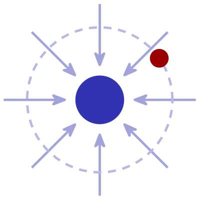
Gravitational Fields

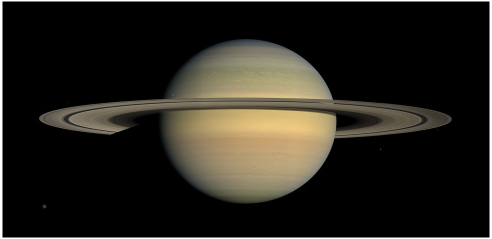
A2 Physics ▪ Topic 13

A-Level Physics

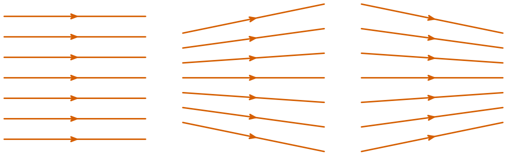


What we will cover

1. Fields & field lines
2. Newton's law of gravitation
3. Field strength $g = GM$;
4. Orbits & Kepler's third law
5. Gravitational potential
6. Recap



Gravitational field strength

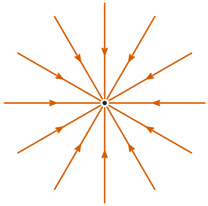


A **gravitational field** 重力场 is a region where a mass feels a force. Its **field strength** 重力场强度 is the force per unit mass on a small **test mass** 检验质量:

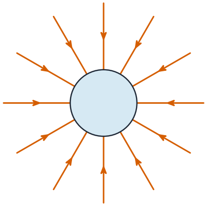
$$g = \frac{F}{m}, \qquad [g] = \mathrm{N\,kg^{-1}}$$

> $\mathrm{N\,kg^{-1}}$ is $\mathrm{m\,s^{-2}}$ – g is also the acceleration of free fall. It is a vector, pointing at the source.

Radial and uniform fields



point mass

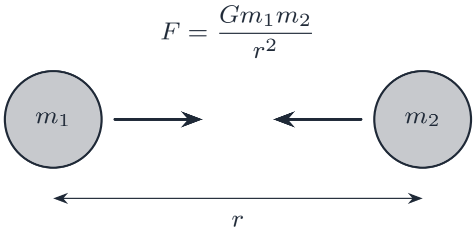


uniform sphere

- round a point mass or sphere: **radial** 径向, pointing inwards
- over a small patch of ground: parallel and evenly spaced – a **uniform field** 匀强场
- closer lines = stronger field

> Outside a uniform sphere the field is identical to a point mass of the same total mass at its centre – which is why you may treat the Earth as a point.

Newton's law of gravitation



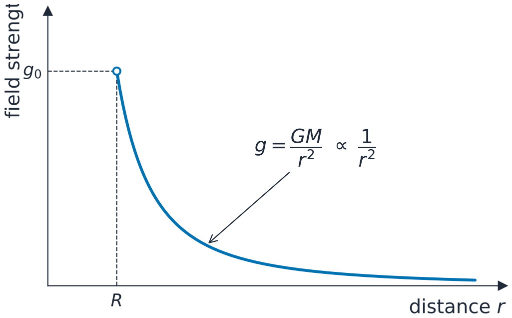
Two point masses attract along the line joining them:

$$F = \frac{Gm_1 m_2}{r^2}$$

with $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$.

> Inverse square: double the separation and the force falls to a quarter, not a half.

Field strength from a point mass



Put $F = GMm/r^2$ into $g = F/m$; the test mass cancels:

$$g = \frac{GM}{r^2}$$

> g depends only on the source mass and the distance – never on the mass you put there. It falls off as $1/r^2$.

Worked example – g at the Earth's surface

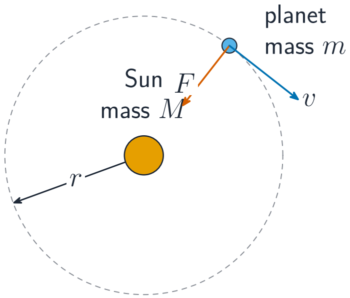
Earth: $M = 6.0 \times 10^{24}\,\mathrm{kg}$, $R = 6.4 \times 10^{6}\,\mathrm{m}$. Find g at the surface.

$$g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})}{(6.4 \times 10^{6})^2}$$

$$\approx 9.8\,\mathrm{N\,kg^{-1}}$$

> Why is g constant in the lab? Climbing $10\,\mathrm{m}$ changes r by one part in a million of $R = 6.4 \times 10^{6}\,\mathrm{m}$.

Orbital motion



Gravity is the centripetal force:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{GM}{r}}$$

> The satellite's mass cancels. A bolt and a space station at the same radius orbit at the same speed – which is why astronauts float alongside their craft.

Worked example – orbital speed

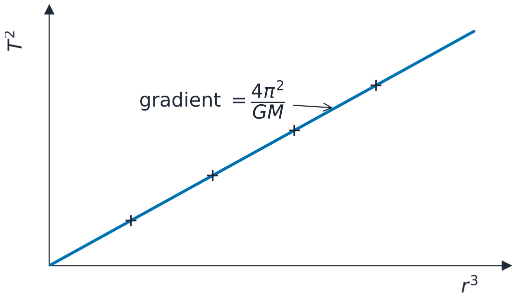
A satellite orbits Earth at $r = 7.0 \times 10^6$ m. Find its speed. ($GM = 4.0 \times 10^{14}\,\mathrm{m^3\,s^{-2}}$)

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{4.0 \times 10^{14}}{7.0 \times 10^6}}$$

$$\approx 7.6 \times 10^3\,\mathrm{m\,s^{-1}}$$

r is measured from the centre of the Earth, not the surface – a height above ground must have R added to it first.

Kepler's third law



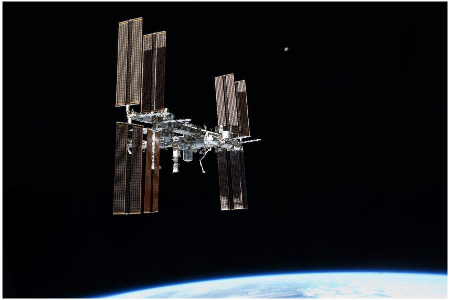
From $T = 2\pi r/v$:

$$T = 2\pi\sqrt{\frac{r^3}{GM}} \;\Rightarrow\; T^2 = \frac{4\pi^2}{GM}\,r^3$$

So $T^2 \propto r^3$ – a straight line through the origin.

The gradient of T^2 against r^3 is $4\pi^2/(GM)$, so orbital data weighs the central body.

Geostationary orbit



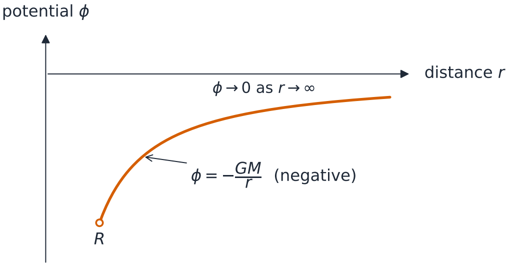
A **geostationary** 地球同步 satellite sits over one spot, so it must have:

- period $24\,\mathrm{h}$ – the Earth's own
- direction west to east
- plane: above the **equator** 赤道

Then $T^2 = 4\pi^2 r^3/(GM)$ fixes $r \approx 4.2 \times 10^7\,\mathrm{m}$.

> Off the equator it would drift north–south each day, so a fixed dish could not track it.

Gravitational potential



Potential 引力势 is the work done per unit mass bringing a test mass in from **infinity** 无穷远:

$$\phi = \frac{W}{m} = -\frac{GM}{r}, \qquad [\phi] = \mathrm{J\,kg^{-1}}$$

> Zero at infinity, so ϕ is negative everywhere else – gravity did the work for you on the way in. ϕ is a scalar: just add them.

Potential energy and escape

A mass m sitting where the potential is ϕ:

$$E_P = m\phi = -\frac{GMm}{r}$$

Raising a satellite from r_1 to r_2:

$$\Delta E_P = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right) > 0$$

To escape, the kinetic energy must match the well's depth:

$$\tfrac{1}{2} m v_{esc}^2 = \frac{GMm}{r} \quad \Rightarrow \quad v_{esc} = \sqrt{\frac{2GM}{r}}$$

$mg\Delta h$ is only the small-height shortcut, where r hardly changes. Over orbital distances you must difference $-GMm/r$ at each radius.

Escape needs no rocket left over: reach v_{esc} and you coast to infinity, arriving with zero speed.

Worked example – escape velocity

Find the escape velocity from the Earth's surface. ($GM = 4.0 \times 10^{14}\,\mathrm{m^3\,s^{-2}}$, $R = 6.4 \times 10^6\,\mathrm{m}$)

$$v_{\mathrm{esc}} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(4.0 \times 10^{14})}{6.4 \times 10^6}}$$

$$\approx 1.1 \times 10^4\,\mathrm{m\,s^{-1}} = 11\,\mathrm{km\,s^{-1}}$$

The mass cancels: a pebble and a rocket need the same $11\,\mathrm{km\,s^{-1}}$. Note the 2 – escape speed is $\sqrt{2}\times$ the circular orbital speed at the same radius.

Topic 13 – key takeaways

1 Force

$F = GMm/r^2$ – inverse square, always attractive

2 Field

$g = GM/r^2$, a vector; the test mass cancels

3 Orbits

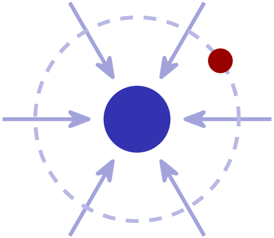
gravity $=$ centripetal $\Rightarrow T^2 \propto r^3$

4 Potential

$\phi = -GM/r$, a scalar, zero at infinity

Check yourself

1. Triple the separation – what happens to F?
2. Why is ϕ always negative?
3. What is the period of a geostationary orbit?
4. Does escape velocity depend on the object's mass?



Answers **1.** falls to $1/9$ **2.** zero is set at infinity **3.** $24\,\mathrm{h}$ **4.** no – m cancels