

Spring stiffness from rotational oscillations

Name: _____ Class: _____ Date: _____

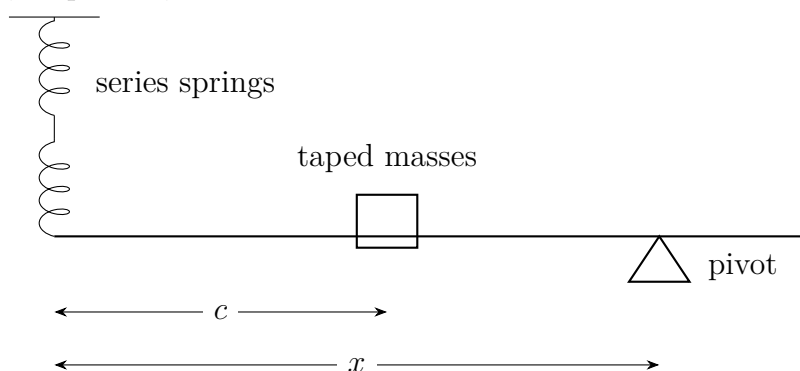
Aim Estimate spring stiffness using a physically explicit oscillation model. Allow 75–90 minutes.

Model A loaded rule rotates through a small angle about a fixed pivot. If two springs in series have effective stiffness k_e , then

$$T^2 x^2 = \frac{4\pi^2 M}{k_e} (x - c)^2 + \frac{4\pi^2 J_c}{k_e}.$$

M is total mass; c is its centre-of-mass distance from the spring line; J_c is its rotational inertia about its centre. This supplied model accounts for the rule's extended mass.

Apparatus Metre rule, two springs in series, secure low-friction pivot, stand/clamp, string, two taped 100 g masses, stopwatch, balance and ruler.



Safety Weight the stand, secure the masses and retain the pivot position during motion. Keep oscillations small and springs within their elastic range. Stop if the rule slips or lifts from the pivot.

1 Set up and time

1. Secure the two masses near the rule's balance point. Measure the combined M and balance again to find c , from the spring attachment point.
2. Choose $x > c$. Set the rule horizontal and springs vertical. Use a pivot approved by the teacher that allows rotation without sliding.
3. Displace the spring end only a few millimetres. Time 10 full oscillations through a fixed reference, starting and ending in the same direction. Take a second timing after resetting the displacement.
4. Use at least six well-spread safe x values. Keep the masses, springs and amplitude unchanged. Record if motion is not a clean single oscillation.

2 Record and process

$M =$ _____ kg $c =$ _____ m $\Delta x =$ _____ m.

x/m	t_1/s	t_2/s	$\bar{t}/\text{s}$	$T = \bar{t}/10/\text{s}$

$X = (x - c)^2/\text{m}^2$	$Y = T^2 x^2/\text{s}^2\text{m}^2$	$\Delta Y/\text{s}^2\text{m}^2$

Uncertainty Estimate ΔT from timing resolution and repeat spread. Use $\Delta Y/Y \simeq 2\Delta T/T + 2\Delta x/x$ as a conservative estimate.

Model check Why must you measure the balance point after adding masses? What observation would invalidate the fixed-pivot assumption?

3 Graph

Plot $Y = T^2 x^2$ against $X = (x-c)^2$. Include ΔY bars and horizontal bars $\Delta X \simeq 2|x-c|(\Delta x + \Delta c)$. Draw best and extreme acceptable lines; do not force a zero intercept.



Choose scales and label both axes with quantities and units. Mark a large gradient triangle on the best-fit line where appropriate.

4 Analyse and evaluate

1. Find gradient s and intercept b , with units. Calculate $k_e = 4\pi^2 M/s$. For identical series springs, each has $k = 2k_e$. Explain why the individual stiffness is larger.

2. Estimate Δs from the extreme line, then $\Delta k_e/k_e \simeq \Delta M/M + \Delta s/|s|$. Explain why small $x - c$ makes the uncertainty in X important.

3. Use $J_c = bk_e/(4\pi^2)$ to estimate rotational inertia. Is a positive intercept expected? Compare your dynamic k_e with a separately measured static force–extension stiffness, if available.

4. Explain two limitations and matched improvements. Consider sliding pivots, damping, spring mass and correlated distance errors. State whether the data support this model over your measured range.

Before you leave Return the apparatus, clear the bench and submit your recorded data.