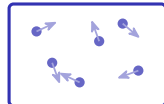


Ideal Gases

A2 Physics ■ Topic 15

A-Level Physics



Ideal Gases

What we will cover

- 1 The mole & N_A
- 2 $pV = nRT = NkT$
- 3 The gas laws
- 4 Kinetic theory
- 5 Molecular KE = $\frac{3}{2}kT$
- 6 Recap



1 The mole

The mole

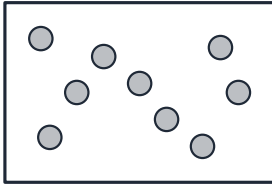
One **mole** 摩尔 contains **Avogadro's number** 阿伏伽德罗常数 of particles:

$$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$$

Number of particles $N = nN_A$. Molar mass in grams is the relative formula mass – 18 g of water is one mole.

Count particles with moles, not with 10^{23} on every line. n is the quantity the gas laws actually use.

One mole, counted



1 mole

$$= 6.02 \times 10^{23} \text{ particles } (N_A)$$

A mole is 6.02×10^{23} particles — the number chosen so that a mole of a substance weighs its relative molecular mass in grams.

2

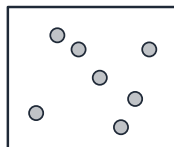
Equation of state

The equation of state

An **ideal gas** 理想气体 obeys $pV \propto T$ exactly:

$$pV = nRT \quad \text{or} \quad pV = NkT$$

- $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$ – per **mole**
- $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$ – per **molecule**
- $N = nN_A$, so $k = R/N_A$



gas: p , V , T , n moles

$$pV = nRT$$

Match the form to your **amount**: moles
 $\rightarrow nRT$, molecules $\rightarrow NkT$. SI throughout – Pa, m^3 , and **kelvin**.

Worked example – how many moles?

A 0.020 m^3 cylinder holds gas at 27°C and $2.0 \times 10^5 \text{ Pa}$. Find n . ($R = 8.31$)

First to kelvin: $T = 27 + 273 = 300 \text{ K}$.

$$n = \frac{pV}{RT} = \frac{(2.0 \times 10^5)(0.020)}{8.31 \times 300} \approx 1.6 \text{ mol}$$

The kelvin conversion is the **first** line, every time. 27 in place of 300 would inflate n by a factor of **11**.

3

The gas laws

A fixed mass changing state

For a fixed amount of gas:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

Hold one quantity fixed and a named law drops out.

Boyle

T fixed: pV constant

Charles

p fixed: V/T constant

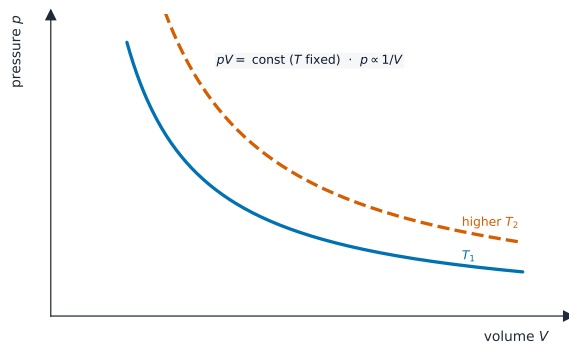
pressure law

V fixed: p/T constant

$pV \propto T$ holds **only** in kelvin – °C here is the classic lost mark.

Boyle's law

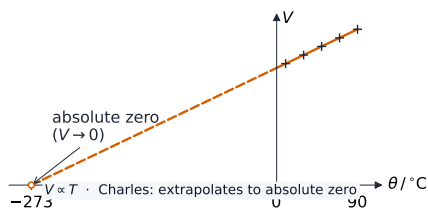
At constant temperature pV is constant, so p against $1/V$ is a **straight line through the origin** — the usual way an exam asks you to verify it.



Charles's law and absolute zero

At constant pressure the volume climbs **linearly** with temperature. Extrapolate the line back to **zero volume** and it meets $\approx -273^\circ\text{C}$.

Every gas points at the **same** intercept — which is what makes absolute zero a property of **temperature**, not of any one gas.



Worked example – heating at constant pressure

A fixed mass at 300 K fills 0.50 m³. Heat it to 450 K at constant pressure. Find the new volume.

p is fixed, so V/T is constant:

$$V_2 = V_1 \frac{T_2}{T_1} = 0.50 \times \frac{450}{300} = 0.75 \text{ m}^3$$

Cancel what is **constant** before substituting – here p drops out and only the temperature **ratio** matters.

4

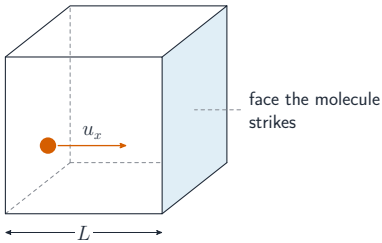
Kinetic theory

Kinetic theory: the assumptions

- 1 many identical molecules in **random motion** 无规则运动
- 2 their own volume is negligible
- 3 collisions are brief; straight lines between
- 4 no **intermolecular** 分子间 forces except in a collision
- 5 collisions are **elastic** 弹性碰撞
- 6 Newton's laws apply

Elastic is what keeps a gas from **cooling itself**. The model fails at **high** p (volume matters) and **low** T (forces matter).

Where the pressure comes from



One molecule, one wall, in a cube of side L :

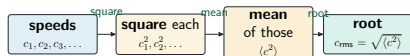
- bounce: $\Delta p_x = -2mu_1$, so the wall takes $+2mu_1$
- next hit after $\Delta t = 2L/u_1$
- so $F_1 = \frac{\Delta p}{\Delta t} = \frac{mu_1^2}{L}$

Sum over N , then use $\langle u_x^2 \rangle = \frac{1}{3}\langle c^2 \rangle$:

$$pV = \frac{1}{3}Nm\langle c^2 \rangle$$

The $\frac{1}{3}$ is **symmetry**: no direction is special, so each axis carries a third of the motion.

Root-mean-square speed



read the name **backwards**: root of the mean of the squares - squaring weights fast molecules more, so c_{rms} is slightly above the mean speed

r.m.s. speed 均方根 $= \sqrt{\langle c^2 \rangle}$: square each speed, average, then square-root. It sits slightly **above** the mean speed – squaring weights the fast molecules more.

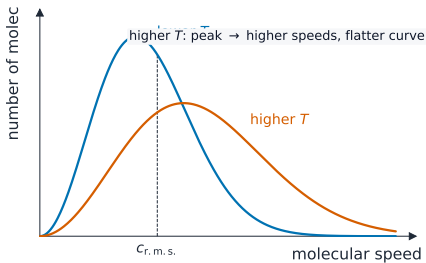
The gas needs $\langle c^2 \rangle$, not $\langle c \rangle$, because **energy** goes as c^2 . Averaging the speeds first gives the wrong answer.

Why it matters under water



A diver's air is compressed: at 10 m the pressure is doubled, so a given mass of gas takes half the volume.

The spread of speeds



Molecules do not share one speed – they occupy a **distribution**. Raise the temperature and the curve **broadens** and shifts to faster speeds.

The area is fixed (the molecules still all exist), so a broader curve must also be a **lower** one.

5 Molecular kinetic energy

Average translational KE

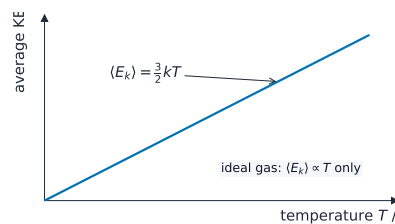
Two expressions for the same pV :

$$pV = NkT, \quad pV = \frac{1}{3}Nm\langle c^2 \rangle$$

Set equal, cancel N , multiply by $\frac{3}{2}$:

$$\langle E_k \rangle = \frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT$$

The average molecular KE depends on **temperature alone** – not the gas, not the pressure. Squeezing a gas at fixed T gives more wall hits, **not** faster molecules.



Worked example – r.m.s. speed of oxygen

Find $c_{\text{r.m.s.}}$ for O_2 at 300 K.

$$m = 5.3 \times 10^{-26} \text{ kg}, \quad k = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

$$\text{From } \frac{1}{2} m \langle c^2 \rangle = \frac{3}{2} kT: \quad \langle c^2 \rangle = 3kT/m$$

$$c_{\text{r.m.s.}} = \sqrt{\frac{3(1.38 \times 10^{-23})(300)}{5.3 \times 10^{-26}}} \\ \approx 480 \text{ m s}^{-1}$$

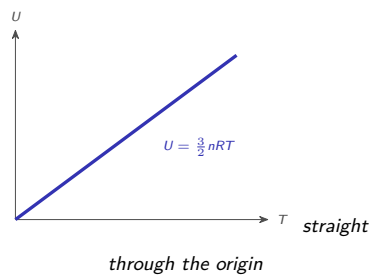
Faster than a jet – yet the gas goes nowhere, because the **directions** are random. Lighter gas, same $T \Rightarrow$ **faster**: hydrogen outruns oxygen in the same room.

Internal energy of an ideal gas

Ideal molecules are points: no intermolecular PE, no rotation or vibration in this model. So the **internal energy** 内能 is **all** kinetic:

$$U = \frac{3}{2} NkT = \frac{3}{2} nRT$$

$U \propto T$ – double the thermodynamic temperature and you double the internal energy.



6

Recap

Topic 15 – key takeaways

1 The mole

$$N = nN_A, N_A = 6.02 \times 10^{23}$$

2 State

$$pV = nRT = NkT, \text{ always in kelvin}$$

3 Kinetic theory

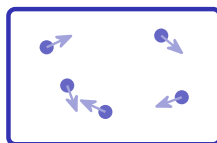
$$pV = \frac{1}{3}Nm\langle c^2 \rangle \text{ from elastic bounces}$$

4 Energy

$$\langle E_k \rangle = \frac{3}{2}kT - \text{temperature only}$$

Check yourself

- 1 How many molecules in 2.0 mol?
- 2 Which form suits a count of molecules?
- 3 Why is there a $\frac{1}{3}$ in $pV = \frac{1}{3}Nm\langle c^2 \rangle$?
- 4 At the same T , which is faster: H_2 or O_2 ?



Answers 1. 1.2×10^{24} 2. $pV = NkT$ 3. symmetry – a third per axis 4. H_2