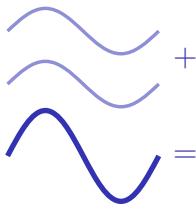


Superposition

AS Physics ■ Topic 8

A-Level Physics



What we will cover

- 1 Stationary waves 8.1
- 2 Diffraction 8.2
- 3 Interference 8.3
- 4 The diffraction grating 8.4
- 5 Recap



1

Stationary waves

Sheet 8.1 – what you need to know

You must be able to:

- state the **principle of superposition**
- explain how a **stationary wave** forms, and compare it with a progressive wave
- find λ from the positions of nodes and antinodes
- describe the microwave, string and air-column experiments

What the exam asks

“State the principle of superposition.” [2]

“Explain how a stationary wave is formed between the vibration generator and the wall.” [2]

“Determine the height of the top end of the tube above the surface of the water when a stationary wave is first produced in the tube.” [3]

Practise on exercise sheet 8.1, then try the real exam questions on past-paper sheet 8.1.

Principle of superposition

Where waves overlap, the displacement is the **vector sum** of each wave's displacement – and they then **pass through each other unchanged**.

in phase

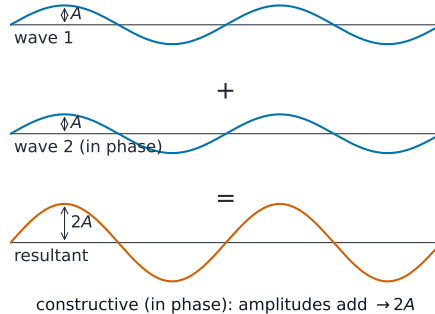
$A_1 + A_2$ – **constructive interference** 相长干涉

out of phase (π)

amplitude $|A_1 - A_2|$ (destructive)

Two equal waves in phase: $I \propto (2A)^2 = 4A^2$ – **four times** the intensity.

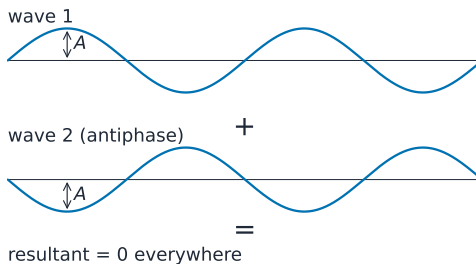
In phase: add



in phase: add

Crest 波峰 on crest: the two add, and the result is **constructive interference** 相长干涉.

Out of phase: cancel

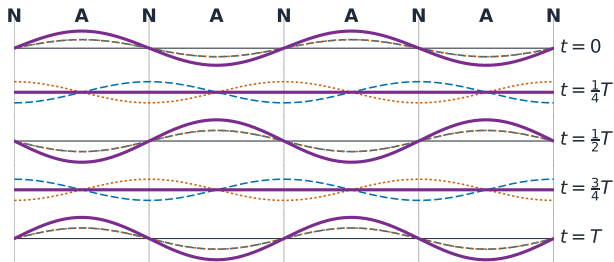


destructive (out of phase by $\lambda/2$): cancel if equal A

out of phase: cancel

Crest on **trough** 波谷 – exactly in **antiphase** 反相 – gives **destructive interference** 相消干涉: they cancel.

Stationary (standing) waves



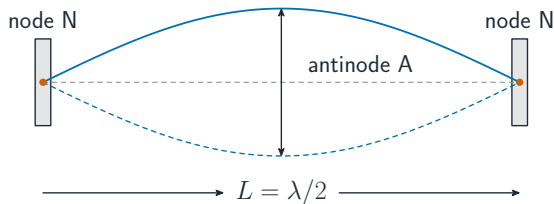
— moving right ... moving left — resultant

stationary wave: nodes and antinodes from two opposite waves

Two identical **progressive waves** 行波 travelling **opposite ways** overlap – usually a wave and its own **reflected** 反射 copy from a fixed end.

It stores energy but does **not** carry it along – the pattern stays put.

Nodes and antinodes



- **node** 波节 – always zero displacement
- **antinode** 波腹 – largest amplitude
- node to node = $\lambda/2$; node to antinode = $\lambda/4$

String 0.80 m, fixed ends, $v = 240 \text{ m s}^{-1}$

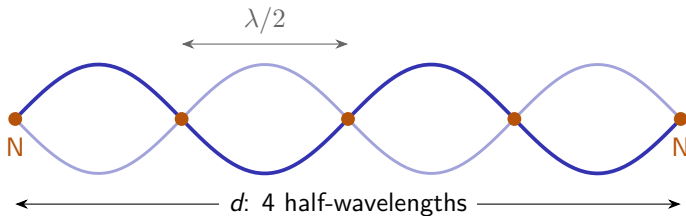
fundamental:

$$\lambda = 2L = 2(0.80 \text{ m}) = 1.60 \text{ m}$$

$$f = \frac{v}{\lambda} = \frac{240 \text{ m s}^{-1}}{1.60 \text{ m}} = 150 \text{ Hz}$$

Measuring wavelength from node spacing

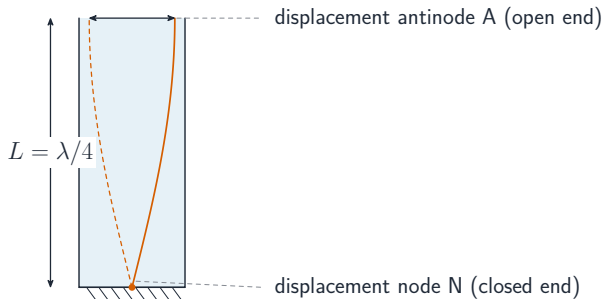
Drive a string with a vibrator at frequency f until a stationary pattern appears.



$$\lambda = \frac{2d}{n}$$

n = number of half-wavelengths between the two nodes

Resonance in pipes



Resonance tube 共鸣管, closed one end:

- closed end = node, open end = antinode
- **fundamental** 基频: $L = \lambda/4$; next $L = 3\lambda/4$
- open both ends: fundamental $L = \lambda/2$

Then $v = f\lambda$ gives the wave speed.

Two stationary-wave experiments

Microwaves: neighbouring minima 25 mm apart

node to node is half a wavelength:

$$\lambda = 2 \times 25 \text{ mm} = 0.050 \text{ m}$$

$$f = \frac{c}{\lambda} = \frac{3.0 \times 10^8 \text{ m s}^{-1}}{0.050 \text{ m}} = 6.0 \times 10^9 \text{ Hz}$$

A 512 Hz fork resonates at 16.6 cm, then 50.0 cm

between resonances the column grows by half a wavelength:

$$\frac{1}{2}\lambda = 50.0 \text{ cm} - 16.6 \text{ cm} = 33.4 \text{ cm, so}$$
$$\lambda = 0.668 \text{ m}$$

$$v = f\lambda = (512 \text{ Hz})(0.668 \text{ m}) = 342 \text{ m s}^{-1}$$

Using the **difference** between two resonances cancels the end correction: the antinode sits a little beyond the open end.

2

Diffraction

Sheet 8.2 – what you need to know

You must be able to:

- explain **diffraction** and where it happens
- describe how the gap width, compared with the wavelength, changes the spreading
- use $\lambda = \frac{v}{f}$ to decide how much a wave will diffract

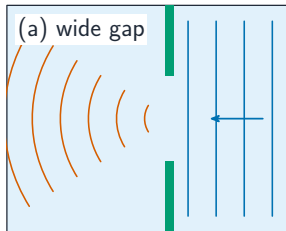
Practise on exercise sheet 8.2, then try the real exam questions on past-paper sheet 8.2.

What the exam asks

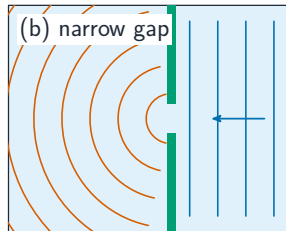
“State what is meant by the diffraction of a wave.” **[2]**

“State what is meant by diffraction.” **[1]**

Diffraction



gap $\gg \lambda$: little spreading



gap $\approx \lambda$: strong spreading

Diffraction 衍射: a wave spreads after a gap or obstacle. Spread depends on λ vs gap width:

- gap $\gg \lambda$: little spread
- gap $\approx \lambda$: large spread

In a **ripple tank** 水波槽 the spread grows as the gap narrows. Same idea: you hear round a corner but cannot see round it.

How much does a wave spread?

Sound through a 0.90 m doorway

at 440 Hz: $\lambda = \frac{v}{f} = \frac{330 \text{ m s}^{-1}}{440 \text{ Hz}} = 0.75 \text{ m}$

about the width of the gap: it spreads a lot, and you hear round the corner

Light through the same doorway

$\lambda \approx 5 \times 10^{-7} \text{ m}$, a million times smaller than the gap:

hardly any spreading, so you cannot see round the corner

Diffraction is greatest when the gap is about the same size as the wavelength. It changes the direction, never the wavelength or the speed.

3

Interference

Sheet 8.3 – what you need to know

You must be able to:

- define **interference** and **coherence**
- decide from the path difference whether waves meet in phase or in antiphase
- describe two-source interference, and the conditions for clear fringes
- use $\lambda = \frac{ax}{D}$, and predict how the fringes change

What the exam asks

“State what is meant by coherent.” [1]

“Explain how the pattern of bright and dark fringes is formed.” [3]

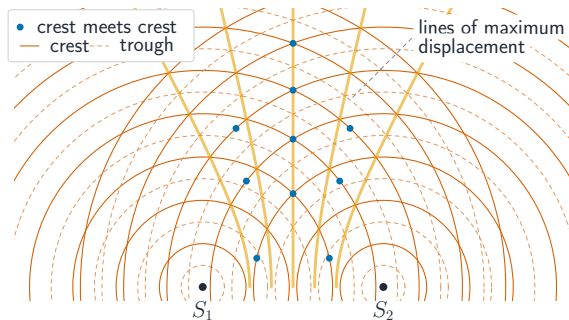
“Calculate the separation a of the slits.” [3]

Practise on exercise sheet 8.3, then try the real exam questions on past-paper sheet 8.3.

Interference and coherence

Interference 干涉: two **coherent** 相干 waves give a **steady** pattern. Coherent = constant phase difference (so the same frequency). Two lamps are not.

Make coherent light by passing **one** source through a **double-slit** 双缝 pair of **slits** 狭缝: both are lit by the same wavefront.



Four conditions for a clear fringe pattern

- 1 two **coherent** 相干 sources – a constant phase difference
- 2 roughly **equal amplitudes**, or the dark fringes are not dark
- 3 the waves **overlap** where you are looking
- 4 for light, the same plane of **polarisation** 偏振

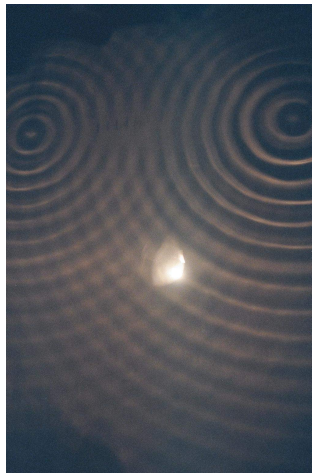
Miss any one and the fringes wash out – which is why two lamps show nothing.

Path difference

What happens at a point depends on the **path difference** 路程差 Δx :

$$\text{bright: } \Delta x = n\lambda$$

$$\text{dark: } \Delta x = (n + \frac{1}{2})\lambda$$



Path difference: a worked example

Two dippers in phase make waves of wavelength 3.0 cm. P is 12.0 cm from one and 16.5 cm from the other.

Whole wavelengths: in phase, loud.
Odd half wavelengths: antiphase, quiet.

What happens at P?

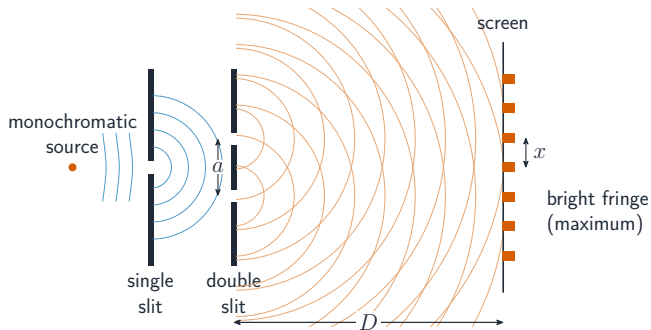
path difference:

$$\Delta x = 16.5 \text{ cm} - 12.0 \text{ cm} = 4.5 \text{ cm}$$

in wavelengths: $\frac{\Delta x}{\lambda} = \frac{4.5 \text{ cm}}{3.0 \text{ cm}} = 1.5$

an odd number of half wavelengths: the waves arrive in **antiphase**, so **destructive** interference: P hardly moves

Young's double-slit experiment



Fringe spacing 条纹间距:

$$x = \frac{\lambda D}{a}$$

Smaller fringes: bigger a ,
smaller D , or bluer light.

$$a = 0.50 \text{ mm}, \lambda = 600 \text{ nm}, D = 2.0 \text{ m}$$

$$x = \frac{\lambda D}{a} = \frac{(600 \times 10^{-9} \text{ m})(2.0 \text{ m})}{0.50 \times 10^{-3} \text{ m}} = 2.4 \text{ mm}$$

Two loudspeakers

Two loudspeakers 1.5 m apart, driven by one signal generator at 680 Hz; a student walks along a line 5.0 m away.

One generator keeps the sources **coherent**: a constant phase difference.

Spacing of the loud regions

$$\lambda = \frac{v}{f} = \frac{340 \text{ m s}^{-1}}{680 \text{ Hz}} = 0.50 \text{ m}$$

the double-slit geometry:

$$x = \frac{\lambda D}{a} = \frac{(0.50 \text{ m})(5.0 \text{ m})}{1.5 \text{ m}} = 1.7 \text{ m}$$

4

The diffraction grating

Sheet 8.4 – what you need to know

You must be able to:

- use $d \sin \theta = n\lambda$ both ways: find an angle, or find the grating
- find the slit spacing from the lines per mm, and the **highest order**
- describe how a grating measures a wavelength

What the exam asks

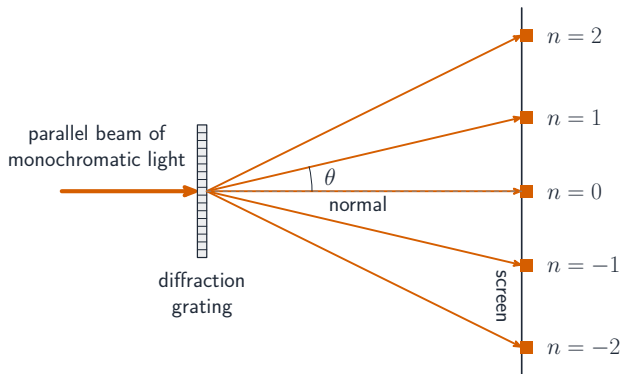
“Describe the diffraction of the light waves at the grating.” [2]

“Calculate the number of lines per unit length in the diffraction grating.” [3]

“Determine the value of θ at which the second-order bright fringe is formed.” [1]

Practise on exercise sheet 8.4, then try the real exam questions on past-paper sheet 8.4.

The diffraction grating



Many slits, spacing d – maxima where

$$d \sin \theta = n\lambda$$

500 lines/mm, $\lambda =$
600 nm, $n = 1$

$$d = \frac{1}{N} = \frac{1}{500 \text{ mm}^{-1}}$$

$$= 2.0 \times 10^{-6} \text{ m}$$

$$\sin \theta = \frac{n\lambda}{d} = \frac{600 \times 10^{-9} \text{ m}}{2.0 \times 10^{-6} \text{ m}}$$

$$= 0.30, \text{ so } \theta \approx 17^\circ$$

Working with a grating

slit spacing

$$N \text{ lines/mm} \Rightarrow d = 10^{-3}/N \text{ m}$$

highest order

$$n_{\text{max}} = \lfloor d/\lambda \rfloor: \text{ at } d/\lambda = 3.27 \\ \text{you see up to } n = 3, \text{ never } n = 4$$

sharper maxima

more slits add $\Rightarrow$ narrower,
brighter lines than two slits

To **find** an unknown λ : shine it straight at the grating, measure θ_1 to the first-order **maximum** 极大, then $\lambda = d \sin \theta_1$. Averaging over higher orders cuts the error.

Working backwards: find the grating

Exam questions often give an angle and ask for the grating. Rearrange:

$$d = \frac{n\lambda}{\sin \theta}, \quad N = \frac{1}{d}$$

Highest order: $\sin \theta \leq 1$, so $n_{\max}$ is $\frac{d}{\lambda}$ rounded down.

Second order at 35° for 730 nm

$$d = \frac{n\lambda}{\sin \theta} = \frac{2(730 \times 10^{-9} \text{ m})}{\sin 35^\circ} = 2.55 \times 10^{-6} \text{ m}$$

$$N = \frac{1}{d} = \frac{1}{2.55 \times 10^{-3} \text{ mm}} \approx$$

390 lines per mm

5

Recap

Exam tips

- Two-source interference: **constructive** when path difference $= n\lambda$, **destructive** when $= (n + \frac{1}{2})\lambda$; the sources must be **coherent**.
- Double slit: $\lambda = ax/D$; diffraction grating: $d\sin\theta = n\lambda$ – know every symbol.
- On a **stationary wave** mark nodes and antinodes; adjacent nodes are $\lambda/2$ apart; it stores energy but does not transfer it.
- A stationary wave needs two waves of the **same frequency travelling in opposite directions**.

Topic 8 – key takeaways

1

Superposition

displacements add; in phase $4\times$
the intensity

2

Stationary waves

nodes & antinodes a $\lambda/2$ apart

3

Interference

coherent sources; $x = \lambda D/a$

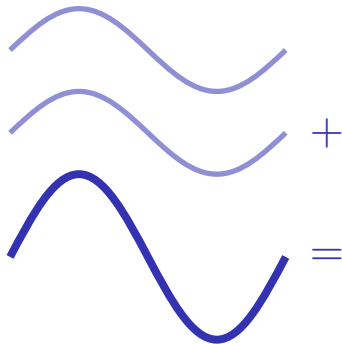
4

Grating

$d \sin \theta = n\lambda$, sharp maxima

Check yourself

- 1 Distance between two next-door nodes?
- 2 What does “coherent” mean?
- 3 Path difference for a bright fringe?
- 4 Grating $d \sin \theta = n\lambda$ – which n is straight on?



Answers 1. $\lambda/2$ 2. constant phase difference 3. $\Delta x = n\lambda$ 4. $n = 0$