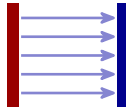


Capacitance

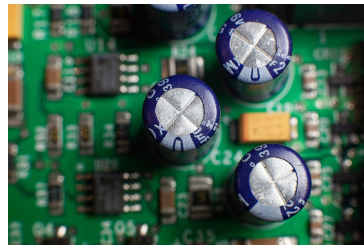
A2 Physics ■ Topic 19

A-Level Physics



What we will cover

- 1 Capacitors and capacitance 19.1
- 2 Energy stored 19.2
- 3 Discharging a capacitor 19.3
- 4 Recap



1

Capacitance

Sheet 19.1 – what you need to know

You must be able to:

- define **capacitance** and use $C = Q/V$
- derive and use the series and parallel rules
- reduce a network; share charge between capacitors

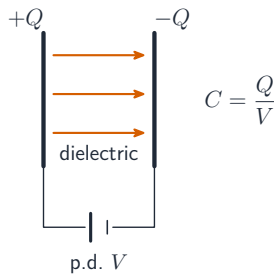
What the exam asks

“Complete Table 6.1 to indicate how Q_S , Q_1 and Q_2 relate to each other, and how V_S , V_1 and V_2 relate.” **[4]**

“Complete Table 7.1 to show expressions, in terms of Q and V , for the final p.d.s across, and the final charges on, the two capacitors.” **[3]**

“Sketch the variation of the total energy E stored in the capacitors with C_Q .” **[2]**

What a capacitor does



A **capacitor** 电容器 stores charge: two **conductor** 导体 plates with an **insulator** 绝缘体 between them – a **dielectric** 电介质, or just a **vacuum** 真空 – with a **potential difference** 电势差 V across the gap.

- connected to a battery, $+Q$ builds on one plate, $-Q$ on the other

The two charges are **equal and opposite** – the capacitor stores charge **separation**, not net charge.

Capacitance $C = Q/V$

$$C = \frac{Q}{V}, \quad [C] = \text{F} = \text{C V}^{-1}$$

- set by size and dielectric, so C is **constant**
- double Q and V doubles too – the **ratio** holds
- an isolated sphere: $C = 4\pi\epsilon_0 r$

A farad is **huge** – real capacitors run from pF to mF.



Worked example – charge stored

**A $100\ \mu\text{F}$ capacitor is charged to $12\ \text{V}$.
Find the charge stored.**

$$\begin{aligned}Q &= CV = (100 \times 10^{-6})(12) \\&= 1.2 \times 10^{-3}\ \text{C} = 1.2\ \text{mC}\end{aligned}$$

Put the μF in as $\times 10^{-6}\ \text{F}$.
A stored mC is a **lot** of separated charge for such a small device.

Parallel: the charges add

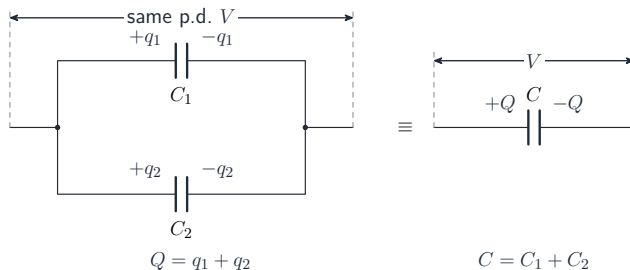
Same p.d. V across each, so the charges sum:

$$C_{\text{parallel}} = C_1 + C_2 + \dots$$

A parallel bank has **larger** capacitance than any one.

More plate area $\Rightarrow$ more room for charge – adding in parallel is like widening the plates.

Parallel: the charges add, in detail

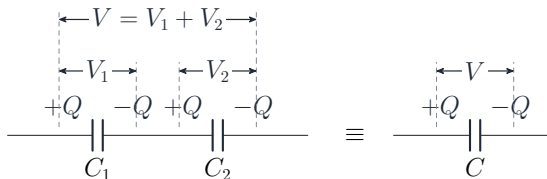


Same p.d. V across each, so the charges sum:

$$C_{\text{parallel}} = C_1 + C_2 + \dots$$

A parallel bank has **larger** capacitance than any one.

Series: the p.d.s add



Same charge Q through each, so the p.d.s sum:

$$\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

A series chain has **smaller** capacitance than any one.

The **opposite** of resistors – because $C = Q/V$ has V on the bottom, while $R = V/I$ has I on the bottom.

Defining C , and deriving the series rule

“Define capacitance.” [2]

The **charge** stored **per unit potential difference**: $C = Q/V$, the charge on one plate over the p.d. between the plates.

	series	parallel
charge	$Q_S = Q_1 = Q_2$	$Q_P = Q_1 + Q_2$
p.d.	$V_S = V_1 + V_2$	$V_P = V_1 = V_2$

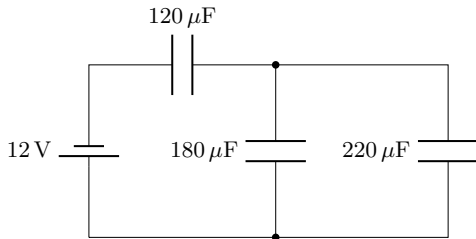
“Show that $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$ in series.” [3]

The joined inner plates are isolated, so each capacitor holds the **same charge** Q .

The p.d.s add: $V = V_1 + V_2$.

With $V = Q/C$: $\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2}$; divide by Q .

Reducing a network



Reduce the **innermost** group first: parallel pair $\rightarrow$ one capacitor, then the series rule.

Which capacitor stores the most charge?

$$C_P = 180 + 220 = 400 \mu\text{F}$$

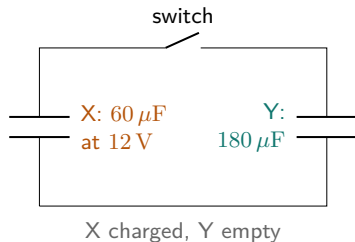
$$\frac{1}{C} = \frac{1}{120} + \frac{1}{400} \Rightarrow C = 92.3 \mu\text{F}$$

$$Q = CV = 92.3 \mu\text{F} \times 12 \text{ V} = 1110 \mu\text{C}$$

All 1110 μC sits on the 120 μF (in series with the rest). The pair has $V = 1110/400 = 2.8 \text{ V}$, so 180 μF holds 500 μC and 220 μF holds 610 μC.

Most: the **120 μF**.

Sharing charge: where the energy goes



The switch is closed. Find the final p.d. and the energy lost.

$$Q = 60 \mu\text{F} \times 12 \text{ V} = 720 \mu\text{C} \quad (\text{conserved})$$

$$V = \frac{720 \mu\text{C}}{(60 + 180) \mu\text{F}} = 3.0 \text{ V}$$

$$E_{\text{before}} = \frac{1}{2} (60 \times 10^{-6}) (12)^2 = 4.32 \text{ mJ}$$

$$E_{\text{after}} = \frac{1}{2} (240 \times 10^{-6}) (3.0)^2 = 1.08 \text{ mJ}$$

Lost: **3.24 mJ**, three quarters, as heat in the wires.

Charge is conserved; energy is **not**. Same p.d. at the end: they are in parallel.

2

Energy stored

Sheet 19.2 – what you need to know

You must be able to:

- use $W = \frac{1}{2}QV = \frac{1}{2}CV^2$, the area under V against Q
- find an energy **change** between two p.d.s
- read Q_0 , C and R from a $\ln Q$ - t graph

What the exam asks

“Calculate the increase in energy ΔE stored in the capacitor.” [2]

“Determine the time constant, in ms, of the smoothing circuit.” [3]

“Sketch a line to show the variation of $\ln(Q/\mu C)$ with t between $t = 0$ and $t = 5.0$ s.” [2]

Practise on exercise sheet 19.2, then try the real exam questions on past-paper sheet 19.2.

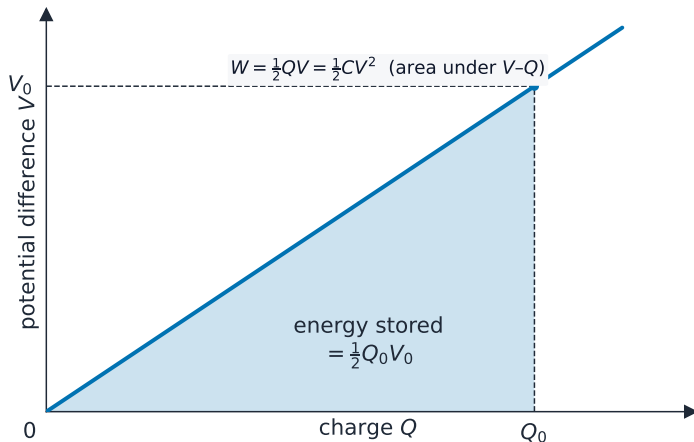
Energy stored in a capacitor

Each extra dq is pushed against the p.d. q/C already there, so

$$W = \int_0^Q \frac{q}{C} dq = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{Q^2}{2C}$$

On the V – Q line the energy is the **area** beneath – the triangle $\frac{1}{2} QV$. The $\frac{1}{2}$ is the **average** p.d. during charging.

Energy stored in a capacitor, in detail



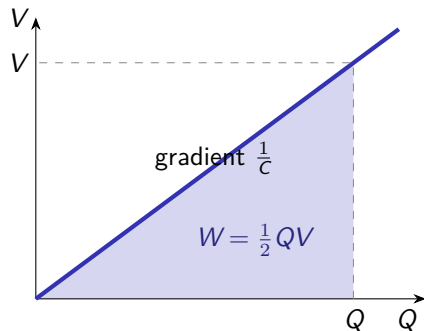
The energy stored is the area under the potential–charge line (the triangle $\frac{1}{2}QV$)

Reading the Q - V graph

V against Q is a straight line through the origin, gradient $1/C$.

$$W = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{Q^2}{2C}$$

the **area** under the line



Worked example – energy stored

Find the energy in a $100\ \mu\text{F}$ capacitor charged to $12\ \text{V}$.

$$\begin{aligned} W &= \frac{1}{2} CV^2 = \frac{1}{2} (100 \times 10^{-6}) (12)^2 \\ &= 7.2 \times 10^{-3} \text{ J} = 7.2 \text{ mJ} \end{aligned}$$

Charging through a wire wastes the **other half**:
the battery gives out $QV = CV^2$, the capacitor keeps $\frac{1}{2} CV^2$, and the rest heats the wire – whatever its resistance.

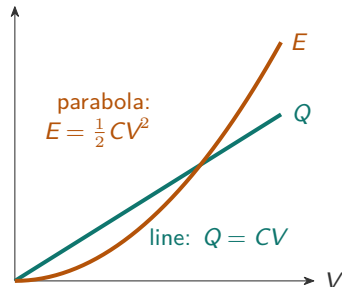
Energy changes: the ΔE trap and $E \propto V^2$

A $470\ \mu\text{F}$ capacitor is charged from $6.0\ \text{V}$ to $9.0\ \text{V}$. Find the increase in energy.

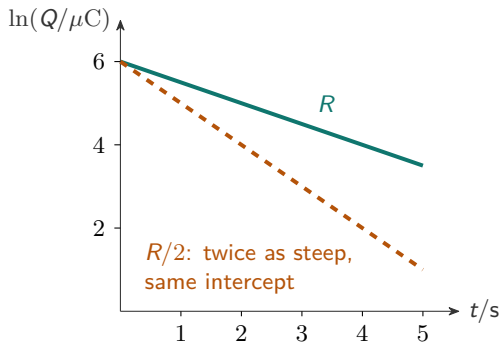
$$\begin{aligned}\Delta E &= \frac{1}{2}C(V_2^2 - V_1^2) \\ &= \frac{1}{2} \times 470 \times 10^{-6} \times (81 - 36) = 10.6\ \text{mJ}\end{aligned}$$

Not $\frac{1}{2}C(\Delta V)^2 = 2.1\ \text{mJ}$: energy is not linear in V .

V falls by 20%, so E becomes $0.8^2 = 0.64$ of its value: a 36% fall.



Reading a $\ln Q$ against t graph



$Q = Q_0 e^{-t/RC}$, so $\ln Q = \ln Q_0 - \frac{t}{RC}$: a straight line.

The capacitor was charged to 12 V. Find C and R .

$$\ln Q_0 = 6.0 \Rightarrow Q_0 = e^{6.0} = 403 \mu\text{C}$$

$$C = \frac{Q_0}{V} = \frac{403 \mu\text{C}}{12 \text{ V}} = 34 \mu\text{F}$$

$$\text{gradient} = -\frac{2.5}{5.0} = -0.50 \text{ s}^{-1} = -\frac{1}{RC}$$

$$R = \frac{2.0 \text{ s}}{3.36 \times 10^{-5} \text{ F}} = 60 \text{ k}\Omega$$

3

Discharging

Sheet 19.3 – what you need to know

You must be able to:

- use $x = x_0 e^{-t/RC}$ for Q , V and I , and $\tau = RC$
- explain why the decay is exponential
- solve for a time with logarithms; read R and C off graphs

What the exam asks

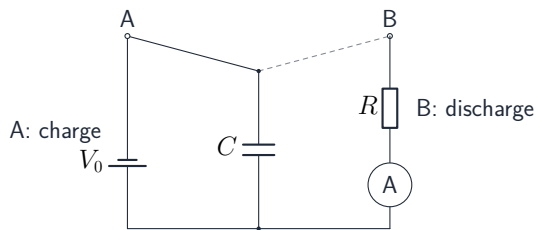
“Explain why the variation of Q with t is exponential in nature.”
[3]

“State the purpose of the capacitor C in the circuit of Fig. 6.1.” [1]

“Explain the shape of the line in Fig. 6.2.” [3]

Practise on exercise sheet 19.3, then try the real exam questions on past-paper sheet 19.3.

Setting up the discharge



Close the switch and C drives current through R . By **Kirchhoff's second law** 基尔霍夫第二定律 around the loop $V_C = V_R$; using $V_C = Q/C$, $V_R = IR$ and $I = -dQ/dt$:

$$\frac{Q}{C} = -R \frac{dQ}{dt}$$

The rate of loss is **proportional** to what is left – and that always gives an **exponential decay** 指数衰减.

The discharge equations

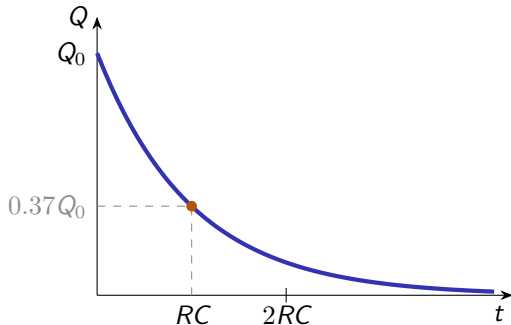
Q , V and I all decay with the **same** time constant RC :

$$Q = Q_0 e^{-t/RC}$$

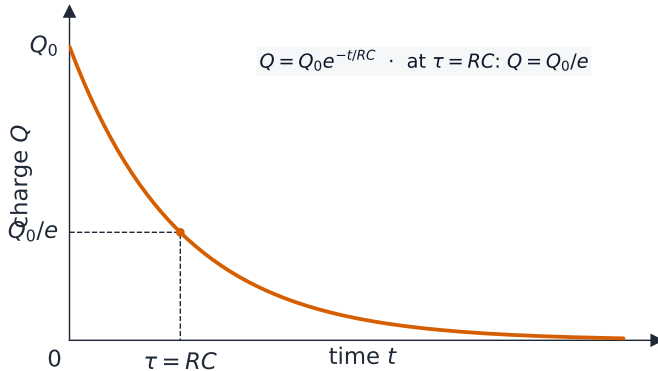
$$V = V_0 e^{-t/RC}, \quad I = I_0 e^{-t/RC}$$

with $I_0 = V_0/R$.

After one RC : down to $1/e \approx 37\%$.



The discharge equations, in detail



Charge decays exponentially during discharge, falling to Q_0/e after one time constant RC

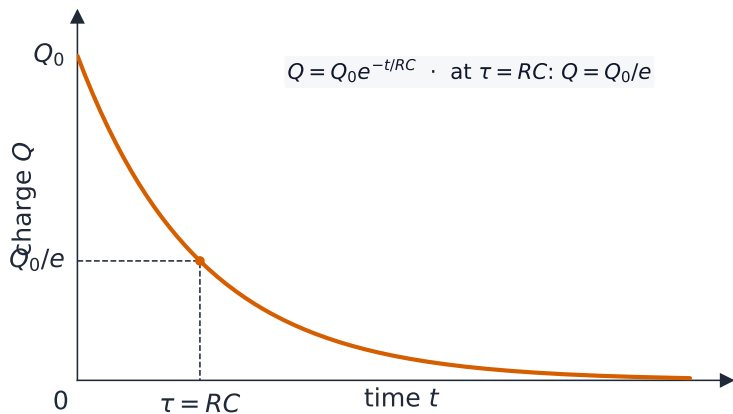
The time constant and half-life

$$\tau = RC, \quad t_{1/2} = RC \ln 2 \approx 0.69 RC$$

- units check: $\Omega \times F = s$
- bigger R or $C \Rightarrow$ slower discharge
- every RC , the charge falls by the same factor $1/e$

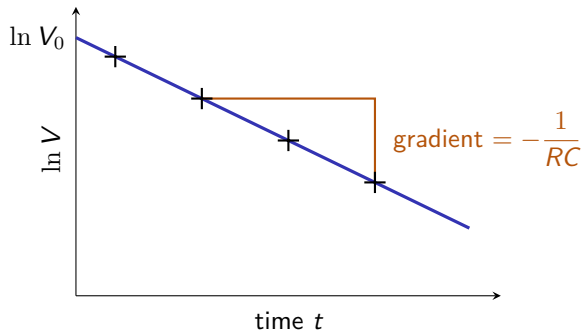
ln turns the exponential into a straight line: plotting $\ln Q$ against t gives a gradient of $-1/RC$.

The time constant and half-life, in detail



$$\tau = RC, \quad t_{1/2} = RC \ln 2 \approx 0.69 RC$$

Time constant



$$V = V_0 e^{-t/RC}$$

discharge

$$\ln V = \ln V_0 - \frac{t}{RC}$$

take logs

A graph of $\ln V$ against t is a **straight line** of gradient $-1/(RC)$: read the time constant from it.

Reading a discharge curve in the exam

Start value	Read V_0 or Q_0 off the graph at $t = 0$.
Time constant	Read the time taken to fall to V_0/e (about 37% of the start): that time <i>is</i> $\tau = RC$.
The missing component	Given R , find $C = \tau/R$ – or given C , find R .
A later value	Predict any later value with $V = V_0 e^{-t/RC}$; or plot $\ln V$ against t for a straight line of gradient $-1/RC$.

Every discharge question is one of these four. Identify which before you touch the algebra.

Worked example – solving for the time: take logs

A $470\ \mu\text{F}$ capacitor charged to $12\ \text{V}$ discharges through $100\ \text{k}\Omega$. How long until the p.d. is $3.0\ \text{V}$?

$$\tau = RC = 1.0 \times 10^5 \times 470 \times 10^{-6} = 47\ \text{s}$$

$$V = V_0 e^{-t/RC} \Rightarrow \frac{V_0}{V} = e^{t/RC}$$

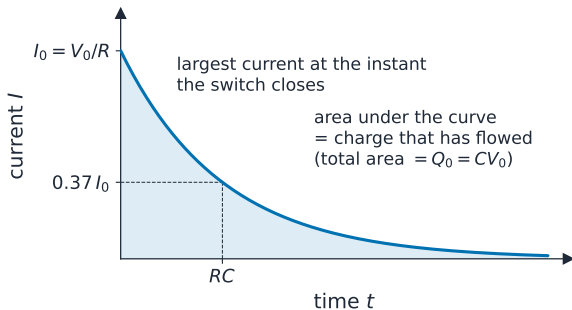
$$t = RC \ln \frac{V_0}{V} = 47 \ln \frac{12}{3.0}$$

$$t = 47 \times \ln 4 = 65\ \text{s}$$

Check: $3.0\ \text{V}$ is a quarter, two **half-lives**:
 $2 \times 0.69 \times 47 = 65\ \text{s}$.

Current down to 15%: $t = -\tau \ln 0.15 = 1.9\ \tau$. The unit of $\tau = RC$ is the **second**.

The current–time curve: shape, R , C and area



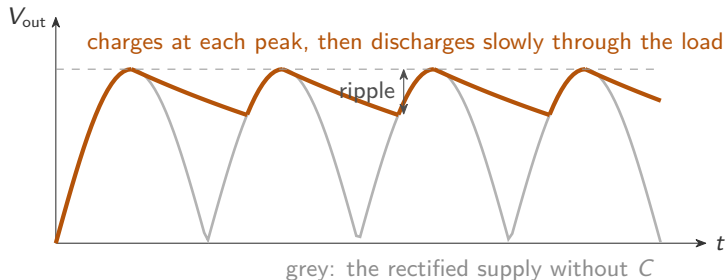
$R = V_0/I_0$ from the intercept; τ where $I = 0.37I_0$;
 $C = \tau/R$.

“Explain the shape of the line.” [3]

As charge leaves, the p.d. across the capacitor **falls**,
 so the p.d. across R , and the **current**, fall,
 so charge leaves **more slowly**: the rate of loss is proportional to the charge left, so the decay is **exponential**.

The area under I – t is the charge:
 the whole area is $Q_0 = CV_0$.

A capacitor smoothing a rectified supply



“State the purpose of the capacitor.” [1]

To **smooth** the output: it discharges through the load between peaks, so the p.d. never falls to zero.

Larger RC (compared with the time between peaks): slower decay, **smaller ripple**. Full-wave rectification gives peaks twice as often, so less ripple too.

4

Recap

Exam tips

- $C = Q/V$; energy stored $W = \frac{1}{2}QV = \frac{1}{2}CV^2$ (the $\frac{1}{2}$ is the area under the Q - V graph).
- Combine capacitors the **opposite way to resistors** (parallel add, series reciprocal).
- Discharge is exponential: $Q = Q_0 e^{-t/RC}$; the time constant $\tau = RC$ is when the charge falls to 37%.

Topic 19 – key takeaways

1 Capacitance

$C = Q/V$, in farads – a constant of the device

2 Combining

parallel $C_1 + C_2$; series
 $1/C_1 + 1/C_2$

3 Energy

$\frac{1}{2} QV = \frac{1}{2} CV^2$ – area under V - Q

4 Discharge

$Q = Q_0 e^{-t/RC}$ – time constant
 $\tau = RC$

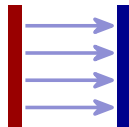
Check yourself

1 Two $2\ \mu\text{F}$ caps in parallel: total C ?

2 Same two in series: total C ?

3 Why the factor $\frac{1}{2}$ in the energy?

4 What fraction is left after one RC ?



Answers 1. $4\ \mu\text{F}$ 2. $1\ \mu\text{F}$ 3. average p.d. is $V/2$ 4. $1/e \approx 37\%$