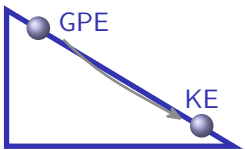
Work, Energy & Power

AS Physics ▪ Topic 5

A-Level Physics



What we will cover

1. Work and energy 5.1
2. Efficiency and power 5.1.1
3. Potential and kinetic energy 5.2
4. Energy methods 5.2.1
5. Recap



1 Work and energy

Sheet 5.1 – what you need to know

You must be able to:

- define work done and use $W = Fs\cos\theta$
- say when a force does positive, negative or zero work
- state the principle of conservation of energy
- account for every joule, friction included, to find a speed or a height

What the exam asks

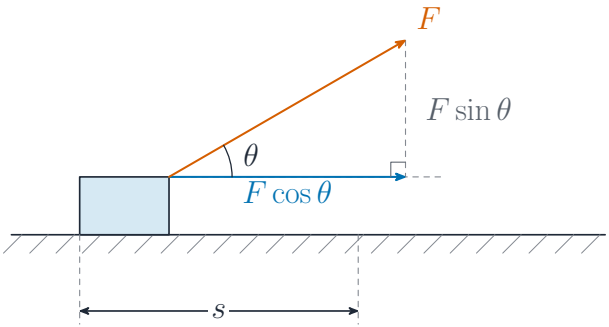
"State what is meant by the work done by a force." **[1]**

"Calculate the work done by F for the movement of the trolley from X to Y." **[1]**

"Use your answer and conservation of energy to determine the speed of the block at point A." **[3]**

Practise on exercise sheet 5.1, then try the real exam questions on past-paper sheet 5.1.

Work done by a force



Work 功 = force × distance along the force:

$$W = Fs\cos\theta$$

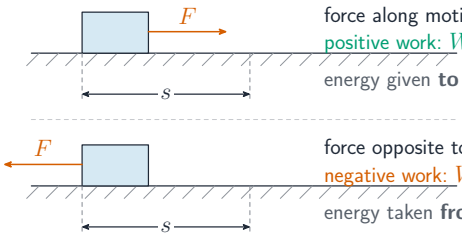
unit: J = N m

Work is a **scalar** 标量.

Sledge: 20 N **at** 60°, 5.0 m

$$\begin{aligned} W &= Fs\cos\theta \\ &= (20\,\mathrm{N})(5.0\,\mathrm{m})\cos 60^\circ \\ &= 50\,\mathrm{J} \end{aligned}$$

Positive, negative and zero work

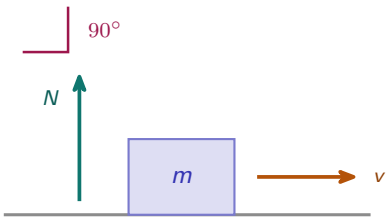


force along motion
positive work: $W = +Fs$
energy given **to** the object

force opposite to motion
negative work: $W = -Fs$
energy taken **from** the object

- $\theta = 0$: $W = +Fs$ – energy in
- $\theta = 90°$: $W = 0$ – no work (normal force on a flat road)
- $\theta = 180°$: $W = -Fs$ – energy out (**friction** 摩擦力)

A force at $90°$ does no work

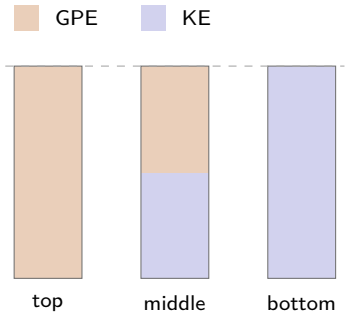


N is vertical. The motion is horizontal.

$$W = Fs\cos\theta$$

At $90°$, $\cos\theta = 0$, so nothing is transferred. The **normal contact force** 法向接触力 on a flat road does no work.

Conservation of energy



The total stays the same.

Energy is never made or destroyed. It only changes form.

> In a **closed system** 封闭系统 the total is constant.

On a frictionless **ramp** 斜坡:

$$mgh = \tfrac{1}{2}mv^2, \quad v = \sqrt{2gh}.$$

With friction, some becomes **thermal energy** 热能.

Pushing against friction

A box is pushed $3.0\,\mathrm{m}$ at a constant speed against a friction force of $40\,\mathrm{N}$.

> Constant speed: no resultant force, so the push equals the friction.

Where the work goes

work done by the push:
$W = Fs = (40\,\mathrm{N})(3.0\,\mathrm{m}) = 120\,\mathrm{J}$

the kinetic energy does not change, so all $120\,\mathrm{J}$ becomes thermal energy of the box and the floor

friction does $-120\,\mathrm{J}$ of work on the box

Down a slope with friction

A $2.0\,\mathrm{kg}$ block slides $4.0\,\mathrm{m}$ down a slope from rest, falling $1.5\,\mathrm{m}$ vertically, against a friction force of $3.0\,\mathrm{N}$.

In and out

released:

$$\Delta E_\mathrm{P} = mg\Delta h = (2.0\,\mathrm{kg})(9.81\,\mathrm{m\,s^{-2}})(1.5\,\mathrm{m}) = 29.4\,\mathrm{J}$$

against friction:

$$W = Fs = (3.0\,\mathrm{N})(4.0\,\mathrm{m}) = 12.0\,\mathrm{J}$$

What is left, and the speed

$$E_\mathrm{K} = \Delta E_\mathrm{P} - W = 29.4\,\mathrm{J} - 12.0\,\mathrm{J} = 17.4\,\mathrm{J}$$

$$v = \sqrt{\frac{2E_\mathrm{K}}{m}} = \sqrt{\frac{2 \times 17.4\,\mathrm{J}}{2.0\,\mathrm{kg}}} = 4.2\,\mathrm{m\,s^{-1}}$$

Friction uses the distance along the slope; potential energy uses the vertical height.

A lift at constant speed

A $600\,\mathrm{kg}$ lift rises $15\,\mathrm{m}$ at a constant speed. Its motor transfers $1.20 \times 10^5\,\mathrm{J}$.

> Constant speed: the kinetic energy does not change, so the useful gain is potential energy only.

Account for every joule

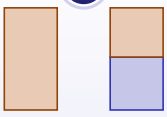
useful gain:
$\Delta E_\mathrm{P} = mg\Delta h = (600\,\mathrm{kg})(9.81\,\mathrm{m\,s^{-2}})(15\,\mathrm{m}) = 8.83 \times 10^4\,\mathrm{J}$

the rest, by conservation of energy:
$E_\mathrm{wasted} = E_\mathrm{in} - \Delta E_\mathrm{P}$
$= 1.20 \times 10^5\,\mathrm{J} - 8.83 \times 10^4\,\mathrm{J} =$
$3.2 \times 10^4\,\mathrm{J}$

as thermal energy in the motor and cables, and sound

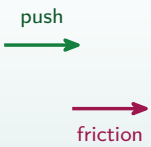
Using energy methods

1



Write the energy at the start and at the end.

2



A push puts energy in. Friction takes energy out.

3

$$E_{\text{start}} + W_{\text{in}} = E_{\text{end}} + W_{\text{lost}}$$

2 Efficiency and power

Sheet 5.1.1 – what you need to know

You must be able to:

- calculate an efficiency from energies or from powers
- define power and use $P = \dfrac{W}{t}$
- derive $P = Fv$ from the definitions of power and work
- use $P = Fv$ on the level, up a slope, and when drag grows with speed

What the exam asks

"Use the definition of power to show that the power P transferred by the force is given by $P = Fv$." **[2]**

"Calculate the percentage efficiency of the solar panel." **[1]**

"Show that the resistive force opposing the forward motion of the lorry is $5200\,\mathrm{N}$." **[1]**

Practise on exercise sheet 5.1.1, then try the real exam questions on past-paper sheet 5.1.1.

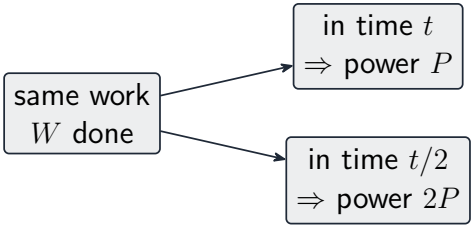
Power – the rate of energy transfer

$$P = \frac{W}{t} = \frac{\Delta E}{\Delta t}, \qquad [P] = \mathrm{W} = \mathrm{J\,s^{-1}}.$$

Power is a **scalar** 标量.

same work
in less time → **more power**

Same work, different power

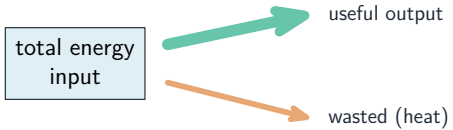


$$P = \frac{W}{t}$$ — less time, more power

Power is work per second: climbing the stairs twice as fast does the *same* work against gravity, at twice the power.

Efficiency

$$\text{efficiency} = \frac{\text{useful output}}{\text{total input}}$$



$$\text{efficiency} = \frac{\text{useful output}}{\text{total input}} \times 100\%$$

A pump lifts $600\,\text{kg}$ of water $12\,\text{m}$ in $30\,\text{s}$, drawing $3.0\,\text{kW}$

useful power, the gain in potential energy per second:

$$P = \frac{mg\Delta h}{t} = \frac{(600\,\text{kg})(9.81\,\text{m s}^{-2})(12\,\text{m})}{30\,\text{s}} = 2350\,\text{W}$$

$$\text{efficiency} = \frac{2350\,\text{W}}{3000\,\text{W}} = 0.78, \text{ or } 78\%$$

The rest is wasted, usually as thermal energy. Real machines stay under 100%.

Power at a steady speed is force times velocity

$$P = F v$$

In a time Δt the force moves through $v\,\Delta t$, doing $F v \Delta t$ of work. Divide by Δt.

Car at $25\,\mathrm{m\,s^{-1}}$ against $600\,\mathrm{N}$

At constant speed the driving force equals the resistive force, so $P = Fv = (600\,\mathrm{N})(25\,\mathrm{m\,s^{-1}}) = 15\,\mathrm{kW}$.

flat road
$P = F_{\text{resist}}\, v$

drag 阻力 grows like v^2
double speed: force $\times 4$
power $P = Fv \propto v^3$, so $\times 8$

steady lift
$P = mg\, v$

Up a slope at a steady speed

A $1200\,\mathrm{kg}$ car climbs a 5.0° slope at a steady $20\,\mathrm{m\,s^{-1}}$ against a resistive force of $600\,\mathrm{N}$.

Along the slope the driving force balances the resistive force and the component of the weight.

Driving force, then power

$mg\sin\theta = (1200\,\mathrm{kg})(9.81\,\mathrm{m\,s^{-2}})\sin 5.0^\circ = 1030\,\mathrm{N}$

$F = F_\mathrm{r} + mg\sin\theta = 600\,\mathrm{N} + 1030\,\mathrm{N} = 1630\,\mathrm{N}$

$P = Fv = (1630\,\mathrm{N})(20\,\mathrm{m\,s^{-1}}) = 3.3 \times 10^4\,\mathrm{W}$

Drag that grows with speed

The resistive force on a car is proportional to the square of its speed: $800\,\mathrm{N}$ at $20\,\mathrm{m\,s^{-1}}$.

> Double the speed: four times the force, eight times the power.

A steady $40\,\mathrm{m\,s^{-1}}$

$F \propto v^2$: $F_2 = F_1 \left(\frac{v_2}{v_1}\right)^2 =$

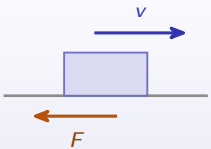
$800\,\mathrm{N} \times \left(\frac{40\,\mathrm{m\,s^{-1}}}{20\,\mathrm{m\,s^{-1}}}\right)^2 = 3200\,\mathrm{N}$

$P_2 = F_2 v_2 = (3200\,\mathrm{N})(40\,\mathrm{m\,s^{-1}}) = 1.28 \times 10^5\,\mathrm{W}$

at $20\,\mathrm{m\,s^{-1}}$:
$P_1 = F_1 v_1 = (800\,\mathrm{N})(20\,\mathrm{m\,s^{-1}}) = 1.6 \times 10^4\,\mathrm{W}$

Reading $P = Fv$ three ways

Force



$F = P/v$

a steady speed gives the opposing force

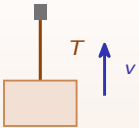
Lifting speed



$\eta VI = mgv$

voltage 电压 V, **current** 电流 I

Tension



$T = P_{\text{useful}}/v$

tension 张力 is useful power over speed

3 Potential and kinetic energy

Sheet 5.2 – what you need to know

You must be able to:

- derive $\Delta E_{\mathrm{P}} = mg\Delta h$ from $W = Fs$
- derive $E_{\mathrm{K}} = \frac{1}{2}mv^2$ from the equations of motion
- solve problems where potential and kinetic energy exchange
- follow the energy through a chain of transfers

What the exam asks

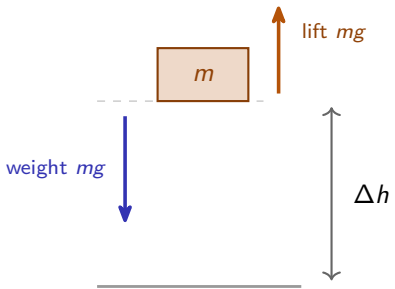
"Use the concept of work done to show that $\Delta E_{\mathrm{P}} = mg\Delta h$." **[2]**

"Calculate the kinetic energy of the block as it leaves the spring." **[2]**

"Determine the gain in kinetic energy of the truck between A and B." **[3]**

Practise on exercise sheet 5.2, then try the real exam questions on past-paper sheet 5.2.

Where $mg\Delta h$ comes from



Raising a **mass** 质量 m stores **gravitational potential energy** 重力势能.

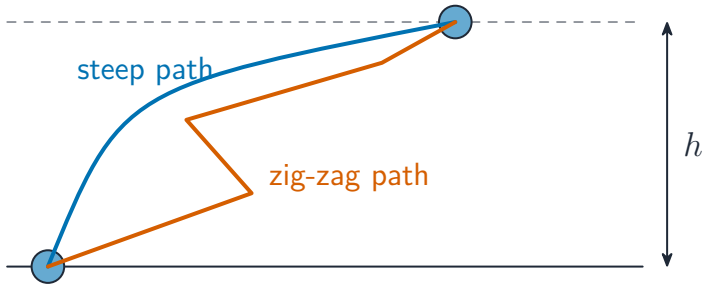
$$\Delta E_{\mathrm{P}} = mg\,\Delta h$$

Lift it slowly, so no kinetic energy is gained. The upward force is mg, the work it does is $mg \cdot \Delta h$, and that work *is* the stored energy.

> You need m, Δh and g – never speed or time.

Only the height matters, not the path

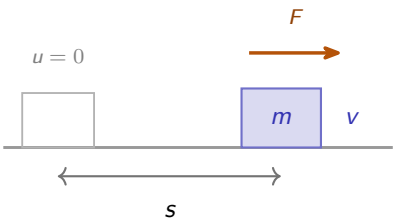
same $\Delta E_P = mgh$ – only the height h matters



Steep or zig-zag, the gain is *mgh* either way: gravitational PE counts only the height risen.

exercises.lol exercises.com.cn

Kinetic energy: where $\frac{1}{2}mv^2$ comes from



A push from rest, through a distance s, up to speed v.

$v^2 = u^2 + 2as$ with $u = 0$
so $s = v^2/(2a)$

↓

$W = Fs = ma \cdot s$
the a cancels

↓

$E_K = \frac{1}{2}mv^2$

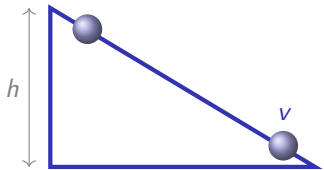
Worked example – speed down a ramp

From rest, smooth ramp, $h = 1.2\,\mathrm{m}$

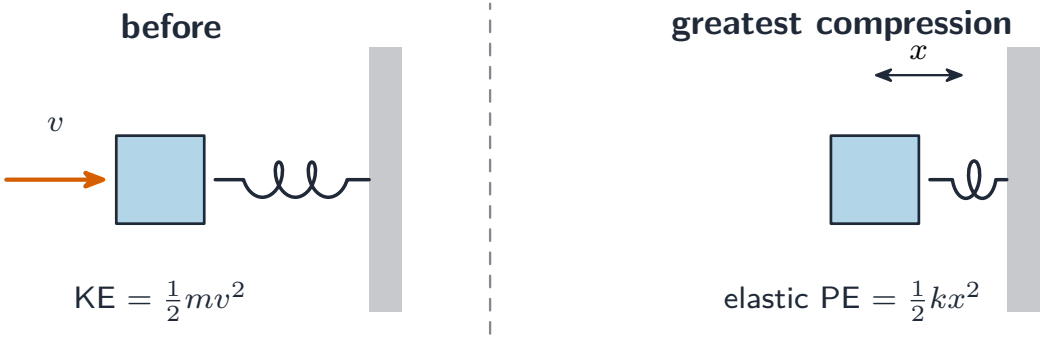
All the GPE becomes KE. Mass cancels.

$$mgh = \tfrac{1}{2}mv^2$$

$$\begin{aligned} v &= \sqrt{2gh} \\ &= \sqrt{2(9.81\,\mathrm{m\,s^{-2}})(1.2\,\mathrm{m})} \approx 4.9\,\mathrm{m\,s^{-1}} \end{aligned}$$



Using energy methods, in detail



A block's kinetic energy becomes elastic potential energy as it compresses the spring

exercises.lol exercises.com.cn

4 Energy methods

Sheet 5.2.1 – what you need to know

You must be able to:

- use the work done by the resultant force as the change in E_K
- compare kinetic energies, including with the same momentum
- find a speed from energy whatever the direction of motion
- find the work done against air resistance, and the average force

What the exam asks

"Calculate the maximum kinetic energy of the car." **[2]**

"Determine the ratio of the kinetic energy of the child at Y with the resistive force to that without it." **[1]**

"Explain why the work done by the upthrust is not equal to the gain in elastic potential energy of the spring." **[1]**

Practise on exercise sheet 5.2.1, then try the real exam questions on past-paper sheet 5.2.1.

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When you know the momentum, not the speed

$$p = mv \quad \text{and} \quad E_\mathrm{K} = \tfrac{1}{2}mv^2$$

$$\Downarrow$$

$$E_\mathrm{K} = \frac{p^2}{2m}$$

A momentum change from p_1 to p_2 at constant mass changes the kinetic energy by $\dfrac{p_2^2 - p_1^2}{2m}$.

> Reach for this whenever a question gives you **momentum** 动量 and asks for energy – there is no need to find v first.

Same momentum, different kinetic energy

$$E_{\mathrm{K}} = \tfrac{1}{2}mv^2 = \frac{p^2}{2m}$$

The same momentum with four times the mass: one quarter of the kinetic energy.

A $1000\,\mathrm{kg}$ car at $20\,\mathrm{m\,s^{-1}}$; a $4000\,\mathrm{kg}$ truck, same p

car: $p = mv = (1000\,\mathrm{kg})(20\,\mathrm{m\,s^{-1}}) = 2.0 \times 10^4\,\mathrm{kg\,m\,s^{-1}}$

$E_{\mathrm{K}} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1000\,\mathrm{kg})(20\,\mathrm{m\,s^{-1}})^2 = 2.0 \times 10^5\,\mathrm{J}$

truck: $v = \dfrac{p}{m} = \dfrac{2.0 \times 10^4\,\mathrm{kg\,m\,s^{-1}}}{4000\,\mathrm{kg}} = 5.0\,\mathrm{m\,s^{-1}}$

$E_{\mathrm{K}} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(4000\,\mathrm{kg})(5.0\,\mathrm{m\,s^{-1}})^2 = 5.0 \times 10^4\,\mathrm{J}$

A speed from energy, whatever the direction

A stone is thrown horizontally at $8.0\,\mathrm{m\,s^{-1}}$ from a cliff $20\,\mathrm{m}$ high; no air resistance.

> Energy gives the speed, not the direction or the time: use the equations of motion for those.

Conservation of energy; the mass cancels

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}mu^2 + mg\Delta h$$

$$v = \sqrt{u^2 + 2g\Delta h} = \sqrt{(8.0\,\mathrm{m\,s^{-1}})^2 + 2(9.81\,\mathrm{m\,s^{-2}})(20\,\mathrm{m})}$$

$$= 21.4\,\mathrm{m\,s^{-1}}$$

Air resistance takes its share

A $0.20\,\mathrm{kg}$ ball is dropped from $5.0\,\mathrm{m}$ and lands with $8.0\,\mathrm{J}$ of kinetic energy.

With air resistance the total energy is not constant: the lower the ball, the less it has left.

The missing energy is the work done against air resistance

$E_\mathrm{P} = mgh = (0.20\,\mathrm{kg})(9.81\,\mathrm{m\,s^{-2}})(5.0\,\mathrm{m}) = 9.81\,\mathrm{J}$

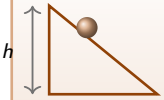
$W = E_\mathrm{P} - E_\mathrm{K} = 9.81\,\mathrm{J} - 8.0\,\mathrm{J} = 1.8\,\mathrm{J}$

$F = \dfrac{W}{s} = \dfrac{1.81\,\mathrm{J}}{5.0\,\mathrm{m}} = 0.36\,\mathrm{N}$, on average

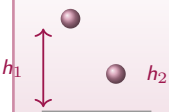
Four places to use it



Spring 弹簧. At full **compression** 压缩, $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$.



Constant speed. The push pays for the height and the friction.



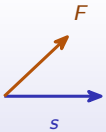
Bounce. The fraction of the height kept is h_2/h_1.



Same height, any angle: it lands at the same speed.

5 Recap

Exam tips



$W = Fs\cos\theta$

Use the distance along the force.



Energy stays

Loss in GPE = gain in KE, plus work against resistance.

$P = W/t = Fv$

P

Efficiency is useful output divided by total input.



Vertical height

GPE uses vertical height, not slope length.

Topic 5 – key takeaways

1 Work

$W = Fs\cos\theta$; energy in or out

2 Energy

conserved; only changes form

3 Power

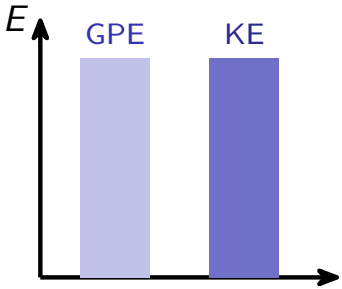
$P = W/t = Fv$; efficiency $< 100\%$

4 Stores

$E_P = mg\Delta h$, $E_K = \frac{1}{2}mv^2$

Check yourself

1. Work done at $90°$ to the motion?
2. Speed after a smooth drop of $5.0\,\mathrm{m}$?
3. Power at $20\,\mathrm{m\,s^{-1}}$ against $400\,\mathrm{N}$?
4. Why is efficiency never 100%?



Answers **1.** zero **2.** $v = \sqrt{2gh} \approx 9.9\,\mathrm{m\,s^{-1}}$ **3.** $P = Fv = 8.0\,\mathrm{kW}$ **4.** some energy always wasted as heat