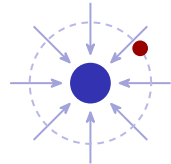


Gravitational Fields

A2 Physics ■ Topic 13

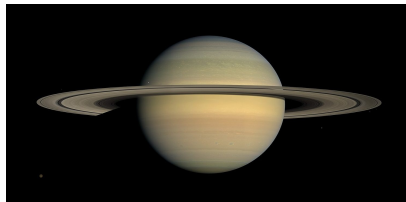
A-Level Physics



Gravitational Fields

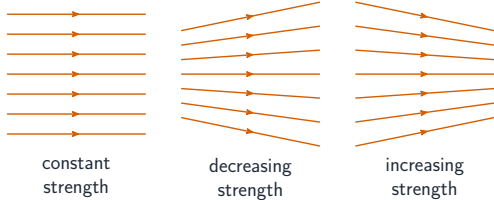
What we will cover

- 1 Fields & field lines
- 2 Newton's law of gravitation
- 3 Field strength $g = GM/r^2$;
- 4 Orbits & Kepler's third law
- 5 Gravitational potential
- 6 Recap



1 Fields and field lines

Gravitational field strength

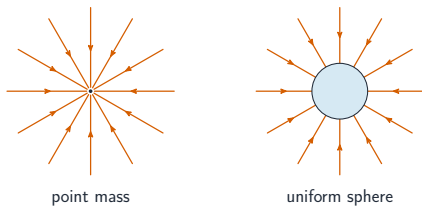


A **gravitational field** 重力场 is a region where a mass feels a force. Its **field strength** 重力场强度 is the force **per unit mass** on a small **test mass** 检验质量:

$$g = \frac{F}{m}, \quad [g] = \text{N kg}^{-1}$$

N kg^{-1} is ms^{-2} – g is also the acceleration of free fall. It is a **vector**, pointing at the source.

Radial and uniform fields



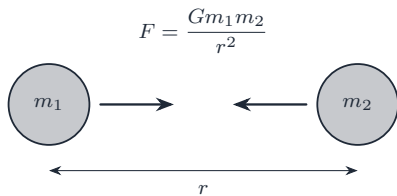
- round a point mass or sphere:
radial 径向, pointing **inwards**
- over a small patch of ground:
parallel and evenly spaced – a **uniform field** 匀强场
- **closer lines** = stronger field

Outside a uniform sphere the field is **identical** to a point mass of the same total mass at its centre – which is why you may treat the Earth as a point.

2

Newton's law of gravitation

Newton's law of gravitation



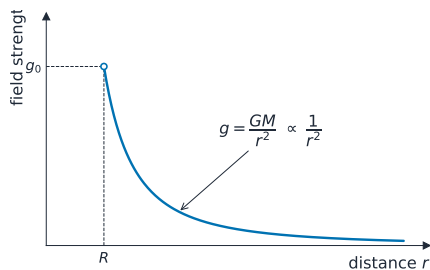
Two point masses attract along the line joining them:

$$F = \frac{Gm_1m_2}{r^2}$$

with $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$.

Inverse square: double the separation and the force falls to a **quarter**, not a half.

Field strength from a point mass



Put $F = GMm/r^2$ into $g = F/m$; the test mass cancels:

$$g = \frac{GM}{r^2}$$

g depends only on the **source** mass and the distance – never on the mass you put there. It falls off as $1/r^2$.

Worked example – g at the Earth's surface

Earth: $M = 6.0 \times 10^{24} \text{ kg}$, $R = 6.4 \times 10^6 \text{ m}$.
Find g at the surface.

$$g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})}{(6.4 \times 10^6)^2}$$

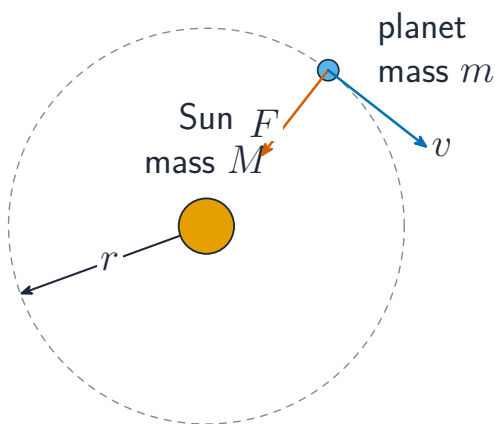
$$\approx 9.8 \text{ N kg}^{-1}$$

Why is g constant in the lab? Climbing 10 m changes r by one part in a **million** of $R = 6.4 \times 10^6 \text{ m}$.

3 Orbits

Gravitational Fields
└ Orbits

Orbital motion



Gravity **is** the centripetal force:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{GM}{r}}$$

The satellite's mass **cancels**.
A bolt and a space station at the same radius orbit at the same speed – which is why astronauts float alongside their craft.

Gravitational Fields
└ Orbits

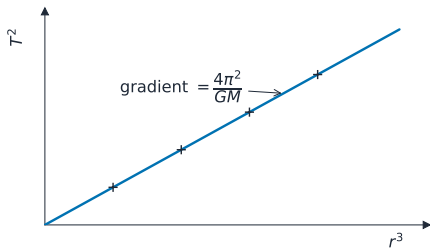
Worked example – orbital speed

A satellite orbits Earth at $r = 7.0 \times 10^6$ m.
Find its speed. ($GM = 4.0 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$)

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{4.0 \times 10^{14}}{7.0 \times 10^6}} \\ \approx 7.6 \times 10^3 \text{ m s}^{-1}$$

r is measured from the **centre** of the Earth, not the surface – a height above ground must have R added to it first.

Kepler's third law



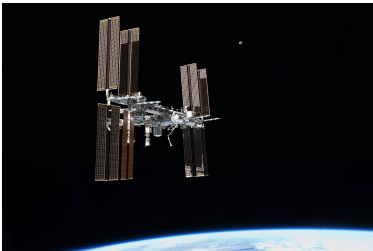
From $T = 2\pi r/v$:

$$T = 2\pi \sqrt{\frac{r^3}{GM}} \Rightarrow T^2 = \frac{4\pi^2}{GM} r^3$$

So $T^2 \propto r^3$ – a straight line through the origin.

The **gradient** of T^2 against r^3 is $4\pi^2/(GM)$, so orbital data **weighs the central body**.

Geostationary orbit



A **geostationary** 地球同步 satellite sits over one spot, so it must have:

- period **24 h** – the Earth's own
- direction **west to east**
- plane: above the **equator** 赤道

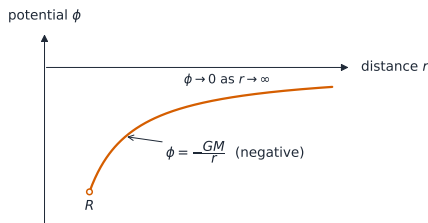
Then $T^2 = 4\pi^2 r^3/(GM)$ fixes $r \approx 4.2 \times 10^7$ m.

Off the equator it would drift north–south each day, so a fixed dish could not track it.

4

Gravitational potential

Gravitational potential



Potential 引力势 is the work done **per unit mass** bringing a test mass in from **infinity** 无穷远:

$$\phi = \frac{W}{m} = -\frac{GM}{r}, \quad [\phi] = \text{J kg}^{-1}$$

Zero at infinity, so ϕ is **negative** everywhere else – gravity did the work for you on the way in. ϕ is a **scalar**: just add them.

Potential energy and escape

A mass m sitting where the potential is ϕ :

$$E_p = m\phi = -\frac{GMm}{r}$$

Raising a satellite from r_1 to r_2 :

$$\Delta E_p = GMm \left(\frac{1}{r_1} - \frac{1}{r_2} \right) > 0$$

To escape, the kinetic energy must match the well's depth:

$$\frac{1}{2}mv_{\text{esc}}^2 = \frac{GMm}{r} \Rightarrow v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$$

$mg\Delta h$ is only the **small-height** shortcut, where r hardly changes. Over orbital distances you must difference $-GMm/r$ at each radius.

Escape needs **no rocket left over**: reach v_{esc} and you coast to infinity, arriving with zero speed.

Worked example – escape velocity

Find the escape velocity from the Earth's surface. ($GM = 4.0 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$, $R = 6.4 \times 10^6 \text{ m}$)

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(4.0 \times 10^{14})}{6.4 \times 10^6}} \\ \approx 1.1 \times 10^4 \text{ m s}^{-1} = 11 \text{ km s}^{-1}$$

The mass **cancels**: a pebble and a rocket need the same 11 km s^{-1} . Note the **2** – escape speed is $\sqrt{2} \times$ the circular orbital speed at the same radius.

5 Recap

Gravitational Fields
└ Recap

Topic 13 – key takeaways

1 Force

$F = GMm/r^2$ – inverse square, always attractive

2 Field

$g = GM/r^2$, a vector; the test mass cancels

3 Orbits

gravity = centripetal $\Rightarrow T^2 \propto r^3$

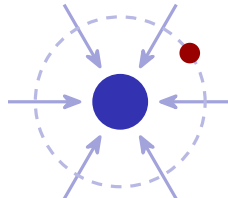
4 Potential

$\phi = -GM/r$, a scalar, zero at infinity

Gravitational Fields
└ Recap

Check yourself

- 1 Triple the separation – what happens to F ?
- 2 Why is ϕ always negative?
- 3 What is the period of a geostationary orbit?
- 4 Does escape velocity depend on the object's mass?



Answers 1. falls to 1/9 2. zero is set at infinity 3. 24 h 4. no – m cancels