{-27} & E_{\text{mol}} &= 4.54 \times 10^{-12} \times 6.02 \times 10^{23} \\
 &= 5.04 \times 10^{-29} \text{ kg} & &= 2.7 \times 10^{12} \text{ J}
 \end{aligned}$$

Keep **all six decimal places** until the subtraction: the mass defect is a small difference of large numbers.

Two more routes to the energy released

Polonium-210 decays: $^{210}\text{Po} \rightarrow ^{206}\text{Pb} + ^4\text{He}$.
Binding energies per nucleon: 7.834, 7.875, 7.074 MeV. **Find the energy released, in eV.**

$$\text{before} = 210 \times 7.834 = 1645.14 \text{ MeV}$$

$$\text{after} = 206 \times 7.875 + 4 \times 7.074 = 1650.55 \text{ MeV}$$

$$E = 1650.55 - 1645.14 = 5.4 \times 10^6 \text{ eV}$$

Multiply each B/A by its A first. Released energy = binding energy **after** – binding energy **before**.

The Sun loses mass at $4.3 \times 10^9 \text{ kg s}^{-1}$. Find its luminosity.

$$L = \frac{\Delta m}{\Delta t} c^2 = 4.3 \times 10^9 \times (3.00 \times 10^8)^2 \\ = 3.9 \times 10^{26} \text{ W}$$

Then $L = 4\pi R^2 \sigma T^4$ (topic 25) gives the surface temperature.

2

The binding-energy curve

Sheet 23.1.1 – what you need to know

You must be able to:

- define **mass defect** and **binding energy** exactly
- read the B/A curve: fusion and fission both move towards iron
- decide from the masses whether energy is released or absorbed

What the exam asks

“State the name of this type of nuclear reaction.” [1]

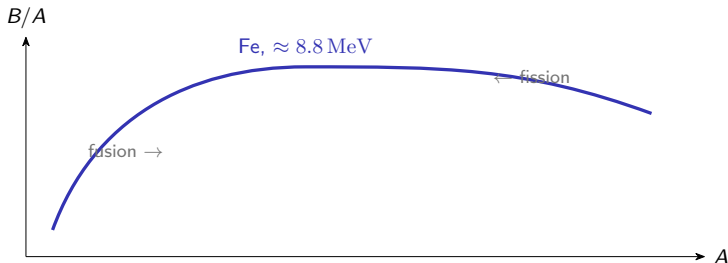
“Show that the energy released when one nucleus of helium-4 is formed from deuterium is $3.47 \times 10^{-12} \text{ J}$.” [3]

“State what is meant by the mass defect of a nucleus.” [2]

Practise on exercise sheet 23.1.1, then try the real exam questions on past-paper sheet 23.1.

Binding energy per nucleon peaks at iron

B/A (MeV per nucleon) measures how tightly each nucleon is held.



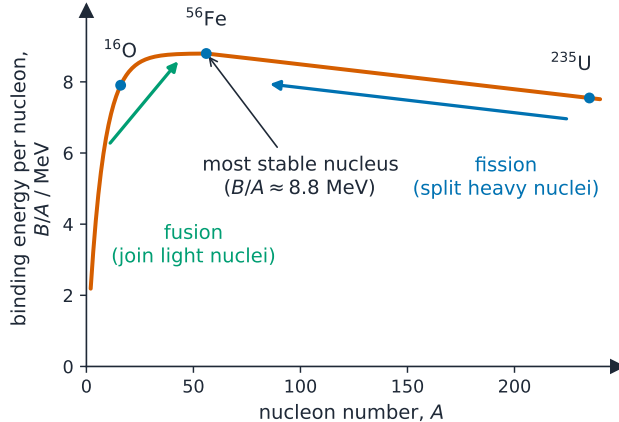
Iron-56 is the most stable nucleus. Light nuclei sit to the left; heavy nuclei sit to the right.

Reading the binding-energy curve

the shape	dome-shaped: rising steeply from hydrogen, peaking at iron , then falling slowly
why it peaks	the strong nuclear force 强核力 is short-range, so beyond about iron the electrostatic repulsion of the protons wins
to the left of iron	fusion moves nuclei up the curve and releases energy
to the right	fission moves them up too – both directions run downhill in energy
mass-energy equivalence 质能等价	$E = \Delta mc^2$ converts the mass defect into the energy released
in every equation	nucleon number, conservation of charge 电荷守恒 and energy must all balance

Binding energy per nucleon measures **stability**, not total energy. Iron is the most stable nuclide, which is why stars stop there.

Binding energy per nucleon vs nucleon number



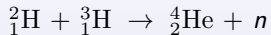
Binding energy per nucleon peaks near iron ($A \approx 56$); lighter and heavier nuclei are less tightly bound

Energy is released towards the iron peak

fusion

核聚变

two light nuclei join



needs millions of kelvin to
beat electrostatic repulsion

fission

核裂变

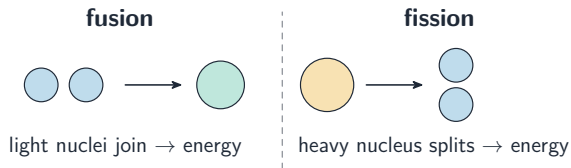
a heavy nucleus splits

extra neutrons can cause
more fissions – a **chain**

reaction 链式反应

needs a **critical mass** 临界质量

Both directions run downhill to iron



Fusion 聚变 joins light nuclei;
fission 裂变 splits heavy ones.
Both move nucleons **towards** the
iron peak, and both release the
difference in binding energy.

Definitions, and released or absorbed

“State what is meant by the mass defect of a nucleus.” [2]

The **difference** between the total mass of its separate nucleons and the mass of the nucleus.

Binding energy

The energy **needed to separate** a nucleus into its individual nucleons. It is **not** energy stored in the nucleus.

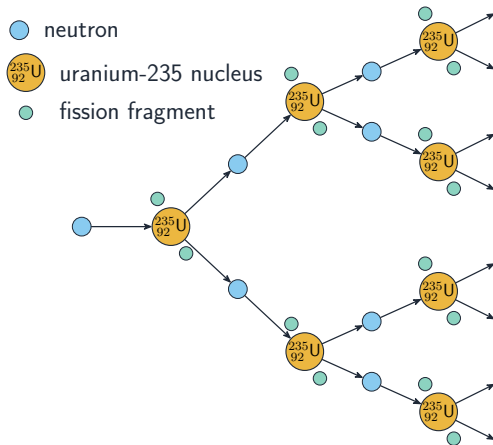
Show that $\approx 3.8 \times 10^{-12}$ J is released when two deuterium nuclei fuse to helium-4 (B/A : 1.11 and 7.07 MeV).

$$\begin{aligned} E &= 4 \times 7.07 - 4 \times 1.11 = 23.8 \text{ MeV} \\ &= 23.8 \times 10^6 \times 1.60 \times 10^{-19} \\ &= 3.8 \times 10^{-12} \text{ J} \end{aligned}$$

This is **fusion**.

Products heavier than reactants? Then energy must be **supplied**. The steep left side of the curve is why fusion gives more energy per nucleon than fission.

A chain reaction



In an uncontrolled chain reaction each fission of uranium-235 releases neutrons that cause more fissions.

Fission in a power station



A nuclear power station releases energy by nuclear fission.

3

Radioactive decay

Sheet 23.2 – what you need to know

You must be able to:

- use $A = \lambda N$, $\lambda = \ln 2/t_{1/2}$, $x = x_0 e^{-\lambda t}$; turn a mass into N
- explain why decay is exponential; compare two isotopes

What the exam asks

“Determine the rate at which positrons are produced in a sample of oxygen-15 ... of mass 2.85×10^{-6} kg.” [4]

“Explain why the variation with time of the activity ... is exponential in nature.” [3]

“Determine, in terms of T , the time at which the two samples will have equal activities.” [3]

Practise on exercise sheet 23.2, then try the real exam questions on past-paper sheet 23.2.

Activity, decay constant, exponential

$$A = \lambda N$$

A is **activity** 活度 (decays per unit time), in **becquerel** 贝可勒尔 (Bq). λ is the **decay constant** 衰变常数 – probability per unit time that a nucleus decays.

$$N = N_0 e^{-\lambda t}, \quad A = A_0 e^{-\lambda t}$$

The same **fraction** decays in each interval – that is what makes it **exponential decay** 指数衰减.

Half-life

Half-life 半衰期 $t_{1/2}$ is the time for N (or A , or the count rate) to fall to half:

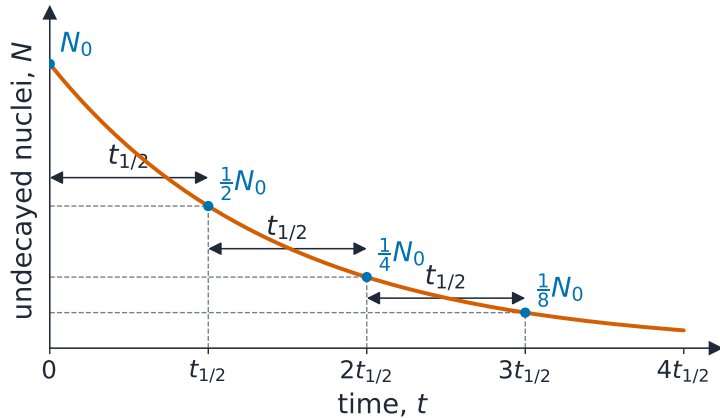
$$\lambda = \frac{\ln 2}{t_{1/2}} \approx \frac{0.693}{t_{1/2}}$$

After n half-lives the surviving fraction is $(1/2)^n$. After 5 half-lives, about 3% remains.

$$t_{1/2} = 6.0 \text{ hours}$$

$$6.0 \text{ h} = 21600 \text{ s, so } \lambda = 0.693/21600 \approx 3.2 \times 10^{-5} \text{ s}^{-1}$$

Half-life, in detail



$$N = N_0 e^{-\lambda t} \cdot T_{1/2} = \ln 2 / \lambda$$

The number of undecayed nuclei falls by half in each half-life

Activity, decay constant and half-life

activity

$A = \lambda N$ – decays per second, in becquerel

λ

the decay constant: the **probability per second** that any one nucleus decays

fixed per nuclide

λ never changes for a given **radionuclide** 放射性核素 – so a **larger sample has proportionally larger activity**

the link

$$\lambda = \frac{\ln 2}{t_{1/2}}$$

the units

convert the half-life to **seconds** first: 6.0 h = 21 600 s

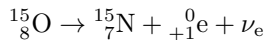
the decay law

$N = N_0 e^{-\lambda t}$, and the same form for A and for count rate

Decay is **random and spontaneous**: unaffected by temperature, pressure or chemical state.
Only the number of nuclei present changes the activity.

A positron emitter: from mass to activity

β^+ decay of the PET tracer oxygen-15:



- a proton becomes a neutron: Z falls by 1, A stays
- the **electron neutrino** carries off some energy, so β energies are spread
- a **tracer**: a radioactive isotope followed through the body by its emissions

Find the rate of positron production in 2.85×10^{-6} kg of oxygen-15 (half-life 122 s).

$$N = \frac{m}{15u} = \frac{2.85 \times 10^{-6}}{15 \times 1.66 \times 10^{-27}} = 1.14 \times 10^{20}$$

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{122 \text{ s}} = 5.68 \times 10^{-3} \text{ s}^{-1}$$

$$A = \lambda N = 6.5 \times 10^{17} \text{ s}^{-1}$$

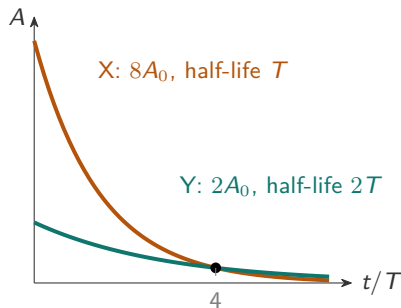
Why decay is exponential, and two isotopes compared

“Explain why the variation with time of the activity is exponential.”
[3]

Each nucleus has a **constant probability** of decaying per unit time (λ);
so the activity is proportional to the number left: $A = \lambda N$;
so a **constant fraction** decays in each equal time.

From 180 to 120 Bq in 5.0 h:

$$\lambda = \frac{1}{5.0} \ln \frac{180}{120} = 0.081 \text{ h}^{-1}.$$



$$8 \cdot 2^{-t/T} = 2 \cdot 2^{-t/2T} \Rightarrow 2^{t/2T} = 4 \Rightarrow t = 4T$$

4

Count rate and log graphs

Sheet 23.2.1 – what you need to know

You must be able to:

- say what **random** and **spontaneous** each mean
- explain why count rate is less than activity; correct for background
- read λ and A_0 off a $\ln A-t$ graph

What the exam asks

“Explain why the count rate measured by the detector is less than the activity of the sample.”

[2]

“Define activity of a radioactive sample.” **[1]**

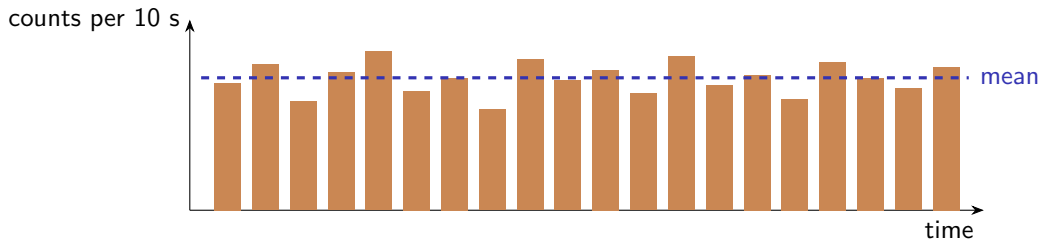
“Determine the half-life, in min, of the radioactive isotope.” **[1]**

Practise on exercise sheet 23.2.1, then try the real exam questions on past-paper sheet 23.2.

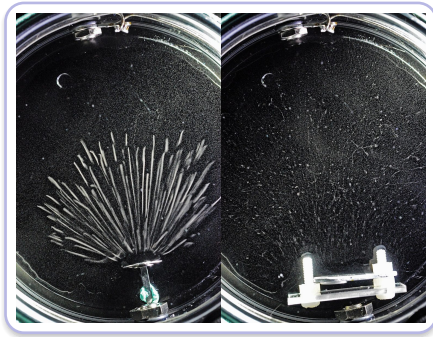
Random and spontaneous

Spontaneous – no outside trigger. Rate is not changed by temperature, pressure or chemical state.

Random – you cannot predict when a given nucleus will decay, only the probability. Evidence: the count rate **fluctuates**.



Seeing single decays



Each track is one decay – unpredictable individually, precisely regular in bulk. A **Geiger counter** 盖革计数器 beside a source clicks at uneven intervals, never a steady stream, which is the evidence that decay is random.

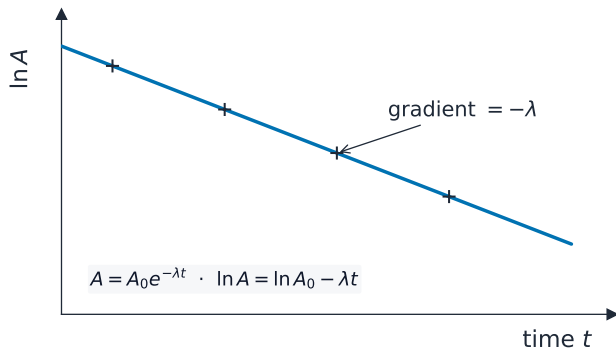
Finding λ from data

Take logs of $A = A_0 e^{-\lambda t}$:

$$\ln A = \ln A_0 - \lambda t$$

Compare $y = c + mx$: intercept $\ln A_0$, gradient $-\lambda$. From two readings,

$$\lambda = \frac{1}{t} \ln \frac{A_0}{A}$$



Read λ off the **gradient**, not from one point.

Count rate is not activity

“Define activity.” [1]

The number of nuclei that **decay per unit time**.

“Explain why the count rate is less than the activity.” [2]

Emissions go in **all directions**, and the detector catches only those heading into it;
some are **absorbed** on the way, and the detector does not register every one.

A detector catches 2.0% of the emissions. It reads 36.5 s^{-1} ; background is 0.5 s^{-1} . Find the activity.

$$A = \frac{36.5 - 0.5}{0.020} = 1.8 \times 10^3 \text{ Bq}$$

Subtract the **background** before taking logs. Left in, it makes the **ln graph level off** at late times. The intercept of $\ln A$ against t is $\ln A_0$.

5

Recap

Exam tips

- $E = mc^2$; **the mass defect** is the mass lost when nucleons bind, released as binding energy.
- **Binding energy per nucleon** peaks near iron – both fusion (light nuclei) and fission (heavy nuclei) move towards it and release energy.
- Activity $A = \lambda N$; decay is exponential $N = N_0 e^{-\lambda t}$; half-life $t_{1/2} = \ln 2 / \lambda$.
- Balance nuclear equations by conserving **nucleon number and proton number**.

Topic 23 – key takeaways

1 $E = mc^2$

$1 \text{ u} \leftrightarrow 931 \text{ MeV}$; balance A and Z

2 Binding

B/A peaks at iron; fusion and fission move towards it

3 Decay

$$A = \lambda N; N = N_0 e^{-\lambda t}$$

4 Half-life

$t_{1/2} = \ln 2 / \lambda$; random and spontaneous

Check yourself

- 1 Why does a small Δm release a lot of energy?
- 2 Which nucleus sits at the peak of the B/A curve?
- 3 Fusion and fission both release energy – why?
- 4 After 3 half-lives, what fraction remains?

Answers 1. c^2 is huge 2. iron-56 3. both move towards the iron peak 4. $1/8$