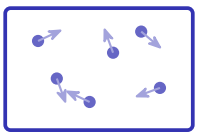
Ideal Gases

A2 Physics ▪ Topic 15

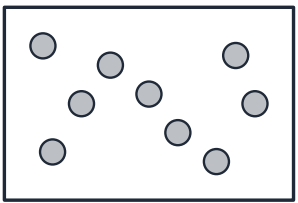
A-Level Physics



What we will cover

1. The mole & N_A
2. $pV = nRT = NkT$
3. The gas laws
4. Kinetic theory
5. Molecular KE $= \frac{3}{2}kT$
6. Recap

The mole



1 mole

$= 6.02 \times 10^{23}$ particles (N_{A})

scuba

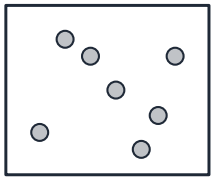
The equation of state

An **ideal gas** 理想气体 obeys $pV \propto T$ exactly:

$$pV = nRT \qquad \text{or} \qquad pV = NkT$$

- $R = 8.31\,\mathrm{J\,mol^{-1}\,K^{-1}}$ – per mole
- $k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$ – per molecule
- $N = nN_\mathrm{A}$, so $k = R/N_\mathrm{A}$

> Match the form to your amount: moles $\rightarrow nRT$, molecules $\rightarrow NkT$. SI throughout – Pa, m^3, and kelvin.



gas: p, V, T, n moles

$$pV = nRT$$

Worked example – how many moles?

A $0.020\,\mathrm{m}^3$ cylinder holds gas at $27\,^\circ\mathrm{C}$ and $2.0\times10^5\,\mathrm{Pa}$. Find n. ($R = 8.31$)

First to kelvin: $T = 27 + 273 = 300\,\mathrm{K}$.

$$n = \frac{pV}{RT} = \frac{(2.0\times10^5)(0.020)}{8.31\times300} \approx 1.6\,\mathrm{mol}$$

The kelvin conversion is the first line, every time. 27 in place of 300 would inflate n by a factor of 11.

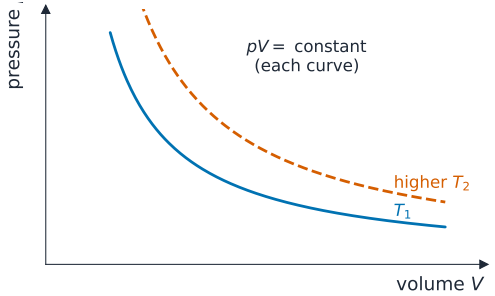
A fixed mass changing state

For a fixed amount of gas:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

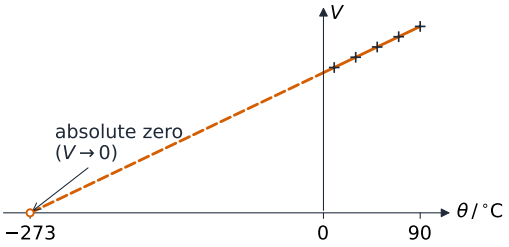
Hold one quantity fixed and a named law drops out:

- T fixed – **Boyle** 玻意耳定律: $p_1 V_1 = p_2 V_2$
- p fixed – **Charles** 查理定律: V/T constant
- V fixed – **pressure law** 气体压强定律: p/T constant



$pV \propto T$ holds only in kelvin – °C here

Charles's law and absolute zero



At constant pressure the volume climbs linearly with temperature. Extrapolate the line back to zero volume and it meets $\approx -273\,^{\circ}\mathrm{C}$.

> Every gas points at the same intercept – which is what makes absolute zero a property of temperature, not of any one gas.

Worked example – heating at constant pressure

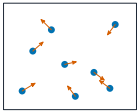
A fixed mass at $300\,\mathrm{K}$ fills $0.50\,\mathrm{m}^3$. Heat it to $450\,\mathrm{K}$ at constant pressure. Find the new volume.

p is fixed, so V/T is constant:

$$V_2 = V_1 \frac{T_2}{T_1} = 0.50 \times \frac{450}{300} = 0.75\,\mathrm{m}^3$$

Cancel what is constant before substituting – here p drops out and only the temperature ratio matters.

Kinetic theory: the assumptions



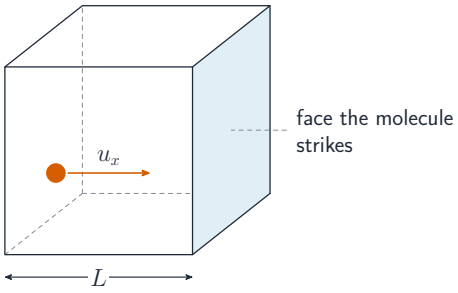
1. many identical molecules, in continuous **random motion**
2. their own volume is **tiny** next to the container
3. straight lines between hits - intermolecular forces **ignored**
4. collisions are **elastic**: no kinetic energy is lost

poor at very high pressure (volume matters) or very low temperature (forces matter)

- many identical molecules, **random motion** 无规则运动
- their own volume is negligible
- collisions are brief; straight lines between
- no **intermolecular** 分子间 forces except in a collision
- collisions are **elastic** 弹性碰撞
- Newton's laws apply

Elastic is what keeps a gas from cooling itself. The model fails at high p (volume matters) and low T

Where the pressure comes from



One molecule, one wall, in a cube of side L:

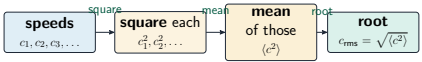
- bounce: $\Delta p_x = -2mu_1$, so the wall takes $+2mu_1$
- next hit after $\Delta t = 2L/u_1$
- so $F_1 = \dfrac{\Delta p}{\Delta t} = \dfrac{mu_1^2}{L}$

Sum over N, then use $\langle u_x^2 \rangle = \frac{1}{3}\langle c^2 \rangle$:

$$pV = \tfrac{1}{3} Nm \langle c^2 \rangle$$

The $\frac{1}{3}$ is symmetry: no direction is special, so each axis carries a third of the motion.

Root-mean-square speed

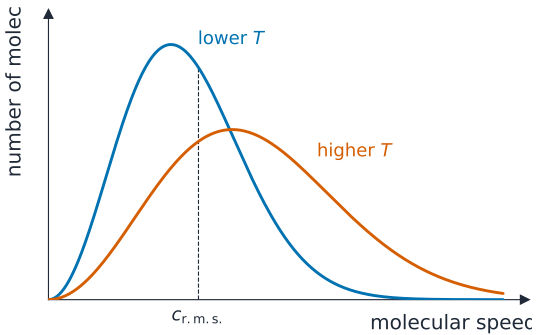


read the name **backwards**: **r**oot of the **m**ean of the **s**quares - squaring weights fast molecules more, so c_{rms} is slightly above the mean speed

r.m.s. speed 均方根 $= \sqrt{\langle c^2 \rangle}$: square each speed, average, then square-root. It sits slightly above the mean speed – squaring weights the fast molecules more.

> The gas needs $\langle c^2 \rangle$, not $\langle c \rangle$, because energy goes as c^2. Averaging the speeds first gives the wrong answer.

The spread of speeds



Molecules do not share one speed – they occupy a distribution. Raise the temperature and the curve broadens and shifts to faster speeds.

> The area is fixed (the molecules still all exist), so a broader curve must also be a lower one.

Average translational KE

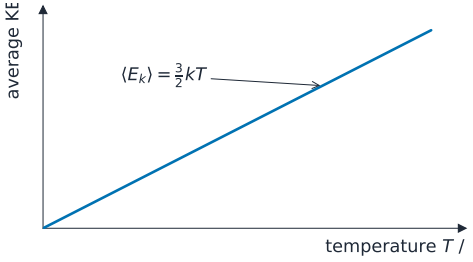
Two expressions for the same pV:

$$pV = NkT, \qquad pV = \tfrac{1}{3}Nm\langle c^2\rangle$$

Set equal, cancel N, multiply by $\frac{3}{2}$:

$$\langle E_\mathrm{k}\rangle = \tfrac{1}{2}m\langle c^2\rangle = \tfrac{3}{2}kT$$

The average molecular KE depends on temperature alone – not the gas, not the pressure. Squeezing a gas at fixed T gives more wall hits, not faster molecules.



Worked example – r.m.s. speed of oxygen

Find $c_{\text{r.m.s.}}$ for $\mathrm{O_2}$ at $300\,\mathrm{K}$.

$m = 5.3 \times 10^{-26}\,\mathrm{kg}$, $k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$

From $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$: $\langle c^2\rangle = 3kT/m$

$$c_{\text{r.m.s.}} = \sqrt{\frac{3(1.38 \times 10^{-23})(300)}{5.3 \times 10^{-26}}}$$

$\approx 480\,\mathrm{m\,s^{-1}}$

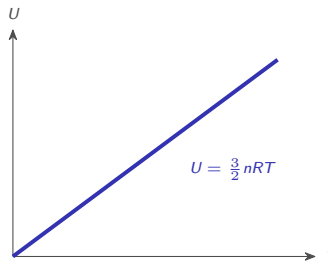
Faster than a jet – yet the gas goes nowhere, because the directions are random. Lighter gas, same $T \Rightarrow$ faster: hydrogen outruns oxygen in the same room.

Internal energy of an ideal gas

Ideal molecules are points: no intermolecular PE, no rotation or vibration in this model. So the **internal energy** 内能 is all kinetic:

$$U = \frac{3}{2}NkT = \frac{3}{2}nRT$$

> $U \propto T$ – double the thermodynamic temperature and you double the internal energy.



straight through the origin

Topic 15 – key takeaways

1 The mole

$N = nN_\mathrm{A}$, $N_\mathrm{A} = 6.02 \times 10^{23}$

2 State

$pV = nRT = NkT$, always in kelvin

3 Kinetic theory

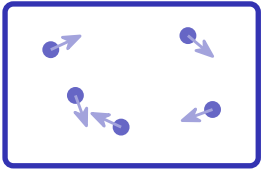
$pV = \frac{1}{3}Nm\langle c^2 \rangle$ from elastic bounces

4 Energy

$\langle E_\mathrm{k} \rangle = \frac{3}{2}kT$ – temperature only

Check yourself

1. How many molecules in $2.0\,\mathrm{mol}$?
2. Which form suits a count of molecules?
3. Why is there a $\frac{1}{3}$ in $pV = \frac{1}{3}Nm\langle c^2\rangle$?
4. At the same T, which is faster: $\mathrm{H_2}$ or $\mathrm{O_2}$?



Answers **1.** 1.2×10^{24} **2.** $pV = NkT$ **3.** symmetry – a third per axis **4.** $\mathrm{H_2}$