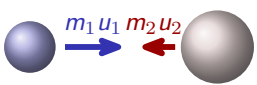
Dynamics

AS Physics ▪ Topic 3

A-Level Physics

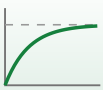


What we will cover

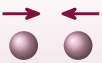
1. **Momentum and force**
   sheet 3.1: $F = \Delta p/\Delta t$, graphs

2. **Newton's laws in use**
   sheet 3.1.1: $F = ma$, lifts, pairs

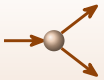
3. **Terminal velocity**
   sheet 3.2: drag, falling



4. **Conserving momentum**
   sheet 3.3: collisions, decays



5. **Elastic and 2-D**
   sheet 3.3.1: relative speed, 2-D



6. **Putting it together**
   joined topics, definitions, tips

1 Momentum and force

Sheet 3.1 – what you need to know

You must be able to:

- define **mass**, **momentum** and **force**, and state Newton's three laws
- find a change in momentum with signs, and the average force $F = \Delta p / \Delta t$
- read a p–t graph (gradient = force) and an F–t graph (area = Δp)
- find the force from a stream of water or air

What the exam asks

"Define force." **[1]**

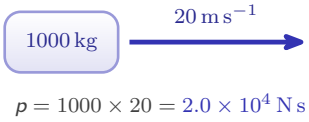
"Calculate the change in momentum of the ball during the collision." **[2]**

"Sketch the variation with time of the force F_X exerted on ball X by ball Y." **[3]**

Practise on exercise sheet 3.1, then try the real exam questions on past-paper sheet 3.1.

Mass and momentum

- **mass** 质量: how hard it is to **change an object's motion**; a scalar, in kg
- **linear momentum** 动量 $p = mv$: a **vector**, pointing along the velocity
- unit of p: $\mathrm{kg\,m\,s^{-1}}$, which is the same as $\mathrm{N\,s}$



A light car moving fast and a heavy lorry moving slowly can carry the **same momentum**.

Force is the rate of change of momentum

Newton's second law in its general form: the **resultant force** 合力 equals the rate of change of momentum.

$$F = \frac{\Delta p}{\Delta t}$$

For a constant mass $\Delta p = m\,\Delta v$, so $F = m\dfrac{\Delta v}{\Delta t} = ma$.

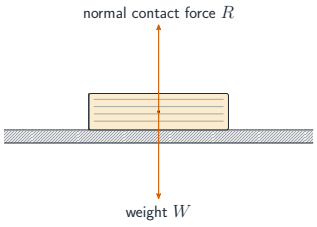
In a **collision** 碰撞: average force = change in momentum ÷ **contact time**.

A crash changes the momentum in a very short time, so the force is very large.

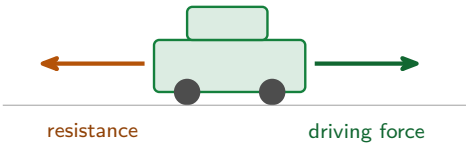
Newton's first law

An object stays **at rest**, or keeps moving at **constant velocity** in a straight line, unless a resultant **external force** 外力 acts on it.

> Zero resultant force ⇔ zero acceleration: the object's **inertia** keeps its motion unchanged.

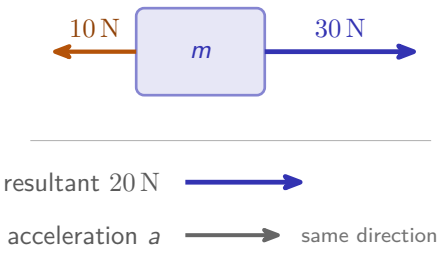


at rest: R balances W



constant velocity: the two forces balance

Newton's second law



The resultant force equals the **rate of change of momentum**, and acts in the direction of that change:

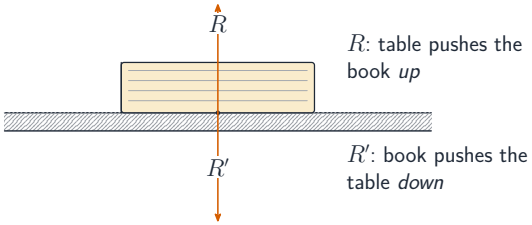
$$F = \frac{\Delta p}{\Delta t} = ma \quad \text{(constant mass)}$$

> Acceleration and resultant force always point the **same way**. Add the forces as vectors first.

Newton's third law

When body A pushes on body B, body B pushes back on A with an **equal and opposite** force. The two forces:

- act on **different objects**
- are the **same type**: both contact, both gravitational, both **electrostatic** 静电
- **same size**, **opposite** directions



A book's **weight** 重力 and the table's **normal contact force** 支持力 are **not** a pair: same object, different types. The weight's partner is the book pulling the **Earth**.

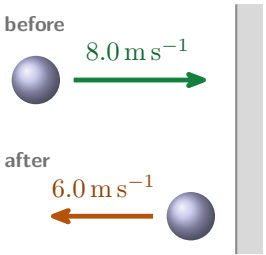
Worked example – a ball bounces off a wall

Average force on a bouncing ball

$m = 0.20\,\text{kg}$, in at $8.0\,\text{m s}^{-1}$, back out at $6.0\,\text{m s}^{-1}$, contact $0.050\,\text{s}$. Rebound direction positive: $u = -8.0$, $v = +6.0$.

$$\begin{aligned}\Delta p &= m(v-u)\\ &= (0.20\,\text{kg})\big(6.0-(-8.0)\big)\,\text{m s}^{-1} = 2.8\,\text{kg m s}^{-1}\end{aligned}$$

$$\begin{aligned}F &= \frac{\Delta p}{\Delta t}\\ &= \frac{2.8\,\text{kg m s}^{-1}}{0.050\,\text{s}} = 56\,\text{N}\end{aligned}$$

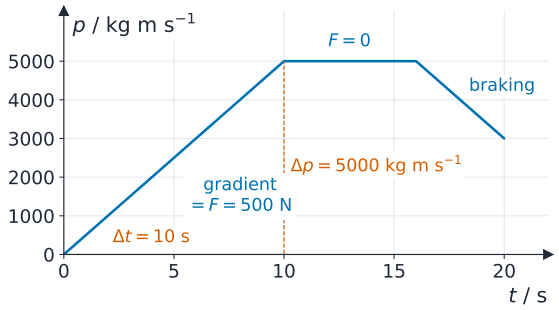


Forget the minus sign: $0.20 \times (6.0 - 8.0)$ gives the wrong size *and* direction.

Momentum–time graphs

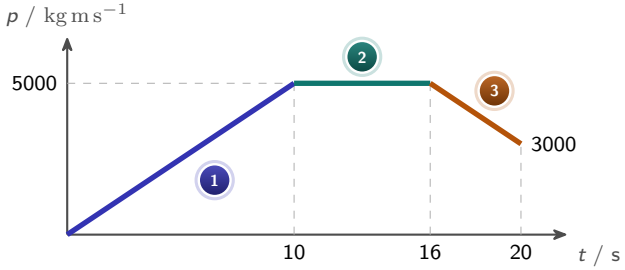
The **gradient** of a momentum–time graph is the **resultant force**, just as the gradient of a velocity–time graph is the acceleration.

- straight line:
  a **constant** resultant force
- horizontal line:
  zero resultant force
- sloping down:
  a force **against** the motion



Speed changing but the resultant force **constant**? Then that force is not air resistance: drag changes as the speed changes.

Worked example – a motorcycle's momentum



1 0 **to** 10 s

$F = \frac{\Delta p}{\Delta t} = \frac{5000\,\mathrm{N\,s}}{10\,\mathrm{s}}$

$+500\,\mathrm{N}$

2 10 **to** 16 s

momentum constant

0: **the forces balance**

3 16 **to** 20 s

$F = \frac{\Delta p}{\Delta t} = \frac{(3000-5000)\,\mathrm{N\,s}}{4\,\mathrm{s}}$

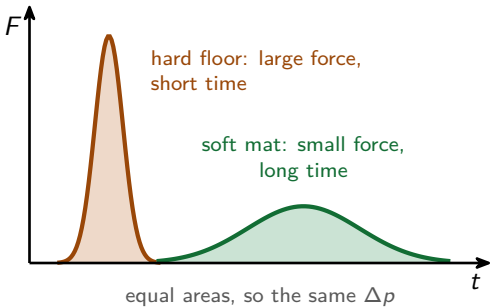
$-500\,\mathrm{N}$, **backwards**

Impulse – the area under a force–time graph

Impulse 冲量 $= F\Delta t = \Delta p$, the **area** under a force–time graph.

The same change in momentum comes from a large force for a short time, or a **small force for a long time**.

> A crumple zone or a soft landing makes Δt longer, so the force is smaller.



Rockets – pushing out mass

A rocket pushes out gas at speed u with a **mass-flow rate** 质量流率 $\dot{m}$ (kilograms per second). The thrust is

$$F = \dot{m}\,u$$

It comes from $F = \Delta p/\Delta t$: the momentum given to the gas each second equals the thrust on the rocket, by Newton's **third law**.

$90\,\mathrm{kg}$ of gas per second at $190\,\mathrm{m\,s^{-1}}$

$$F = \dot{m}\,u = (90\,\mathrm{kg\,s^{-1}})(190\,\mathrm{m\,s^{-1}})$$
$$= 1.7 \times 10^4\,\mathrm{N}$$

Worked example – thrust at lift-off

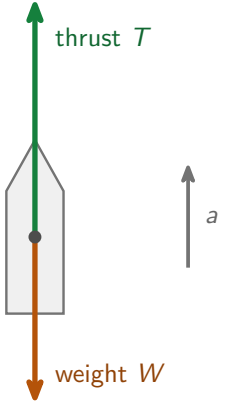
Thrust needed at lift-off

A rocket of weight W lifts off with acceleration a. Upwards positive: $T - W = ma$, and $m = W/g$, so $T = W + Wa/g$:

$$T = W\left(1 + \frac{a}{g}\right)$$

For $W = 2.0 \times 10^6\,\mathrm{N}$ and $a = 4.0\,\mathrm{m\,s^{-2}}$:

$$T = (2.0 \times 10^6\,\mathrm{N})\left(1 + \frac{4.0\,\mathrm{m\,s^{-2}}}{9.81\,\mathrm{m\,s^{-2}}}\right)$$
$$= 2.8 \times 10^6\,\mathrm{N}$$



The thrust must be **larger than the weight** before the rocket moves at all.

2 Newton's laws in use

Sheet 3.1.1 – what you need to know

You must be able to:

- use $W = mg$, and say how mass and weight differ
- draw the forces on **one** object, then use $F = ma$
- find the contact force in a lift, and the force between joined objects
- name the other force of a third-law pair

What the exam asks

"Which arrow represents the other force of this pair?" **[1]**

"Calculate the acceleration of the car." **[1]**

"On Fig. 3.3, sketch the variation of force X with time." **[3]**

Practise on exercise sheet 3.1.1, then try the real exam questions on past-paper sheet 3.1.1.

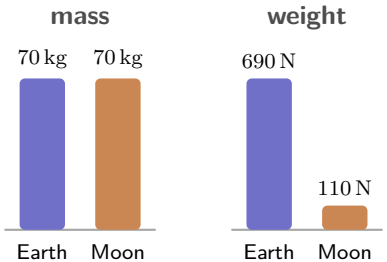
Weight – a gravitational force

Weight 重力 is the force on an object from a **gravitational field** 重力场:

$W = mg, \qquad g \approx 9.81\,\mathrm{m\,s^{-2}}$ near the Earth

It points towards the **centre of the Earth**. Mass is the same everywhere; weight **changes with place**.

> Multiple choice: weight is "the force on an object due to a gravitational field", not "mass times acceleration" and not "the resultant force".

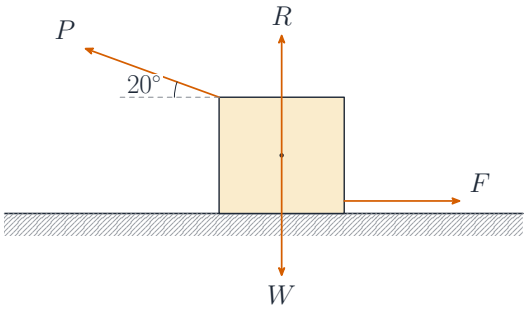


on the Moon $g \approx 1.6\,\mathrm{m\,s^{-2}}$, about $\frac{1}{6}$ as much

Free-body diagrams

A **free-body diagram** shows every force acting **on one object**:

- draw each force as an arrow from the object
- label every arrow: weight W, contact force R, friction F, pull P
- forces the object exerts on *other* bodies do **not** belong on it



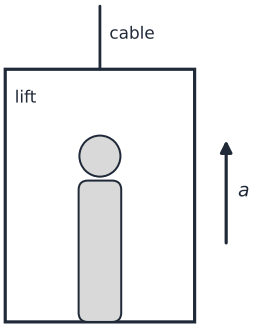
The pull P at $20°$ has a horizontal and a vertical component: resolve it before you add the forces.

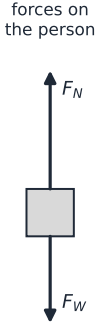
Finding a third-law partner: swap the two bodies

the **back wheel** pushes the **road** backwards	↔	the **road** pushes the **back wheel** forwards
the **Earth** pulls the **Moon**	↔	the **Moon** pulls the **Earth**, just as hard
the **engine** pushes the **gas** down	↔	the **gas** pushes the **engine** up: the **thrust** 推力
the **Earth** pulls the **book** down: its weight	↔	the **book** pulls the **Earth** up

The road's forward push is what drives the bicycle. Weight and air resistance are not part of a rocket's thrust pair.

A person in a lift





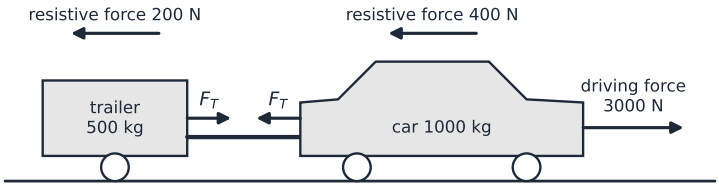
A $60\,\mathrm{kg}$ person: weight $mg = 589\,\mathrm{N}$ down, **normal contact force** 支持力 R up. Up positive, Newton's second law:

$$R - mg = ma \quad \Rightarrow \quad R = m(g + a)$$

motion of the lift	a / $\mathrm{m\,s^{-2}}$	R / N
up, speeding up	$+1.5$	680
steady, up or down	0	589
up and slowing, or down and speeding up	-1.5	500

Bathroom scales read R, not the weight.

Two objects joined together



Whole system for a, then one object for the force between them

1 Both as one object, $F = ma$; the tension is inside it and cancels:

$$a = \frac{F_{\text{net}}}{m_{\text{total}}} = \frac{3000\,\text{N} - 400\,\text{N} - 200\,\text{N}}{1000\,\text{kg} + 500\,\text{kg}} = 1.6\,\text{m s}^{-2}$$

2 The trailer alone, $F = ma$, with T forwards and $200\,\text{N}$ backwards: $T - F_\text{r} = m_\text{t} a$, so

$$T = m_\text{t} a + F_\text{r} = (500\,\text{kg})(1.6\,\text{m s}^{-2}) + 200\,\text{N} = 1000\,\text{N}$$

3 Friction, drag and terminal velocity

Sheet 3.2 – what you need to know

You must be able to:

- describe friction and drag: drag is zero at rest and grows with speed
- describe and explain a fall from rest to **terminal velocity**
- read the graphs: the terminal velocity, a tangent for a, the area for the distance
- find the acceleration when a parachute opens, or on the way up

What the exam asks

"Explain, with reference to forces, the shape of the curve." **[3]**

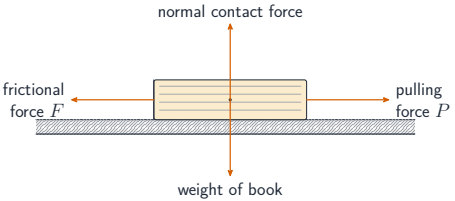
"By drawing a suitable line, determine the acceleration at $t = 9.0\,\mathrm{s}$." **[2]**

"Sketch the variation with t of the acceleration a of the stone." **[3]**

Practise on exercise sheet 3.2, then try the real exam questions on past-paper sheet 3.2.

Friction and drag

- **friction** 摩擦力: between two solid surfaces, opposing the sliding
- **drag** 阻力 or **viscous** 黏性 force: from a **fluid** 流体 on an object moving through it
- drag grows with **speed**, **cross-sectional area** 横截面积 and the fluid's density; it is zero at zero speed



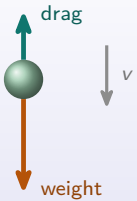
Both act **against the motion** and turn kinetic energy into thermal energy. No **coefficient** 系数 is needed.

Falling through air – three stages

1 just released

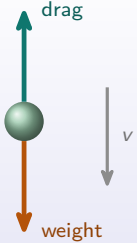
only the weight acts:
$a = g$

2 speeding up



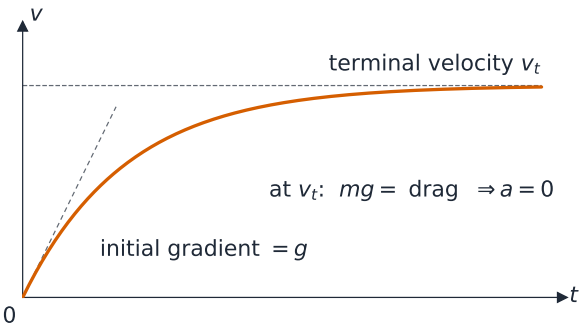
drag grows, so the resultant force shrinks: $a < g$

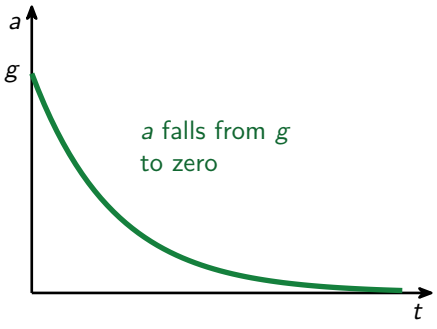
3 terminal velocity



drag = weight:
$a = 0$, constant speed

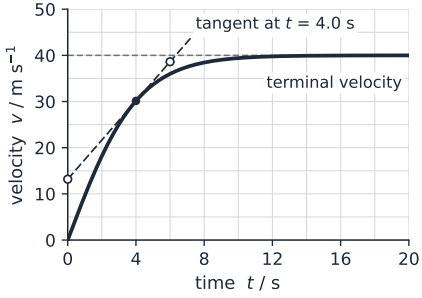
The graphs of a fall through air

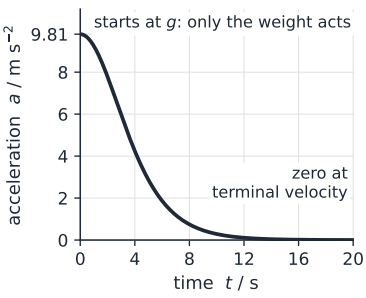




Free fall in a vacuum, in a **uniform gravitational field** 匀强重力场, is a straight line on the v–t graph. With air resistance the line **bends and flattens** at the terminal velocity.

Reading the graph of a fall



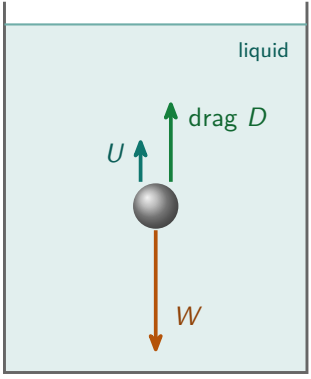


$v_T = 40\,\mathrm{m\,s^{-1}}$ where the curve levels off. **Tangent** at $4.0\,\mathrm{s}$: $a = \dfrac{\Delta v}{\Delta t} = \dfrac{(39-13)\,\mathrm{m\,s^{-1}}}{6.0\,\mathrm{s}} = 4.3\,\mathrm{m\,s^{-2}}$. **Area** up to $10\,\mathrm{s}$: about 29 squares of $10\,\mathrm{m}$, $s \approx 290\,\mathrm{m}$; never $\frac{1}{2}at^2$, because a changes.

Falling through a liquid: three forces

- **weight** W, down
- **upthrust** 浮力 U, up: **hydrostatic pressure** 流体静压强 grows with depth, so the liquid pushes **harder on the bottom**; $U = \rho_{\text{liquid}} V g$, the weight of the liquid displaced
- **viscous drag** D, up: it grows with speed

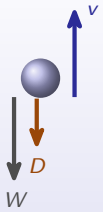
Terminal speed when $W = U + D$. A "draw labelled arrows" question wants exactly these three.



at terminal speed: $U + D = W$

Moving up, moving down

going up



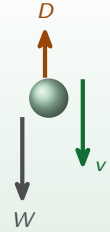
$W + D$: $a > g$

at the top

$v = 0,\ D = 0$

W only: $a = g$

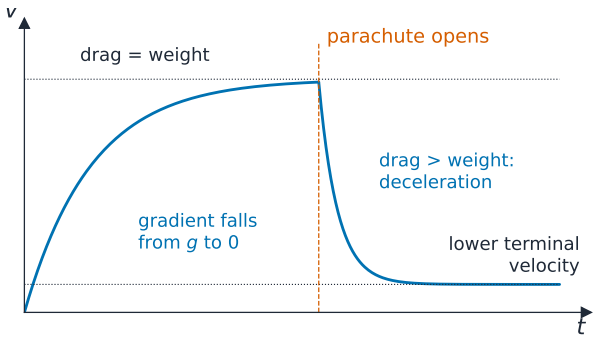
coming down



$W - D$: $a < g$

On the way up the weight and the drag **both** point down, so the deceleration is more than g; as the ball slows, the drag and the resultant force decrease.

Opening a parachute



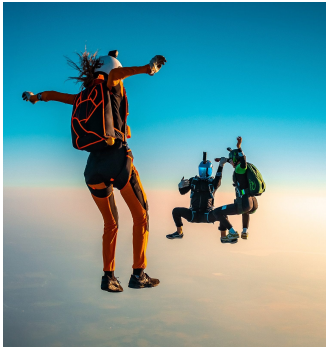
1. the parachute opens: the drag **suddenly exceeds** the weight
2. the resultant force is **upwards**, so the skydiver **decelerates**: acceleration up, velocity still down
3. the drag falls with the speed until it equals the weight again, at a **much lower** terminal velocity

What sets the terminal velocity?

Terminal velocity 收尾速度 is reached when the drag balances the weight.

- **a denser sphere** of the same size has a larger weight, so it needs a larger drag, which it reaches only at a **higher** speed
- **a larger area** gives more drag at every speed, so the drag balances the weight at a **lower** speed

> A parachute works by making the area, and so the drag, much larger.



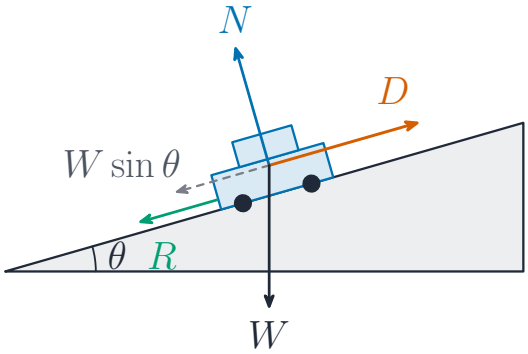
Spread out: the largest area, the most drag, a lower terminal velocity.

Vehicles at constant speed

Flat road: the resultant force is zero, so the **driving force** 驱动力 equals the total resistive force (friction, air resistance, rolling resistance).

Faster ⇒ more drag ⇒ a larger driving force, and more **power** 功率.

Up a slope: $D = R + W\sin\theta$. Going down, the component $W\sin\theta$ helps the motion.



Energy during a terminal-velocity fall

At terminal velocity a **parachutist** 跳伞者 falls at a **constant** speed, so the kinetic energy stays the same while the height, and the **gravitational potential energy** 重力势能, keep falling.

The lost potential energy becomes **thermal energy** of the air, *not* extra kinetic energy.

4 Conserving momentum

Sheet 3.3 – what you need to know

You must be able to:

- state the principle of **conservation of momentum**
- solve collisions along a line: bouncing apart or sticking together
- use an explosion or a decay from rest: the total momentum stays zero
- find the kinetic energy lost, and say whether a collision is elastic

What the exam asks

"State the principle of conservation of momentum." **[2]**

"Calculate the percentage of the total initial kinetic energy that is transferred." **[2]**

"Determine the magnitude and direction of the final velocity of trolley A." **[3]**

Practise on exercise sheet 3.3, then try the real exam questions on past-paper sheet 3.3.

The principle of conservation of momentum

Conservation of momentum 动量守恒: learn it word for word

The total momentum of a system of objects remains constant provided no resultant external force acts on the system.

mark 1: what stays constant

mark 2: the condition

collisions

explosions 爆炸

recoil 反冲

"Total momentum is constant" alone scores **one** mark. In two dimensions momentum is conserved along each direction on its own.

Why momentum is conserved

equal size, opposite direction
(forces act on different objects)

Newton's third law: the forces on A and B are **equal and opposite**

↓

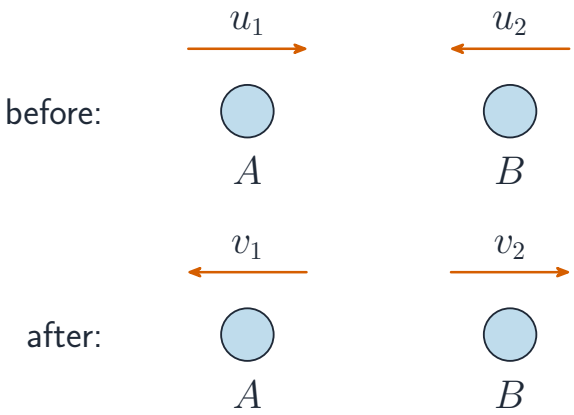
they act for the **same time**, so $F\Delta t$ gives $\Delta p_A = -\Delta p_B$

↓

the total momentum does not change

Whatever momentum one object gains from the other, the other loses: with **no resultant external force**, the total cannot change.

Head-on collisions: pick one positive direction



For a **head-on** 正面 collision, momentum along that line is conserved:

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

Every velocity carries a **sign**: one to the left is **negative**. Choose the direction once and keep it.

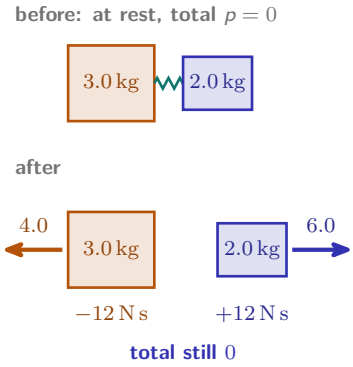
Worked example – two trolleys pushed apart

Two trolleys pushed apart

A $2.0\,\mathrm{kg}$ and a $3.0\,\mathrm{kg}$ trolley are held against a compressed spring, then released from rest. The $2.0\,\mathrm{kg}$ trolley leaves at $6.0\,\mathrm{m\,s^{-1}}$. The total momentum was **zero**, so it stays zero: $m_1 v_1 = m_2 v_2$.

$$v_2 = \frac{m_1 v_1}{m_2} = \frac{(2.0\,\mathrm{kg})(6.0\,\mathrm{m\,s^{-1}})}{3.0\,\mathrm{kg}} = 4.0\,\mathrm{m\,s^{-1}}$$

in the **opposite** direction.



Elastic and inelastic collisions

In **every** collision momentum is conserved. Kinetic energy is the difference:

Elastic collision 弹性碰撞

kinetic energy is **also** conserved

relative speed 相对速率 of approach = relative speed of separation

Inelastic collision 非弹性碰撞

kinetic energy **falls**: some becomes thermal, sound or **deformation** 形变 energy

if they stick together: perfectly inelastic

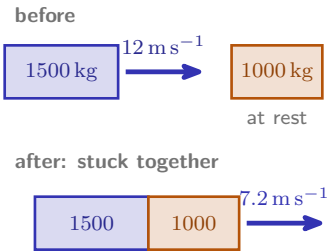
Worked example – two cars lock together

Common velocity after the crash

A $1500\,\mathrm{kg}$ car at $12\,\mathrm{m\,s^{-1}}$ runs into a stationary $1000\,\mathrm{kg}$ car, and they lock together.

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2)\, v$$

$$\begin{aligned} v &= \frac{m_1 u_1}{m_1 + m_2} \\ &= \frac{(1500\,\mathrm{kg})(12\,\mathrm{m\,s^{-1}})}{1500\,\mathrm{kg} + 1000\,\mathrm{kg}} = 7.2\,\mathrm{m\,s^{-1}} \end{aligned}$$

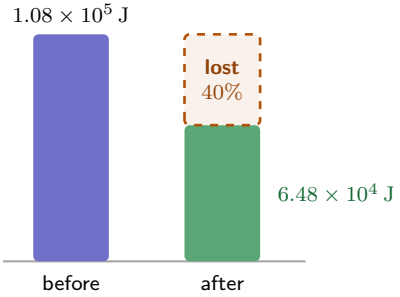


Kinetic energy in the car crash

Momentum is conserved; kinetic energy is **not**.

$$
\begin{aligned}
E_{\text{before}} &= \tfrac{1}{2} m_1 u_1^2 \\
&= \tfrac{1}{2}(1500\,\text{kg})(12\,\text{m s}^{-1})^2 = 1.08 \times 10^5\,\text{J} \\
E_{\text{after}} &= \tfrac{1}{2}(m_1 + m_2)v^2 \\
&= \tfrac{1}{2}(2500\,\text{kg})(7.2\,\text{m s}^{-1})^2 = 6.48 \times 10^4\,\text{J}
\end{aligned}
$$

$4.3 \times 10^4\,\text{J}$, which is **40%** of the kinetic energy, becomes thermal energy, sound and deformation of the cars.



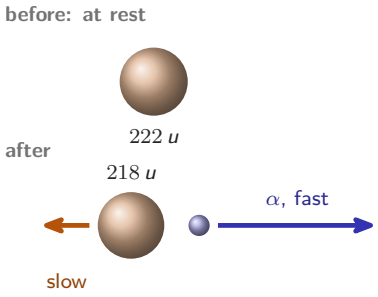
Worked example – a nucleus emits an alpha particle

Recoil of the nucleus left behind

A stationary nucleus of mass $222\,u$ emits an **alpha particle** α 粒子 of mass $4\,u$ at $1.6 \times 10^7\,\mathrm{m\,s^{-1}}$. The total momentum stays zero: $m_\mathrm{D} v_\mathrm{D} = m_\alpha v_\alpha$.

$$v_\mathrm{D} = \frac{m_\alpha v_\alpha}{m_\mathrm{D}} = \frac{(4\,u)(1.6 \times 10^7\,\mathrm{m\,s^{-1}})}{218\,u} = 2.9 \times 10^5\,\mathrm{m\,s^{-1}}$$

opposite to the alpha particle. The unit u cancels, so its value is never needed.



In a decay, momentum, charge, nucleon number and mass–energy are all conserved (topic 11).

Kinetic energy from momentum

$$E_\mathrm{k} = \tfrac{1}{2}mv^2$$

$$= \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$

$$\Delta E_\mathrm{k} = \frac{p_2^2 - p_1^2}{2m}$$

Use it when you know the **momentum** but not the speed: the change in **kinetic energy** 动能 comes straight from the two momenta.

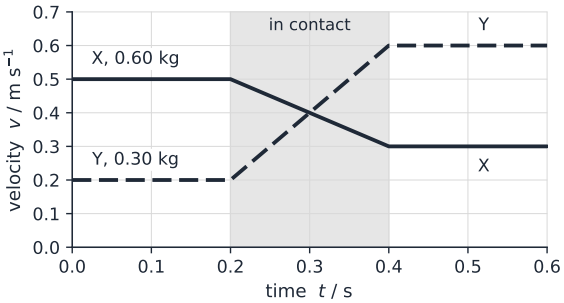
The bouncing ball again

$$p_1 = mv_1 = (0.20\,\mathrm{kg})(8.0\,\mathrm{m\,s^{-1}}) = 1.6\,\mathrm{N\,s}$$
$$p_2 = mv_2 = (0.20\,\mathrm{kg})(6.0\,\mathrm{m\,s^{-1}}) = 1.2\,\mathrm{N\,s}$$

$$\Delta E_\mathrm{k} = \frac{p_2^2 - p_1^2}{2m} = \frac{(1.2\,\mathrm{N\,s})^2 - (1.6\,\mathrm{N\,s})^2}{2 \times 0.20\,\mathrm{kg}} = -2.8\,\mathrm{J}$$

The ball loses $2.8\,\mathrm{J}$ in the bounce.

A collision on a velocity–time graph



Read the graph

Y's gradient during contact:

$$a = \frac{\Delta v}{\Delta t} = \frac{(0.60 - 0.20)\,\mathrm{m\,s^{-1}}}{0.20\,\mathrm{s}} = 2.0\,\mathrm{m\,s^{-2}}$$

$$F = m_\mathrm{Y} a = (0.30\,\mathrm{kg})(2.0\,\mathrm{m\,s^{-2}}) = 0.60\,\mathrm{N}$$

X feels $0.60\,\mathrm{N}$ back, for the **same** $0.20\,\mathrm{s}$.

The same collision: conserve momentum

X ($0.60\,\mathrm{kg}$, $0.50\,\mathrm{m\,s^{-1}}$) catches Y ($0.30\,\mathrm{kg}$, $0.20\,\mathrm{m\,s^{-1}}$); Y leaves at $0.60\,\mathrm{m\,s^{-1}}$

$$m_\mathrm{X} u_\mathrm{X} + m_\mathrm{Y} u_\mathrm{Y} = m_\mathrm{X} v_\mathrm{X} + m_\mathrm{Y} v_\mathrm{Y}$$

$$(0.60\,\mathrm{kg})(0.50\,\mathrm{m\,s^{-1}}) + (0.30\,\mathrm{kg})(0.20\,\mathrm{m\,s^{-1}}) = (0.60\,\mathrm{kg})\,v_\mathrm{X} + (0.30\,\mathrm{kg})(0.60\,\mathrm{m\,s^{-1}})$$

$$v_\mathrm{X} = 0.30\,\mathrm{m\,s^{-1}}$$

$E_\mathrm{k} = \sum \frac{1}{2} m v^2$ is $0.081\,\mathrm{J}$ before and after, so the collision is **elastic**.

5 Elastic and 2-D collisions

Sheet 3.3.1 – what you need to know

You must be able to:

- use the **relative-speed rule** with momentum to solve an elastic collision
- resolve momenta, and conserve each component on its own
- find the missing fragment of an explosion
- test a two-dimensional collision using the full speeds

What the exam asks

"What is the speed and direction of ball X after the collision?" **[1]**

"By considering the components of the momenta along the line AB, calculate θ." **[3]**

"By calculation of kinetic energies, state whether the collision is inelastic." **[2]**

Practise on exercise sheet 3.3.1, then try the real exam questions on past-paper sheet 3.3.1.

An elastic collision gives you a second equation

Momentum alone leaves two unknowns. If the collision is **elastic**, kinetic energy is conserved as well:

$$\tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2$$

or, the same thing in a form that is easier to solve:

$$u_1 - u_2 = -(v_1 - v_2)$$

Relative speed of approach = relative speed of separation. Now there are two equations for the two unknowns.

A head-on elastic collision with a stationary mass

Mass m at speed u hits a **stationary** 静止 mass M head-on, elastically:

$$v_m = \frac{m - M}{m + M} u \qquad v_M = \frac{2m}{m + M} u$$

equal masses, $m = M$

$v_m = 0$ and $v_M = u$: they swap velocities

light on heavy, $m \ll M$

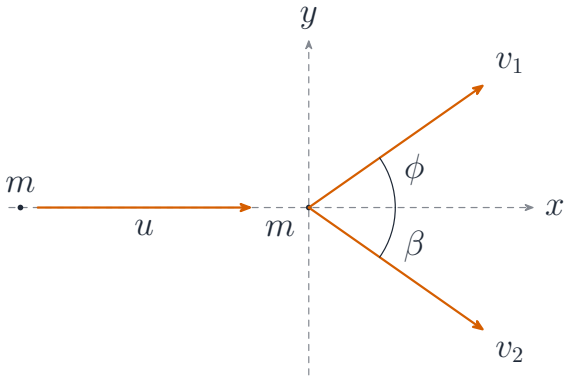
$v_m \approx -u$: it bounces straight back

heavy on light, $m \gg M$

$v_m \approx u$ and $v_M \approx 2u$

Collisions in two dimensions

Split the velocities into **perpendicular** 垂直 **components** 分量 and conserve momentum along **each direction** on its own. Choose one axis along the first object's motion and one across it.



Worked example – momentum in two dimensions

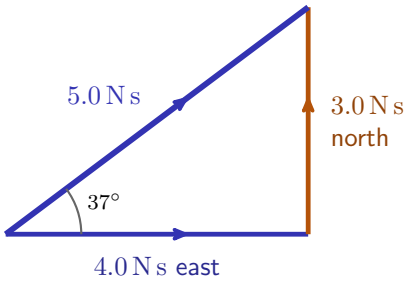
Two balls collide and stick

On a frictionless surface ball A has $4.0\,\mathrm{N\,s}$ due east and ball B $3.0\,\mathrm{N\,s}$ due north. Along each axis momentum is conserved, so after sticking it is still 4.0 east and 3.0 north:

$$p = \sqrt{p_{\mathrm{E}}^2 + p_{\mathrm{N}}^2} = \sqrt{(4.0\,\mathrm{N\,s})^2 + (3.0\,\mathrm{N\,s})^2}$$
$$= 5.0\,\mathrm{N\,s}$$

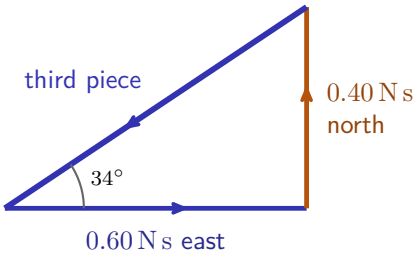
$$\theta = \tan^{-1}\frac{p_{\mathrm{N}}}{p_{\mathrm{E}}} = \tan^{-1}\frac{3.0\,\mathrm{N\,s}}{4.0\,\mathrm{N\,s}} = 37^\circ$$

north of east



A multiple-choice diagram of two momentum arrows asks exactly this: add the arrows **tip to tail**.

An explosion in three pieces



A firework at rest breaks into three pieces

$0.20\,\mathrm{kg}$ at $3.0\,\mathrm{m\,s^{-1}}$ east and $0.10\,\mathrm{kg}$ at $4.0\,\mathrm{m\,s^{-1}}$ north; the third has $0.10\,\mathrm{kg}$. From rest the momenta add to **zero**, so the third closes the triangle: $p_1 = m_1 v_1 = 0.60\,\mathrm{N\,s}$ east; $p_2 = m_2 v_2 = 0.40\,\mathrm{N\,s}$ north

$$p = \sqrt{p_1^2 + p_2^2} = \sqrt{(0.60\,\mathrm{N\,s})^2 + (0.40\,\mathrm{N\,s})^2}$$
$$= 0.72\,\mathrm{N\,s}$$
$$v = \frac{p}{m} = \frac{0.72\,\mathrm{N\,s}}{0.10\,\mathrm{kg}} = 7.2\,\mathrm{m\,s^{-1}}$$

directed $34°$ **south of west**, opposite to the other two together.

6 Putting it together

Worked example – a car climbing a hill

Driving force and power up a slope

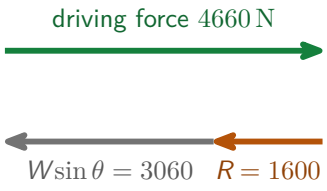
$m = 1500\,\text{kg}$, slope $12°$, constant $30\,\text{m s}^{-1}$, resistive force $1600\,\text{N}$.

$$\begin{aligned} mg\sin\theta &= (1500\,\text{kg})(9.81\,\text{m s}^{-2})\sin 12° \\ &= 3060\,\text{N} \end{aligned}$$

$$\begin{aligned} D &= R + mg\sin\theta \\ &= 1600\,\text{N} + 3060\,\text{N} = 4660\,\text{N} \approx 4700\,\text{N} \end{aligned}$$

Power is force times velocity (topic 5):

$$\begin{aligned} P &= Dv \\ &= (4660\,\text{N})(30\,\text{m s}^{-1}) = 1.4 \times 10^5\,\text{W} \end{aligned}$$



Worked example – a steel ball falling through oil

A steel ball of radius $r = 2.0\,\mathrm{mm}$ and weight $W = 2.56 \times 10^{-3}\,\mathrm{N}$ falls through oil of density $\rho = 850\,\mathrm{kg\,m^{-3}}$. The viscous drag is $F = 6\pi\eta rv$, where η is the **viscosity** 黏度 of the oil.

a. The base units of η

rearrange $F = 6\pi\eta rv$:

$$\eta = \frac{F}{6\pi rv}$$

units:

$$\frac{\mathrm{kg\,m\,s^{-2}}}{\mathrm{m} \times \mathrm{m\,s^{-1}}} = \mathrm{kg\,m^{-1}\,s^{-1}}$$

b. The upthrust

$$V = \tfrac{4}{3}\pi r^3 = \tfrac{4}{3}\pi(2.0 \times 10^{-3}\,\mathrm{m})^3 = 3.35 \times 10^{-8}\,\mathrm{m^3}$$

$$U = \rho g V = (850\,\mathrm{kg\,m^{-3}})(9.81\,\mathrm{m\,s^{-2}})(3.35 \times 10^{-8}\,\mathrm{m^3})$$
$$= 2.8 \times 10^{-4}\,\mathrm{N}$$

Worked example – the steel ball's terminal speed

At terminal speed the resultant force is zero, so the drag balances the weight less the upthrust:

$$F = W - U$$

Terminal speed: **no** resultant force, so **no** acceleration.

c. Terminal speed

the oil's viscosity is $\eta = 0.20\,\mathrm{kg\,m^{-1}\,s^{-1}}$

$F = W - U = 2.56 \times 10^{-3}\,\mathrm{N} - 0.28 \times 10^{-3}\,\mathrm{N} = 2.28 \times 10^{-3}\,\mathrm{N}$

rearrange $F = 6\pi\eta r v$:

$$v = \frac{F}{6\pi\eta r} = \frac{2.28 \times 10^{-3}\,\mathrm{N}}{6\pi(0.20\,\mathrm{kg\,m^{-1}\,s^{-1}})(2.0 \times 10^{-3}\,\mathrm{m})}$$

$$= 0.30\,\mathrm{m\,s^{-1}}$$

Definitions the examiner accepts

A definition is marked against **fixed wording**: learn these exactly, and give one answer only.

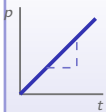
mass	质量	the property of an object that resists a change in its motion
linear momentum	动量	the product of an object's mass and its velocity
force	力	the rate of change of momentum
Newton's first law	牛顿第一定律	an object remains at rest or moves at constant velocity unless acted on by a resultant force
Newton's second law	牛顿第二定律	the resultant force on an object is proportional to the rate of change of its momentum, and acts in the direction of the change
Newton's third law	牛顿第三定律	when two bodies interact, the force on one is equal in magnitude and opposite in direction to the force on the other

Definitions: weight, collisions and terminal velocity

Five more, marked the same way:

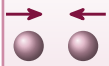
weight	重力	the force on an object due to a gravitational field; equal to the product of its mass and the acceleration of free fall
principle of conservation of momentum	动量守恒	the total momentum of a system of objects remains constant provided no resultant external force acts on the system
elastic collision	弹性碰撞	a collision in which total kinetic energy is conserved (and the relative speed of approach equals the relative speed of separation)
inelastic collision	非弹性碰撞	a collision in which momentum is conserved but some kinetic energy is transferred to other forms
terminal velocity	收尾速度	the constant velocity reached when the resistive force on a moving object equals the force driving it, so the resultant force is zero

Exam tips

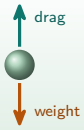


Newton's second law is $F = \Delta p/\Delta t$: $F = ma$ is the special case for constant mass.

Third-law pairs: the same type of force, on **two different bodies**, never the balanced forces on one body.



Momentum is conserved in every collision; kinetic energy only in an **elastic** one.



Terminal velocity: drag rises with speed until drag = weight, so the acceleration is zero.

Common mistakes

✗ The mistake	✓ What is right
"The forces are balanced" or "the forces cancel out".	Say "the **resultant force** on the object is zero".
Adding both speeds in a head-on collision because momentum is "total".	Choose a positive direction, give the opposing velocity a **minus sign**, then add.
Kinetic energy from a component of the velocity.	Kinetic energy uses the **full speed** of each body.
Weight and the normal contact force on one object as a third-law pair.	A pair acts on **two different objects** and is one type of force.
Conservation of momentum stated without the condition.	It holds only when **no resultant external force** acts on the system.

7 Recap

Topic 3 – key takeaways

1 Momentum

$p = mv$ and $F = \Delta p/\Delta t$; impulse = area under F–t

2 Newton's laws

$F = ma$; a third-law pair acts on two different bodies

3 Terminal velocity

drag grows with speed until the resultant force is zero

4 Conservation

momentum always, with no external force; kinetic energy only if elastic

Check yourself

1. The unit of momentum, written with the newton?
2. A book's weight and the table's push: a third-law pair?
3. At terminal velocity, what is the resultant force?
4. Two trolleys stick together: is momentum or kinetic energy conserved?

Try all four first – the answers appear on the next slide.

Check yourself

1. The unit of momentum, written with the newton?

   ⇒ $\mathrm{N\,s}$, the same as $\mathrm{kg\,m\,s^{-1}}$

2. A book's weight and the table's push: a third-law pair?

   ⇒ **no**: they act on the same object and are different types

3. At terminal velocity, what is the resultant force?

   ⇒ **zero**: the drag balances the weight

4. Two trolleys stick together: is momentum or kinetic energy conserved?

   ⇒ **momentum**; some kinetic energy becomes thermal energy