

Capacitance

A2 Physics ■ Topic 19

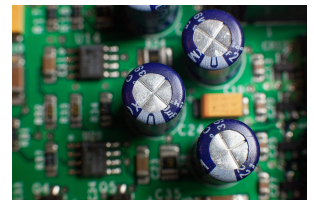
A-Level Physics



Capacitance

What we will cover

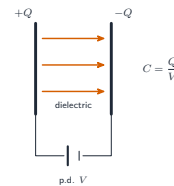
- 1 Capacitance $C = Q/V$
- 2 Combining capacitors
- 3 Energy stored
- 4 Discharge through a resistor
- 5 The time constant RC
- 6 Recap



1 Capacitance

Capacitance

What a capacitor does



A **capacitor** 电容器 stores charge: two **conductor** 导体 plates with an **insulator** 绝缘体 (a **dielectric** 电介质) between them.

- connected to a battery, $+Q$ builds on one plate, $-Q$ on the other
- a p.d. V appears across the gap

The two charges are **equal and opposite** – the capacitor stores charge **separation**, not net charge.

Capacitance

Capacitance $C = Q/V$

$$C = \frac{Q}{V}, \quad [C] = F = CV^{-1}$$

- set by size and dielectric, so C is **constant**
- double Q and V doubles too – the **ratio** holds
- an isolated sphere: $C = 4\pi\epsilon_0 r$

A farad is **huge** – real capacitors run from pF to mF.



Capacitance

Worked example – charge stored

A $100\ \mu\text{F}$ capacitor is charged to 12 V. Find the charge stored.

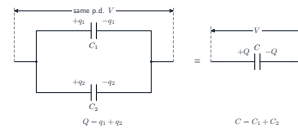
$$Q = CV = (100 \times 10^{-6})(12) \\ = 1.2 \times 10^{-3} \text{ C} = 1.2 \text{ mC}$$

Put the μF in as $\times 10^{-6} \text{ F}$. A stored mC is a **lot** of separated charge for such a small device.

2 Combining capacitors

Capacitance Combining capacitors

Parallel: the charges add



Same p.d. V across each, so the charges sum:

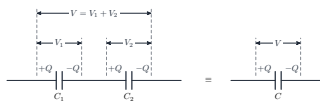
$$C_{\text{parallel}} = C_1 + C_2 + \dots$$

A parallel bank has **larger** capacitance than any one.

More plate area $\Rightarrow$ more room for charge – adding in parallel is like widening the plates.

Capacitance Combining capacitors

Series: the p.d.s add



Same charge Q through each, so the p.d.s sum:

$$\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

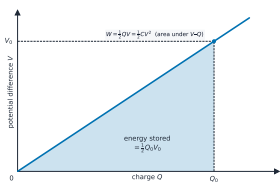
A series chain has **smaller** capacitance than any one.

The **opposite** of resistors – because $C = Q/V$ has V on the bottom, while $R = V/I$ has I on the bottom.

3 Energy stored

Capacitance Energy stored

Energy stored in a capacitor



Each extra dq is pushed against the p.d. q/C already there, so

$$W = \int_0^Q \frac{q}{C} dq = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{Q^2}{2C}$$

On the V - Q line the energy is the **area** beneath – the triangle $\frac{1}{2} QV$. The $\frac{1}{2}$ is the **average** p.d. during charging.

Capacitance Energy stored

Worked example – energy stored

Find the energy in a $100 \mu\text{F}$ capacitor charged to 12 V.

$$W = \frac{1}{2} CV^2 = \frac{1}{2} (100 \times 10^{-6}) (12)^2 = 7.2 \times 10^{-3} \text{ J} = 7.2 \text{ mJ}$$

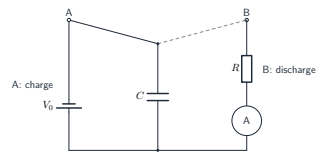
Charging through a wire wastes the **other half**: the battery gives out $QV = CV^2$, the capacitor keeps $\frac{1}{2} CV^2$, and the rest heats the wire – whatever its resistance.

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Discharge through a resistor

Capacitance
Discharge through a resistor

Setting up the discharge



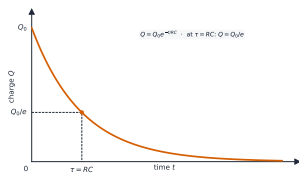
Close the switch and C drives current through R . By **Kirchhoff's second law** 基尔霍夫第二定律 $V_C = V_R$, with $I = -dQ/dt$:

$$\frac{Q}{C} = -R \frac{dQ}{dt}$$

The rate of loss is **proportional** to what is left – and that always gives an **exponential decay** 指数衰减.

Capacitance
Discharge through a resistor

The discharge equations



Charge, p.d. and current all decay with the **same** time constant RC :

$$Q = Q_0 e^{-t/RC}$$

$$V = V_0 e^{-t/RC}, \quad I = I_0 e^{-t/RC}$$

with $I_0 = V_0/R$.

After one time constant RC the charge is down to $Q_0/e \approx 37\%$ of the start.

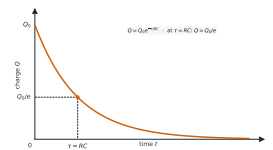
Capacitance
Discharge through a resistor

The time constant and half-life

$$\tau = RC, \quad t_{1/2} = RC \ln 2 \approx 0.69 RC$$

- units check: $\times F = s$
- bigger R or $C \Rightarrow$ **slower** discharge
- every RC , the charge falls by the **same factor** $1/e$

It turns the exponential into a straight line: plotting $\ln Q$ against t gives a gradient of $-1/RC$.



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Recap

Capacitance
Recap

Topic 19 – key takeaways

1 Capacitance

$C = Q/V$, in farads – a constant of the device

2 Combining

parallel $C_1 + C_2$; series $1/C_1 + 1/C_2$

3 Energy

$\frac{1}{2} QV = \frac{1}{2} CV^2$ – area under $V-Q$

4 Discharge

$Q = Q_0 e^{-t/RC}$ – time constant $\tau = RC$

Check yourself

- 1 Two $2\ \mu\text{F}$ caps in parallel: total C ?
- 2 Same two in series: total C ?
- 3 Why the factor $\frac{1}{2}$ in the energy?
- 4 What fraction is left after one RC ?



Answers 1. $4\ \mu\text{F}$ 2. $1\ \mu\text{F}$ 3. average p.d. is $V/2$ 4. $1/e \approx 37\%$