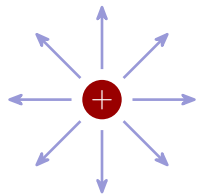
Electric Fields

A2 Physics ▪ Topic 18

A-Level Physics



What we will cover

1. Fields and field lines 18.1
2. Uniform fields 18.2
3. Coulomb's law 18.3
4. Field of a point charge 18.4
5. Electric potential 18.5
6. Recap

1 Fields and field lines

Sheet 18.1 – what you need to know

You must be able to:

- define an electric field and field strength $E = F/q$
- draw field lines for plates, point charges and pairs
- draw a charge's path between plates and after it leaves them

What the exam asks

"Define electric field." **[2]**

"On Fig. 5.1, draw the path of the electron as it moves between and beyond the plates." **[2]**

"Explain, with reference to the forces exerted by the two fields on the electron, why the path of the electron is undeviated." **[2]**

Practise on exercise sheet 18.1, then try the real exam questions on past-paper sheet 18.1.

Electric field strength

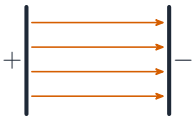
An **electric field** 电场 is a region where a charge feels a force. Its **field strength** 电场强度 is the force per unit positive charge:

$$E = \frac{F}{q} \Rightarrow F = qE, \qquad [E] = \mathrm{N\,C^{-1}}$$

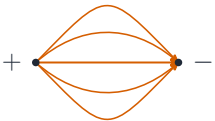
> Defined on a positive test charge – so a negative charge feels a force against the field.

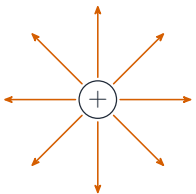
Field-line patterns

a) parallel plates (uniform field)

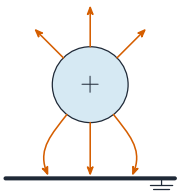


b) dipole (+ and −)





c) point charge (radial)



d) sphere above earthed plate

- they run from + to − (or off to infinity)
- they never cross
- closer lines = stronger field

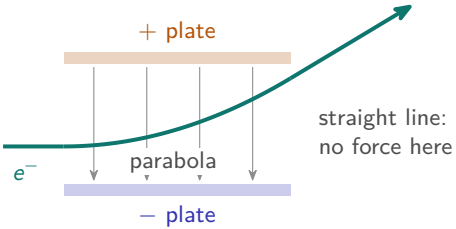
Two crossing lines would give the force at that point two directions at once – impossible, which is why it never happens.

Four field patterns worth recognising

Point charge	Radial lines pointing *out* from a positive **point charge** 点电荷, and *in* towards a negative one.
A dipole	Two opposite charges – a **dipole** 偶极子 – give lines curving from $+$ to $-$, densest in the gap between them.
Parallel plates	Two parallel charged plates a distance d apart with **potential difference** 电势差 V give a **uniform field** 匀强电场 of size $E = V/d$, apart from edge effects.
Several charges	For more than one charge, add the separate fields as a **vector sum** 矢量和 at each point.

A charge in a uniform field feels a constant force $F = qE$ and so a constant acceleration $a = qE/m$ – exactly like a mass in a uniform **gravitational field** 引力场.

Between the plates, and beyond them



Field lines between plates: parallel, evenly spaced, from + to −. The path is a parabola, not a circle: the force keeps one direction.

"Define electric field." [2]

A region where a charge experiences a force.

"With a magnetic field added, explain why the electron is undeviated." [2]

The electric force eE and the magnetic force Bev are equal in size and opposite in direction, so the resultant force is zero: $v = E/B$.

2 Uniform fields

Sheet 18.2 – what you need to know

You must be able to:

- use $E = V/d$ between parallel plates
- find the force and acceleration of a charge: $a = qE/m$
- explain the velocity selector, $u = V/Bd$

What the exam asks

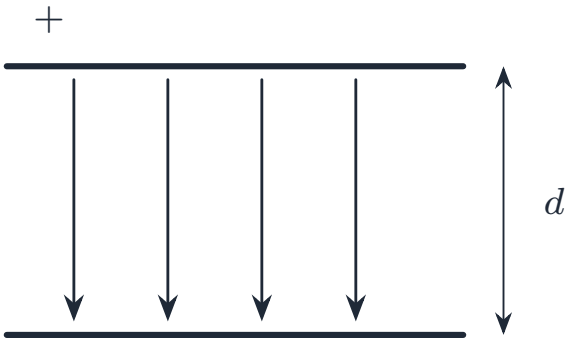
"Use the symbols + and − to indicate which of plates X and Y is the positive plate." **[1]**

"Determine the acceleration of the electron." **[2]**

"Show that the speed u of ions that pass undeviated through the velocity selector is given by $u = V/Bd$." **[2]**

Practise on exercise sheet 18.2, then try the real exam questions on past-paper sheet 18.2.

The uniform field between plates



Plates a distance d apart with p.d. V across them:

$$E = \frac{V}{d}$$

pointing from the high-potential plate to the low.

This is where $\mathrm{V\,m^{-1}}$ comes from – and it is the same unit as $\mathrm{N\,C^{-1}}$.

Worked example – plates and an electron

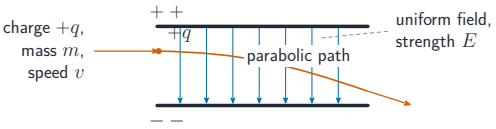
Plates $5.0\,\mathrm{mm}$ apart carry a p.d. of $200\,\mathrm{V}$. Find E, and the force on an electron between them. ($e = 1.6 \times 10^{-19}\,\mathrm{C}$)

$$E = \frac{V}{d} = \frac{200}{5.0 \times 10^{-3}} = 4.0 \times 10^{4}\,\mathrm{V\,m^{-1}}$$

$$F = qE = (1.6 \times 10^{-19})(4.0 \times 10^{4}) = 6.4 \times 10^{-15}\,\mathrm{N}$$

Convert mm to m first – forgetting it scales E by a thousand. The force is the same everywhere in the gap: that is what uniform means.

A charge crossing a uniform field



Constant force ⇒ constant acceleration $a = qE/m$:

- released at rest: speeds up along the field, gaining KE
- entering at right angles: a **parabolic** 抛物线 path

Exactly a **projectile** 抛体 problem – steady velocity across, uniform acceleration down. This is how a **cathode-ray tube** 阴极射线管 steered its beam.

Two ways a charge is set moving

Released at rest	It accelerates along the field (positive charge) or against it (negative), gaining **kinetic energy** 动能 qV after falling through a potential difference V.
Fired across	Constant velocity along the plates, constant acceleration across them – the same parabola as a projectile.
In an atom	An **electron** 电子 orbiting a **nucleus** 原子核 of charge $+Ze$ at radius r: the Coulomb attraction supplies the **centripetal force** 向心力, $\frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}$.

Energy method or force method – both work. Use energy when the question gives a potential difference, force when it gives a field.

Worked example – acceleration in a uniform field

Two plates $2.0\,\mathrm{cm}$ apart have a p.d. of $500\,\mathrm{V}$ across them. Find the acceleration of an electron between them.

$$E = \frac{V}{d} = \frac{500\,\mathrm{V}}{0.020\,\mathrm{m}} = 2.5 \times 10^4\,\mathrm{V\,m^{-1}}$$

$$F = eE = 1.6 \times 10^{-19} \times 2.5 \times 10^4 = 4.0 \times 10^{-15}\,\mathrm{N}$$

$$a = \frac{F}{m_\mathrm{e}} = \frac{4.0 \times 10^{-15}\,\mathrm{N}}{9.11 \times 10^{-31}\,\mathrm{kg}} = 4.4 \times 10^{15}\,\mathrm{m\,s^{-2}}$$

towards the positive plate.

Halve d at the same V and E, F and a all double. Gravity on the electron ($\sim 10^{-29}\,\mathrm{N}$) is negligible.

The velocity selector

"Show that the speed u of ions that pass undeviated is $u = V/Bd$." [2]

Undeviated: the forces balance,

$$qE = Bqu$$
$$u = \frac{E}{B} = \frac{V/d}{B} = \frac{V}{Bd}$$

q cancels, and m never appears: an alpha particle at the same speed also passes straight through.

$V = 150\,\mathrm{V}$, $d = 5.0\,\mathrm{mm}$, $B = 0.060\,\mathrm{T}$. **Find u.**

$$u = \frac{150}{0.060 \times 5.0 \times 10^{-3}}$$
$$= 5.0 \times 10^{5}\,\mathrm{m\,s^{-1}}$$

Faster ions?

Bqv grows with v but qE does not, so a faster ion is deflected in the direction of the magnetic force.

3 Coulomb's law

Sheet 18.3 – what you need to know

You must be able to:

- state Coulomb's law in words and use $F = Q_1 Q_2 / 4\pi\varepsilon_0 r^2$
- balance a Coulomb force against weight and tension
- use the Coulomb force as a centripetal force in an atom

What the exam asks

"State Coulomb's law." **[2]**

"Determine the ratio electric field strength due to the other electron / electric field strength due to the nucleus." **[2]**

"Determine the mass of sphere Y." **[4]**

Practise on exercise sheet 18.3, then try the real exam questions on past-paper sheet 18.3.

Coulomb's law

like charges

repel — pushed apart

opposite charges

attract — pulled together

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$$

with the **permittivity of free space** 真空介电常数 $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$ and $\dfrac{1}{4\pi\varepsilon_0} \approx 9.0 \times 10^{9}\,\mathrm{N\,m^2\,C^{-2}}$.

Inverse square, along the joining line – like repels, opposite attracts. That sign choice is the one thing gravity cannot do.

Worked example – two point charges

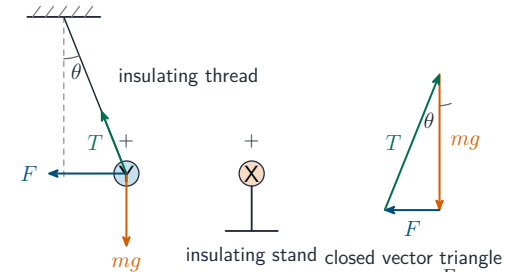
Find the force between $+2.0\,\mathrm{nC}$ and $+3.0\,\mathrm{nC}$ set $4.0\,\mathrm{cm}$ apart.

$$F = \frac{(9.0 \times 10^9)(2.0 \times 10^{-9})(3.0 \times 10^{-9})}{(0.040)^2}$$

$\approx 3.4 \times 10^{-5}\,\mathrm{N}$, and repulsive (both $+$)

State the direction as well as the size – "repulsive" is part of the answer. And r goes in as metres, squared.

A charged sphere hanging in equilibrium



Y ($+20\,\mathrm{nC}$) hangs on a $1.2\,\mathrm{m}$ thread, pushed $0.080\,\mathrm{m}$ sideways by X ($+20\,\mathrm{nC}$), $0.060\,\mathrm{m}$ away. Find the mass of Y.

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2} = \frac{8.99\times10^9 \times (20\times10^{-9})^2}{0.060^2}$$
$$= 1.0\times10^{-3}\,\mathrm{N}$$
$$\tan\theta = \frac{F}{mg} \approx \frac{0.080}{1.2}$$
$$m = \frac{F}{g\tan\theta} = \frac{1.0\times10^{-3}\times1.2}{9.81\times0.080}$$
$$= 1.5\times10^{-3}\,\mathrm{kg}$$

Coulomb forces in an atom

Hydrogen: the electron orbits at $r = 5.3 \times 10^{-11}\,\mathrm{m}$. Find its speed.

$$F = \frac{e^2}{4\pi\varepsilon_0 r^2} = 8.2 \times 10^{-8}\,\mathrm{N}$$

$$F = \frac{mv^2}{r} \;\Rightarrow\; v = \sqrt{\frac{Fr}{m}}$$

$$v = \sqrt{\frac{8.2 \times 10^{-8} \times 5.3 \times 10^{-11}}{9.11 \times 10^{-31}}}$$

$$= 2.2 \times 10^{6}\,\mathrm{m\,s^{-1}}$$

Helium: nucleus $+2e$; the two electrons are on opposite sides, each at r. Find $E_{\text{other electron}}/E_{\text{nucleus}}$.

$$\frac{e/(2r)^2}{2e/r^2} = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

It points outwards, so it reduces the net inward force.

"State Coulomb's law." [2]

The force between two point charges is proportional to the product of the charges and inversely proportional to the square of their separation.

4 Field of a point charge

Sheet 18.4 – what you need to know

You must be able to:

- use $E = Q/4\pi\varepsilon_0 r^2$, measuring r from the centre of a sphere
- sketch E against distance, with its sign
- link V, E, Q and C for a charged sphere

What the exam asks

"Explain why the electric potential near an isolated proton is positive." **[3]**

"Sketch the variation of the electric field E at point P with x for values of x between $x = -3R$ and $x = 3R$." **[3]**

"Determine the capacitance of the sphere." **[2]**

Practise on exercise sheet 18.4, then try the real exam questions on past-paper sheet 18.4.

Spheres as point charges

A charged spherical **conductor** 导体 of total charge Q:

- outside: identical to a point charge Q at its centre (measure r from the centre)
- inside a hollow one: the field is zero

> Zero field inside makes the whole conductor one **equipotential** 等势面 – and is why a car shields you from lightning.

$$E = F/q \quad (\mathrm{N\,C^{-1}})$$

The field of a point charge

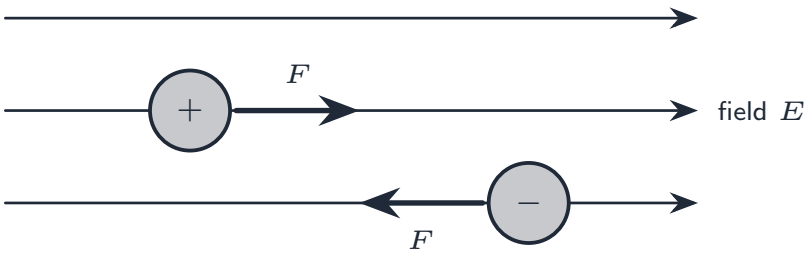
Coulomb's law gives the force between two charges. Divide by the test charge and only the source is left:

$$E = \frac{F}{q} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Out of a positive Q, into a negative one, falling as $1/r^2$.

> The same shape as $g = GM/r^2$ – but charges can cancel, so for several of them add the fields as a vector sum.

The field of a point charge, in detail



The field of a point charge Coulomb's law gives the force between two charges.

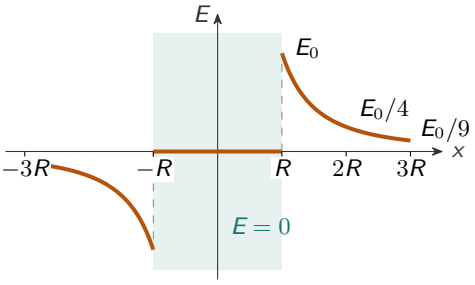
Signs: why $V > 0$ near a proton, and E either side of a sphere

"Explain why the electric potential near an isolated proton is positive." [3]

Potential is the work done per unit positive charge bringing it from infinity.

A positive test charge is repelled by the proton,

so work must be done on it: V is positive.



Left of the sphere the field points the other way, so E is negative there.

Worked example – charge and capacitance of a sphere

An isolated charged sphere has potential $45\,\mathrm{kV}$ and field strength $3.0 \times 10^5\,\mathrm{V\,m^{-1}}$ at its surface. Find its radius, its charge and its capacitance.

$V = \dfrac{Q}{4\pi\varepsilon_0 r}$ and $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$, so $V = Er$:

$$r = \frac{V}{E} = \frac{4.5 \times 10^4}{3.0 \times 10^5} = 0.15\,\mathrm{m}$$

$$\begin{aligned} Q &= 4\pi\varepsilon_0 rV \\ &= 4\pi \times 8.85 \times 10^{-12} \times 0.15 \times 4.5 \times 10^4 \\ &= 7.5 \times 10^{-7}\,\mathrm{C} \end{aligned}$$

$$C = \frac{Q}{V} = 1.7 \times 10^{-11}\,\mathrm{F}$$

$C = 4\pi\varepsilon_0 r$ for an isolated sphere: it depends only on the radius.

5 Electric potential

Sheet 18.5 – what you need to know

You must be able to:

- define electric potential and use $V = Q/4\pi\varepsilon_0 r$
- add potentials from several charges, with signs
- sketch V along a line and link E to its gradient

What the exam asks

"Define electric potential at a point." **[2]**

"Show that the electric potential V at point P when $x = 10\,\text{pm}$ is equal to $130\,\text{V}$." **[2]**

"Sketch the variation of V with x between $x = 10\,\text{pm}$ and $x = 110\,\text{pm}$." **[3]**

Practise on exercise sheet 18.5, then try the real exam questions on past-paper sheet 18.5.

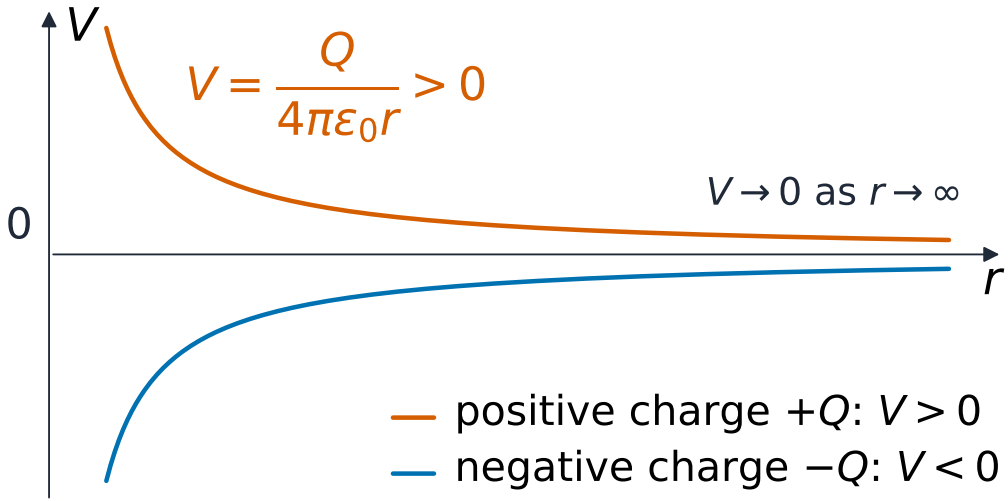
Electric potential

Electric potential 电势 is the work done per unit positive charge bringing a test charge in from **infinity** 无穷远:

$$V = \frac{W}{q} = \frac{Q}{4\pi\varepsilon_0 r}$$

Note $1/r$, not $1/r^2$. V is a scalar and takes the sign of Q – so several charges just add, signs and all, with no directions to resolve.

Electric potential, in detail



Worked example – potential near a charge

Find V at $4.0\,\mathrm{cm}$ from a point charge of $+3.0\,\mathrm{nC}$.

$$V = \frac{Q}{4\pi\varepsilon_0 r} = \frac{(9.0\times10^{9})(3.0\times10^{-9})}{0.040}$$

$$\approx 6.8\times10^{2}\,\mathrm{V}$$

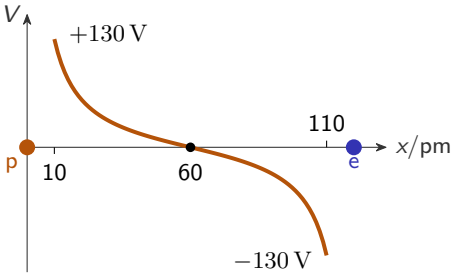
Same charge, same distance as the Coulomb example – but r appears to the first power here. Mixing up $1/r$ and $1/r^2$ is the standard slip.

Adding potentials: a proton and an electron

A proton is at $x = 0$ and an electron at $x = 120\,\mathrm{pm}$. Show that $V = 130\,\mathrm{V}$ at $x = 10\,\mathrm{pm}$.

Potential is a scalar: add with signs.

$$V = \frac{e}{4\pi\varepsilon_0}\left(\frac{1}{r_\mathrm{p}} - \frac{1}{r_\mathrm{e}}\right)$$
$$= 1.44 \times 10^{-9}\left(\frac{1}{10^{-11}} - \frac{1}{1.1 \times 10^{-10}}\right)$$
$$= 130\,\mathrm{V}$$



- $V = 0$ at the midpoint ($60\,\mathrm{pm}$): equal and opposite terms
- positive near the proton, negative near the electron

Where $V = 0$ is not where $E = 0$

Charges $+5.0\,\mathrm{nC}$ and $-2.0\,\mathrm{nC}$ are $0.70\,\mathrm{m}$ apart. Where between them is $V = 0$?

$$\frac{5.0}{x} = \frac{2.0}{0.70 - x}$$

$$5.0(0.70 - x) = 2.0x$$

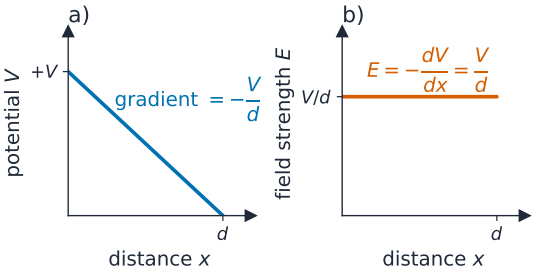
$$x = \frac{3.5}{7.0} = 0.50\,\mathrm{m} \text{ from the } +5.0\,\mathrm{nC}$$

Is the field strength also zero there?

No. Between unlike charges both fields point the same way (towards the negative charge), so they add.

V adds as a scalar; E adds as a vector. $E = 0$ needs the fields to cancel, which happens between like charges.

Field is the potential gradient



$E = -dV/dr$ · field = negative gradient of potential

The field is the negative potential gradient:

$$E = -\frac{dV}{dx}$$

- between plates V falls evenly $\Rightarrow$ $E = V/d$, constant
- point charge:
$$E = -\frac{dV}{dr} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Minus sign: E points downhill, high V to low.

Electric potential energy

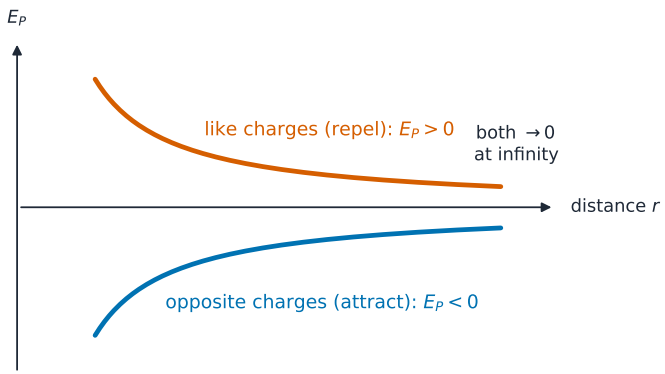
A charge q where the potential is V has $E_P = qV$; for two point charges

$$E_P = \frac{Qq}{4\pi\varepsilon_0 r}$$

- like: $E_P > 0$ – energy released if they fly apart
- opposite: $E_P < 0$ – a **bound** 束缚 well

Both tend to zero as $r \to \infty$ – that is the choice of zero, exactly as for gravity.

Electric potential energy, in detail



A charge q sitting where the potential is V has $E_P = qV$, so for two point charges

$$E_P = \frac{Qq}{4\pi\varepsilon_0 r}.$$

Like charges give $E_P > 0$, opposite charges $E_P < 0$; both tend to zero at infinity.

Gravitational vs electric

Quantity	**Gravitational**	**Electric**
Source	mass M (always $+$)	charge Q (either sign)
Field strength	$g = GM/r^2$	$E = Q/(4\pi\varepsilon_0 r^2)$
Force	$F = mg$	$F = qE$
Gravitational potential 引力势 vs potential	$\phi = -GM/r$	$V = Q/(4\pi\varepsilon_0 r)$
PE of two	$-GMm/r$	$Qq/(4\pi\varepsilon_0 r)$
Nature	always attractive	attract *or* repel

Identical machinery, one difference: gravity's minus sign is permanent (mass only attracts), while the electric sign is inherited from the charge.

6 Recap

Exam tips

- **Coulomb's law** $F = Q_1 Q_2 / 4\pi\varepsilon_0 r^2$ (inverse-square, but can attract or repel).
- Distinguish a **uniform field** ($E = V/d$, between plates) from a **radial field** ($E = Q/4\pi\varepsilon_0 r^2$).
- Electric potential $V = Q/4\pi\varepsilon_0 r$ (sign follows the charge); the field points from high to low potential.

Topic 18 – key takeaways

1 Coulomb

$F = Q_1 Q_2 / 4\pi\varepsilon_0 r^2$ – attract or repel

2 Two fields

uniform $E = V/d$; radial $E = Q/4\pi\varepsilon_0 r^2$

3 Potential

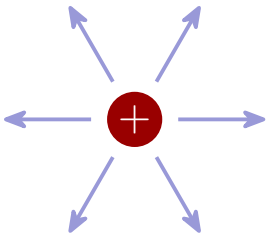
$V = Q/4\pi\varepsilon_0 r$ – scalar, signed, $1/r$

4 The link

$E = -dV/dx$ – the field points downhill in V

Check yourself

1. Which way does a field line point?
2. Halve the plate gap at fixed V – what happens to E?
3. Is potential a vector or a scalar?
4. What is the field inside a hollow charged sphere?



Answers **1.** the force on a + charge **2.** doubles **3.** scalar **4.** zero