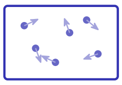
Ideal Gases

A2 Physics ▪ Topic 15

A-Level Physics



What we will cover

1. The mole & N_A
2. $pV = nRT = NkT$
3. The gas laws
4. Kinetic theory
5. Molecular KE $= \frac{3}{2}kT$
6. Recap

1 The mole

The mole

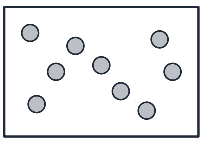
One **mole** 摩尔 contains **Avogadro's number** 阿伏伽德罗常数 of particles:

$$N_A = 6.02 \times 10^{23}\,\mathrm{mol^{-1}}$$

Number of particles $N = nN_A$. Molar mass in grams is the relative formula mass – $18\,\mathrm{g}$ of water is one mole.

> Count particles with moles, not with 10^{23} on every line. n is the quantity the gas laws actually use.

One mole, counted



$= 6.02 \times 10^{23}$ particles (N_A)

1 mole

A mole is 6.02×10^{23} particles — the number chosen so that a mole of a substance weighs its relative molecular mass in grams.

2 Equation of state

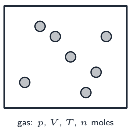
The equation of state

An **ideal gas** 理想气体 obeys $pV \propto T$ exactly:

$$pV = nRT \qquad \text{or} \qquad pV = NkT$$

- $R = 8.31\,\mathrm{J\,mol^{-1}\,K^{-1}}$ – per mole
- $k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$ – per molecule
- $N = nN_A$, so $k = R/N_A$

Match the form to your amount: moles $\to nRT$, molecules $\to NkT$. SI throughout – Pa, $\mathrm{m^3}$, and kelvin.



$pV = nRT$

Worked example – how many moles?

A $0.020\,\mathrm{m^3}$ cylinder holds gas at $27\,^\circ\mathrm{C}$ and $2.0 \times 10^5\,\mathrm{Pa}$. Find n. ($R = 8.31$)

First to kelvin: $T = 27 + 273 = 300\,\mathrm{K}$.

$$n = \frac{pV}{RT} = \frac{(2.0 \times 10^5)(0.020)}{8.31 \times 300} \approx 1.6\,\mathrm{mol}$$

The kelvin conversion is the first line, every time. 27 in place of 300 would inflate n by a factor of 11.

3 The gas laws

A fixed mass changing state

For a fixed amount of gas:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

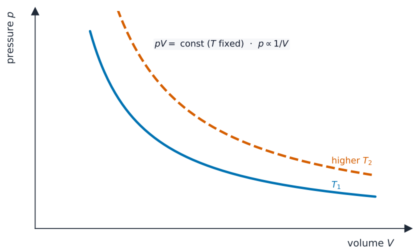
Hold one quantity fixed and a named law drops out.

Boyle	Charles	pressure law
T fixed: pV constant	p fixed: V/T constant	V fixed: p/T constant

$pV \propto T$ holds only in kelvin – °C here is the classic lost mark.

Boyle's law

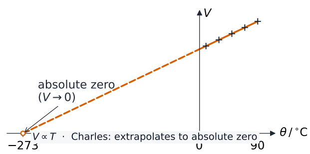
At constant temperature pV is constant, so p against $1/V$ is a straight line through the origin — the usual way an exam asks you to verify it.



Charles's law and absolute zero

At constant pressure the volume climbs linearly with temperature. Extrapolate the line back to zero volume and it meets $\approx -273\,^\circ\mathrm{C}$.

Every gas points at the same intercept – which is what makes absolute zero a property of temperature, not of any one gas.



Worked example – heating at constant pressure

A fixed mass at $300\,\mathrm{K}$ fills $0.50\,\mathrm{m}^3$. Heat it to $450\,\mathrm{K}$ at constant pressure. Find the new volume.

p is fixed, so V/T is constant:

$$V_2 = V_1\frac{T_2}{T_1} = 0.50 \times \frac{450}{300} = 0.75\,\mathrm{m}^3$$

Cancel what is constant before substituting – here p drops out and only the temperature ratio matters.

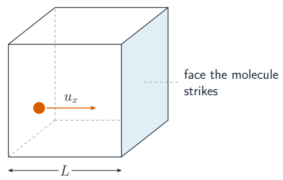
4 Kinetic theory

Kinetic theory: the assumptions

1. many identical molecules in **random motion** 无规则运动
2. their own volume is negligible
3. collisions are brief; straight lines between
4. no **intermolecular** 分子间 forces except in a collision
5. collisions are **elastic** 弹性碰撞
6. Newton's laws apply

Elastic is what keeps a gas from cooling itself. The model fails at high p (volume matters) and low T (forces matter).

Where the pressure comes from



One molecule, one wall, in a cube of side L:

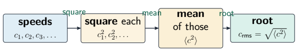
- bounce: $\Delta p_x = -2mu_1$, so the wall takes $+2mu_1$
- next hit after $\Delta t = 2L/u_1$
- so $F_1 = \dfrac{\Delta p}{\Delta t} = \dfrac{mu_1^2}{L}$

Sum over N, then use $\langle u_x^2\rangle = \frac{1}{3}\langle c^2\rangle$:

$$pV = \tfrac{1}{3}Nm\langle c^2\rangle$$

The $\frac{1}{3}$ is symmetry: no direction is special, so each axis carries a third of the motion.

Root-mean-square speed



read the name **backwards**: **r**oot of the **m**ean of the **s**quares - squaring weights fast molecules more, so c_{rms} is slightly above the mean speed

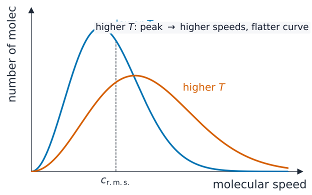
r.m.s. speed 均方根 $= \sqrt{\langle c^2\rangle}$: square each speed, average, then square-root. It sits slightly above the mean speed – squaring weights the fast molecules more.

The gas needs $\langle c^2\rangle$, not $\langle c\rangle$, because energy goes as c^2. Averaging the speeds first gives the wrong answer.

Why it matters under water

A diver's air is compressed: at $10\,\mathrm{m}$ the pressure is doubled, so a given mass of gas takes half the volume.

The spread of speeds



Molecules do not share one speed – they occupy a distribution. Raise the temperature and the curve broadens and shifts to faster speeds.

> The area is fixed (the molecules still all exist), so a broader curve must also be a lower one.

5 Molecular kinetic energy

Average translational KE

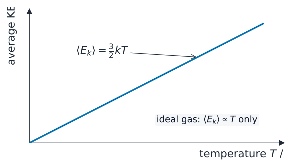
Two expressions for the same pV:

$$pV = NkT, \qquad pV = \tfrac{1}{3}Nm\langle c^2\rangle$$

Set equal, cancel N, multiply by $\frac{3}{2}$:

$$\langle E_k\rangle = \tfrac{1}{2}m\langle c^2\rangle = \tfrac{3}{2}kT$$

> The average molecular KE depends on temperature alone – not the gas, not the pressure. Squeezing a gas at fixed T gives more wall hits, not faster molecules.



Worked example – r.m.s. speed of oxygen

Find $c_{\text{r.m.s.}}$ for O_2 at $300\,\mathrm{K}$.

$m = 5.3 \times 10^{-26}\,\mathrm{kg}$, $k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$

From $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$: $\langle c^2\rangle = 3kT/m$

$$c_{\text{r.m.s.}} = \sqrt{\frac{3(1.38\times10^{-23})(300)}{5.3\times10^{-26}}}$$

$\approx 480\,\mathrm{m\,s^{-1}}$

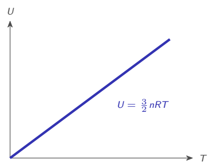
> Faster than a jet – yet the gas goes nowhere, because the directions are random. Lighter gas, same $T \Rightarrow$ faster: hydrogen outruns oxygen in the same room.

Internal energy of an ideal gas

Ideal molecules are points: no intermolecular PE, no rotation or vibration in this model. So the **internal energy** 内能 is all kinetic:

$$U = \tfrac{3}{2}NkT = \tfrac{3}{2}nRT$$

> $U \propto T$ – double the thermodynamic temperature and you double the internal energy.



straight

through the origin

6 Recap

Topic 15 – key takeaways

1 The mole

$N = nN_A$, $N_A = 6.02 \times 10^{23}$

2 State

$pV = nRT = NkT$, always in kelvin

3 Kinetic theory

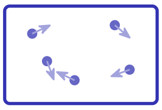
$pV = \frac{1}{3}Nm\langle c^2\rangle$ from elastic bounces

4 Energy

$\langle E_k\rangle = \frac{3}{2}kT$ – temperature only

Check yourself

1. How many molecules in $2.0\,\mathrm{mol}$?
2. Which form suits a count of molecules?
3. Why is there a $\frac{1}{3}$ in $pV = \frac{1}{3}Nm\langle c^2\rangle$?
4. At the same T, which is faster: H_2 or O_2?



Answers **1.** 1.2×10^{24} **2.** $pV = NkT$ **3.** symmetry – a third per axis **4.** H_2