

Magnetic Fields

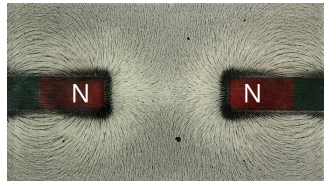
A2 Physics ■ Topic 20

A-Level Physics



What we will cover

- 1 Magnetic fields 20.1
- 2 Force on a conductor 20.2
- 3 Force on a moving charge 20.3
- 4 Hall effect and selectors 20.3.1
- 5 Fields due to currents 20.4
- 6 Electromagnetic induction 20.5
- 7 E.m.f. graphs, rotating coils 20.5.1
- 8 Recap



1

Magnetic fields

Sheet 20.1 – what you need to know

You must be able to:

- say what a **magnetic field** is and draw its lines
- draw the fields of a wire, a coil, a solenoid and a magnet
- find the force on each side of a coil, and its **torque**

What the exam asks

“State what is meant by a magnetic field.” [2]

“Draw four field lines to represent the magnetic field around wire P due to the current in the wire.” [3]

“Show that the torque τ on the coil is given by
 $\tau = 1.6 \times 10^{-3} \cos \theta \text{ N m}.$ ” [2]

Practise on exercise sheet 20.1, then try the real exam questions on past-paper sheet 20.1.

A field that acts on moving charge

A **magnetic field** 磁场 is a region where a moving charge (or a **current** 电流) feels a **force** 力. There are two ways to make one.

moving charges

usually a current in a wire

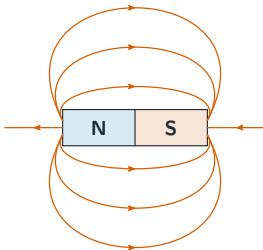
permanent magnets

tiny atomic currents
inside the material

Both are the same thing underneath: a magnetic field is made by **moving charge**, whether it flows along a wire or circulates inside an atom.

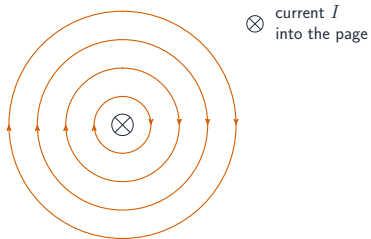
The field of a bar magnet

field lines run from **N** to **S** outside the magnet



Field lines run from **N** to **S** outside the magnet, and never cross. Where they crowd together the field is **strongest** – at the poles.

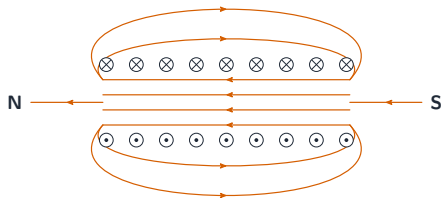
Field around a long straight wire



circular field lines (right-hand grip rule)

Field around a long straight wire – right-hand grip rule
rule 右手螺旋定则

field nearly uniform inside, spreads out like a bar magnet outside

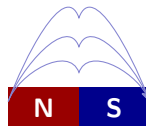


Field of a solenoid

The field is **circles** around the wire, closer together near it. Grip the wire with the right hand, thumb along the current, and the fingers curl the way the field points.

Field-line rules

- **field lines** 场线 run **N to S** outside a magnet (S to N inside – closed loops)
- lines never cross
- closer lines $\Rightarrow$ a stronger field – **flux density** 磁通密度 B , in **tesla** 特斯拉
- a **long solenoid** 长螺线管 has a uniform field inside and looks like a bar magnet outside



Four patterns you must recognise

bar magnet

curved $N \rightarrow S$ outside; strongest near the poles

straight wire

circles; **right-hand grip** 右手定则:
thumb = current, fingers = field

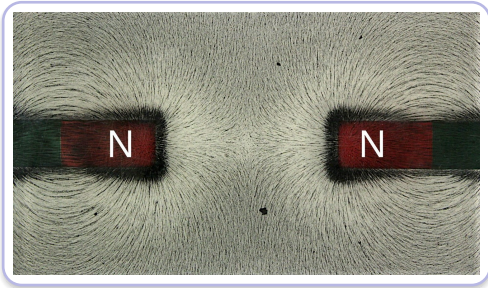
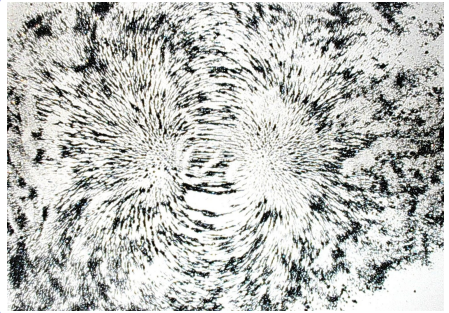
flat coil

field through the centre is at right angles; acts like a small bar magnet

solenoid

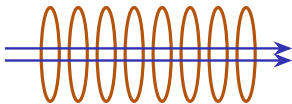
nearly **uniform** inside along the axis; outside falls off fast

The same patterns, in iron filings

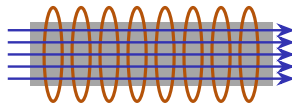


Each filing turns to lie **along** the field, and they crowd where it is **strongest**. Between two like poles they part, leaving a **neutral point**.

An iron core multiplies the field



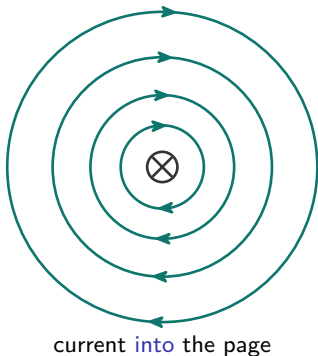
air core



iron core

An **iron core** 铁芯 **multiplies** the field: the iron's atomic magnets line up and add to it. That is why **electromagnets** 电磁铁 and **transformers** 变压器 have iron cores.

A magnetic field, in words and in lines



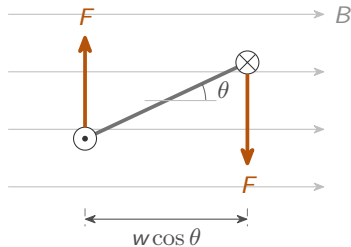
“State what is meant by a magnetic field.” [2]

A region of space where a **moving charge** (or a current-carrying conductor, or a magnetic material) experiences a **force**.

“Draw four field lines around the wire.” [3]

Concentric circles centred on the wire;
spacing **increasing** outwards (the field weakens);
direction **clockwise** for a current into the page (right-hand grip).

The torque on a coil



end view: plane at θ to B

A 50-turn coil, sides $L = 0.050$ m in the field and width $w = 0.080$ m, carries 0.20 A in $B = 0.040$ T. Show that $\tau = 1.6 \times 10^{-3} \cos \theta$ N m.

$$F = NBIL = 50 \times 0.040 \times 0.20 \times 0.050 = 0.020 \text{ N}$$

$$\begin{aligned} \tau &= F \times w \cos \theta = 0.020 \times 0.080 \cos \theta \\ &= 1.6 \times 10^{-3} \cos \theta \text{ N m} \end{aligned}$$

τ against θ is a **cosine**: largest with the plane along B , zero across it. An iron core raises B , so the torque.

2

Force on a conductor

Sheet 20.2 – what you need to know

You must be able to:

- define magnetic flux density and use $F = BIL \sin \theta$
- find the force's direction with Fleming's left-hand rule
- measure B with a current balance

What the exam asks

“Define magnetic flux density.”

[2]

“Calculate the magnitude of the force on side QR of the coil.” **[3]**

“Draw a line to show the path of the positrons through the magnetic field.” **[3]**

Practise on exercise sheet 20.2, then try the real exam questions on past-paper sheet 20.2.

Force on a current-carrying wire

$$F = BIL \sin \theta$$

θ is the angle between the wire and the field. Largest when the wire is at right angles ($F = BIL$); zero when the wire is along the field. This also **defines** the **magnetic flux density** 磁通密度 B :

$$B = \frac{F}{IL} \quad (\text{wire} \perp \text{field}), \quad [B] = \text{T} = \text{N A}^{-1} \text{m}^{-1}$$

Fleming's left-hand rule

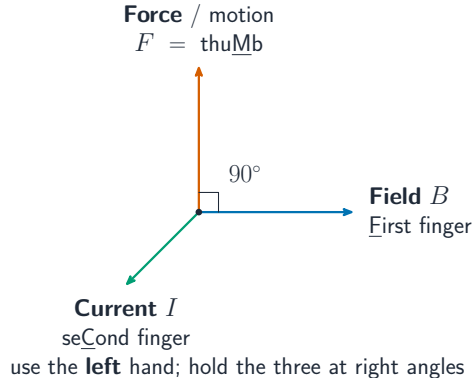
Use the **left** hand. Three fingers at right angles:

- **F**irst finger – **F**ield
- se**C**ond finger – **C**urrent
- **t**humb – **t**hrust (force)

$$L = 0.20 \text{ m}, I = 3.0 \text{ A}, B = 0.50 \text{ T}, \\ \theta = 90^\circ$$

$$F = BIL = 0.50 \times 3.0 \times 0.20 \\ = 0.30 \text{ N}$$

Fleming's left-hand rule: thumb = force



Hold the three at **right angles**: **F**orce on the thumb, **F**ield on the first finger, **C**urrent on the second. It is the **left** hand for a motor – the right hand is for a generator.

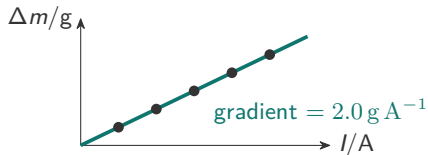
Defining B , and measuring it with a balance

“Define magnetic flux density.” [2]

The force per unit current per unit length on a straight conductor placed at right angles to the field.

$$F = BIL \sin \theta$$

$$1 \text{ T} = 1 \text{ N A}^{-1} \text{ m}^{-1}$$



A wire with $L = 0.050 \text{ m}$ in the field of a magnet resting on a top-pan balance. The reading changes by Δm as I rises. Find B .

By Newton's third law the wire pushes the magnet as hard as the magnet pushes the wire:

$$\Delta m g = BIL \Rightarrow \frac{\Delta m}{I} = \frac{BL}{g}$$

$$B = \frac{\text{gradient} \times g}{L} = \frac{2.0 \times 10^{-3} \times 9.81}{0.050}$$

$$= 0.39 \text{ T}$$

3

Force on a moving charge

Sheet 20.3 – what you need to know

You must be able to:

- use $F = BQv\sin\theta$ and $r = mv/BQ$
- draw and explain circular paths, with their direction
- combine electric and magnetic fields

What the exam asks

“Explain how an electric field can be used with the magnetic field to ensure that the particle now passes through point Z.” [3]

“Show that the speed u of ions that pass undeviated through the velocity selector is given by $u = V/(Bd)$.” [2]

“By reference to the forces acting on a positive ion, determine the direction of the magnetic field. Explain your reasoning.” [3]

Force on a moving charge

$$F = BQv\sin\theta$$

Same left-hand rule – the second finger is the motion of a **positive** charge (reverse it for a negative one). In a wire that motion is the electrons' **drift velocity** 漂移速度. Largest when $v \perp B$; zero when v is along B .

The force is always perpendicular to the velocity, so it does **no work** – it changes direction, never speed.

Circular path in a uniform field

$BQv = mv^2/r$ gives

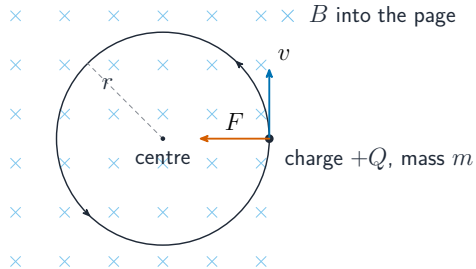
$$r = \frac{mv}{BQ}, \quad T = \frac{2\pi m}{BQ}$$

Here BQv is the **centripetal force** 向心力. T does **not** depend on speed – a faster particle goes in a bigger circle but takes the same time per turn, and its **kinetic energy** 动能 never changes because the force does no work. A component of v along B is unchanged, so the path is a **helix** 螺旋.

Proton: $m = 1.7 \times 10^{-27} \text{ kg}$,
 $v = 2.0 \times 10^6 \text{ m s}^{-1}$, $B = 0.50 \text{ T}$

$$r = \frac{mv}{BQ} \approx 0.043 \text{ m}$$

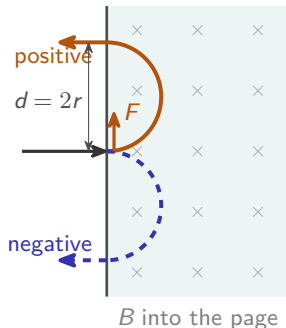
Circular motion in a uniform field



$$BQv = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{BQ}$$

Circular path of a charged particle in a magnetic field

Drawing the path: semicircles, signs and speeds



- force at entry: **at right angles** to v , towards the centre
- opposite charge (positron vs electron): curves the **other way**
- faster or heavier: **larger** r ; larger B or q : smaller r

A proton enters at $2.0 \times 10^6 \text{ m s}^{-1}$ at right angles to $B = 0.10 \text{ T}$. Where does it leave the field?

$$r = \frac{mv}{Bq} = \frac{1.67 \times 10^{-27} \times 2.0 \times 10^6}{0.10 \times 1.60 \times 10^{-19}} = 0.21 \text{ m}$$

$$d = 2r = 0.42 \text{ m from the entry point}$$

Adding an electric field: the selector answers

“Explain how an electric field can be used with the magnetic field so that the particle passes straight through.”
[3]

Apply E at right angles to both B and v , directed so the electric force is opposite the magnetic force,
with E adjusted so that $qE = Bqv$, i.e. $E = Bv$.

The period $T = 2\pi m/Bq$ does not depend on v : faster particles go round bigger circles in the same time.

Protons of kinetic energy 2.0 keV enter a selector with $B = 0.050$ T, plates 10 mm apart. Find the p.d. needed.

$$v = \sqrt{\frac{2E_K}{m}} = \sqrt{\frac{2 \times 3.2 \times 10^{-16}}{1.67 \times 10^{-27}}} \\ = 6.2 \times 10^5 \text{ m s}^{-1}$$

$$V = Ed = Bvd = 0.050 \times 6.2 \times 10^5 \times 0.010 \\ = 310 \text{ V}$$

4

Hall effect

Sheet 20.3.1 – what you need to know

You must be able to:

- derive $V_H = BI/ntq$ step by step
- explain why a Hall probe is a **semiconductor**
- use a velocity selector, then $r = mv/Bq$ to find a mass

What the exam asks

“Derive an expression for v in terms of B and the electric field strength E .” [2]

“State how the expression in (a) predicts that the alpha particle also passes undeviated through the velocity selector.” [1]

“The positive ions enter with greater kinetic energy. Sketch the path of these ions.” [2]

Practise on exercise sheet 20.3.1, then try the real exam questions on past-paper sheet 20.3.

Hall effect – a voltage across a current

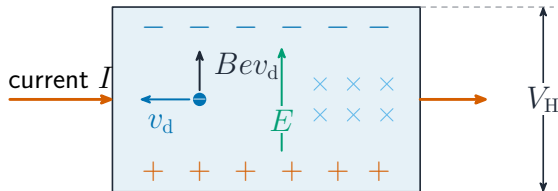
A slab carrying I , in a field B at right angles, develops a **Hall voltage** 霍尔电压. Charges feel BQv_d , pile up, and make an electric field that opposes further build-up. At steady state $E = Bv_d$:

$$V_H = \frac{BI}{ntq}$$

A **Hall probe** 霍尔探头 measures B : known current through a thin semiconductor, read V_H . Largest when the slab is at right angles to B .

Inside the Hall probe

Charges are pushed sideways until the electric field they build up balances the magnetic force – so V_H measures B .



× B into the page; electrons drift against I

$$V_H = \frac{BI}{n t q}$$

Deriving the Hall voltage

“Show that $V_H = \frac{BI}{ntq}$.” [4]

balance: $Bqv = qE_H = q\frac{V_H}{d} \Rightarrow V_H = Bvd$

current: $I = nAvq = n(td)vq \Rightarrow v = \frac{I}{ntdq}$

so $V_H = Bd \frac{I}{ntdq} = \frac{BI}{ntq}$

d (the width across which V_H is measured) **cancels**; t is the thickness **along** B .

A semiconductor probe: $n = 1.0 \times 10^{22} \text{ m}^{-3}$, $t = 0.50 \text{ mm}$, $I = 30 \text{ mA}$, $B = 0.20 \text{ T}$. **Find V_H .**

$$ntq = (1.0 \times 10^{22})(5.0 \times 10^{-4}) \times (1.6 \times 10^{-19}) = 0.80$$

$$V_H = \frac{BI}{ntq} = \frac{0.20 \times 0.030}{0.80} = 7.5 \text{ mV}$$

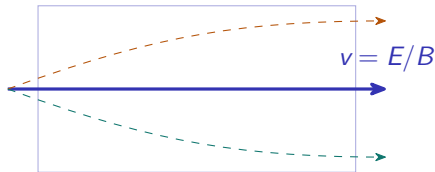
Why a **semiconductor**? Small n , so a large V_H . In copper it would be about 1 nV .

Velocity selector – only one speed gets through

Crossed E and B fields. The electric force qE and magnetic force qvB oppose. Net force is zero only when

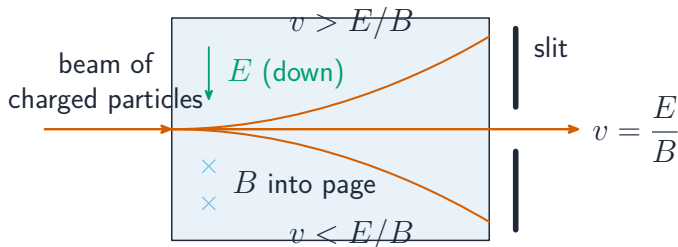
$$v = \frac{E}{B}$$

Particles at that speed go straight; faster or slower ones are deflected.



The selector, drawn

Only a charge with $v = E/B$ passes straight through: for it the electric and magnetic forces cancel exactly, whatever its charge or mass.



goes straight when $qE = qvB$, i.e. $v = E/B$

After the selector: measuring a mass

“State how $v = E/B$ predicts that an alpha particle also passes undeviated.” [1]

$v = E/B$ contains no charge and no mass: any particle at that speed passes.

“The ions enter with greater kinetic energy. Sketch their path.” [2]

Faster, so $Bqv > qE$: they curve in the direction of the magnetic force, gently at first.

Ions (+e) leave a selector at $5.0 \times 10^5 \text{ m s}^{-1}$ and enter $B = 0.20 \text{ T}$, where they move in a circle of radius 5.2 cm. Find their mass.

$$m = \frac{rBq}{v} = \frac{0.052 \times 0.20 \times 1.6 \times 10^{-19}}{5.0 \times 10^5} \\ = 3.3 \times 10^{-27} \text{ kg } (\approx 2 \text{ u})$$

Same v , B and q : $r \propto m$. Double the mass, double the radius.

5

Fields due to currents

Sheet 20.4 – what you need to know

You must be able to:

- draw the fields of a wire, a flat coil and a solenoid, with direction
- explain the force between two currents from field and Fleming
- use $F/L = BI$ and Newton's third law

What the exam asks

“Calculate the magnetic force per unit length exerted on wire Q by wire P.” [2]

“State the direction of the force exerted on wire Q by wire P.” [1]

“Determine the magnitude of the current in wire P. Explain your reasoning.” [2]

Practise on exercise sheet 20.4, then try the real exam questions on past-paper sheet 20.4.

Two wires, each in the other's field

Using Fleming's left-hand rule:

same direction

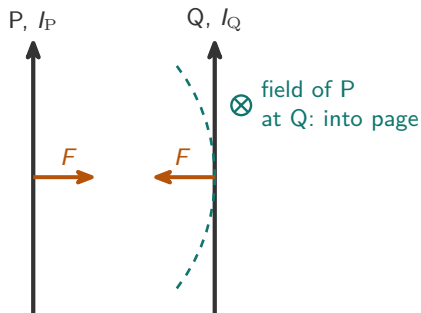
currents **attract**

opposite directions

currents **repel**

This is the physics behind the SI definition of the **ampere** 安培.

Two wires: field of one, force on the other



same direction: attract

More B from a solenoid at the same I : more turns per unit length, or an iron core.

Field of P at Q: $4.0 \times 10^{-5} \text{ T}$; $I_Q = 5.0 \text{ A}$.
Field of Q at P: $2.5 \times 10^{-5} \text{ T}$. Find F/L on Q, and I_P .

$$\begin{aligned} \frac{F}{L} &= B_P I_Q = 4.0 \times 10^{-5} \times 5.0 \\ &= 2.0 \times 10^{-4} \text{ N m}^{-1} \end{aligned}$$

$$\frac{F}{L} = B_Q I_P \Rightarrow I_P = \frac{2.0 \times 10^{-4}}{2.5 \times 10^{-5}} = 8.0 \text{ A}$$

The two forces are equal and opposite (Newton's third law). Explain a direction in two steps: the field of P at Q (right-hand grip), then Fleming's left-hand rule on Q.

6

Induction

Sheet 20.5 – what you need to know

You must be able to:

- use flux $\Phi = BA$ and flux linkage $N\Phi$
- state Faraday's and Lenz's laws in words
- use $E = BLv$ and the idea of flux cut

What the exam asks

“State Faraday's law of electromagnetic induction.” [2]

“Use Lenz's law to explain which of the wingtips P and Q is at the higher induced potential.” [3]

“By reference to electromagnetic induction and to conservation of energy, explain why the oscillations are damped.” [3]

Practise on exercise sheet 20.5, then try the real exam questions on past-paper sheet 20.5.

Flux and flux linkage

Magnetic flux 磁通量 through a flat area A :

$$\Phi = BA \cos \theta$$

Unit: **weber** 韦伯, $\text{Wb} = \text{T m}^2$. For a **coil** 线圈 of N turns the **flux linkage** 磁链 is $N\Phi$. Flux changes if you change B , change A (a rod sliding on rails), or change orientation (a coil turning – the a.c. generator).

Faraday and Lenz

Faraday 法拉第: the induced e.m.f. equals the rate of change of flux linkage

$$|\varepsilon| = N \frac{d\Phi}{dt}$$

Lenz 楞次: the induced e.m.f. **opposes the change** that makes it – conservation of energy. Combined:

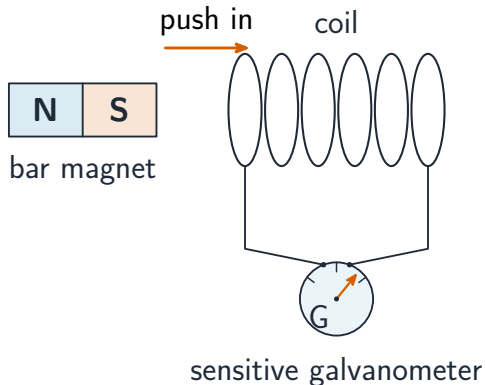
$$\varepsilon = - \frac{d(N\Phi)}{dt}$$

$N = 200$, $A = 0.010 \text{ m}^2$, B falls
from 0.50 T to 0 in 0.20 s

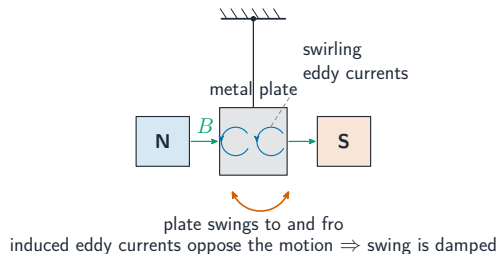
$$\Delta(N\Phi) = NBA = 1.0 \text{ Wb}$$

$$|\varepsilon| = \frac{1.0}{0.20} = 5.0 \text{ V}$$

Seeing induction happen



Faster motion, bigger deflection; reverse the motion, reverse the needle.



Eddy currents 涡流 in the plate oppose its motion, so it stops quickly.

What makes the induced e.m.f. larger

From $\varepsilon = N d\Phi/dt$ with $\Phi = BA$, four things raise it:

more turns N

stronger B

larger area A

faster change

It is the **rate of change** of flux linkage that matters, not the flux itself.

Faraday's law in words, and flux cutting

“State Faraday's law of electromagnetic induction.” [2]

The induced e.m.f. is **proportional to the rate of change of magnetic flux linkage** (or the rate of cutting of flux).

A rod of length L at speed v cuts flux BLv each second:

$$E = BLv$$

An aircraft of wingspan 40 m flies north at 250 m s^{-1} ; the vertical field is $4.0 \times 10^{-5} \text{ T}$ downwards.

$$E = BLv = 4.0 \times 10^{-5} \times 40 \times 250 \\ = 0.40 \text{ V}$$

flux cut in 15 s = $Et = 0.40 \times 15 = 6.0 \text{ Wb}$

The magnetic force on the free electrons pushes them to the **east** wingtip, so the **west** wingtip is at the higher potential.

A rod accelerating from rest: $v = at$, so $E = BLat$, a straight line through the origin.

7

E.m.f. from graphs

Sheet 20.5.1 – what you need to know

You must be able to:

- read the e.m.f. as minus the gradient of $N\Phi$ against t
- describe the pulses from a magnet falling through a coil
- use $E_0 = 2\pi fNBA$ for a rotating coil

What the exam asks

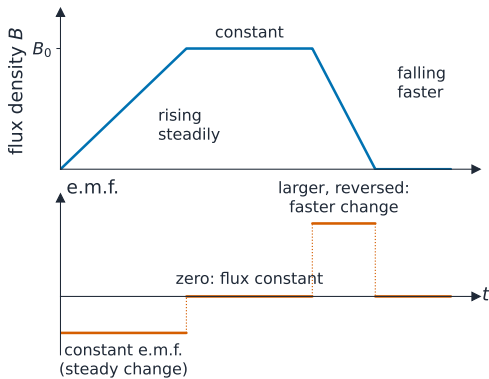
“Calculate the magnetic flux cut by the wings of the aircraft in a time of 15 s. Give a unit with your answer.” [2]

“Explain how Fig. 7.2 shows that E is proportional to the velocity v of the rod.” [2]

“Suggest, with a reason, the effect of this change on the oscillations.” [2]

Practise on exercise sheet 20.5.1, then try the real exam questions on past-paper sheet 20.5.

E.m.f. is minus the gradient of $N\Phi$ against t



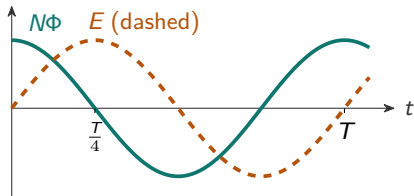
$$E = -\frac{\Delta(N\Phi)}{\Delta t}$$

- steady rise: **constant** e.m.f.
- flat: **zero** e.m.f., however large the flux
- steeper fall: **larger** e.m.f., opposite sign

A magnet falls through a coil. Describe the e.m.f.–time graph.

Two pulses of **opposite sign**; the second **taller and narrower** (it is moving faster); **equal areas** (the same change of flux linkage).

A coil rotating in a uniform field



E is greatest when $N\Phi$ is **zero** (plane along B , cutting fastest): a **quarter cycle** apart.

$N\Phi = NBA \cos \omega t$ gives $E = NBA\omega \sin \omega t$:

$$E_0 = 2\pi f NBA$$

$N = 100$, $A = 4.0 \times 10^{-3} \text{ m}^2$, $B = 0.050 \text{ T}$, $f = 50 \text{ Hz}$.

$$E_0 = 2\pi \times 50 \times 100 \times 0.050 \times 4.0 \times 10^{-3} = 6.3 \text{ V}$$

Double f : twice the peak **and** twice as many cycles. Lenz: a magnet falls slowly through a **copper** tube (eddy currents oppose it), freely through plastic.

8

Recap

Exam tips

- Force on a current $F = BIL \sin \theta$; on a moving charge $F = BQv$; find the direction with **Fleming's left-hand rule**.
- A charge moving perpendicular to B moves in a **circle** ($BQv = mv^2/r$).
- **Electromagnetic induction**: e.m.f. = rate of change of flux linkage (Faraday); its direction opposes the change (Lenz's law).

Topic 20 – key takeaways

1 On a wire

$$F = BIL \sin \theta - \text{left-hand rule}$$

2 On a charge

$$r = mv/BQ; \text{ Hall } V_H = Bl/ntq$$

3 Two wires

same way attract, opposite repel

4 Induction

$$\varepsilon = -d(N\Phi)/dt - \text{Faraday and Lenz}$$

Check yourself

- 1 When is $F = BIL$ a maximum?
- 2 Why does $T = 2\pi m/BQ$ not depend on speed?
- 3 In a velocity selector, $v = ?$
- 4 What does the minus sign in Faraday's law mean?

Answers 1. wire $\perp B$ 2. faster $\Rightarrow$ bigger r , same time per turn 3. E/B 4. the induced e.m.f. opposes the change (Lenz)