Mass–Energy Equivalence

A2 Physics ▪ Topic 23

A-Level Physics

$$E = mc^2$$

What we will cover

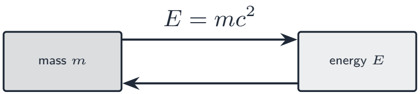
1. Mass–energy $E = mc^2$
2. Nuclear reactions
3. Mass defect & binding energy
4. Fusion & fission
5. Radioactive decay
6. Recap

Mass and energy are the same thing

$$E = mc^2, \qquad \Delta E = \Delta m\, c^2$$

- c^2 is huge, so a tiny mass carries vast energy
- mass and energy are conserved together

> $1\,\mathrm{u}$ of mass equals about $931\,\mathrm{MeV}$ – the currency that turns a mass change into an energy release.



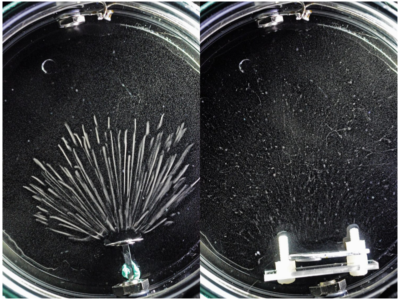
Nuclear reactions

Balance nucleon number (top) and proton number (bottom) on both sides:

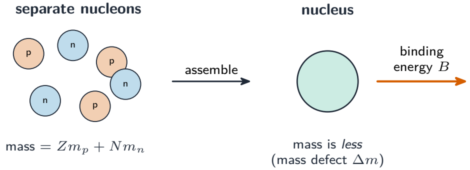
$$^{A}_{Z}\mathrm{X} \to {}^{A'}_{Z'}\mathrm{Y} + \ldots$$

- α: loses $^{4}_{2}\mathrm{He}$
- β^{-}: a neutron $\to$ proton $+$ electron

> Any mass lost reappears as the kinetic energy of the products, via $E = mc^2$.



Mass defect and binding energy



A nucleus weighs less than its separate nucleons. That **mass defect** 质量亏损 Δm is the **binding energy** 结合能:

$$E_B = \Delta m\, c^2$$

> Binding energy is the work to pull the nucleus apart – a bigger E_B per nucleon means a more stable nucleus.

Worked example – binding energy

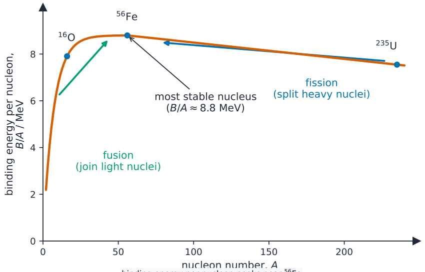
A nucleus has a mass defect of $0.030\,\mathrm{u}$. Find its binding energy. ($1\,\mathrm{u} = 931\,\mathrm{MeV}$)

$$E_B = \Delta m\,c^2 = 0.030 \times 931$$

$$\approx 28\,\mathrm{MeV}$$

Using $1\,\mathrm{u} = 931\,\mathrm{MeV}$ skips the joules entirely. Divide by the nucleon number to compare stability.

Fusion and fission



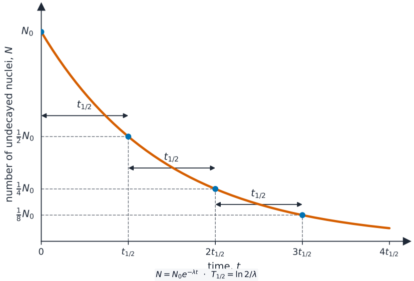
binding energy per nucleon peaks near ^{56}Fe

The binding-energy-per-nucleon curve peaks near iron:

- **fusion** 核聚变: join light nuclei (below iron)
- **fission** 核裂变: split heavy nuclei (above iron)

> Both move up the curve to more binding energy per nucleon – and the difference is released.

Radioactive decay



Decay is random but has a fixed probability λ per second:

$$A = \lambda N, \qquad N = N_0 e^{-\lambda t}$$

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

> Every **half-life** 半衰期 the count halves. Plotting $\ln N$ against t gives a straight line of gradient $-\lambda$.

Topic 23 – key takeaways

1 $E = mc^2$

mass and energy interconvert; $1\,\mathrm{u} = 931\,\mathrm{MeV}$

2 Binding

$E_B = \Delta m\,c^2$ – higher per nucleon = stabler

3 Fusion/fission

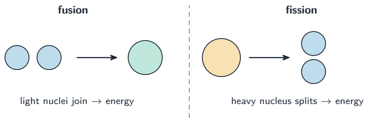
both climb toward iron, releasing energy

4 Decay

$N = N_0\,e^{-\lambda t}$ – half-life $\ln 2/\lambda$

Check yourself

1. Where does the energy of decay come from?
2. What does a big E_B per nucleon mean?
3. Is the Sun powered by fusion or fission?
4. After 3 half-lives, what fraction is left?



Answers **1.** lost mass ($E = mc^2$) **2.** more stable **3.** fusion **4.** $1/8$