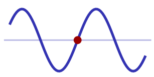
Oscillations

A2 Physics ▪ Topic 17

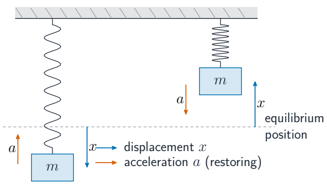
A-Level Physics



What we will cover

1 Defining SHM

Simple harmonic motion



- **proportional to its displacement** from a fixed **equilibrium** 平衡 point, and
- **directed back towards that point** – opposite in sign to the displacement.

The words you must use precisely

displacement x – from equilibrium, signed	**amplitude** x_0 – the largest $\|x\|$, always +
period T – one full oscillation	**frequency** $f = 1/T$, in Hz
angular frequency $\omega = 2\pi f = 2\pi/T$	**phase difference** – a fraction of a cycle, in rad

Given any one of ω, f, T you can reach the other two – start every question by getting ω.

Worked example – maximum acceleration

A mass oscillates with amplitude $0.050\,\mathrm{m}$ at $2.5\,\mathrm{Hz}$. Find its maximum acceleration.

$\omega = 2\pi f = 2\pi \times 2.5 = 15.7\,\mathrm{rad\,s^{-1}}$

$|a|$ is largest at the extremes, where $|a| = \omega^2 x_0$:

$a_{\max} = 15.7^2 \times 0.050 \approx 12\,\mathrm{m\,s^{-2}}$

Fastest at the centre, most accelerated at the ends – the two extremes are opposite. Confusing them is the classic error.

2 x, v and a

Displacement, velocity, acceleration

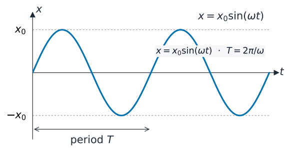
Starting from $x = 0$ moving positive:

$$x = x_0 \sin \omega t$$

$$v = x_0\omega \cos \omega t = v_0 \cos \omega t$$

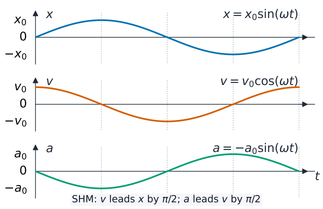
$$a = -x_0\omega^2 \sin \omega t = -\omega^2 x$$

so $v_0 = x_0\omega$ and $a_{\max} = \omega^2 x_0$.



> Starting at the extreme instead? Use $x = x_0 \cos \omega t$. Pick the one that matches $t = 0$.

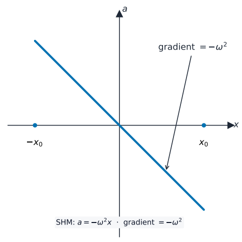
The three graphs together



- x: sine, amplitude x_0
- v: cosine, leads x by $\pi/2$, amplitude ωx_0
- a: $-$sine, π out of phase with x, amplitude $\omega^2 x_0$

> Each differentiation advances the phase by $\pi/2$ and multiplies the amplitude by ω – that is the whole pattern.

Reading ω off the a–x graph



$a = -\omega^2 x$ is a straight line through the origin with gradient $-\omega^2$. So

$$\omega = \sqrt{|\text{gradient}|}, \qquad T = \frac{2\pi}{\omega}$$

> A straight line through the origin with negative gradient *is* the test for SHM – and a very common exam route to T.

Velocity without time

$$v = \pm\omega\sqrt{x_0^2 - x^2}$$

- at $x = 0$: $v = \pm\omega x_0$ – the maximum speed
- at $x = \pm x_0$: $v = 0$ – at rest for an instant

> The $\pm$ is real: the particle crosses equilibrium twice a cycle, once each way.

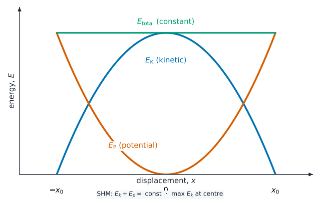
$x_0 = 0.050\,\text{m}$, $\omega = 15.7\,\text{rad}\,\text{s}^{-1}$. Find v at $x = 0.030\,\text{m}$.

$v = 15.7\sqrt{0.050^2 - 0.030^2}$

$= 15.7 \times 0.040 \approx 0.63\,\text{m}\,\text{s}^{-1}$

3 Energy

Energy in SHM



$$E_K = \frac{1}{2}m\omega^2(x_0^2 - x^2)$$
$$E_P = \frac{1}{2}m\omega^2 x^2$$

E_K is a downward parabola, E_P an upward one – and their sum is flat:

$$E_{\text{total}} = \frac{1}{2}m\omega^2 x_0^2$$

> $E \propto x_0^2$: double the amplitude for four times the energy.

Worked example – total energy

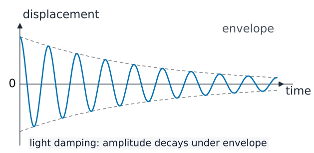
A $0.20\,\text{kg}$ mass oscillates with $x_0 = 0.050\,\text{m}$ and $\omega = 15.7\,\text{rad}\,\text{s}^{-1}$. Find the total energy.

The total equals the maximum KE, as it passes $x = 0$ at $v_{\max} = \omega x_0$:
$E = \frac{1}{2}m\omega^2 x_0^2 = \frac{1}{2}(0.20)(15.7)^2(0.050)^2$
$\approx 0.062\,\text{J}$

> Evaluate the total where one form vanishes – at $x = 0$ it is all kinetic, at $x = x_0$ all potential. Either gives the same number.

4 Damping and resonance

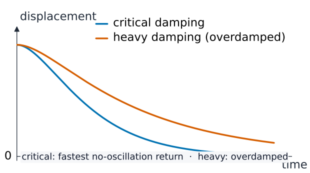
Damped oscillations



A resistive force (**drag** 阻力, **friction** 摩擦力) removes energy as heat, so the amplitude shrinks. **Light damping** 轻阻尼: the amplitude dies away slowly over many cycles, still near the natural frequency.

> A car's suspension is light-to-medium damped – bumps die away, but the ride stays smooth.

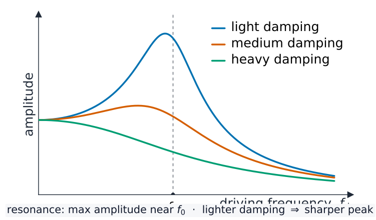
Critical and heavy damping



- **critical damping** 临界阻尼: back to equilibrium in the shortest time, no overshoot, no oscillation
- **heavy damping** 过阻尼: no oscillation either, but slower than critical

> A **galvanometer** 检流计 needle is critically damped so it settles at once; a strong door closer is heavily damped.

Forced oscillations and resonance



A **forced oscillation** 受迫振动 runs at the driving frequency f_d, not its own. **Resonance** 共振: $f_d = f_0$, the **natural frequency** 固有频率 – largest amplitude, most efficient transfer.

> Damping shapes the peak: lighter → sharper and higher; heavier → broader, lower, and nudged to a lower frequency.

Resonance in the world

- a swing pushed at the right rate builds a big amplitude
- a wine glass shattered by its own ringing note
- a building shaken by an earthquake at a matching frequency

> Engineers design a building's natural frequencies to avoid the main earthquake range – resonance is as often something to dodge as to exploit.

Resonance that brought a bridge down

Tacoma Narrows, 1940: a driving frequency close to a natural frequency, with too little damping to limit the amplitude.

5 Recap

Topic 17 – key takeaways

1 The condition
$a = -\omega^2 x$ – proportional, and back to equilibrium

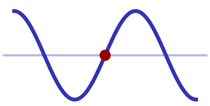
2 Maxima
$v_{\max} = \omega x_0$ at the centre; $a_{\max} = \omega^2 x_0$ at the ends

3 Energy
$E = \frac{1}{2} m\omega^2 x_0^2$, constant; $E \propto x_0^2$

4 Resonance
peak at $f_d = f_0$; damping lowers and broadens it

Check yourself

1. Where in the cycle is the speed greatest?
2. What is the gradient of an a–x graph?
3. Triple the amplitude – what happens to E?
4. Which damping settles fastest with no overshoot?



> **Answers** **1.** at equilibrium, $x = 0$ **2.** $-\omega^2$ **3.** $\times 9$ **4.** critical