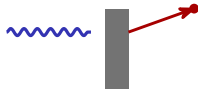


Quantum Physics

A2 Physics ■ Topic 22

A-Level Physics



What we will cover

- 1 Photons: $E = hf$, $p = h/\lambda$ 22.1
- 2 The photoelectric effect 22.2
- 3 Stopping potential 22.2.1
- 4 Wave-particle duality 22.3
- 5 Energy levels and spectra 22.4
- 6 Recap

1

Photons

Sheet 22.1 – what you need to know

You must be able to:

- say what a **photon** is; use $E = hf$, the electronvolt and $p = h/\lambda$
- count photons per second: $N = P/E$
- find the force of a beam on an absorber or a mirror

What the exam asks

“State what is meant by a photon.” [2]

“Calculate the number of photons emitted per unit time by the laser.” [1]

“Determine the average force exerted by the beam of photons on the surface.” [4]

Practise on exercise sheet 22.1, then try the real exam questions on past-paper sheet 22.1.

Light comes in packets

A **photon** 光子 is a quantum of EM radiation, travelling at c :

$$E = hf = \frac{hc}{\lambda}, \quad h = 6.63 \times 10^{-34} \text{ J s}$$

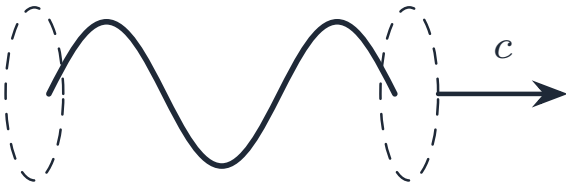
where h is the **Planck constant** 普朗克常数. Higher frequency (shorter wavelength) $\Rightarrow$ more energy per photon. Brightness is the **number** of photons per second; colour is the energy of **each** one.

Green light, $\lambda = 500 \text{ nm}$

$$E = hc/\lambda \approx 4.0 \times 10^{-19} \text{ J } (\approx 2.5 \text{ eV})$$

One packet, one electron

A photon is absorbed **whole** or not at all – which is why the effect depends on the photon's frequency and not on how many arrive.



$$E = hf$$

a photon (a packet of light)

The electronvolt, and photon momentum

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

The KE an electron gains moving through 1 V.

eV $\rightarrow$ J: multiply by 1.60×10^{-19} ; J
 $\rightarrow$ eV: divide.

A photon has zero rest mass but non-zero momentum:

$$p = \frac{E}{c} = \frac{h}{\lambda}$$

Radiation pressure follows from this.

Photons: counting them, and the force they exert

“State what is meant by a photon.” [2]

A discrete packet (quantum) of electromagnetic energy, of energy hf .

$$E = hf = \frac{hc}{\lambda} \text{ and } E = pc \text{ give}$$

$$p = \frac{h}{\lambda}$$

Same intensity, red or blue: same force.
Blue photons carry more momentum each, but there are fewer of them.

A 5.0 mW laser (630 nm) shines on a surface that absorbs it. Find the photons per second and the force.

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^8}{630 \times 10^{-9}} \\ = 3.16 \times 10^{-19} \text{ J}$$

$$N = \frac{P}{E} = \frac{5.0 \times 10^{-3}}{3.16 \times 10^{-19}} = 1.6 \times 10^{16} \text{ s}^{-1}$$

$$F = \frac{\Delta p}{\Delta t} = Np = \frac{P}{c} = 1.7 \times 10^{-11} \text{ N}$$

A mirror reverses each photon: $\Delta p = 2p$, so $F = 2P/c$.

2

Photoelectric effect

Sheet 22.2 – what you need to know

You must be able to:

- describe the effect and use $hf = \Phi + \frac{1}{2}mv_{\max}^2$
- read h , f_0 and Φ off an $E_{K,\max}-f$ graph
- find the fastest electron's speed

What the exam asks

“State what is meant by the photoelectric effect.” [2]

“Sketch the variation of E_K with f for each plate.” [4]

“State three quantitative conclusions that can be drawn from the results.” [3]

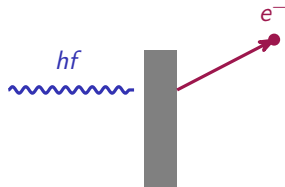
Practise on exercise sheet 22.2, then try the real exam questions on past-paper sheet 22.2.

One photon frees one electron

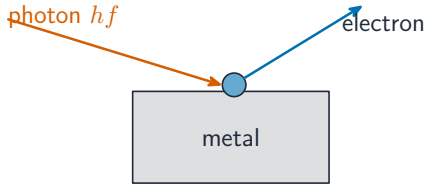
EM radiation of high enough frequency hitting a metal emits **photoelectrons** 光电子.

$$hf = \Phi + \frac{1}{2}mv_{\max}^2$$

Work function 逸出功 $\Phi = hf_0$ is the least energy to free an electron. Below the **threshold frequency** 极限频率 f_0 , **no** electrons come out, however bright.

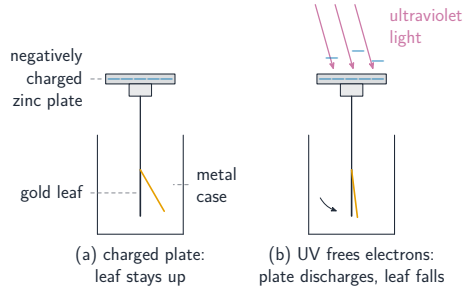


Photoelectric effect



$$hf = \Phi + KE_{\max}$$

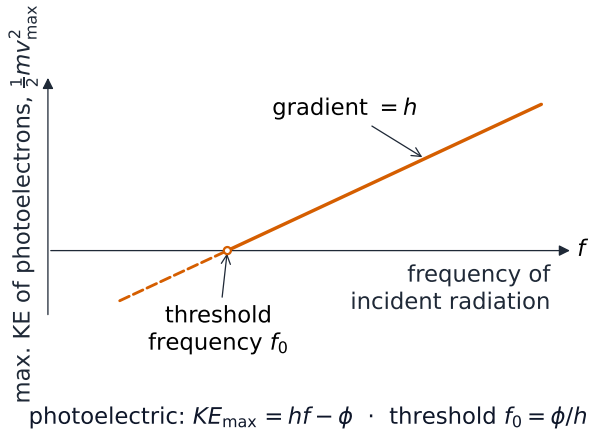
(escape energy Φ + leftover KE)



A charged zinc plate loses its charge (the gold leaf falls) when ultraviolet light shines on it.

One photon gives its energy hf to one electron: part frees it (the work function Φ), the rest is the electron's KE

Einstein's photoelectric equation



The maximum kinetic energy of photoelectrons rises linearly with frequency, reaching zero at the threshold frequency f_0

Worked example – maximum KE

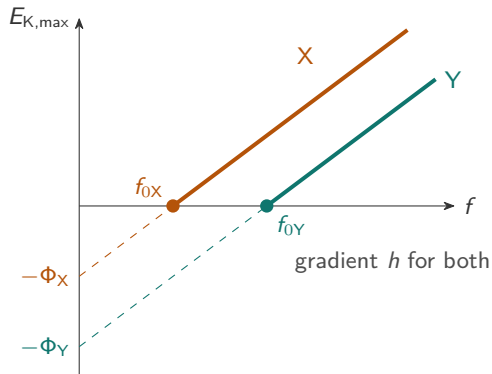
Work function 2.0 eV; photons of 3.5 eV

$$\begin{aligned}\frac{1}{2}mv_{\text{max}}^2 &= hf - \Phi = 3.5 - 2.0 \\ &= 1.5 \text{ eV}\end{aligned}$$

Maximum KE depends **linearly on frequency**, not on brightness.

Convert Φ to joules before subtracting if hf is in joules. A negative answer means you are below threshold.

Two metals on one graph



“State what is meant by the photoelectric effect.” [2]

The emission of **electrons** from a metal surface when **electromagnetic radiation** of high enough frequency falls on it.

Sketch marks:

- lines start on the axis at f_0 , with $E_K = 0$ below
- **parallel**: both gradients are h
- extended back, intercepts $-\Phi$: if $f_{0Y} = 2f_{0X}$, then $\Phi_Y = 2\Phi_X$

Worked example – work function and fastest electron

Magnesium's threshold frequency is 8.9×10^{14} Hz. Find its work function, and the maximum speed of electrons emitted by ultraviolet of 1.1×10^{15} Hz.

$$\begin{aligned}\Phi &= hf_0 = 6.63 \times 10^{-34} \times 8.9 \times 10^{14} \\ &= 5.9 \times 10^{-19} \text{ J (3.7 eV)}\end{aligned}$$

$$hf = 6.63 \times 10^{-34} \times 1.1 \times 10^{15} = 7.3 \times 10^{-19} \text{ J}$$

$$E_{K,\max} = hf - \Phi = 1.4 \times 10^{-19} \text{ J}$$

$$\begin{aligned}v_{\max} &= \sqrt{\frac{2E_{K,\max}}{m_e}} = \sqrt{\frac{2 \times 1.4 \times 10^{-19}}{9.11 \times 10^{-31}}} \\ &= 5.5 \times 10^5 \text{ m s}^{-1}\end{aligned}$$

Work in **joules** before using $\frac{1}{2}mv^2$: $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$.

A field of solar panels



Solar cells use the photoelectric effect to turn light into electricity.

3

Stopping potential

Sheet 22.2.1 – what you need to know

You must be able to:

- explain each observation with photons, and why waves fail
- use $eV_S = hf - \Phi$ and the threshold wavelength
- separate what intensity changes from what frequency changes

What the exam asks

“Explain, with reference to photons, why V_S depends on the frequency of the incident electromagnetic radiation.” [3]

“Calculate the work function energy of magnesium.” [2]

“For ultraviolet radiation with a frequency of 11×10^{14} Hz, calculate the maximum speed of the emitted electrons.” [3]

Practise on exercise sheet 22.2.1, then try the real exam questions on past-paper sheet 22.2.

Why the wave model fails

A wave model says brightness should set the KE, and any frequency should work given enough time. Experiment says otherwise:

no emission below f_0 , however bright

immediate emission at or above f_0 , even when dim

max KE depends on frequency, not brightness

the *number* of photoelectrons (the current) depends on brightness

Why max KE is fixed but current grows with brightness

A brighter beam of the **same frequency** has **more photons per second**, but each still carries hf .

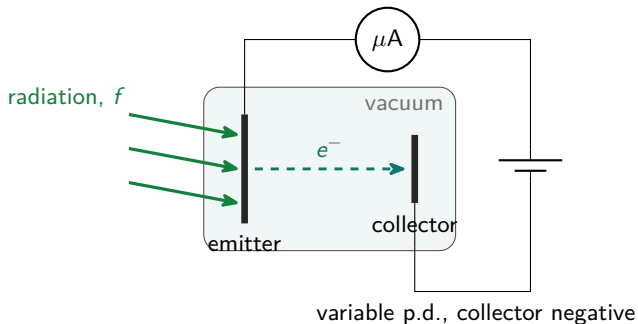
$$KE_{\max} = hf - \Phi$$

set by f only

double the brightness

photons per second	$\times 2$
current	$\times 2$
energy per photon hf	unchanged
$KE_{\max}$	unchanged

Stopping potential



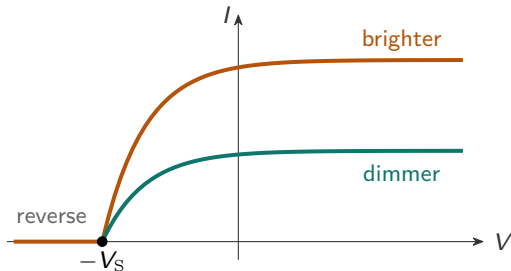
Raise the reverse p.d. until the current is zero: that p.d. is V_S , and $eV_S = E_{K,max}$.

$$eV_S = hf - \Phi$$

“Explain, with reference to photons, why V_S depends on the frequency.” [3]

Photon energy hf rises with frequency;
one photon gives all its energy to **one** electron;
so the maximum KE ($hf - \Phi$) rises, and a larger p.d. is needed to stop the fastest electrons.

What intensity changes, and what frequency changes



Brighter, same f : more photons, more current, the same V_S .

- higher f : each electron gets more energy, so a larger V_S
- V_S against f : a straight line of gradient h/e , crossing the f -axis at f_0
- threshold wavelength $\lambda_0 = hc/\Phi$; for $\Phi = 3.7 \text{ eV}$, $\lambda_0 = 340 \text{ nm}$: longer wavelengths emit nothing

4

Wave-particle duality

Sheet 22.3 – what you need to know

You must be able to:

- give evidence for waves and for particles, for light and for electrons
- use $\lambda = h/p$ with $p = \sqrt{2mqV}$
- explain annihilation into two photons

What the exam asks

“Explain why two gamma-ray photons are produced, rather than just one.” [1]

“Determine an expression, in terms of m , q and V , for the momentum p of an electron in the beam.” [3]

“The speed of the electrons is increased. Suggest, with a reason, how this change affects the pattern observed on the screen.” [2]

Light is both – and so is matter

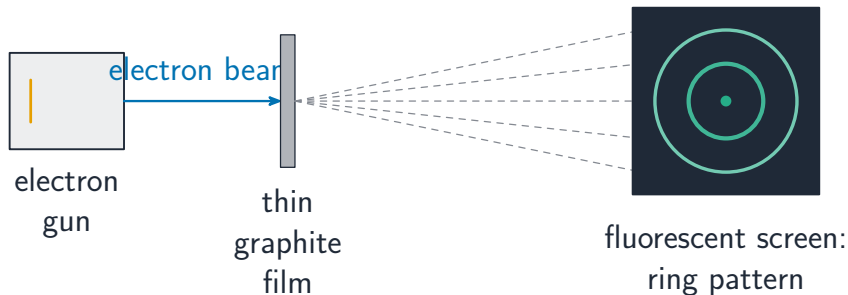
Photoelectric effect $\Rightarrow$ particle. Interference and diffraction $\Rightarrow$ wave. That is **wave-particle duality** 波粒二象性. de Broglie: any moving particle has a wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Electrons fired at graphite make a ring **diffraction** 衍射 pattern – only waves diffract, yet electrons do. Faster electron $\Rightarrow$ larger $p \Rightarrow$ shorter $\lambda \Rightarrow$ rings closer together.

Electrons diffract, so electrons are waves

Fire electrons at a thin **crystal lattice** 晶格 of graphite and they produce **diffraction rings** – a wave effect, from particles with mass. That is **wave-particle duality** 波粒二象性: the lattice acts as a natural **diffraction grating** 衍射光栅.



Worked example – de Broglie wavelength

Electron accelerated through 2500 V. Find λ .

$E_K = eV$, so $p = \sqrt{2m_e eV}$ and $\lambda = h/p$:

$$\lambda = \frac{h}{\sqrt{2m_e eV}} \approx 2.5 \times 10^{-11} \text{ m}$$

Close to atomic spacings – which is why the electrons diffract off graphite.

Annihilation, and the momentum of an accelerated electron

“Explain why two gamma-ray photons are produced, rather than just one.” [1]

The pair has (almost) **zero momentum**; one photon would carry momentum away, so two go off in **opposite directions**.

Each photon from an electron and positron at rest:

$$E = m_e c^2 = 9.11 \times 10^{-31} \times (3.00 \times 10^8)^2 \\ = 8.2 \times 10^{-14} \text{ J}$$

$$\lambda = \frac{hc}{E} = 2.4 \times 10^{-12} \text{ m}$$

“Determine an expression, in terms of m , q and V , for the momentum p .” [3]

$$qV = \frac{1}{2}mv^2 = \frac{p^2}{2m} \\ p = \sqrt{2mqV}$$

At 5.0 kV: $p = 3.8 \times 10^{-23} \text{ N s}$, so $\lambda = h/p = 1.7 \times 10^{-11} \text{ m}$.

Faster electrons: shorter λ , less diffraction, so the rings get **closer together**. p against $1/\lambda$ is a line of gradient h .

5

Energy levels

Sheet 22.4 – what you need to know

You must be able to:

- explain line spectra with discrete energy levels
- use $hf = E_1 - E_2$ in eV and in joules, forwards and backwards
- explain why the levels are negative

What the exam asks

“Calculate the energy, in J, of the ground state.” [1]

“Determine the energy, in eV, of the energy level from which the electron transition originates.” [2]

“Determine the wavelength, in nm, of this radiation as detected by the observer on the Earth.” [2]

Practise on exercise sheet 22.4, then try the real exam questions on past-paper sheet 22.4.

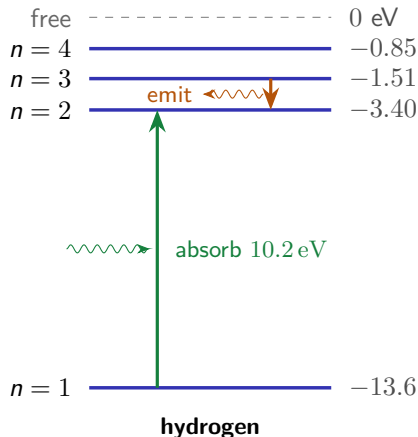
Electrons sit on discrete rungs

Electrons occupy **discrete energy levels** 能级: the lowest is the **ground state** 基态, the rest are **excited states** 激发态.

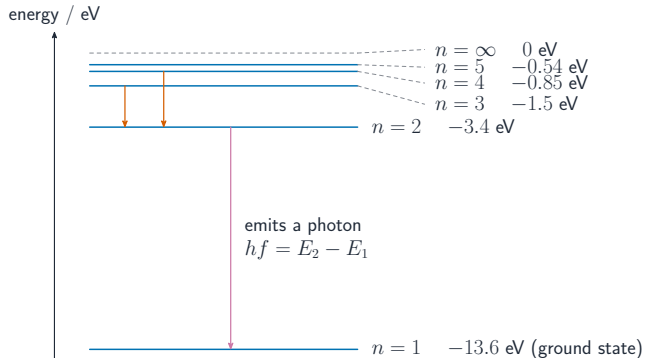
$$hf = E_2 - E_1$$

photon energy = gap between the levels

- **down** a level: **emit** (bright line)
- **up** a level: **absorb** (dark line)



Energy levels in atoms



The electron energy levels of hydrogen are discrete and negative, with the ground state at -13.6 eV

Line spectra are a fingerprint

Because the levels are fixed, only certain photon energies appear – a **line spectrum** 线状光谱.

emission

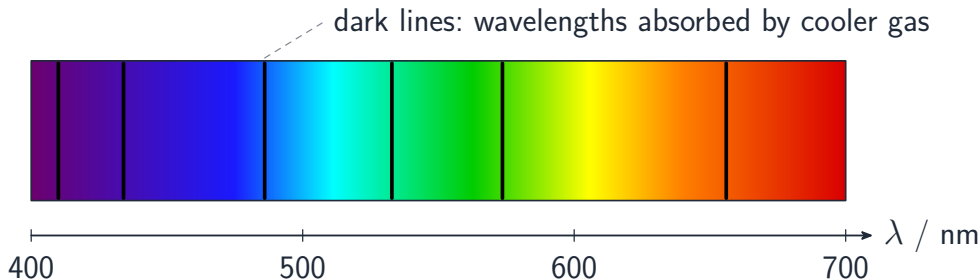
sharp **bright** lines on
a dark background

absorption

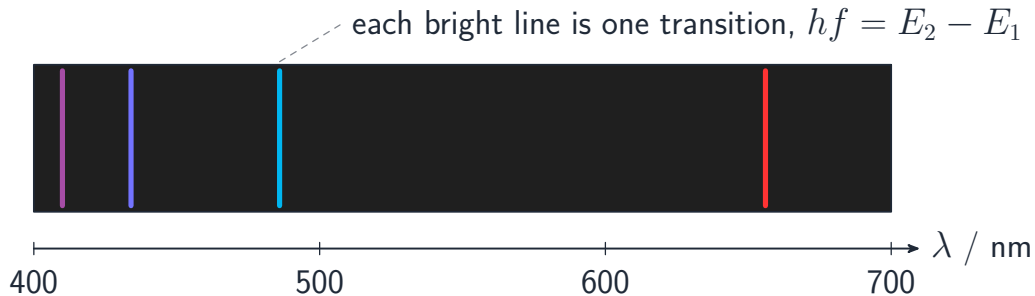
sharp **dark** lines on a
continuous background

Emission and absorption are the same fingerprint

Hot gas **emits** at its own frequencies; the same cool gas in front of a continuous source gives an **absorption spectrum** 吸收光谱 at exactly those frequencies – the dark **Fraunhofer lines** 夫琅禾费线 in sunlight name the elements in the Sun.



Emission spectrum



The emission spectrum of hydrogen is a set of sharp bright lines on a dark background

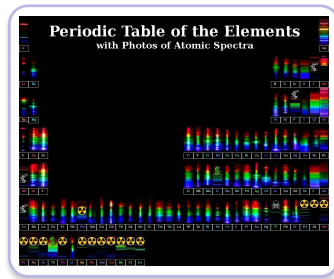
Worked example – a line from a transition

Hydrogen: $n = 3 \rightarrow n = 2$

$$E_3 = -1.51 \text{ eV}, \quad E_2 = -3.40 \text{ eV}$$

$$\Delta E = E_3 - E_2 = 1.89 \text{ eV}$$

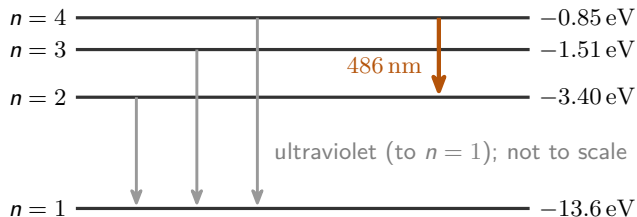
$$\lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV nm}}{1.89 \text{ eV}} \approx 656 \text{ nm}$$



The red hydrogen line, at 656 nm.

Using $hc = 1240 \text{ eV nm}$ turns a level difference straight into a wavelength.

Working back to an energy level



A 486 nm line comes from a drop to the -3.40 eV level. Find the upper level.

$$E = \frac{hc}{\lambda} = 4.09 \times 10^{-19} \text{ J} = 2.56 \text{ eV}$$

$$E_{\text{upper}} = -3.40 + 2.56 = -0.84 \text{ eV}$$

Levels are **negative**: zero is a **free** electron at infinity, so a bound one needs energy to escape.

Ground state in J:

$$-13.6 \times 1.60 \times 10^{-19} = -2.18 \times 10^{-18} \text{ J.}$$

Line spectra: answers that score

“Explain how a line spectrum shows that electrons have discrete energy levels.” [3]

Each line is **one wavelength**, so one photon energy ($E = hc/\lambda$);
each photon comes from an electron dropping between **two levels**;
only certain energy differences occur, so only **certain levels** exist.

A galaxy recedes at $1.5 \times 10^7 \text{ m s}^{-1}$. Where is the 656.3 nm hydrogen line seen on Earth?

$$\Delta\lambda = \lambda \frac{v}{c} = 656.3 \times \frac{1.5 \times 10^7}{3.00 \times 10^8}$$
$$= 32.8 \text{ nm}$$

$$\lambda_{\text{seen}} = \text{689 nm (redshifted)}$$

Every line shifts by the **same fraction**:
that is how a known pattern is spotted
in a moving source (topic 25).

6

Recap

Exam tips

- Photon energy $E = hf = hc/\lambda$; the **photoelectric equation** is $hf = \Phi + \frac{1}{2}mv_{max}^2$.
- The photoelectric effect shows light is **quantised**: below the threshold frequency, no electrons are emitted whatever the intensity.
- **de Broglie** $\lambda = h/p$ gives particles a wavelength; electron diffraction is the evidence.
- Energy levels are discrete; an emitted photon's energy equals the **difference between two levels**.

Topic 22 – key takeaways

1 Photon

$$E = hf = hc/\lambda; p = h/\lambda$$

2 Photoelectric

$$hf = \Phi + E_{k,\max}; \text{ threshold } f_0$$

3 Duality

$$\lambda = h/p - \text{electrons diffract too}$$

4 Levels

$$hf = E_2 - E_1 - \text{line spectra}$$

Check yourself

- 1 Below threshold, does brighter light help?
- 2 What does doubling brightness (same f) do to max KE and to current?
- 3 Faster electron: longer or shorter de Broglie λ ?
- 4 Why is an emission spectrum a set of sharp lines?

Answers 1. no 2. KE unchanged; current doubles 3. shorter 4. energy levels are discrete