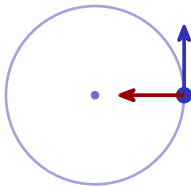


Circular Motion

A2 Physics ■ Topic 12

A-Level Physics



What we will cover

- 1 Radians and angular speed 12.1
- 2 Centripetal force 12.2
- 3 Recap



1

Radians and angular speed

Sheet 12.1 – what you need to know

You must be able to:

- define the **radian** and convert between degrees and radians
- use $\omega = \frac{\theta}{t} = \frac{2\pi}{T}$ and $v = r\omega$
- link wheels: same v along a chain, same ω when fixed together
- find the circle a point on the Earth moves in:
 $r = R \cos \lambda$

What the exam asks

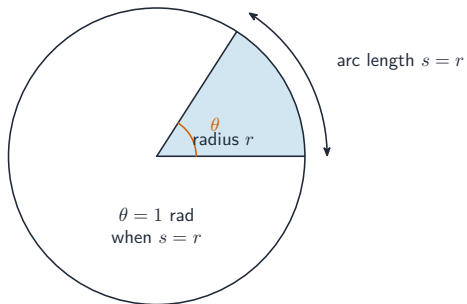
“Define the radian.” [1]

“Calculate the angular speed of the rear wheel.” [2]

“Calculate the speed at which Cambridge moves around the circle.” [3]

Practise on exercise sheet 12.1, then try the real exam questions on past-paper sheet 12.1.

Angles in radians



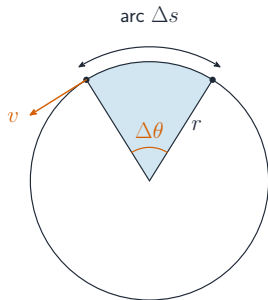
One **radian** 弧度 is the angle whose **arc** 弧 equals the radius:

$$\theta = \frac{s}{r}$$

- a ratio of lengths $\Rightarrow$ **no unit**
- full circle = 2π rad; half = π
- $1 \text{ rad} = 180^\circ/\pi \approx 57.3^\circ$

Put the calculator in **radian** mode for this whole topic – degree mode silently gives wrong answers.

Angular speed



Angular displacement 角位移 θ is the angle the radius turns through; **angular speed** 角速度 is its rate of change:

$$\omega = \frac{\theta}{t}, \quad [\omega] = \text{rad s}^{-1}$$

One turn ($2\pi \text{ rad}$) takes the **period** 周期 T :

$$\omega = \frac{2\pi}{T} = 2\pi f$$

ω measures how fast the **angle** grows
– not how fast the object moves
through space.

Linear and angular speed

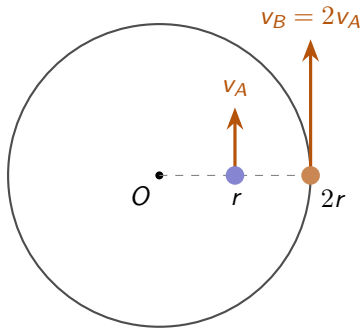
In one period the object covers the circumference $2\pi r$.

$$v = \frac{2\pi r}{T} = r\omega$$

- v is the **tangential** 切向 speed
- same ω , bigger $r \Rightarrow$ bigger v

On a merry-go-round both children share ω – but the one at the **edge** moves faster, because $v = r\omega$.

Linear and angular speed, in detail



Both riders turn through the same angle each second: the **same** ω .

$$v = r\omega$$

linear speed

The outer rider covers twice the distance, so moves **twice as fast**.

In one period the object covers the circumference $2\pi r$, so $v = \frac{2\pi r}{T} = r\omega$.

Worked example – a fairground ride

A ride of radius 4.0 m turns once every 8.0 s. Find ω and the rider's linear speed.

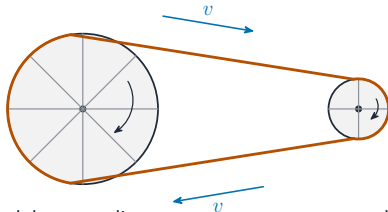
$$\omega = \frac{2\pi}{T} = \frac{2\pi}{8.0} = 0.79 \text{ rad s}^{-1}$$

$$v = r\omega = 4.0 \times 0.79 = 3.1 \text{ m s}^{-1}$$

Go through ω **first**: it is the quantity every rider shares. Then $v = r\omega$ gives each rider's own speed from their own radius.

Wheels linked by a chain or a belt

every link of the chain moves at the same speed v



pedal cog: radius r_1 ,
angular speed ω_1

rear cog: radius r_2 ,
angular speed ω_2

$$v = r_1\omega_1 = r_2\omega_2 \quad \Rightarrow \quad \frac{\omega_2}{\omega_1} = \frac{r_1}{r_2}$$

A **chain** 链条 or belt that does not slip moves at **one speed** v all the way round.

- two linked **cogs** 齿轮: same **rim speed** $v = r_1\omega_1 = r_2\omega_2$
- two parts fixed together (a cog on its wheel): same ω

Ask first: are the two parts **linked** (same v) or **fixed together** (same ω)? The smaller cog always turns faster.

Worked example – a bicycle

A bicycle moves at 12 m s^{-1} . The rear wheel (radius 0.35 m) carries a cog of radius 0.040 m ; the chain runs to the pedal cog, radius 0.12 m . Find the pedals' angular speed.

1. Rear wheel

its rim moves at the road speed

$$\begin{aligned}\omega_w &= \frac{v}{r_w} = \frac{12 \text{ m s}^{-1}}{0.35 \text{ m}} \\ &= 34.3 \text{ rad s}^{-1}\end{aligned}$$

2. Rear cog

fixed to the wheel: same ω

$$\begin{aligned}v_c &= r_c \omega_w = 0.040 \times 34.3 \\ &= 1.37 \text{ m s}^{-1}\end{aligned}$$

3. Pedal cog

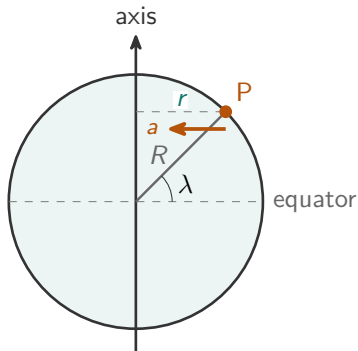
linked by the chain: same v

$$\begin{aligned}\omega_p &= \frac{v_c}{r_p} = \frac{1.37 \text{ m s}^{-1}}{0.12 \text{ m}} \\ &= 11 \text{ rad s}^{-1}\end{aligned}$$

Check: $\omega_p/\omega_w = r_c/r_p = 1/3$. A smaller rear cog makes the pedals turn **more slowly** for the same road speed.

A point on the spinning Earth

A point at **latitude** 纬度 λ circles the **axis**, not the centre:



$$r = R \cos \lambda$$

radius of its circle

Latitude 30° , $R = 6.37 \times 10^6$ m, one turn in 24 h. Find its speed.

$$r = R \cos \lambda = (6.37 \times 10^6 \text{ m}) \cos 30^\circ = 5.52 \times 10^6 \text{ m}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{24 \times 3600 \text{ s}} = 7.27 \times 10^{-5} \text{ rad s}^{-1}$$

$$v = r\omega = (5.52 \times 10^6 \text{ m})(7.27 \times 10^{-5} \text{ rad s}^{-1}) = 401 \text{ m s}^{-1}$$

2

Centripetal force

Sheet 12.2 – what you need to know

You must be able to:

- describe uniform circular motion in terms of **velocity** and **acceleration**
- use $a = \frac{v^2}{r} = r\omega^2$ and $F = \frac{mv^2}{r} = mr\omega^2$
- say **which real force** provides the centripetal force
- resolve forces for a banked track, a cone or a vertical circle

What the exam asks

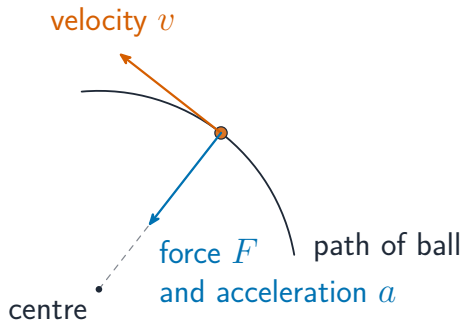
“In terms of velocity and acceleration, describe uniform circular motion of an object.” [2]

“Determine the magnitude of the resultant force that acts to cause the circular motion of the student.” [2]

“Describe how the forces acting on the ball cause its acceleration to be centripetal.” [2]

Practise on exercise sheet 12.2, then try the real exam questions on past-paper sheet 12.2.

Describing uniform circular motion



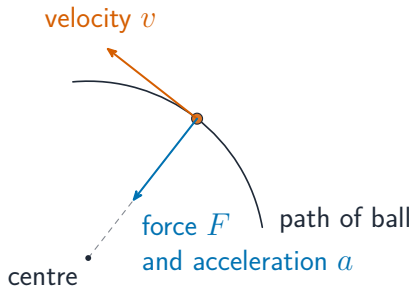
“In terms of velocity and acceleration, describe uniform circular motion.” [2]

The **speed** is constant, but the **velocity** keeps changing direction: it is always along the **tangent** 切线.

The acceleration has a constant size and always points to the **centre**, at 90° to the velocity.

“Constant velocity” scores nothing: the direction changes, so the velocity changes.

Centripetal acceleration



Constant **speed**, changing **velocity** – the direction keeps turning, so there *is* an acceleration:

$$a = \frac{v^2}{r} = r\omega^2$$

(equal, because $v = r\omega$ – pick the form matching your data).

a is **perpendicular** 垂直 to v at every instant. Any part **along** the motion would change the speed.

Centripetal force



Circular motion at constant speed is still a **changing velocity** – the direction turns – so there is an acceleration, and Newton's second law radially gives:

$$F = ma = \frac{mv^2}{r} = mr\omega^2$$

The **centripetal force** 向心力 is **not a new force**. It is the **net result** of the real forces – always say **which** one provides it.

What provides the centripetal force?

ball on a string:
the **tension**

car on a flat corner:
friction

banked track:
the **normal force**

satellite in orbit:
gravity

electron in an atom:
electrostatic pull

too fast $\Rightarrow$ it **runs out**
and the car skids

The same equation, four different forces

On a string

The tension: $T = mv^2/r$.

Planet or satellite

A **planet or satellite** 行星或卫星 in **orbit** 轨道: the **gravitational attraction** 万有引力, $GMm/r^2 = mv^2/r$.

Electron in an atom

The **electrostatic** 静电 attraction between the electron and the positive **nucleus** 原子核.

Charge in a field

A charged particle in a **magnetic field** 磁场: $Bqv = mv^2/r$, which is how a mass spectrometer sorts by mass.

One **revolution** 转 is 2π radians in a **period** 周期 T , so $\omega = 2\pi/T$ – and the centripetal force is never a new force, only the name of whichever one points inwards.

Worked example – a ball on a string

A 0.20 kg ball is whirled in a horizontal circle of radius 0.50 m at 3.0 m s⁻¹. Find the tension.

The tension *is* the centripetal force:

$$F = \frac{mv^2}{r} = \frac{0.20 \times 3.0^2}{0.50} = 3.6 \text{ N}$$

Name the real force first (*tension*), then set it equal to mv^2/r . Never add a separate “centripetal force” on top of the tension – you would count it twice.

Worked example – what speed does the force allow?

An electric force of $8.2 \times 10^{-8} \text{ N}$ keeps an electron (mass $9.1 \times 10^{-31} \text{ kg}$) in a circle of radius $5.3 \times 10^{-11} \text{ m}$. Find its speed.

The electric force **is** the centripetal force:

$$F = \frac{mv^2}{r} \quad \Rightarrow \quad v = \sqrt{\frac{Fr}{m}}$$

$$v = \sqrt{\frac{8.2 \times 10^{-8} \text{ N} \times 5.3 \times 10^{-11} \text{ m}}{9.1 \times 10^{-31} \text{ kg}}} = 2.2 \times 10^6 \text{ m s}^{-1}$$

Same three steps every time: **name** the real force, set it equal to mv^2/r (or $m\omega^2 r$), then rearrange for the unknown.

The force that turns you with the Earth

A 60 kg student stands at latitude 30° : $r = 5.52 \times 10^6 \text{ m}$, $\omega = 7.27 \times 10^{-5} \text{ rad s}^{-1}$. Find the resultant force on the student.

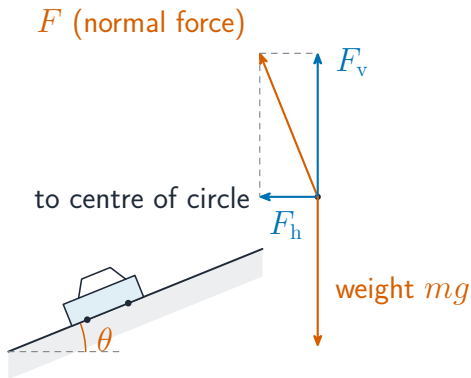
Resultant force = centripetal force:

$$\begin{aligned} F &= m r \omega^2 \\ &= 60 \text{ kg} \times 5.52 \times 10^6 \text{ m} \times (7.27 \times 10^{-5} \text{ rad s}^{-1})^2 \\ &= 1.75 \text{ N towards the axis} \end{aligned}$$

- it comes from the **weight** and the **normal contact force** not quite cancelling
- 1.75 N is only 0.3% of the weight (589 N)
- the arrow is **parallel to the equator**, towards the axis – not towards the Earth's centre

Draw the resultant towards the **centre of the circle** the object actually moves in.

The banked corner



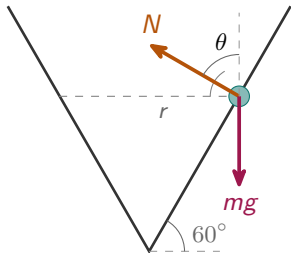
Tilt the track and the **normal contact force** 支持力 leans inwards:

- vertical part balances the weight
- horizontal part turns the car

$$\tan \theta = \frac{v^2}{rg}$$

At this angle the car needs **no friction** to turn.

A ball inside a smooth cone



Two forces: weight and **normal contact force** 支持力 N , at 90° to the surface, so $\theta = 60^\circ$.

The ball circles at $r = 0.20$ m. Find its speed.

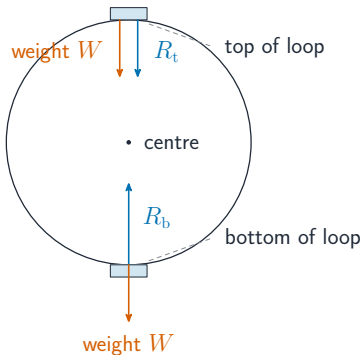
$$N \cos \theta = mg, \quad N \sin \theta = \frac{mv^2}{r}$$

$$\tan \theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg \tan \theta}$$

$$v = \sqrt{0.20 \text{ m} \times 9.81 \text{ m s}^{-2} \times \tan 60^\circ} \\ = 1.8 \text{ m s}^{-1}$$

Faster ball: θ is fixed by the cone, so r must **grow** – it rides higher.

Vertical circles



The speed is **not** constant (gravity does work), but the **net inward** force is still mv^2/r .

- bottom: $T - mg = \frac{mv^2}{r}$ – tension **largest**
- top: $T + mg = \frac{mv^2}{r}$ – tension **smallest**

At the top both the weight **and** the tension point to the centre – they add, they do not cancel.

Worked example – the top of a loop

A car loops a vertical circle of radius 2.0 m. Find the minimum speed at the top that keeps it on the track. ($g = 9.81 \text{ m s}^{-2}$)

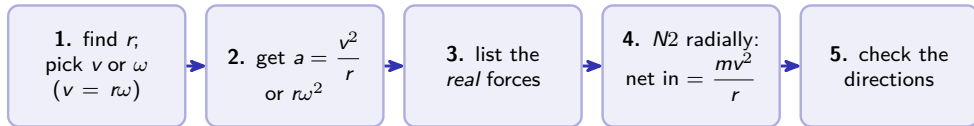
Slowest $\Rightarrow$ the track pushes with **zero** force, so gravity alone turns the car:

$$mg = \frac{mv_{\min}^2}{r} \Rightarrow v_{\min} = \sqrt{gr}$$

$$v_{\min} = \sqrt{9.81 \times 2.0} \approx 4.4 \text{ m s}^{-1}$$

“Just keeps contact” always means **set the contact force to zero**. The mass cancels – the answer is the same for a car or a coin.

How to structure the answer



For a period or frequency, reach for $\omega = 2\pi/T$ or $T = 2\pi r/v$.

3

Recap

Exam tips

- Work in **radians**; angular speed $\omega = 2\pi/T = v/r$.
- **Centripetal acceleration** $a = v^2/r = \omega^2 r$; **the net force acts towards the centre** – it is provided by tension/gravity/friction, not an extra force.
- Always state **what provides the centripetal force** in the situation given.

Topic 12 – key takeaways

1 Radians

$\theta = s/r$, work in rad, not degrees

2 Speeds

$\omega = 2\pi/T$ and $v = r\omega$

3 Acceleration

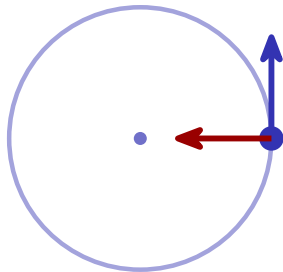
$a = v^2/r = r\omega^2$, always to the centre

4 Force

$F = mv^2/r$, provided by a *real* force

Check yourself

- 1 How many radians in a half-circle?
- 2 A wheel turns once in 0.50 s. Find ω .
- 3 Which way does the centripetal acceleration point?
- 4 What provides the centripetal force on a satellite?



Answers 1. π 2. $\omega = 2\pi/0.50 \approx 13 \text{ rad s}^{-1}$ 3. to the centre 4. gravity