Course Companion

A-Level Physics

Every topic handout in one volume

全部专题讲义合集

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Physical quantities and units

A-Level Physics

Physical quantities

A **physical quantity** 物理量 has two parts: a number (its **magnitude** 大小) and a **unit** 单位. The number on its own tells you nothing. You must also say what is measured and in which unit.

Example: "the length is 1.5" is not complete. "The length is 1.5 m" is a physical quantity.

Making estimates

You should be able to **estimate** 估算 the size of the physical quantities in this syllabus. Paper 1 usually opens with a question like this. Learn these rough values:

- **mass** 质量 of an adult human: ~ 70 kg; of a car: ~ 1000 kg; of an apple: ~ 0.1 kg
- **weight** 重力 of an adult human: ~ 700 N; of an apple: ~ 1 N
- height of an adult human: ~ 1.7 m; height of a room: ~ 3 m
- walking speed: ~ 1.5 m s^{-1}; a car on a fast road: ~ 30 m s^{-1}
- speed of sound in air: ~ 340 m s^{-1}; speed of light in a **vacuum** 真空: 3.0×10^8 m s^{-1}
- **acceleration** 加速度 of **free fall** 自由落体: $g \approx 9.81$ m s^{-2}
- **density** 密度 of water: 1000 kg m^{-3}; of air: ~ 1.2 kg m^{-3}; of steel: ~ 8000 kg m^{-3}
- **atmospheric pressure** 大气压强: $\sim 1.0 \times 10^5$ Pa
- room temperature: ~ 20 °C ≈ 293 K
- power of a kettle: ~ 2 kW; of a person climbing stairs: ~ 300 W
- wavelength of visible light: $\sim 5 \times 10^{-7}$ m; diameter of an atom: $\sim 10^{-10}$ m; of a nucleus: $\sim 10^{-15}$ m

A good estimate has the right **order of magnitude** 数量级 (the right power of ten). For a human, 70 kg is a good guess; 7 kg is not.

To estimate a quantity that is not in the list, build it from ones that are. The kinetic energy of a moving car is $\frac{1}{2}mv^2 \approx \frac{1}{2} \times 1000 \times 30^2 \approx 5 \times 10^5$ J. The pressure under a standing person is 700 N/0.02 m^2 $\approx 4 \times 10^4$ Pa. Always check that the power of ten looks sensible before you move on.

SI units 国际单位制

Base units

The SI system has seven **base quantities** 基本量; five of them are used in this topic. You must know these five and their units:

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- **mass** – kilogram, kg
- **length** 长度 – metre, m
- time – second, s
- **current** 电流 – ampere 安培, A
- **temperature** 温度 – kelvin 开尔文, K

Almost every other unit in this syllabus is built from these five. The exception is the **mole** 摩尔 (mol), the SI base unit of amount of substance, which you meet with the Avogadro constant and the gas laws.

Derived units

A **derived unit** 导出单位 is made by multiplying or dividing base units. You should be able to write any quantity in this syllabus in base units.

Build a derived unit from the equation that defines it:

- **speed** 速率 = distance / time, so its unit is m s^{-1}
- **acceleration** = change in **velocity** 速度 / time, so its unit is m s^{-2}
- **force** 力 = mass × acceleration, so its unit is kg m s^{-2}. The **newton** 牛顿 is 1 N = 1 kg m s^{-2}.
- **work** 功 and **energy** 能量 = force × distance, so the unit is kg m^2 s^{-2}. The **joule** 焦耳 is 1 J = 1 kg m^2 s^{-2}.
- **power** 功率 = energy / time, so the unit is kg m^2 s^{-3}. The **watt** 瓦特 is 1 W = 1 kg m^2 s^{-3}.
- **pressure** 压强 and **stress** 应力 = force / area, so the unit is kg m^{-1} s^{-2}. The **pascal** 帕斯卡 is 1 Pa = 1 kg m^{-1} s^{-2}.

When a question asks for the SI base units of a quantity, replace each named unit with its base units, then simplify. Example: the SI base units of the watt are kg m^2 s^{-3}.

Checking that the units match

An equation is **homogeneous** 量纲一致 when both sides have the same base units. In plain words: the units on both sides match.

Write each side in base units and compare. Take the equation $v^2 = u^2 + 2as$:

- left side: (m s^{-1})^2 = m^2 s^{-2}
- right side, first term: (m s^{-1})^2 = m^2 s^{-2}
- right side, second term: m s^{-2} · m = m^2 s^{-2}

Both sides give m^2 s^{-2}, so the units match.

Be careful: matching units do **not** prove the whole equation is correct. It could still have a wrong number, or a missing factor of 2. But if the units do **not** match, the equation is wrong for sure.

Worked example. The drag force on a falling ball is given by $F = kv^2$, where v is the speed. Find the SI base units of the constant k.

Rearrange: $k = F/v^2$. In base units, F is kg m s^{-2} and v^2 is m^2 s^{-2}, so

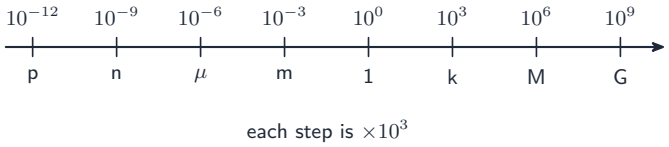
$$k : \frac{\text{kg m s}^{-2}}{\text{m}^2\ \text{s}^{-2}} = \text{kg m}^{-1}.$$

A multiple-choice question often asks "which equation could be correct?". Check the base units of each option; only a homogeneous equation can be correct. A pure number (like 2, π or $\frac{1}{2}$) has no unit, so it never changes the check.

Prefixes

A **prefix** 词头 is a letter put in front of a unit to make it bigger or smaller by powers of ten. You must know these:

Prefix	Symbol	Factor
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}



The SI prefixes climb in steps of a thousand, from pico to giga

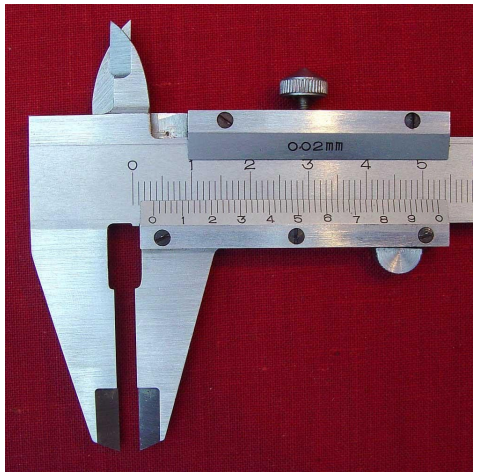
To change a prefixed unit into base units, replace the prefix with its factor, then simplify. Example: change 0.25 kN mm^{-2} into N m^{-2}:

$$0.25\ \text{kN mm}^{-2} = 0.25 \times \frac{10^3\ \text{N}}{(10^{-3}\ \text{m})^2} = 0.25 \times \frac{10^3}{10^{-6}}\ \text{N m}^{-2} = 2.5 \times 10^8\ \text{N m}^{-2}.$$

Take special care with squared units like mm^2: you must square the factor too.

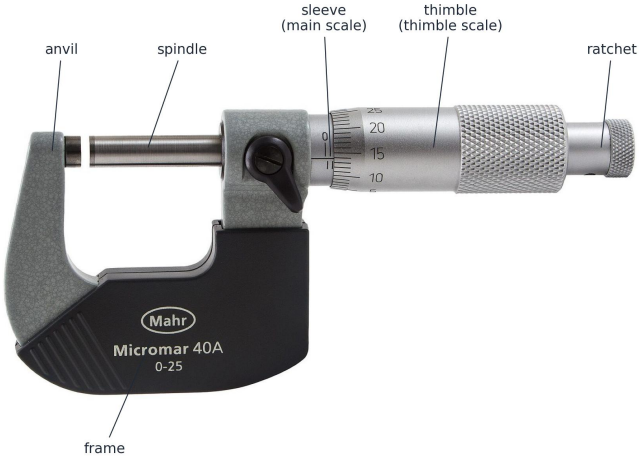
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Errors and uncertainties

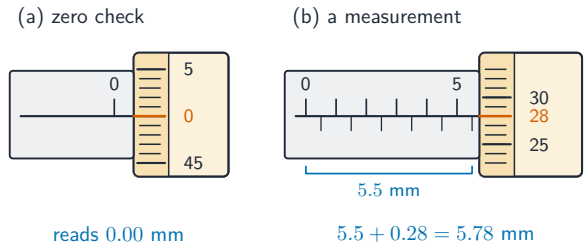


A vernier caliper measures length precisely, with a small uncertainty.

Every **measurement** 测量 has some **uncertainty** 不确定度 – we are never fully sure of the value. A good experimenter knows where the uncertainty comes from, makes a fair estimate of it, and carries it through to the final answer.



The main parts of a real micrometer screw gauge, which measures to the nearest 0.01 mm



Reading a micrometer: add the main scale reading to the thimble reading

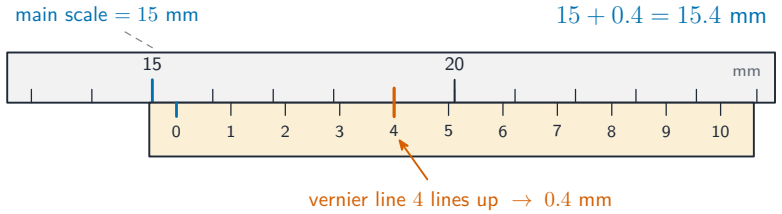


Reading a real micrometer: read the mm and half-mm on the sleeve, then add the thimble scale



Vernier calipers measure to the nearest 0.02 mm – the sliding scale gives the extra digit

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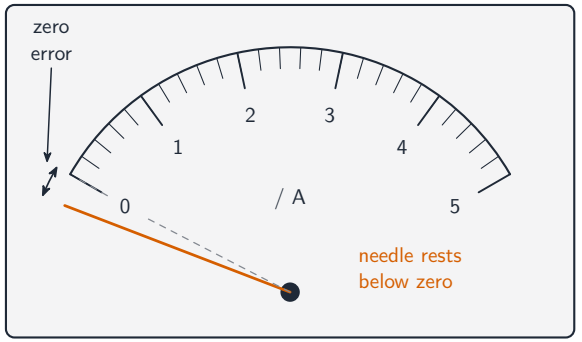


Reading a vernier caliper: whole millimetres from the main scale, plus the tenths from the vernier line that lines up

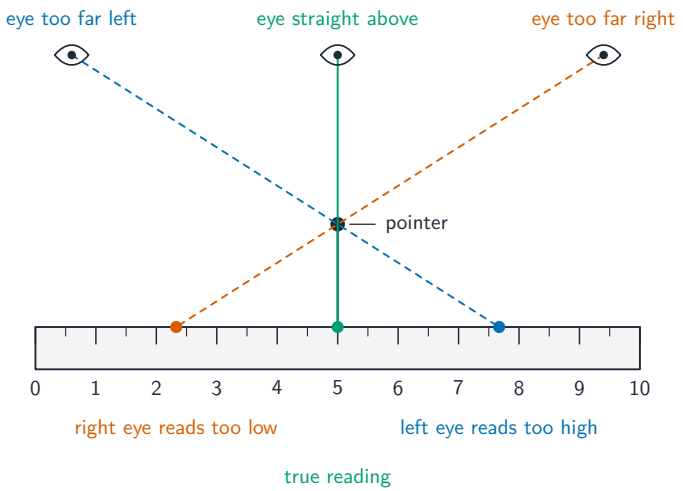
Systematic and random errors

A **systematic error** 系统误差 changes every reading by the same amount, in the same direction. You cannot find it by repeating the measurement. Common causes:

- a **zero error** 零点误差 (the scale does not read zero when the true value is zero)
- a **calibration** 校准 error (the scale itself is wrong)
- **parallax** 视差 (your eye is always to one side of the scale)



An ammeter with a zero error: the needle reads below zero before any current flows



Parallax error: different viewing angles give different scale readings

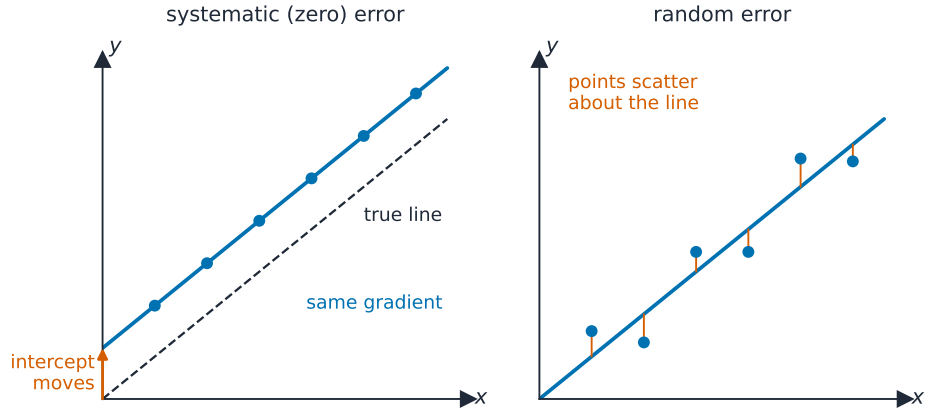
A systematic error makes the **accuracy** 准确度 worse, but it does not change the **precision** 精密度.

A **random error** 随机误差 makes readings jump above and below the true value, with no pattern. Causes include how carefully you read the scale, changing conditions, and the smallest step the **instrument** 仪器 can show. If you repeat the measurement many times and take the **mean** 平均值 (the average), random errors partly cancel out.

A random error makes the precision worse. But with enough repeats, the mean can still be accurate.

On a graph the two errors look different. A systematic error moves every point by the same amount, so the line of best fit keeps its gradient but no longer passes through the origin: the **intercept** 截距 changes. Random errors scatter the points above and below the line; a best-fit line drawn through the middle of the scatter still gives a reliable gradient.

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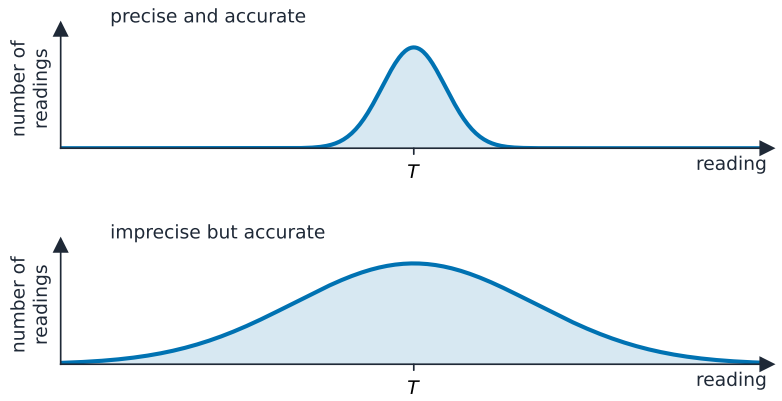
A systematic error shifts the whole line, so the intercept moves; random errors scatter the points about the line

Error	How to reduce it
systematic	check for a zero error and subtract it; calibrate the instrument against a known standard; read the scale from directly in front
random	repeat the reading and take the mean; use an instrument with smaller divisions; time many oscillations instead of one

Precision and accuracy

Precision is how close repeated readings are to each other. Precise readings are grouped very close together.

Accuracy is how close a reading (or the mean of several readings) is to the true value.

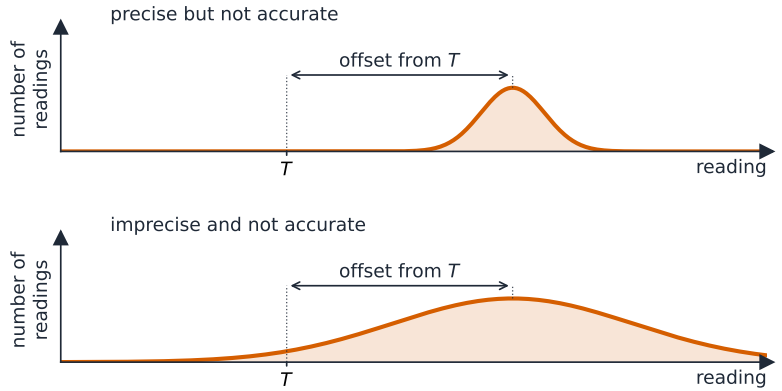


precision: how closely repeated readings agree (random error / spread)

Precision: how narrow the distribution is around the true value T

A set of readings can be:

- precise and accurate – close together and near the true value
- precise but not accurate – close together, but away from the true value (a systematic error)
- accurate but not precise – spread out, but the mean is near the true value
- neither – spread out and away from the true value



accuracy: how close readings are to the true value (systematic error reduces accuracy)

Accuracy: whether the peak of the distribution is centred on the true value T

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When a question gives a table of repeated readings, look at the **spread** (precision) and the **mean** (accuracy) separately.

Estimating the uncertainty in a reading

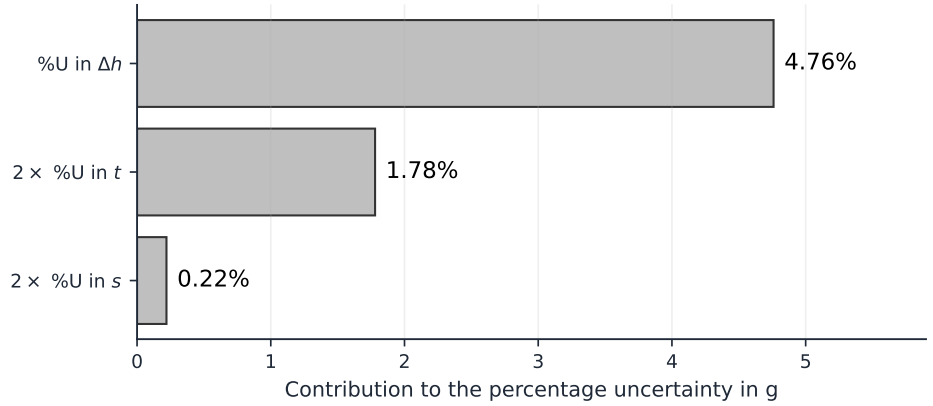
Before you combine uncertainties, you need a sensible uncertainty for each raw reading:

- A single reading on an analogue scale: half the smallest **division** 分度 (a metre rule reads to 1 mm, so one reading is ±0.5 mm). A length needs two readings, one at each end, so its uncertainty is ±1 mm.
- A digital meter: ±1 in the last digit shown (a stopwatch showing 2.47 s is ±0.01 s).
- A hand-timed measurement: your **reaction time** 反应时间 matters more than the display. Allow about ±0.1 to 0.2 s, and time many oscillations instead of one, so the same uncertainty is shared between all of them.
- Repeated readings: the uncertainty is **half the range** 极差的一半. Three timings of 2.42, 2.48 and 2.45 s have a mean of 2.45 s and a range of 0.06 s, so $t = (2.45 \pm 0.03)$ s.

Be honest. If the edge of a shadow is hard to see, the uncertainty in its position is several millimetres, however fine the scale. Examiners do not accept "the smallest division" as the uncertainty of a difficult reading.

Uncertainty in a derived quantity

A measurement is often written as $x \pm \Delta x$. Here Δx is the **absolute uncertainty** 绝对不确定度. The **percentage uncertainty** 百分比不确定度 is



The height difference dominates. Improving the timing would barely change the result; improving Δh would.

$$\text{percentage uncertainty in } x = \frac{\Delta x}{|x|} \times 100\%.$$

A **derived quantity** 导出量 is one you calculate from measured values. Its uncertainty is found by simple rules:

- **Adding or subtracting** – add the absolute uncertainties. If $y = a + b$ or $y = a - b$, then $\Delta y = \Delta a + \Delta b$.
- **Multiplying or dividing** – add the percentage uncertainties. If $y = \dfrac{a \cdot b}{c}$, then

$$\frac{\Delta y}{|y|} = \frac{\Delta a}{|a|} + \frac{\Delta b}{|b|} + \frac{\Delta c}{|c|}.$$

- **Powers** – multiply the percentage uncertainty by the power. If $y = a^n$, then $\dfrac{\Delta y}{|y|} = |n| \cdot \dfrac{\Delta a}{|a|}$.

> **Worked example.** A ball's **diameter** 直径 is measured as $d = (5.26 \pm 0.02)$ cm. The **volume** 体积 of a sphere is $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (d/2)^3$, so $V \propto d^3$ (V depends on d cubed).

The percentage uncertainty in d is

$$\frac{0.02}{5.26} \times 100\% \approx 0.38\%.$$

Because $V \propto d^3$, the percentage uncertainty in V is three times this, about 1.14%. The volume is $\frac{4}{3}\pi(2.63)^3 \approx 76.2$ cm^3. So the absolute uncertainty is $0.0114 \times 76.2 \approx 0.87$ cm^3. The final answer is $V = (76.2 \pm 0.9)$ cm^3.

> **Worked example (a difference).** The ends of a spring are at 54.3 cm and 12.6 cm on a ruler, each read to ±0.1 cm. Find its length with its uncertainty. The length is $54.3 - 12.6 = 41.7$ cm. For a difference, **add** the absolute uncertainties: $0.1 + 0.1 = 0.2$ cm, so $L = (41.7 \pm 0.2)$ cm. The uncertainties never subtract, even though the values do. If the two readings were close together, say 13.0 and 12.6 cm, the length 0.4 ± 0.2 cm would have a 50% uncertainty: avoid measuring a small difference between two large readings.

> **Worked example (several quantities).** The resistivity of a wire is $\rho = RA/L$, with $A = \frac{1}{4}\pi d^2$. The percentage uncertainties are 2.0% in R, 1.0% in d and 0.5% in L. Find the percentage uncertainty in ρ. The area depends on d^2, so its percentage uncertainty is $2 \times 1.0\% = 2.0\%$ (the constant $\frac{1}{4}\pi$ has none). Then add the percentages for the product and quotient: $2.0\% + 2.0\% + 0.5\% = 4.5\%$. The diameter contributes the most once it is squared, which is why it is measured with a micrometer at several points along the wire.

Do two values agree? Practical questions often ask whether two calculated values of a constant support a suggested relationship. Do not just say "they are close". Work out the **percentage difference** 百分比差异 between them,

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$$\text{percentage difference} = \frac{|k_1 - k_2|}{\text{mean of } k_1 \text{ and } k_2} \times 100\%,$$

and compare it with the percentage uncertainty in k (or with the criterion the question gives, often 10%). If the difference is smaller, the two values agree within the uncertainty and the relationship is supported. If it is larger, they do not.

> **Worked example.** It is suggested that $T^2 = kL$. Two sets of readings give $k_1 = 4.05$ s^2 m^{-1} and $k_2 = 4.38$ s^2 m^{-1}. The criterion is 10%. Does the data support the relationship? The mean is 4.215, and the percentage difference is $0.33 \div 4.215 \times 100\% = 7.8\%$. This is less than 10%, so **yes**: the two values of k agree within the criterion, and the data supports the relationship. State the number, the comparison and the conclusion; all three earn marks.

■ Significant figures

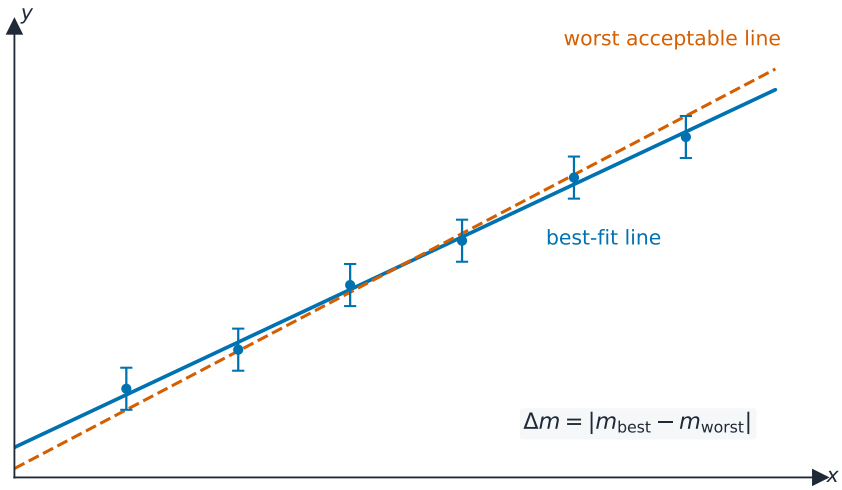
When you write a calculated quantity, give it the **same number of significant figures** 有效数字 as the least precise measurement you used – usually two or three in this syllabus. Too many significant figures makes the answer look more exact than it really is. Too few loses useful information.

Practical papers ask you to **justify** the number you chose. Name the raw readings: "a is given to 2 significant figures because L and t were each measured to 2 significant figures." The phrase "because of the raw data" on its own does not earn the mark. In a table of results, judge each row from the least precise raw reading in that row; do not force every row to the same number of figures.

■ Uncertainties on a graph

Paper 5 asks you to carry uncertainties through a graph:

- Plot each point with an **error bar** 误差棒 whose length shows the absolute uncertainty in that value.
- Draw the **line of best fit** 最佳拟合直线 through the points, then the **worst acceptable line** 最差可接受直线: the steepest (or shallowest) straight line that still passes through every error bar.
- The uncertainty in the gradient is the difference between the two gradients, $\Delta m = |m_{\text{best}} - m_{\text{worst}}|$. The uncertainty in the intercept is found the same way.
- When you plot a logarithm, the absolute uncertainty in $\ln x$ is $\Delta x / x$ (and in $\lg x$ it is $0.434\,\Delta x/x$). Work it out for the largest and smallest values of x separately.



The worst acceptable line still passes through every error bar; the gradient uncertainty is the difference between the two gradients

A percentage uncertainty in the gradient then flows into any quantity you calculate from it, using the rules above.

> **Worked example.** A graph of T^2 against L has a best-fit gradient of $4.02\ \text{s}^2\,\text{m}^{-1}$, and the worst acceptable line has a gradient of $4.31\ \text{s}^2\,\text{m}^{-1}$. The gradient is $4\pi^2/g$. Find g with its uncertainty. The uncertainty in the gradient is $4.31 - 4.02 = 0.29$, which is $0.29 \div 4.02 \times 100\% = 7.2\%$. Then $g = 4\pi^2 \div 4.02 = 9.82\ \text{m s}^{-2}$. Dividing by the gradient keeps the same percentage uncertainty, 7.2%, so $\Delta g = 0.072 \times 9.82 = 0.71$: $g = (9.8 \pm 0.7)\ \text{m s}^{-2}$. Round the uncertainty to one or two significant figures and give the value to the same decimal place.

■ Planning an experiment (Paper 5)

Question 1 of Paper 5 describes a relationship and asks you to plan an experiment to test it. You never do the experiment, so every mark is for what you write: an examiner must be able to follow your plan. Cover four things.

1. **Variables.** Name the **independent variable** 自变量 (the one you change), the **dependent variable** 因变量 (the one you measure) and the most important **control variables** 控制变量 (the ones you keep constant), and say **how** you keep each one constant.
2. **Method.** Draw a labelled diagram of the apparatus. For every quantity, name the instrument that measures it and say how you use it. Say how you change the independent variable.

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3. **Analysis.** Say which graph you plot, what shape shows the relationship is true, and how you find each constant from the gradient or the intercept.
4. **Safety and detail.** Give one precaution for a hazard in **this** experiment, and add the practical detail that makes the readings reliable.

> **Worked example.** It is suggested that the terminal velocity v of a steel ball falling through oil depends on its radius r as $v = kr^n$, where k and n are constants. Plan an experiment to test this relationship and find n and k.

- **Variables.** Independent: r. Dependent: v. Control: the oil (the same oil, kept at the same temperature, because its viscosity changes a lot with temperature: stand the tube in a water bath and check with a thermometer) and the material of the balls (all steel, so the density is the same).
- **Method.** Fill a tall, wide glass tube with oil and put two rubber bands round it, a distance d apart, measured with a metre rule. Put the upper band well below the surface so the ball has reached terminal velocity before it passes. Measure the diameter of each ball with a **micrometer** 千分尺 in several directions, take the mean and halve it to get r. Drop the ball at the centre of the tube, start a stopwatch as it passes the upper band and stop it at the lower band: $v = d/t$. Use at least five different ball sizes.
- **Analysis.** Taking logarithms, $\lg v = n \lg r + \lg k$. Plot $\lg v$ against $\lg r$. If the graph is a straight line, the relationship is true; the gradient is n, and the y-intercept is $\lg k$, so $k = 10^{\text{intercept}}$.
- **Safety and detail.** Stand the tube in a tray to catch spilt oil, which makes the floor slippery. Check that the ball has reached terminal velocity by timing it over two equal distances: the times should be equal. Repeat each timing and take the mean. Use a tube that is wide compared with the balls, so the walls do not slow them, and take the balls out with a magnet.

Exam tip. "Measure the speed" earns nothing on its own: say what you measure, with which instrument, and how the speed is calculated from it. Name the graph with its axes ("plot $\lg v$ on the y-axis against $\lg r$"), not just "draw a graph".

Scalars and vectors

A **scalar** 标量 has size only. A **vector** 矢量 has both size and direction.

Examples from the syllabus:

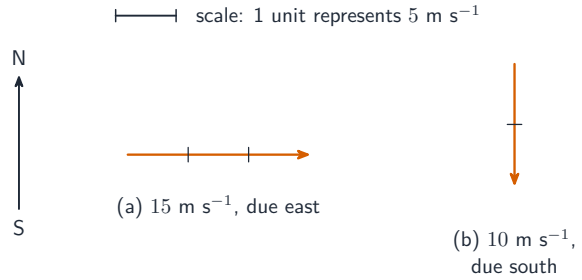
- scalars: mass, time, temperature, energy, work, power, distance, speed, pressure, density, **electric charge** 电荷
- vectors: **displacement** 位移, velocity, acceleration, force (including weight), **momentum** 动量

Quick test: if it makes sense to ask "in which direction?", the quantity is a vector. You cannot ask "in which direction is the temperature?", so temperature is a scalar.

You can ask "in which direction is the velocity?", so velocity is a vector.

Adding and subtracting vectors

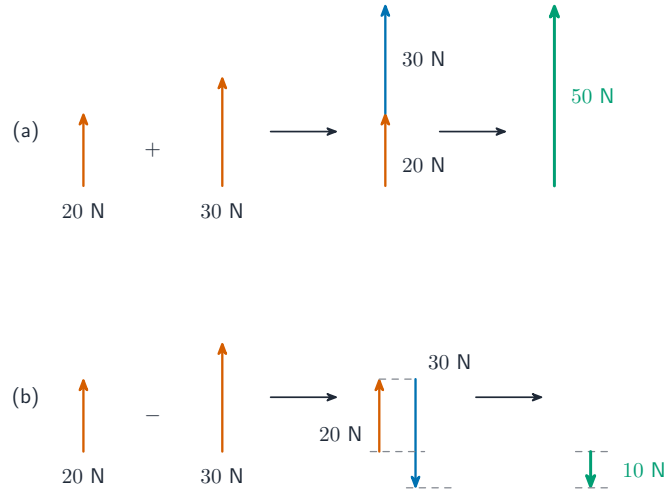
A vector is drawn as an arrow: the **direction** of the arrow gives the direction of the quantity, and the **length** of the arrow (drawn to scale) gives the magnitude.



Vectors represented as arrows drawn to scale

To add two **coplanar** 共面 vectors (vectors in the same flat plane), draw them **tip to tail**. The **resultant** 合矢量 goes from the tail of the first arrow to the tip of the second.

To find $\vec{X} - \vec{Y}$, add the reverse of $\vec{Y}$: $\vec{X} + (-\vec{Y})$. The reverse of $\vec{Y}$ has the same size as $\vec{Y}$ but points the opposite way.



Adding and subtracting parallel vectors

If the two vectors are at right angles (90°), the size of the resultant is

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$$|\vec{R}| = \sqrt{X^2 + Y^2},$$

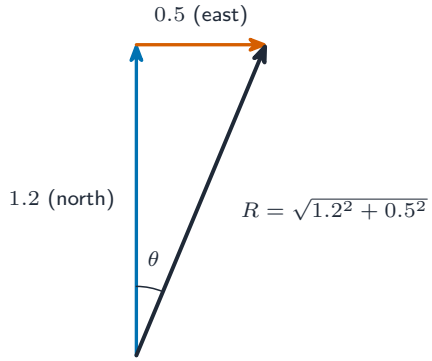
and its direction comes from $\tan\theta = Y/X$.

> **Worked example.** A swimmer heads north at 1.2 m s^{-1} across a river that flows east at 0.5 m s^{-1}. Find the size and direction of the resultant velocity.

The two velocities are perpendicular, so

$$|\vec{R}| = \sqrt{1.2^2 + 0.5^2} = \sqrt{1.69} = 1.3 \text{ m s}^{-1},$$

at an angle $\tan\theta = 0.5/1.2$, giving $\theta \approx 23°$ east of north.



Two perpendicular vectors add to a resultant of size $\sqrt{X^2 + Y^2}$ at angle θ

If two vectors have the same size F with an angle 2α between them, the resultant has size $2F\cos\alpha$ and lies along the line that cuts the angle in half.

> **Worked example.** Two forces of 6.0 N act on an object with 60° between them. Find the resultant.

Here $2\alpha = 60°$, so $\alpha = 30°$ and $R = 2 \times 6.0 \times \cos 30° = 10.4$ N, along the line halfway between the two forces. Resolving gives the same answer: each force has a component $6.0\cos 30°$ along that line, and their components at right angles to it cancel.

For two vectors at any other angle, either make a **scale drawing** 按比例作图 of the vector triangle and measure the resultant, or resolve both vectors into perpendicular components, add the components, and recombine with $\sqrt{X^2 + Y^2}$.

> **Worked example.** A ball moving east at 5.0 m s^{-1} is hit so that it moves north at 5.0 m s^{-1}. Find its change in velocity.

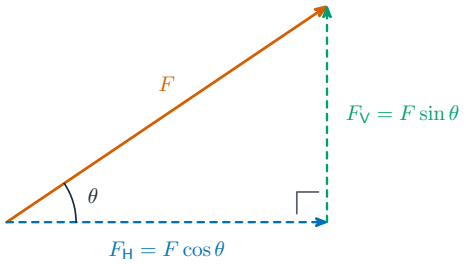
A change in velocity is a vector subtraction: $\Delta\vec{v} = \vec{v}_{\text{final}} - \vec{v}_{\text{initial}}$. Draw 5.0 m s^{-1} north, then add the reverse of the initial velocity, 5.0 m s^{-1} west. The two are perpendicular,

so $|\Delta\vec{v}| = \sqrt{5.0^2 + 5.0^2} = 7.1$ m s^{-1}, pointing north-west. The speed did not change, but the velocity did. That is why a force must have acted on the ball (topic 3).

■ Splitting a vector into perpendicular parts

Any vector can be split into two **perpendicular** 垂直 (at right angles) **components** 分量. Usually these are **horizontal** 水平 and **vertical** 竖直, or along and across a surface. For a vector $\vec{v}$ at angle θ to the horizontal:

$$v_{\mathrm{H}} = v\cos\theta, \qquad v_{\mathrm{V}} = v\sin\theta.$$

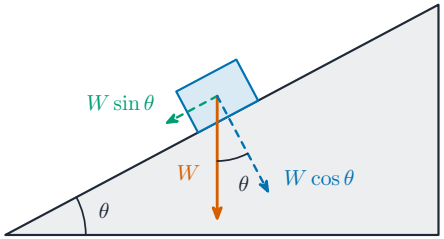


Resolving a vector into horizontal and vertical components

Choose the directions that make the problem easiest. On a slope (an **inclined plane** 斜面), split the weight into one part along the slope and one part at right angles to it:

$$W_{\parallel} = W\sin\theta, \qquad W_{\perp} = W\cos\theta,$$

where θ is the angle of the slope to the horizontal.



On a slope the weight splits into $W\sin\theta$ down the slope and $W\cos\theta$ into the slope; the angle between W and the perpendicular is the slope angle θ

You split a vector into components whenever you need to know how much of it acts in one direction. For example: the part of a force that acts along a slope, or the horizontal and vertical parts of a ball's velocity after it is thrown.

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Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
physical quantity	a numerical magnitude together with a unit
homogeneous equation	an equation in which every term has the same base units
systematic error	an error that shifts every reading in the same direction by the same amount; not reduced by repeating
random error	an error that scatters readings above and below the true value; reduced by repeating and averaging
zero error	the reading an instrument shows when the true value is zero
precision	how close repeated readings are to each other
accuracy	how close a reading, or the mean of several readings, is to the true value
absolute uncertainty	the range within which the true value is expected to lie, in the units of the quantity
percentage uncertainty	the absolute uncertainty divided by the value, multiplied by 100%
scalar	a quantity with magnitude only
vector	a quantity with magnitude and direction

Exam tips

- Give every answer a **unit**, and check **homogeneity** – both sides of an equation must have the same base units.
- Distinguish **random error** (reduce by repeating and averaging) from **systematic error** (a zero or calibration error that repeats do not remove).
- Combine uncertainties: **add absolute** uncertainties when adding/subtracting, **add percentage** uncertainties when multiplying/dividing (and multiply the % by any power).
- Distinguish **precision** (small spread) from **accuracy** (close to the true value).
- Resolve a vector into perpendicular components ($F\cos\theta$, $F\sin\theta$); add vectors tip-to-tail or by components.

■ Common mistakes

- Substituting 5 cm as 5, or 20 g as 20. Convert the prefix first: 0.05 m, 0.020 kg.
- Giving the percentage uncertainty in T^2 as the same as in T. Squaring doubles it; a square root halves it.
- Treating the percentage uncertainty and the absolute uncertainty as the same number.
- Quoting "the smallest division" as the uncertainty of a reading that was hard to take.
- Offering two answers to a definition. Commit to one.

■ Recognising quantities and estimates

Distinguish the quantity from its unit: temperature is measured in kelvin, time in seconds or minutes, potential difference in volts, and spring constant in N/m. A quantity has a measurable numerical magnitude; a unit name or a category alone does not state a quantity. Values may be fractional and units may be non-SI. Visible light has wavelengths roughly 400–700 nm; green is about 540 nm ($1\ \text{nm} = 10^{-9}\ \text{m}$).

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Kinematics

A-Level Physics

Key definitions



A speedometer shows speed: the distance travelled per unit time.

These five quantities come up in almost every **kinematics** 运动学 question. Learn the exact words – the examiner gives marks for precise wording.

- **distance** 距离 – the total length of the path travelled. A **scalar** 标量.
- **displacement** 位移 – the straight-line distance from the start to the end, with a direction. A **vector** 矢量.
- **speed** 速率 – the rate of change of distance with time. A scalar.
- **velocity** 速度 – the rate of change of displacement with time. A vector.
- **acceleration** 加速度 – the rate of change of velocity with time. A vector.

The unit of speed and velocity is m s^{-1}; the unit of acceleration is m s^{-2}.

Two more phrases the examiner uses: **uniform acceleration** 匀加速 means constant acceleration (a straight line on a velocity–time graph), and the **acceleration of free fall** 自由落体加速度 g is the acceleration of an object falling freely in a **uniform gravitational field** 匀强重力场 when air resistance is **negligible** 可忽略的 (small enough to ignore).

A common mistake: **deceleration** 减速度 just means acceleration in the opposite direction to the velocity. It is not a separate quantity.

Speeds are sometimes given in km h^{-1}. Convert before you calculate: $85\ \text{km h}^{-1} = 85 \times 1000/3600 = 23.6\ \text{m s}^{-1}$.



A speed camera measures average speed over a known distance – displacement over time, made into a fine

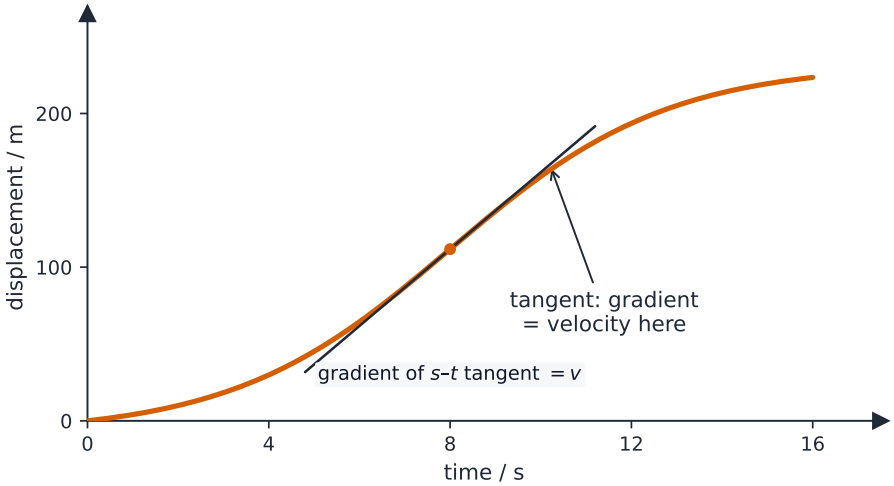
Motion graphs

Many marks come from reading or drawing motion graphs, the **graphical methods** 图像法 the syllabus names. Two graphs matter.

■ Displacement–time graph

The **gradient** 斜率 (steepness) of a displacement–time graph at a point gives the velocity at that moment.

- flat line → the object is at rest.
- straight sloping line → constant velocity (gradient = velocity).
- curved line → changing velocity. Draw a **tangent** 切线 at the point and find its gradient.



Displacement–time graph of a car on a test track

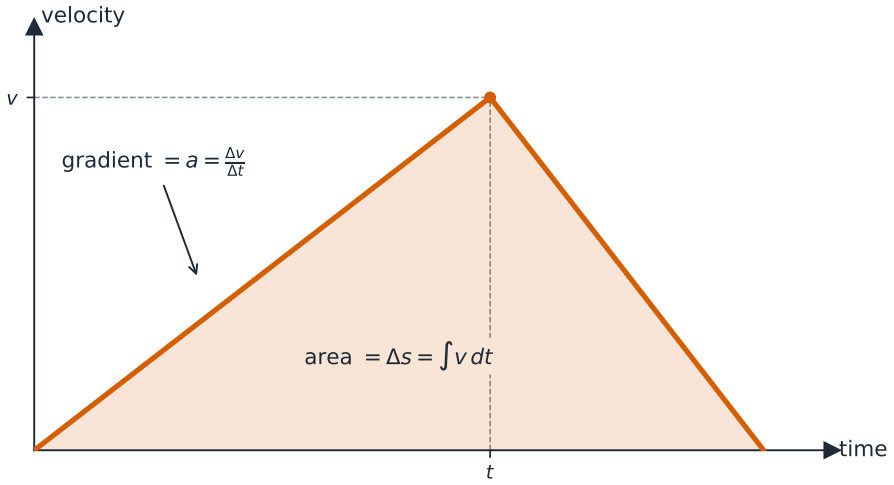
■ Velocity–time graph

The gradient of a velocity–time graph gives the acceleration at that moment.

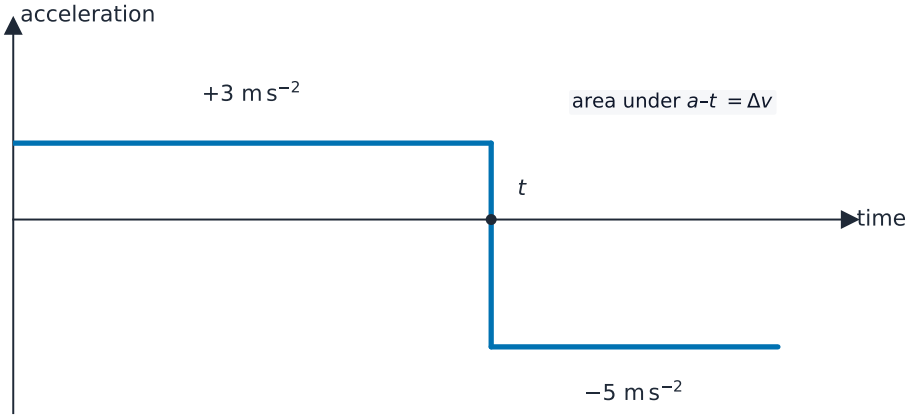
The **area** between the line and the time axis gives the displacement in that time.

- flat line → constant velocity (zero acceleration).
- straight sloping line → uniform acceleration (constant acceleration).
- curved line → changing acceleration.
- area above the time axis is positive displacement; area below is negative (the object moved backwards).

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Velocity–time graph – gradient gives acceleration, area gives displacement

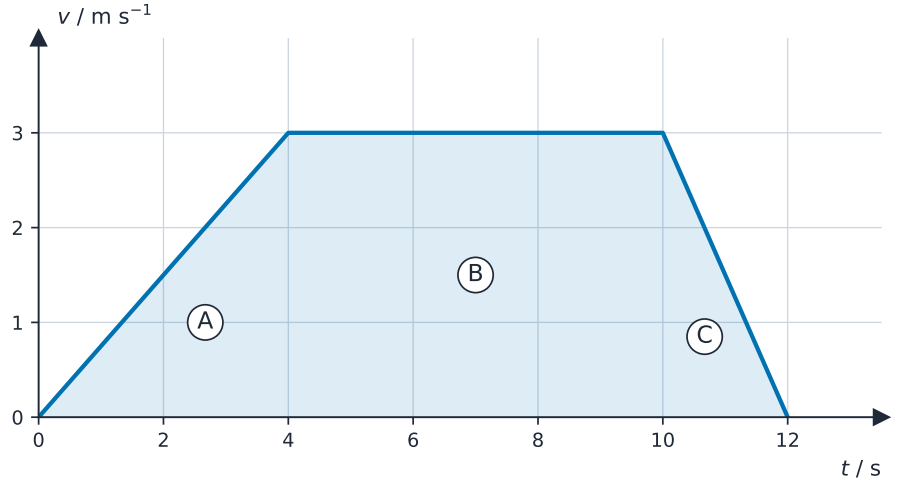


Acceleration–time graph derived from the same motion

To find the displacement, split the area into triangles and rectangles, or count grid squares. Area of a triangle is $\frac{1}{2} \times$ base $\times$ height; area of a rectangle is base $\times$ height.

> **Worked example.** The graph shows the velocity of a lift. Find the acceleration in each stage and the total distance travelled.

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Velocity–time graph of a lift: gradient gives each acceleration, the shaded area gives the distance

- Stage A: gradient $= \dfrac{3.0 - 0}{4.0} = 0.75$ m s^{-2}.
- Stage B: the line is flat, so the acceleration is zero.
- Stage C: gradient $= \dfrac{0 - 3.0}{2.0} = -1.5$ m s^{-2} (a deceleration of 1.5 m s^{-2}).
- Distance: triangle A $= \frac{1}{2} \times 4.0 \times 3.0 = 6.0$ m, rectangle B $= 6.0 \times 3.0 = 18$ m, triangle C $= \frac{1}{2} \times 2.0 \times 3.0 = 3.0$ m; total 27 m.

When the graph is a curve, the area is still the displacement: count the squares under the curve (half a square or more counts as one) and multiply by the value of one square.

The four SUVAT equations

For motion in a straight line with uniform acceleration, we use five symbols: starting velocity u, final velocity v, acceleration a, displacement s, and time t. Four equations link them:

$$v = u + at$$

$$s = ut + \tfrac{1}{2}at^2$$

$$s = \tfrac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

Each equation uses four of the five symbols. To pick the right one: write down what you know and what you want, then choose the equation with exactly those four.

> **Worked example.** A car accelerates uniformly from 8 m s^{-1} to 20 m s^{-1} over a distance of 56 m. Find its acceleration.

We know u, v and s and want a, so use $v^2 = u^2 + 2as$:

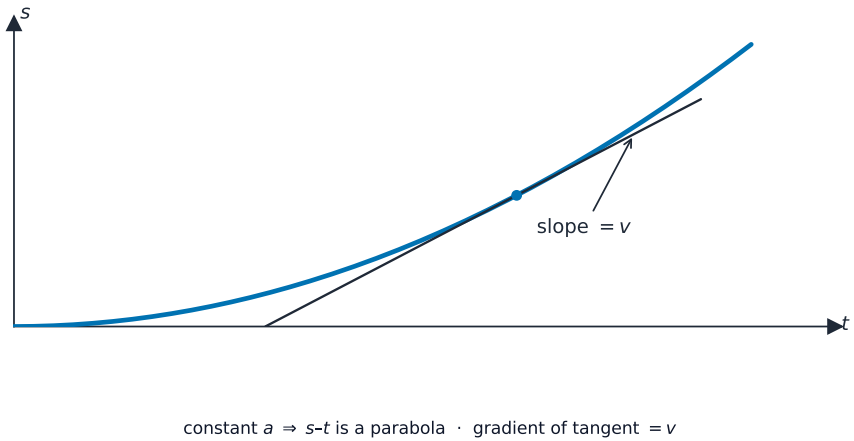
$$20^2 = 8^2 + 2a(56) \quad \Rightarrow \quad 336 = 112\,a \quad \Rightarrow \quad a = 3.0 \text{ m s}^{-2}.$$

Where the SUVAT equations come from

You should be able to get these from the definitions of velocity and acceleration. Think of the velocity–time graph of the motion: a straight line from u at $t = 0$ to v at time t.

- $v = u + at$: acceleration is the gradient, $a = (v - u)/t$, so $v = u + at$.
- $s = \frac{1}{2}(u + v)t$: displacement is the area under the line, a **trapezium** 梯形 with parallel sides u and v and width t, so $s = \frac{1}{2}(u + v)t$.
- $s = ut + \frac{1}{2}at^2$: put $v = u + at$ into the area: $s = \frac{1}{2}(u + u + at)t = ut + \frac{1}{2}at^2$.
- $v^2 = u^2 + 2as$: from the first equation $t = (v - u)/a$; put this into $s = \frac{1}{2}(u + v)t$ to get $2as = (v + u)(v - u) = v^2 - u^2$.

A "derive" question wants exactly these steps, each starting from a definition or an equation already established, with no numbers.



Displacement–time graph for uniform acceleration – the slope at any point equals the instantaneous velocity

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If a question asks "which equation can be found using only the gradient of a velocity–time graph?", the answer is $v = u + at$ (the gradient is the acceleration).

"Show that" questions. When the question gives the answer ("show that the height is 3.2 km"), the marks are for the method. State the equation, substitute every value with its unit, and give the result to one more significant figure than the value quoted (3.24 km, which rounds to 3.2 km). Reaching the quoted value proves nothing on its own.

Choosing a positive direction

Pick a positive direction at the start and keep it. Anything pointing the other way gets a minus sign. For a ball thrown straight up, if "up" is positive: u is positive, $a = -g$ (**gravity** 重力 pulls down), and at the highest point the displacement is positive but the velocity is zero.

Free fall under gravity

When **air resistance** 空气阻力 can be ignored, an object in **free fall** 自由落体 has a constant acceleration $g \approx 9.81$ m s^{-2} downwards. This is the same for every mass.

For a ball dropped from rest and falling a distance h:

$$h = \tfrac{1}{2}gt^2, \qquad v = gt, \qquad v^2 = 2gh.$$

For a ball thrown straight up with speed u:

- greatest height: put $v = 0$ in $v^2 = u^2 - 2gh$, giving $h = u^2/(2g)$.
- time to reach the top: put $v = 0$ in $v = u - gt$, giving $t = u/g$.
- total time to fall back to the start height: $2u/g$ (the motion is **symmetric** 对称).

> **Worked example.** A ball is thrown straight up at 20 m s^{-1}. Find the greatest height it reaches (take $g = 9.81$ m s^{-2}).

At the highest point $v = 0$, so from $h = u^2/(2g)$:

$$h = \frac{20^2}{2 \times 9.81} = \frac{400}{19.62} \approx 20.4 \text{ m}.$$

When air resistance is not negligible

The equations above assume the only force is the weight. With air resistance, the resultant force on a falling object is smaller than its weight, so the acceleration is less than g; and because air resistance grows with speed, the acceleration keeps decreasing as the object speeds up. On a velocity–time graph the line curves, its gradient falling towards zero as the object approaches **terminal velocity** 收尾速度 (topic 3). For a projectile, air resistance shortens the range and lowers the maximum height, and the path is no longer a symmetrical **parabola** 抛物线: the object comes down more

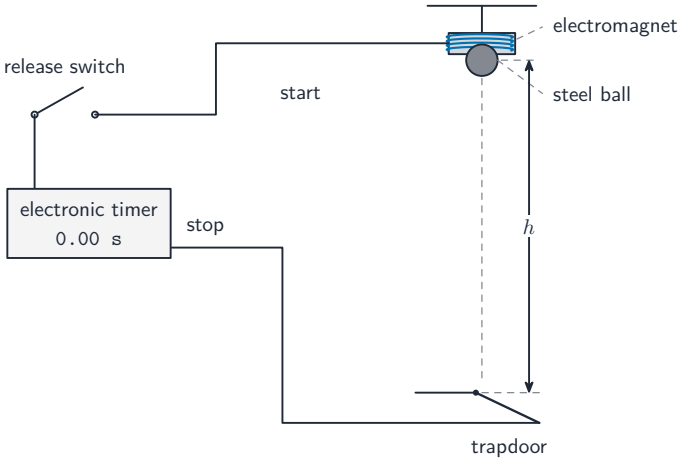
steeply than it went up. A "state and explain" question wants the force first, then its effect on the acceleration, then the effect on the motion.

Experiment to find g

A common method: drop an object from rest, then measure the distance h it falls and the time t it takes. Then

$$g = \frac{2h}{t^2}.$$

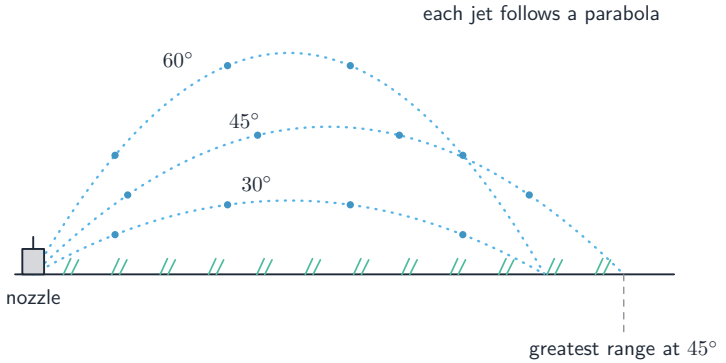
Repeat for several heights and plot h against t^2. The gradient of the best straight line is $g/2$, so g is twice the gradient. Repeating reduces **random error** 随机误差. An electronic timer – using **light gates** 光电门, or a switch the ball hits – removes **reaction-time** 反应时间 error.



Experimental set-up for measuring the acceleration due to free fall

Motion in two directions

When an object moves at constant velocity in one direction (say **horizontal** 水平) and speeds up in a direction at right angles to it (say **vertical** 竖直, under gravity), the two motions do not affect each other. Treat each direction on its own, with its own SUVAT equation.



Water jets from a sprinkler trace parabola paths – a real example of projectile motion

Horizontal throw

An object thrown horizontally with speed u_H from height h, with air resistance ignored:

- **horizontal**: constant velocity u_H. After time t, the horizontal distance is $x = u_H t$.
- **vertical**: starts from rest and speeds up downwards at g. After time t, it has fallen $y = \frac{1}{2}gt^2$ and has vertical velocity $v_V = gt$.

The time to reach the ground depends **only** on the height h, not on u_H. Solve $h = \frac{1}{2}gt^2$ for t; then the horizontal **range** 射程 is $u_H t$.

> **Worked example.** A ball is thrown horizontally at 15 m s^{-1} from the top of a cliff 20 m high. Find the time it takes to land and how far from the base it lands (take $g = 9.81$ m s^{-2}).

Vertical motion gives the time: from $h = \frac{1}{2}gt^2$,

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 20}{9.81}} \approx 2.0 \text{ s}.$$

Horizontal motion then gives the range: $x = u_H t = 15 \times 2.0 \approx 30$ m.

The horizontal-velocity graph is a flat line at u_H. The vertical-velocity graph is a straight line from the origin with gradient g.

Comparing two objects. If object A is dropped and object B is thrown horizontally from the same height at the same moment, both reach the ground at the same time: their vertical motions are identical. If instead B is thrown with an upward component, it takes longer, because it must first rise and come back to the start height before it falls the same distance. Its speed on landing is still found from energy: the kinetic energy gained equals the potential energy lost, whatever the direction of the throw, so a ball launched at any angle with the same speed from the same height lands at the

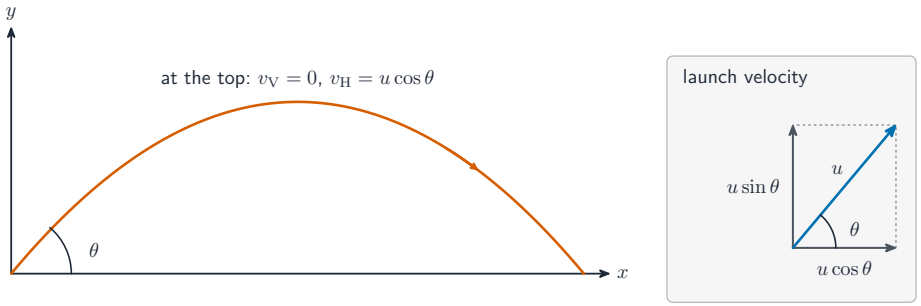
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same speed (topic 5).

Projectile at an angle

A **projectile** 抛体 thrown at speed u at angle θ above the horizontal:

- horizontal **component** 分量 of the starting velocity: $u_{\mathrm{H}} = u\cos\theta$ (stays constant during the flight).
- vertical component of the starting velocity: $u_{\mathrm{V}} = u\sin\theta$ (gets smaller, becomes zero at the top, then grows downwards).

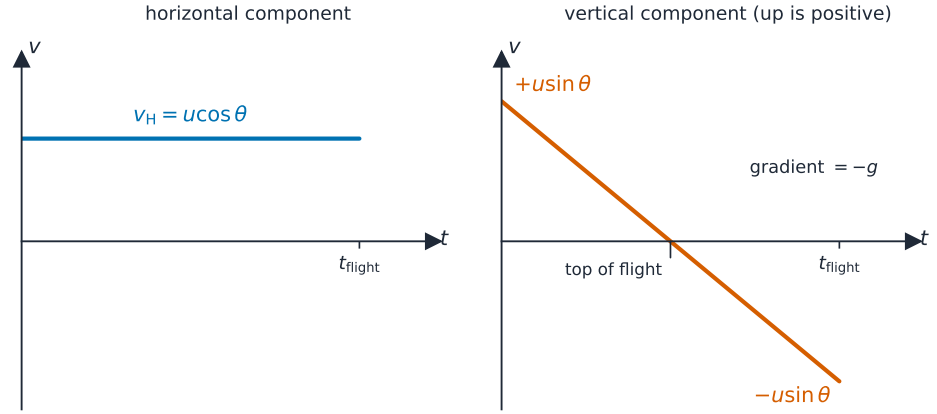


Projectile launched at angle θ – horizontal and vertical motions are independent

At the highest point, $v_{\mathrm{V}} = 0$, but v_{H} is still $u\cos\theta$. The time to the top is $t_{\mathrm{up}} = u\sin\theta/g$; the total flight time (back to the start height) is $2t_{\mathrm{up}}$.

Sketching the velocity graphs. Examiners often ask you to sketch v_{H} and v_{V} against time on the same axes, taking upwards as positive. The horizontal component is a flat line at $u\cos\theta$ for the whole flight. The vertical component is a straight line of gradient $-g$: it starts at $+u\sin\theta$, crosses zero at the top of the flight, and reaches $-u\sin\theta$ on landing at the start height. Label each line, mark the times where the lines start and stop, and make the crossing sit exactly at t_{up}.

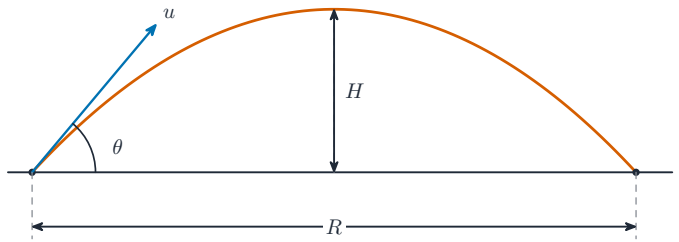
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Velocity–time sketches for a projectile: the horizontal component stays constant; the vertical component falls in a straight line through zero at the top

Worked example. A ball leaves the ground at 18 m s^{-1} at 60° to the horizontal. Show that it reaches its maximum height at $t = 1.6$ s.

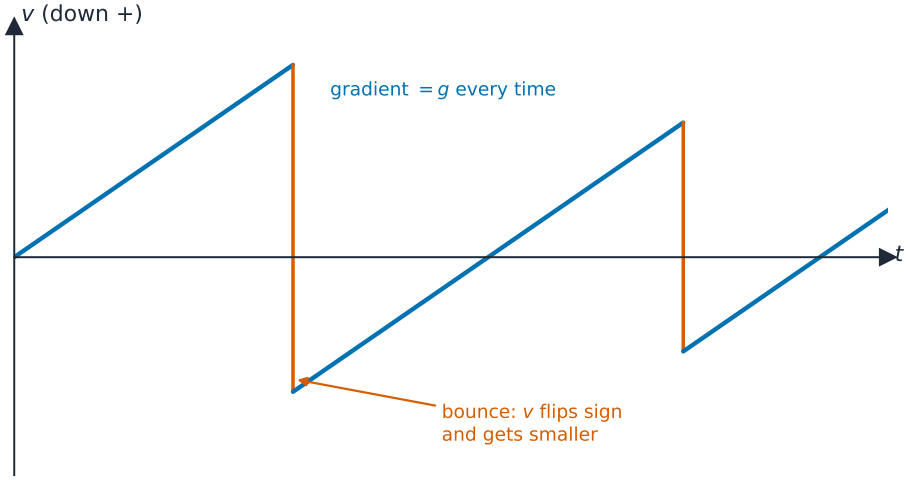
$u_{\mathrm{V}} = 18\sin 60° = 15.6$ m s^{-1}. At the top $v_{\mathrm{V}} = 0$, so from $v = u + at$ with $a = -9.81$ m s^{-2}: $t = 15.6/9.81 = 1.59$ s ≈ 1.6 s.



Range R of a projectile launched from and landing on level ground

Bouncing ball

When a ball bounces, its velocity–time graph is a set of straight sloping lines (constant g) with a sudden jump at each bounce (the velocity flips direction, and gets smaller if some **energy** 能量 is lost). Add up the times and the distances across the bounces.



A bouncing ball: every sloping line has gradient g; the velocity flips sign and shrinks at each bounce

Read such a graph carefully: the gradient of every sloping line is the same g; the ball is at its highest point wherever the line crosses the time axis; and the height of each bounce comes from the area of the triangle above (or below) the axis for that flight.

Two objects meeting

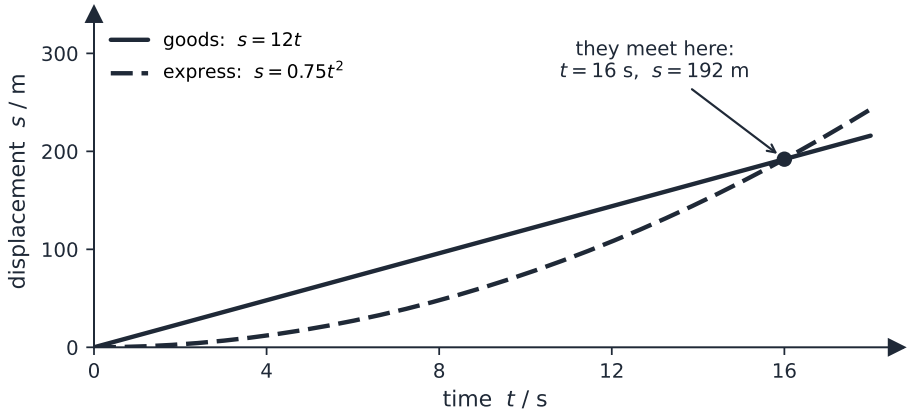
When two objects move along the same line in different ways, write a displacement equation for each. Use the same start time and the same positive direction. Then set the two displacements equal (or set their difference to a given gap).

For a goods train at constant velocity u_{G} and an express train starting from rest with acceleration a, both passing the same point at $t = 0$:

$$s_{\mathrm{G}} = u_{\mathrm{G}} t, \qquad s_{\mathrm{E}} = \tfrac{1}{2} a t^2.$$

They are level again when $s_{\mathrm{G}} = s_{\mathrm{E}}$, giving $t = 2u_{\mathrm{G}}/a$.

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Meet where the displacement–time graphs cross (same s at same t)

Tips for solving problems

1. **Draw a diagram** and mark the positive direction.
2. **List the SUVAT symbols** with their known and unknown values, including signs.
3. **Choose the SUVAT equation** with exactly the four symbols you have, plus the one you want.
4. For projectile motion, **split into horizontal and vertical** SUVAT problems, linked only by the time t.
5. Always **check the units** of your answer, and that its size is sensible.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
distance	the total length of the path travelled (a scalar)
displacement	the distance moved in a stated direction, from start to finish (a vector)
speed	the rate of change of distance with time
velocity	the rate of change of displacement with time
acceleration	the rate of change of velocity with time
uniform acceleration	acceleration that is constant in magnitude and direction
acceleration of free fall	the acceleration of an object falling freely under gravity alone, with air resistance negligible

Exam tips

- Use the **SUVAT** equations only for **constant acceleration**; list s, u, v, a, t and pick the equation missing your unknown.
- Choose one direction as **positive** and keep signs consistent (usually $g = -9.81$ m s^{-2}).
- On a **velocity-time graph**, gradient = acceleration and area = displacement.
- For projectiles, treat **horizontal (constant velocity) and vertical ($a = g$) motion separately**, linked by the same time.

Common mistakes

- Saying the heavier ball hits the ground first, or faster, when air resistance is ignored. Both fall with the same acceleration and land at the same speed.
- Taking the gradient of a v–t graph when the question wants the area, or the area when it wants the gradient. Gradient is acceleration; area is displacement.
- Drawing a curve on a v–t graph for a body under constant acceleration. Constant acceleration is a straight line; at a bounce v jumps to the opposite sign (and to a smaller size if energy is lost), and each new flight is again a straight line.
- Using $g = +9.81$ for a ball thrown upwards with "up" as positive. Once up is positive, $a = -9.81$ m s^{-2} for the whole flight, on the way down too.
- Using a SUVAT equation when the acceleration is changing (air resistance, a curved graph). Then only the graph methods work.

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Dynamics

A-Level Physics

Mass, momentum and force

Mass

Mass 质量 tells you how hard it is to change an object's motion. The larger the mass, the larger the **force** 力 needed to give it a certain **acceleration** 加速度. Mass is measured in kilograms (kg) and is a **scalar** 标量.

Momentum

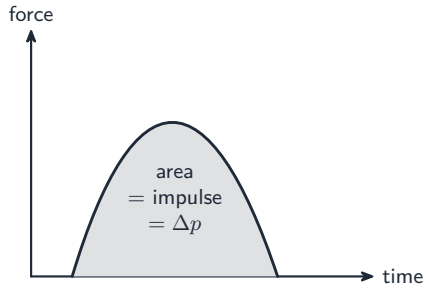
Linear momentum 动量 is the product of mass and velocity:

$$p = mv.$$

Momentum is a **vector** 矢量 – it points the same way as the **velocity** 速度. Its unit is kg m s^{-1}, which is the same as N s.

Force as the rate of change of momentum

Newton's second law, in its general form: the **resultant force** 合力 on an object equals the rate of change of its momentum.



Impulse is the area under a force-time graph, equal to the change in momentum

$$F = \frac{\Delta p}{\Delta t}.$$

When the mass is constant this becomes $F = ma$, because $\Delta p = m\,\Delta v$ and $\Delta v/\Delta t = a$. Cambridge questions often want you to use $F = \Delta p/\Delta t$ directly for a **collision** 碰撞 or an **impulse** 冲量: the average force equals the change in momentum divided by the contact time.

A ball hits a wall with momentum p_1 and bounces back with momentum p_2 in the

opposite direction. The change in momentum is $\Delta p = p_2 - p_1$ (give each direction the correct sign). The average force is $\Delta p/\Delta t$, where Δt is the contact time.

> **Worked example.** A 0.20 kg ball hits a wall at 8.0 m s^{-1} and bounces straight back at 6.0 m s^{-1}. The contact lasts 0.050 s. Find the average force on the ball.

Take the rebound direction as positive, so $u = -8.0$ m s^{-1} and $v = +6.0$ m s^{-1}:

$$\Delta p = m(v - u) = 0.20 \times \big(6.0 - (-8.0)\big) = 2.8 \text{ kg m s}^{-1},$$

$$F = \frac{\Delta p}{\Delta t} = \frac{2.8}{0.050} = 56 \text{ N}.$$

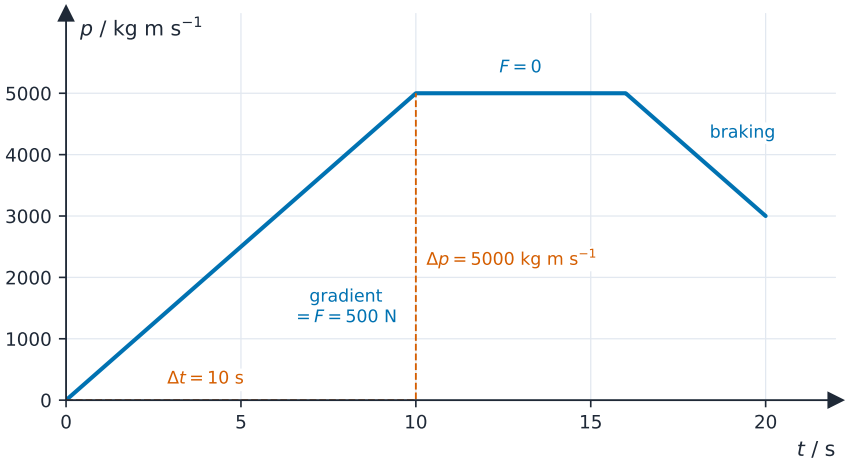
When you know the momentum but not the speed, the change in **kinetic energy** 动能 is

$$\Delta E_\text{k} = \frac{p_2^2 - p_1^2}{2m}.$$

This comes from $E_\text{k} = \frac{1}{2}mv^2 = p^2/(2m)$.

Momentum–time graphs

Because $F = \Delta p/\Delta t$, the **gradient** of a momentum–time graph is the resultant force, just as the gradient of a velocity–time graph is the acceleration. A straight line means a constant resultant force; a horizontal line means zero resultant force; a line sloping down means a force acting against the motion.



The gradient of a momentum–time graph is the resultant force: 500 N while accelerating, zero while the momentum is constant, negative while braking

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> **Worked example.** The graph shows the momentum of a motorcycle. Find the resultant force on it during the first 10 s.

Gradient $= \Delta p/\Delta t = 5000/10 = 500$ N. Between 10 s and 16 s the momentum is constant, so the resultant force is zero: the driving force just balances the resistive forces. From 16 s to 20 s the momentum falls by 2000 kg m s^{-1}, so the resultant force is -500 N, acting backwards.

If a question tells you the speed is changing while the resultant force stays constant, that force cannot be air resistance: drag depends on speed, so it would change as the speed changes, and drag always acts against the motion.

Newton's three laws of motion



A rocket pushes gas down; by Newton's third law the gas pushes the rocket up.

First law

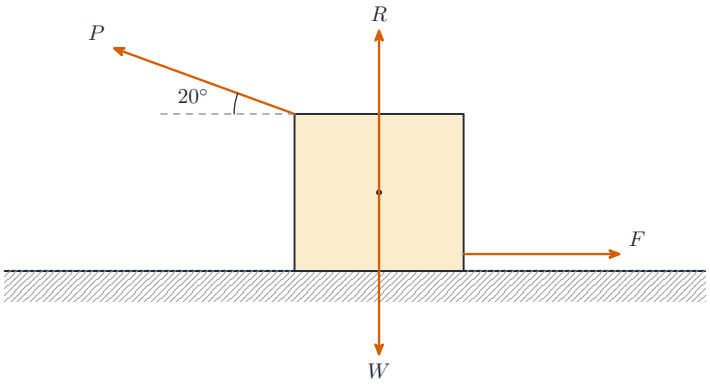
An object stays at rest, or keeps moving at constant velocity in a straight line, unless a resultant **external force** 外力 acts on it. In short: zero resultant force means zero acceleration.

Second law

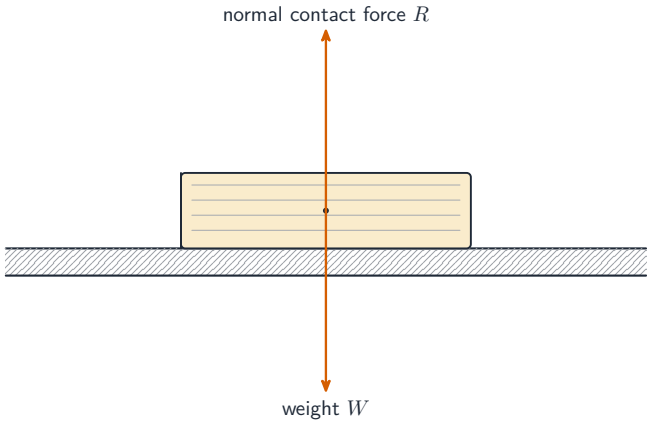
The resultant force on an object equals its rate of change of momentum, and acts in the same direction as that change. In SI units,

$$F = \frac{\Delta p}{\Delta t} = ma \quad \text{(for constant mass)}.$$

Acceleration and resultant force always point the **same way**.



Free-body diagram showing all forces on a block being pulled at an angle



Weight and normal contact force on a book resting on a table

Third law

When body A pushes on body B, body B pushes back on body A with an **equal and opposite** force. The two forces:

- act on **different objects**,
- are of the **same type** (both gravitational, both contact, both **electrostatic** 静电, and so on),
- have the same size and opposite direction.

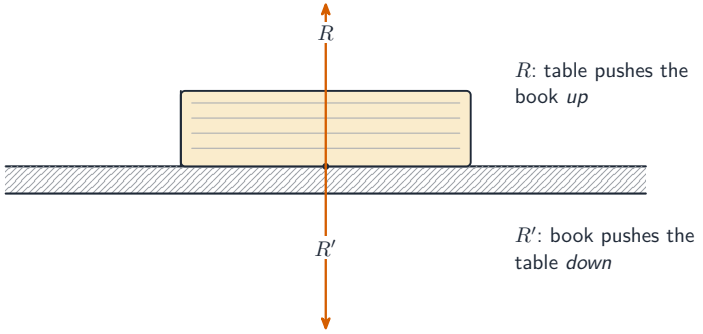
A common trap: the **weight** 重力 of a block on a table and the **normal contact force** 支持力 from the table are **not** a third-law pair (they act on the same object and

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are different types). The third-law partner of the block's weight is the pull the block makes on the Earth. The third-law partner of the table's contact force is the push the block makes on the table.

To name a third-law partner, swap the two bodies in the sentence. "The back wheel of a bicycle pushes backwards on the road" pairs with "the road pushes forwards on the back wheel", and that forward push is what drives the bicycle. "The Earth pulls the Moon" pairs with "the Moon pulls the Earth", with the same size of force.

For a rocket: the **thrust** 推力 on the rocket and the force on the gases are a third-law pair (the engine pushes the gas down, the gas pushes the engine up). Weight and air resistance are not part of this pair.



Newton's third-law pair: R on the book (up) and R' on the table (down)

Weight

Weight is the force on an object from a **gravitational field** 重力场. Near the Earth's surface,

$$W = mg,$$

where $g \approx 9.81\ \text{m s}^{-2}$ is the acceleration of free fall. Weight is a vector that points towards the centre of the Earth. Do not mix it up with mass: mass is the same everywhere, but weight changes with place.

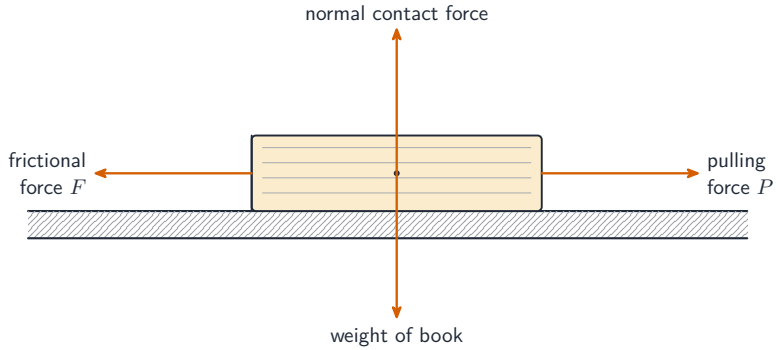
On the Moon g is about $1.6\ \text{m s}^{-2}$, so an astronaut's mass is unchanged but their weight is about one sixth of its value on Earth. When a multiple-choice question asks which statement describes weight, the answer is the one that says "the force on the object due to a gravitational field", not "mass times acceleration" and not "the resultant force".

Non-uniform motion: friction, drag and terminal velocity

Friction and drag forces

A **friction** 摩擦力 force between two solid surfaces acts along the surface and opposes the sliding. A **viscous** 黏性 or **drag** 阻力 force is the resistive force from a **fluid** 流体 (a liquid or gas) on an object moving through it; air resistance is the case for air. You do not need to use any **coefficient** 系数 of friction or viscosity.

A simple model: the drag gets bigger as the speed gets bigger. At zero speed, the drag is zero. Drag also grows with the **cross-sectional area** 横截面积 of the object and with the density of the fluid; a parachute works by making the area, and so the drag, much larger. These frictional forces and drag forces are all resistive forces: they act against the direction of motion and transfer kinetic energy to thermal energy.



Free-body diagram of a book being pulled on a table

An object falling through air

For an object dropped from rest and falling through air:

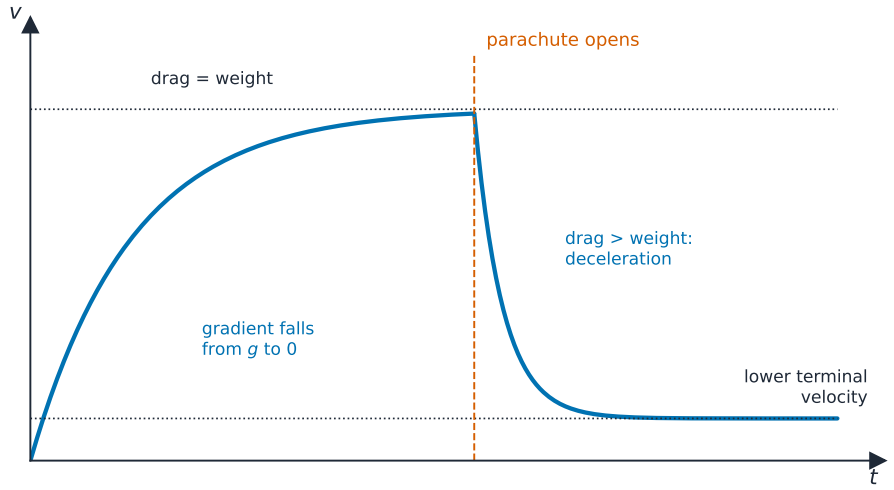
1. At first, only weight acts, so the object speeds up downwards at g.
2. As the speed grows, the upward drag grows. The resultant force gets smaller, so the acceleration gets smaller.
3. In the end, the drag equals the weight. The resultant force is zero, the acceleration is zero, and the speed stays constant – the **terminal velocity** 收尾速度.

On a velocity–time graph, the line starts straight with gradient g, then bends and flattens at the terminal velocity. This shape (fast start, then slowing acceleration, then constant speed) is how "falling with air resistance" differs from "free fall in a vacuum" in a **uniform gravitational field** 匀强重力场. On an acceleration–time graph the acceleration starts at g and decreases, curving down to zero.

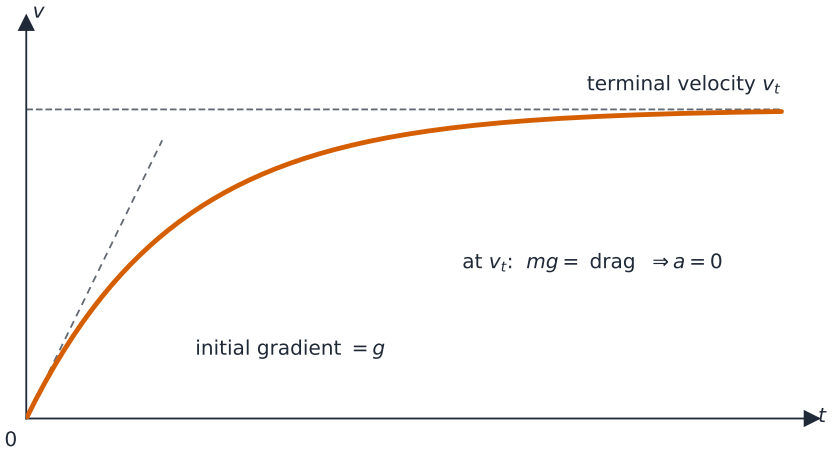
Two spheres of the same size but different density have different terminal velocities: the denser sphere has the larger weight, so it needs a larger drag to balance it, and

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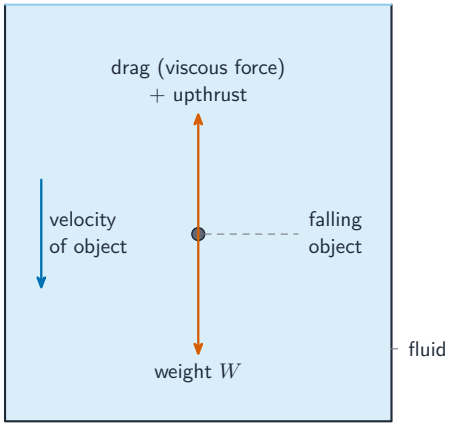
drag only reaches that value at a higher speed. That is also why a parachute changes everything: when it opens, the drag suddenly exceeds the weight, the resultant force is upwards, and the skydiver **decelerates** (the acceleration is upwards while the velocity is still downwards) until the drag has fallen back to equal the weight at a new, much lower terminal velocity.



A skydiver's velocity–time graph: the acceleration falls from g to zero at the first terminal velocity; opening the parachute makes the drag exceed the weight, so the velocity falls to a new, lower terminal velocity



Velocity–time graph for an object falling through air



Forces on a falling object in a fluid

Falling through a liquid: three forces

A ball falling through a liquid has three forces on it: its weight downwards, the **upthrust** 浮力 upwards, and the **viscous drag** upwards. The upthrust comes from **hydrostatic pressure** 流体静压强: the pressure in a liquid increases with depth, so the liquid pushes harder on the bottom of the ball than on the top, and the resultant of these pushes is upwards. Its size is the weight of the liquid the ball displaces, $U = \rho_{\text{liquid}} V g$ (topic 4). The drag grows with speed. When weight = upthrust + drag, the resultant force is zero and the ball falls at its terminal speed. A "draw labelled arrows" question wants exactly these three, with the upthrust and drag both up and the weight down.

> **Worked example.** A steel ball of radius 2.0 mm and weight 2.56×10^{-3} N falls through oil of density 850 kg m^{-3}. The viscous drag on it is $F = 6\pi\eta r v$, where η is the **viscosity** 黏度 of the oil and v the speed. (a) Find the SI base units of η. (b) Show that the upthrust on the ball is 2.8×10^{-4} N. (c) Taking $\eta = 0.20$ in SI units, find the terminal speed.

(a) $\eta = F/(6\pi r v)$, so its units are $\text{N}/(\text{m} \cdot \text{m s}^{-1}) = \text{kg m s}^{-2}/(\text{m}^2\ \text{s}^{-1}) = \text{kg m}^{-1}\ \text{s}^{-1}$.
(b) $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(2.0 \times 10^{-3})^3 = 3.35 \times 10^{-8}$ m^3, so $U = 850 \times 3.35 \times 10^{-8} \times 9.81 = 2.79 \times 10^{-4}$ N $\approx 2.8 \times 10^{-4}$ N.
(c) At the terminal speed, drag $= W - U = 2.56 \times 10^{-3} - 2.8 \times 10^{-4} = 2.28 \times 10^{-3}$ N. So $v = F/(6\pi\eta r) = 2.28 \times 10^{-3}/(6\pi \times 0.20 \times 2.0 \times 10^{-3}) = 0.30$ m s^{-1}.

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Why the spread-eagle pose? Spreading out gives the largest area, so the most drag. The bigger the drag, the sooner drag balances weight – and the lower the steady terminal velocity they fall at

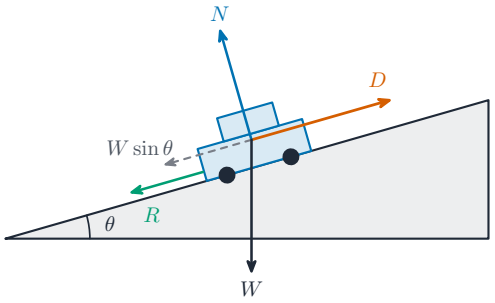
Energy during a terminal-velocity fall

At terminal velocity, a **parachutist** 跳伞者 has constant kinetic energy. But the **gravitational potential energy** 重力势能 keeps falling as they go down. Where does it go? Almost all of it turns into **thermal energy** 热能 of the air around them. It does **not** become kinetic energy of the parachutist – that stays constant.

Cyclist or car at constant speed

A vehicle at constant speed on a flat road has zero resultant force. The forward **driving force** 驱动力 is equal and opposite to the total resistive force (friction, air resistance, and rolling resistance). At higher speed the drag is larger, so the driving force must be larger too – and so the **power** 功率 must be larger.

On a slope the weight has a component along the road. Going up at constant speed, the driving force must balance the resistive force **and** the part of the weight that pulls down the slope, $W \sin\theta$; going down, that same component helps the motion.



A car climbing a slope at constant speed: along the slope the driving force balances the resistive force plus the weight's component $W \sin\theta$

Worked example. A car of mass 1500 kg climbs a road inclined at 12° to the horizontal at a constant 30 m s^{-1}. The total resistive force is 1600 N. Find the driving force and the useful output power of the engine.

Constant speed means zero resultant force along the slope: $D = 1600 + 1500 \times 9.81 \times \sin 12° = 1600 + 3060 = 4660$ N ≈ 4700 N. Power is force times velocity (topic 5): $P = Dv = 4660 \times 30 = 1.4 \times 10^5$ W.

Conservation of linear momentum



In a crash, a large force acts over a very short time to change momentum.

The principle

For a system with no resultant external force, the total momentum stays constant. This is **conservation of momentum** 动量守恒. Stated for the two marks the examiner gives: **the total momentum of a system of objects remains constant provided no resultant external force acts on the system.** Both halves are needed: "total momentum is constant" alone scores one.

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It always holds when there is no outside resultant force – in collisions, **explosions** 爆炸, and **recoil** 反冲. In two dimensions, momentum is conserved along each direction on its own.

Worked example. A 2.0 kg trolley and a 3.0 kg trolley are held together against a compressed spring, then released from rest. The 2.0 kg trolley flies off at 6.0 m s^{-1}. Find the speed of the other trolley.

The total momentum stays zero (it started at rest), so

$$0 = (2.0)(6.0) + (3.0)(-v) \quad \Rightarrow \quad v = \frac{12}{3.0} = 4.0 \text{ m s}^{-1}$$

in the opposite direction.



equal size, opposite direction
(forces act on different objects)

Newton's third law in an isolated two-particle system: equal and opposite forces

Elastic and inelastic collisions

In **every** collision, momentum is conserved (if there is no outside resultant force).

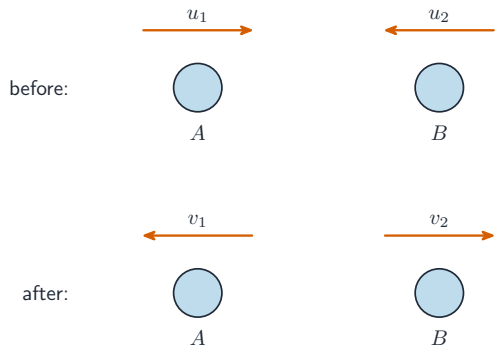
An **elastic collision** 弹性碰撞 is one where the total kinetic energy is **also** conserved. A quick test: in an elastic collision, the **relative speed** 相对速率 of approach equals the relative speed of separation.

In an **inelastic collision** 非弹性碰撞, momentum is conserved but kinetic energy goes down – some becomes thermal, sound, or **deformation** 形变 energy. If the two objects stick together, the collision is perfectly inelastic.

Solving collision problems (one dimension)

For two objects with masses m_1, m_2 and starting velocities u_1, u_2 that hit **head-on** 正面, write

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2.$$



Head-on collision: velocities before and after

Use **signed** velocities (positive in one chosen direction). If the collision is elastic, add the relative-speed equation

$$u_1 - u_2 = -(v_1 - v_2),$$

or, the same thing, $\tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2$. That gives two equations for two unknowns.

> **Worked example.** A 1500 kg car moving at 12 m s^{-1} runs into a stationary 1000 kg car and they lock together. Find their common velocity just after the collision.

Momentum is conserved (the cars stick, so $v_1 = v_2 = v$):

$$1500 \times 12 + 1000 \times 0 = (1500 + 1000)\,v \quad \Rightarrow \quad v = \frac{18\,000}{2500} = 7.2 \text{ m s}^{-1}.$$

Questions often go on to ask what fraction of the kinetic energy is transferred to other forms. Before: $E_\text{k} = \tfrac{1}{2}\times1500\times12^2 = 1.08\times10^5$ J. After: $\tfrac{1}{2}\times2500\times7.2^2 = 6.48\times10^4$ J. So 4.3×10^4 J, which is 40% of the original kinetic energy, becomes thermal energy, sound and deformation of the cars. Momentum is conserved; kinetic energy is not.

> **Worked example.** A stationary nucleus of mass $222\,u$ decays by emitting an **alpha particle** α 粒子 of mass $4\,u$ at 1.6×10^7 m s^{-1}. Find the speed of the remaining nucleus.

Before the decay the total momentum is zero, so afterwards the two momenta are equal and opposite: $218\,u \times v = 4\,u \times 1.6 \times 10^7$, giving $v = 2.9 \times 10^5$ m s^{-1} in the opposite direction to the alpha particle. The unit u cancels, so its value is never needed. In a decay the momentum, the charge (proton number), the nucleon number and the total mass–energy are all conserved (topic 11).

A useful result for a head-on elastic collision of mass m with a **stationary** 静止 mass M:

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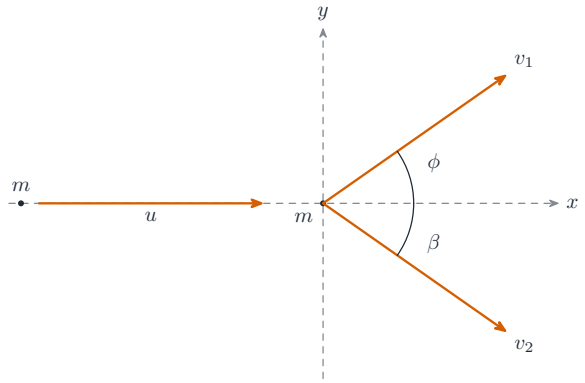
$$v_m = \frac{m - M}{m + M}u, \qquad v_M = \frac{2m}{m + M}u.$$

Collisions in two dimensions

If the objects move in two dimensions, split the velocities into **perpendicular** 垂直 **components** 分量 and apply conservation of momentum along each direction on its own. For a collision where the objects hit at an angle, choose one axis along the first object's motion and one across it. The total momentum is conserved along each axis.

> **Worked example.** On a frictionless surface, ball A has momentum 4.0 N s due east and ball B has momentum 3.0 N s due north. They collide and stick together. Find the momentum of the combined object.

Momentum is conserved along each axis: 4.0 N s east and 3.0 N s north after the collision, exactly as before. The total momentum is the vector sum, $\sqrt{4.0^2 + 3.0^2} = 5.0$ N s at $\tan^{-1}(3.0/4.0) = 37°$ north of east; the combined object moves that way at 5.0 N s divided by the total mass. A multiple-choice diagram of two momentum arrows is asking exactly this: add the arrows tip to tail.



A glancing collision resolved along two perpendicular axes

Rocket / pushing out mass

A rocket pushes out gas at velocity u (relative to itself) at a **mass-flow rate** 质量流率 $\dot{m}$ (kg per second). It feels a thrust

$$F = \dot{m} \cdot u,$$

which comes from $F = \Delta p/\Delta t$. The momentum given to the gas each second equals the thrust on the rocket (Newton's third law: the rocket pushes the gas one way, the

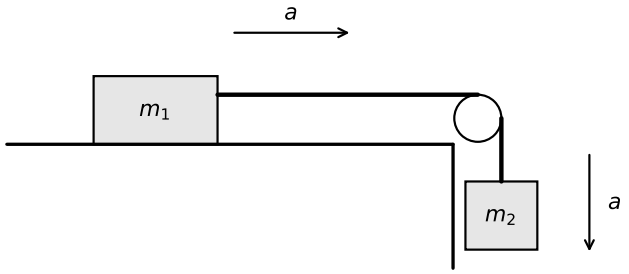
gas pushes the rocket the other way). An engine ejecting 90 kg of gas per second at 190 m s^{-1} produces a thrust of $90 \times 190 = 1.7 \times 10^4$ N.

> **Worked example.** A rocket of weight W leaves the ground with an initial acceleration a. What thrust does its engine produce?

Take upwards as positive. The resultant force is thrust minus weight, so $T - W = ma$. With $m = W/g$, $T = W + Wa/g = W(1 + a/g)$. A rocket of weight 2.0×10^6 N accelerating at 4.0 m s^{-2} needs a thrust of $2.0 \times 10^6 \times (1 + 4.0/9.81) = 2.8 \times 10^6$ N. The thrust must exceed the weight before the rocket moves at all.

■ Connected bodies moving along different lines

Draw a separate force diagram for each body. A light string over a smooth pulley has the same tension on both sides; a taut inextensible string gives the bodies the same acceleration magnitude. Choose a positive direction along each body's motion.



Connected bodies share acceleration magnitude and string tension.

> **Worked example.** A 4.0 kg block on a smooth table is joined to a hanging 1.0 kg mass. For the block, $T = m_1 a$; for the hanging mass, $m_2 g - T = m_2 a$. Adding cancels the internal tension: $a = m_2 g/(m_1 + m_2) = 1.0 \times 9.81/(4.0 + 1.0) = 1.96$ m s^{-2}. Then $T = m_1 a = 4.0 \times 1.96 = 7.85$ N. Normal force and weight balance on the block, but weight and tension do not balance on the accelerating hanging mass.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
mass	the property of an object that resists a change in its motion

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Term	Definition
linear momentum	the product of an object's mass and its velocity
force	the rate of change of momentum
Newton's first law	an object remains at rest or moves at constant velocity unless acted on by a resultant force
Newton's second law	the resultant force on an object is proportional to the rate of change of its momentum, and acts in the direction of the change
Newton's third law	when two bodies interact, the force on one is equal in magnitude and opposite in direction to the force on the other
weight	the force on an object due to a gravitational field; equal to the product of its mass and the acceleration of free fall
principle of conservation of momentum	the total momentum of a system of objects remains constant provided no resultant external force acts on the system
elastic collision	a collision in which total kinetic energy is conserved (and the relative speed of approach equals the relative speed of separation)
inelastic collision	a collision in which momentum is conserved but some kinetic energy is transferred to other forms
terminal velocity	the constant velocity reached when the resistive force on a moving object equals the force driving it, so the resultant force is zero

Exam tips

- **Newton's second law is** $F = \Delta p/\Delta t$ (rate of change of momentum); $F = ma$ is the special case for constant mass.
- Identify **third-law pairs** correctly: the same type of force, acting on **two different bodies** – not the balanced forces on one body.
- **Momentum is conserved** in every collision; kinetic energy is conserved only in an **elastic** collision.
- For **terminal velocity**, explain that drag rises with speed until drag = weight, so the acceleration becomes zero.

Common mistakes

- Writing “the forces are balanced” or “the forces cancel out”. Say “the resultant force on the object is zero”.
- Adding both speeds in a head-on collision because momentum is “total”. Choose a positive direction, give the opposing velocity a minus sign, then add.
- Calculating kinetic energy from a component of the velocity. Kinetic energy uses the full speed of each body.
- Calling the weight and the normal contact force on one object a third-law pair. A pair acts on two different objects and is one type of force.
- Stating conservation of momentum without the condition. It holds only when no resultant external force acts on the system.

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Forces, density and pressure

A-Level Physics

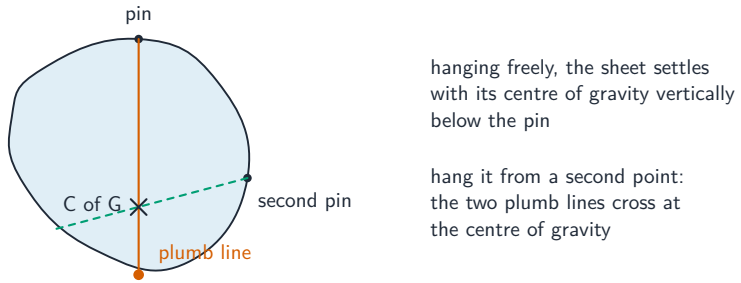
Turning effects of forces

Centre of gravity

The **weight** 重力 of a large object can be treated as acting at one single point, called the **centre of gravity** 重心 (the same as the **centre of mass** 质心 in a uniform gravitational field). For a uniform, regular shape – a rectangle, a sphere, a uniform rod – the centre of gravity is at the middle.

When you draw a **free-body diagram** 受力图, always put the weight arrow at the centre of gravity.

For an irregular shape, find the centre of gravity by experiment. Hang the object freely from a pin: in equilibrium it settles with its centre of gravity **vertically below the pin**, because only then does the weight have no moment about the pin. A **plumb line** 铅垂线 hung from the same pin marks that vertical. Hang the object from a second point and draw the second line; the centre of gravity is where the two lines cross. The same fact answers the exam’s “explain why the sheet hangs like this”: its centre of gravity is directly below the support, so the weight’s line of action passes through the pivot and there is no resultant torque.



Finding a centre of gravity: hung freely, an object settles with its centre of gravity vertically below the pin, so two plumb lines from two pins cross at it

For a **non-uniform** 非均匀 bar, the centre of gravity is not at the middle. Find it by balancing the bar on a pivot, or by taking moments: if the bar’s weight W balances a known weight, the distance of its centre of gravity from the pivot is the one moment that makes the two sides equal.

Moment of a force

The **moment** 力矩 of a force about a point is

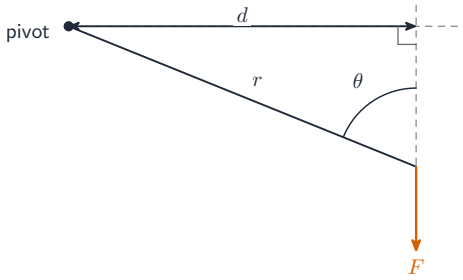
$$M = F \cdot d,$$

where F is the size of the **force** 力 and d is the **perpendicular distance** 垂直距离 from the point to the **line of action** 作用线 of the force. Unit: N m.

The **lever arm** 力臂 is the perpendicular distance d from the pivot to the force's line of action. If the line from the **pivot** 支点 to the point where the force is applied has length r and makes angle θ with the force, then $d = r\sin\theta$, so $M = Fr\sin\theta$. Only the part of the force **perpendicular** 垂直 to this line makes a moment.

> **Worked example.** A force of 50 N is applied to the end of a spanner 0.40 m long, at 30° to the spanner. Find the moment of the force about the nut.

$$M = Fr\sin\theta = 50 \times 0.40 \times \sin 30° = 50 \times 0.40 \times 0.50 = 10 \text{ N m}.$$



The moment depends on the perpendicular distance d from the pivot to the line of action of the force

A moment is either **clockwise** 顺时针 or **anticlockwise** 逆时针 about the chosen point.

Couple and torque

A **couple** 力偶 is a pair of forces that are:

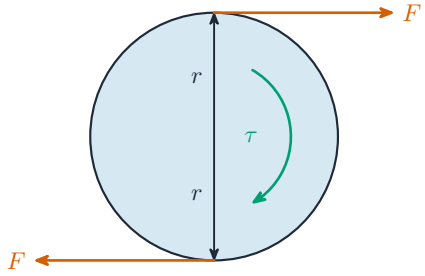
- **equal in size**,
- **opposite in direction**,
- with their lines of action a perpendicular distance apart.

A couple makes the body **turn only** – its resultant force is zero, so it gives no straight-line acceleration.

The **torque** 力偶矩 of a couple is the turning effect it makes:

$$\tau = F \cdot d,$$

where F is the size of one force and d is the perpendicular distance between the two lines of action. Unit: N m. The torque is the same about any point – a special property of couples.



A couple: two equal and opposite forces, a distance apart, producing a torque

A common multiple-choice trap: two equal forces in the **same** direction are not a couple (they have a resultant force and cause **translation** 平动). A couple needs equal size **and** opposite direction.

Watch the distance in the formula. For a couple of two forces F each acting a distance d from a central pivot, on opposite sides, the perpendicular distance between the two lines of action is $2d$, so the torque is $F \times 2d = 2Fd$. Taking moments about the pivot gives the same answer, $Fd + Fd$. The two-mark definition wants both parts: **the product of one of the forces and the perpendicular distance between the lines of action of the forces.**

Equilibrium of forces

Conditions for equilibrium

A body is in **equilibrium** 平衡 when:

1. the **resultant force** 合力 is zero (no straight-line acceleration), AND
2. the resultant moment about any point is zero (no **angular acceleration** 角加速度).

Both must hold. A body with no resultant force can still be turning; a body with no resultant moment can still be moving in a straight line.

The classic multiple-choice case: four forces, two equal pairs pointing in opposite directions but not along the same lines. The resultant force is zero, so the centre of mass does not accelerate, but the pairs form couples with a resultant torque, so the object spins faster and faster while its centre of mass moves at constant velocity. Only when both conditions hold is the body in equilibrium.

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Principle of moments

For a body in rotational equilibrium, **the total clockwise moment about any point equals the total anticlockwise moment about the same point**. This is the **principle of moments** 力矩原理.

To solve a balance problem:

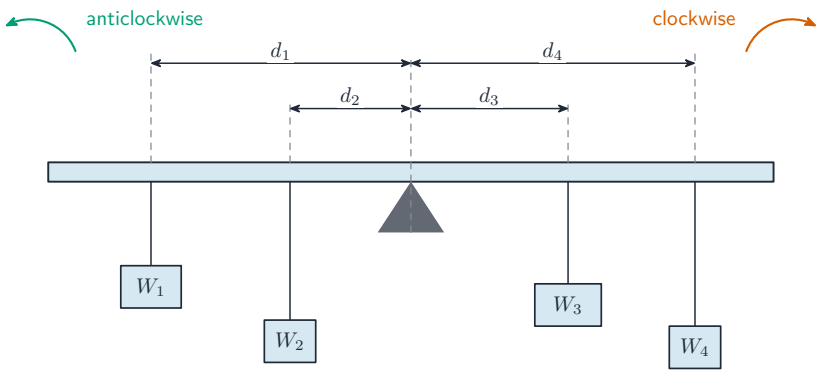
1. Choose a pivot – usually where an unknown force acts, so that force drops out (its distance is zero).
2. List every force and its perpendicular distance from the pivot.
3. Set $\sum M_{\text{clockwise}} = \sum M_{\text{anticlockwise}}$.
4. Use $\sum F = 0$ if you need a second equation.

A ruler balanced on a pivot with masses on each side is solved this way. For a heavy uniform rod, remember to include its weight acting at its centre of gravity. Stated for the marks: **for a body in equilibrium, the sum of the clockwise moments about any point equals the sum of the anticlockwise moments about that same point.**

> **Worked example.** A uniform beam of length 4.0 m is pivoted at its centre. A 40 N weight hangs 1.5 m from the pivot on one side. How far from the pivot, on the other side, must a 30 N weight hang to balance the beam?

The beam's own weight acts at the centre (the pivot), so it has no moment. Setting clockwise = anticlockwise moments:

$$40 \times 1.5 = 30 \times d \quad \Rightarrow \quad d = \frac{60}{30} = 2.0 \text{ m}.$$

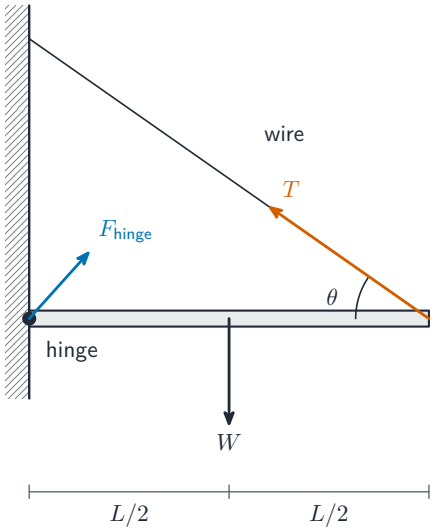


Weights on a rule balanced at a pivot – used to test the principle of moments



A tower crane is a moments problem: the counterweight's moment about the mast balances the moment of the load

> **Worked example.** A uniform rod of length L and weight $W = 28$ N is hinged to a wall at one end and held horizontal by a wire from its other end. The wire makes 40° with the rod. Find the tension T in the wire.



A hinged rod held by a wire: taking moments about the hinge removes the unknown hinge force

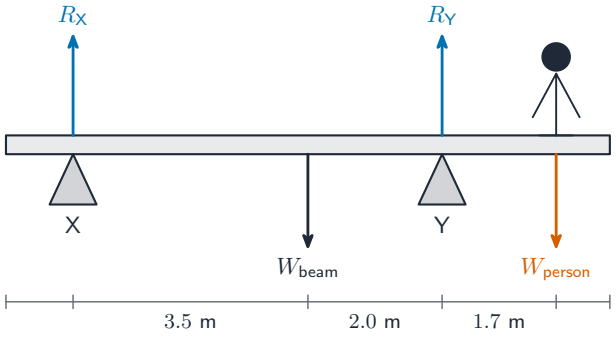
Take moments about the hinge, so the unknown hinge force drops out. The weight acts at the centre, a distance $L/2$ from the hinge: its moment is $W \times L/2$, clockwise. The tension acts at the end, but not at right angles to the rod, so its moment is $T \sin 40° \times L$, anticlockwise (only the component of T perpendicular to the rod turns it). Equate them:

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$$T\sin 40^\circ \times L = W \times \frac{L}{2} \quad \Rightarrow \quad T = \frac{28}{2\sin 40^\circ} = 22 \text{ N}.$$

The length L cancels, which is why the question need not give it. If a load hangs from the rod as well, add its moment on the clockwise side. This is the shape of almost every moments question: choose the pivot where the unknown force acts, use the perpendicular component of any angled force, and let unknown lengths cancel.

> **Worked example.** A uniform beam of weight 200 N and length 9.0 m rests on two supports, X at 1.0 m from the left end and Y at 6.5 m. A person of weight 700 N stands 1.7 m from Y, beyond it. Find the force from each support.



A beam on two supports: moments about one support give the other support's force; the sum of forces then gives the first

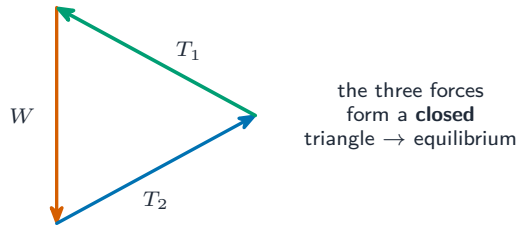
Two unknown forces, R_X and R_Y, so take moments about X to remove R_X. The beam's weight acts at its centre, 3.5 m from X; Y is 5.5 m from X; the person is 7.2 m from X:

$$R_Y \times 5.5 = 200 \times 3.5 + 700 \times 7.2 \quad \Rightarrow \quad R_Y = \frac{700 + 5040}{5.5} = 1040 \text{ N}.$$

Then the resultant force is zero: $R_X + R_Y = 200 + 700$, so $R_X = 900 - 1040 = -140$ N. A negative answer means the beam would lift off X: with the person that far past Y, X must hold the beam down, not up. Questions often ask where a person can stand before the beam tips; that is the position at which R_X becomes zero.

■ Vector triangle

Three forces in the same plane that are in equilibrium can be drawn as a **closed vector triangle** 矢量三角形 – drawn tip-to-tail, the three arrows come back to the start. This is a drawing method instead of splitting into **components** 分量.



Three forces in equilibrium form a closed vector triangle

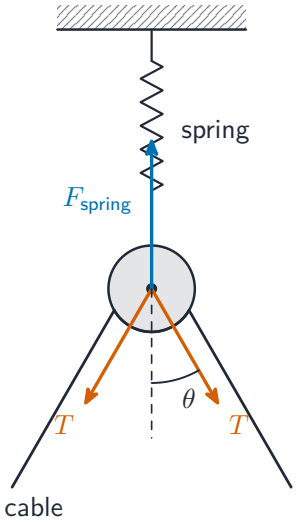
Use the **sine rule** 正弦定理 or the **cosine rule** 余弦定理 on the triangle to find unknown sizes or directions, or draw the triangle to scale on graph paper.

You can also split each force into **horizontal** 水平 and **vertical** 竖直 components and set $\sum F_x = 0$ and $\sum F_y = 0$.

To draw the triangle for the marks: draw the weight first (vertical, to scale), then add the other two forces tip to tail in their real directions, so the third arrow closes the triangle; label every side with its force and mark the angles. For a block resting on a slope the three forces are the weight (vertical), the normal contact force (perpendicular to the slope) and friction (along the slope), so the triangle is right-angled with the weight as the hypotenuse. For a picture hanging from a string over a pin, the two equal tensions make an isosceles triangle with the weight. The test of any answer is that the triangle closes; if it does not, the object is not in equilibrium.

> **Worked example.** A pulley of negligible weight is held by a spring. A single cable passes under the pulley, and each side of the cable makes 30° with the vertical. The tension in the cable is $T = 60$ N. Find the force from the spring and the extension of the spring, given its spring constant is 2000 N m^{-1}.

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A pulley held by a spring: the two tensions in the one cable are equal, and the spring force balances their vertical components

Because it is one continuous cable, the tension is the same on both sides. The two tensions pull down and outwards; their horizontal components cancel and their vertical components add, so the spring must pull up with $F = 2T\cos 30° = 2 \times 60 \times 0.866 = 104$ N. From Hooke's law (topic 6), $x = F/k = 104/2000 = 0.052$ m. If the angle to the vertical grows, $\cos\theta$ falls, so the spring force and extension fall: the cable is pulling more sideways and less upwards.

Density



An iceberg floats with most of its volume hidden: ice is slightly less dense than water.

Density 密度 is the mass per unit volume:

$$\rho = \frac{m}{V}.$$

Unit: kg m^{-3} (or g cm^{-3}; 1 g cm^{-3} = 1000 kg m^{-3}). Density is a **scalar** 标量.

Some useful densities to know:

- water: 1000 kg m^{-3}
- air at room conditions: $\sim$ 1.2 kg m^{-3}
- iron / steel: $\sim$ 7800 kg m^{-3}

Measuring density is a practical question in disguise. For a cuboid, measure the mass on a balance and the three side lengths with a rule or calipers, then $\rho = m/(xyz)$. Because density is a product and quotient, the percentage uncertainty in ρ is the **sum** of the percentage uncertainties in m, x, y and z (topic 1). A zero error on the balance, or a rule whose end is worn, is a systematic error; repeating the readings does not remove it, but subtracting the zero reading does.

> **Worked example.** A block has mass (120 ± 1) g and sides (4.0 ± 0.1) cm, (3.0 ± 0.1) cm and (2.0 ± 0.1) cm. Find its density and the percentage uncertainty.

$\rho = 120/(4.0 \times 3.0 \times 2.0) = 5.0$ g cm^{-3} = 5000 kg m^{-3}. The percentage uncertainties are 0.8%, 2.5%, 3.3% and 5.0%, so the density is uncertain by 12%, about $\pm$600 kg m^{-3}. The shortest side contributes most, which is why a question asks which measurement to improve first.

Pressure

Pressure 压强 is the force per unit area, where the force acts at right angles to the surface:

$$p = \frac{F}{A}.$$

Unit: Pa = N m^{-2}. Pressure is a scalar.

The force is the one pressing at right angles to the surface, and the area is the area of contact. A tree trunk of weight 9.0 kN standing on a post of diameter 0.28 m presses with $p = F/A = 9000/(\pi \times 0.14^2) = 1.5 \times 10^5$ Pa; the same weight on a wider post gives a smaller pressure. Convert diameters to radii, and centimetres to metres, before squaring.

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A precision aneroid barometer measures atmospheric pressure

Hydrostatic pressure

Take a column of **fluid** 流体 with density ρ, **cross-sectional area** 横截面积 A and height Δh. Its weight is

$$W = mg = (\rho \cdot A \cdot \Delta h) \cdot g.$$

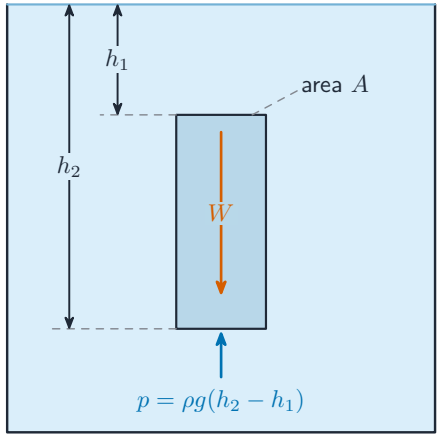
This weight presses down on the area A at the bottom, so the extra pressure at the bottom compared with the top is

$$\Delta p = \frac{W}{A} = \rho g \Delta h.$$

This is the **hydrostatic pressure** 流体静压强 equation. It depends only on the density of the fluid and the **depth** 深度 – the shape of the container does not matter.

> **Worked example.** Find the extra pressure due to the water at the bottom of a swimming pool 2.5 m deep. (Water density 1000 kg m^{-3}, $g = 9.81$ m s^{-2}.)

$$\Delta p = \rho g \Delta h = 1000 \times 9.81 \times 2.5 \approx 2.5 \times 10^4 \text{ Pa}.$$

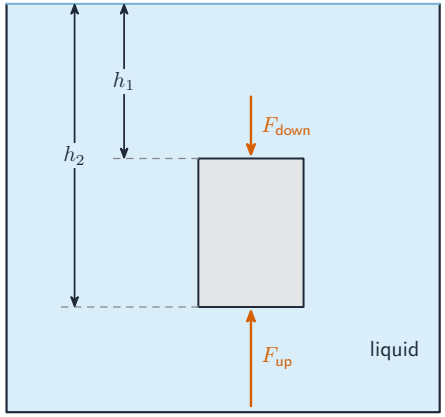


A column of liquid of area A: its weight sets the extra pressure at the depth below

For a submarine at depth h below the surface, the pressure from the water is $\rho_{\text{seawater}}\, g\, h$. For the total pressure, add the **atmospheric pressure** 大气压强 at the surface (about 1.0×10^5 Pa).

Upthrust and Archimedes' principle

When an object is **submerged** 浸没 in a fluid, the pressure at the bottom of the object is greater than the pressure at the top (by $\rho g \Delta h$, where Δh is the object's height). This difference gives a net upward force called the **upthrust** 浮力.



Upthrust arises because the pressure on the bottom of the object is greater than on the top

For an object of volume V (the volume of fluid **displaced** 排开), the upthrust is

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$$F_{\text{upthrust}} = \rho_{\text{fluid}}\, g\, V.$$

This is **Archimedes' principle** 阿基米德原理: the upthrust on a body in a fluid equals the weight of the fluid it pushes aside.

> **Worked example.** A metal block of volume 2.0×10^{-3} m^3 is fully submerged in water. Find the upthrust on it. (Water density 1000 kg m^{-3}, $g = 9.81$ m s^{-2}.)

$$F_{\text{upthrust}} = \rho_{\text{fluid}}\, g\, V = 1000 \times 9.81 \times 2.0 \times 10^{-3} \approx 20 \text{ N}.$$

For a fully submerged object, V is its full volume. For a **floating** 漂浮 object, V is only the volume below the surface – the object floats when the upthrust on the part below the surface equals its weight.

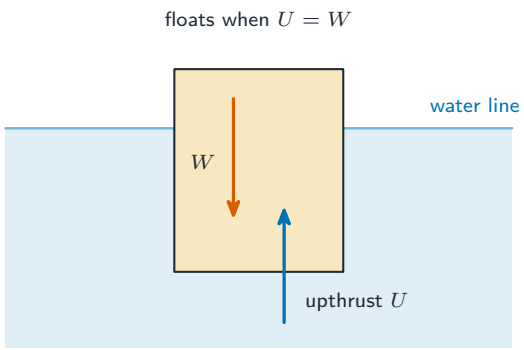
Whether an object floats at all depends on densities: for a fully submerged object the ratio of upthrust to weight is $\rho_{\text{fluid}} V g / \rho_{\text{object}} V g = \rho_{\text{fluid}} / \rho_{\text{object}}$, so it floats if it is less dense than the fluid and sinks if it is denser. Halving the object's density doubles that ratio; changing its volume or the depth changes nothing.

> **Worked example.** A cylinder of mass 11 kg and diameter 0.78 m floats upright in water of density 990 kg m^{-3}. Find the depth y of its base below the surface.

Floating means upthrust = weight. The submerged volume is the base area times the depth, Ay, with $A = \pi \times 0.39^2 = 0.478$ m^2:

$$\rho g A y = mg \quad \Rightarrow \quad y = \frac{m}{\rho A} = \frac{11}{990 \times 0.478} = 0.023 \text{ m}.$$

Each extra newton of load on the cylinder needs an extra $1/(\rho g A)$ of depth, so the depth rises in a straight line with the added weight, wherever on the cylinder it is placed.



A floating object sinks until the upthrust on the submerged part equals its weight

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Force balance with upthrust

A block held under water by a string tied to the bottom of the container is in equilibrium under three vertical forces: weight (down), **tension** 张力 (down), upthrust (up). Set $F_{\text{upthrust}} = W + T$ to find the tension.

A submerged block hanging from a **newton meter** 弹簧测力计 reads less than its weight in air, because of the upthrust: reading $= W - F_{\text{upthrust}}$.

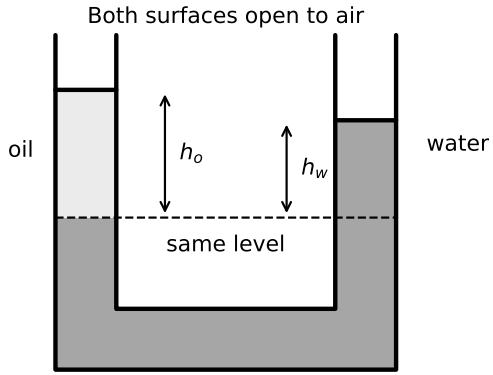
> **Worked example.** A cylinder of weight 25.0 N hangs from a newton meter fully submerged in water, and the meter reads 10.0 N. Find the volume of the cylinder.

The upthrust is the missing 15.0 N, and upthrust $= \rho g V$, so $V = 15.0/(1000 \times 9.81) = 1.53 \times 10^{-3}$ m^3. The same method finds the density of the cylinder: $25.0/(9.81 \times 1.53 \times 10^{-3}) = 1670$ kg m^{-3}.

The upthrust depends on the fluid density and the displaced volume, not on the object's material or depth (for an **incompressible** 不可压缩 fluid). On a planet with smaller g, the upthrust is smaller in the same ratio as the weight, so a floating object still floats with the same fraction below the surface.

connected liquids

In a stationary connected liquid, pressures at the **same horizontal level in the same liquid** are equal. Start at a free surface, add $\rho g h$ when moving down, and subtract it when moving up. Both open surfaces have atmospheric pressure, which cancels.



Compare liquid columns from the same horizontal level.

Worked example. Oil of density 800 kg/m^3 has a 0.10 m column above the oil–water boundary in an open U-tube. The water surface is h_w above that level. $p_{\text{atm}} + \rho_o g h_o = p_{\text{atm}} + \rho_w g h_w$, hence $h_w = \rho_o h_o / \rho_w = 800 \times 0.10 / 1000 = 0.080$ m. Compare heights from the same level, not from the tube's bottom.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
centre of gravity	the point at which the whole weight of an object may be considered to act
moment of a force	the product of the force and the perpendicular distance from the point to the line of action of the force
couple	a pair of equal and opposite forces whose lines of action do not coincide, which produces rotation only
torque of a couple	the product of one of the forces and the perpendicular distance between the lines of action of the forces
principle of moments	for a body in equilibrium, the sum of the clockwise moments about any point equals the sum of the anticlockwise moments about that point
equilibrium	the state of a body on which the resultant force and the resultant torque are both zero
density	mass per unit volume
pressure	force per unit area, where the force acts at right angles to the surface
upthrust	the upward force on a body in a fluid caused by the pressure being greater on its lower surface than on its upper surface
Archimedes' principle	the upthrust on a body in a fluid is equal to the weight of the fluid displaced by the body

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Exam tips

- Take **moments** about a chosen pivot; for equilibrium, total clockwise = total anticlockwise moments **and** the resultant force is zero.
- A body in equilibrium under three forces gives a **closed triangle** of forces.
- Fluid pressure = $\rho g h$; **upthrust = weight of fluid displaced** (Archimedes).
- Distinguish mass, weight and density, and give the base unit each time.

Common mistakes

- Resolving with $\cos\theta$ where the geometry needs $\sin\theta$. Draw the triangle and check which side the angle is next to before choosing.
- Substituting the weight of the object where the question is about the upthrust on it. Upthrust is the weight of the fluid displaced, $\rho_{\text{fluid}} V g$.
- Leaving out the weight of a uniform beam. It acts at the centre, and it has a moment about any pivot that is not at the centre.
- Using the distance along the rod for an angled force. The moment needs the perpendicular distance, or the perpendicular component of the force.
- Taking the torque of a couple as Fd with d measured from the pivot. The distance is between the two lines of action, so a symmetric couple gives $2Fd$.

Work, energy and power

A-Level Physics



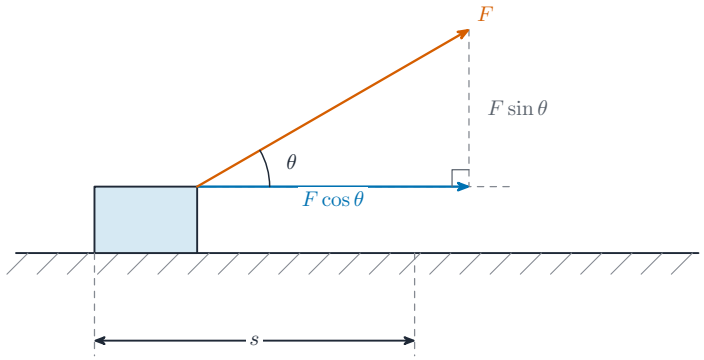
*Wind turbines transfer the kinetic energy of the wind into **electrical energy** 电能.*

Work done by a force

Work 功 is done when a force moves its point of contact along the line of the force. For the one-mark definition write: **work done is the product of the force and the distance moved in the direction of the force.** The work done by a constant force F that causes a **displacement** 位移 s is

$$W = F \cdot s \cdot \cos\theta,$$

where θ is the angle between the force and the displacement. Only the **component** 分量 of the force along the displacement does work.



Only the component of the force along the displacement ($F\cos\theta$) does work

Worked example. A child pulls a sledge 5.0 m across the snow with a rope, using a force of 20 N at 60° to the ground. Find the work done by the rope.

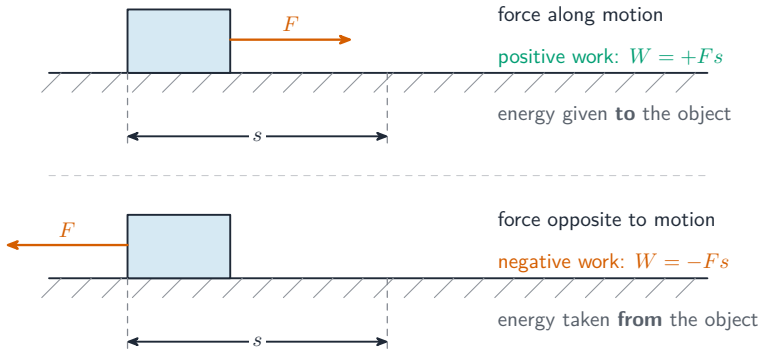
$$W = Fs\cos\theta = 20 \times 5.0 \times \cos 60° = 20 \times 5.0 \times 0.50 = 50 \text{ J}.$$

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Unit: J = N m. Work is a **scalar** 标量.

Special cases:

- force in the same direction as the motion ($\theta = 0$): $W = Fs$, positive work, **energy** 能量 given **to** the object.
- force at right angles to the motion ($\theta = 90°$): $W = 0$. The **normal contact force** 支持力 on a car on a flat road does no work.
- force opposite to the motion ($\theta = 180°$): $W = -Fs$, negative work, energy taken **from** the object (for example **friction** 摩擦力).

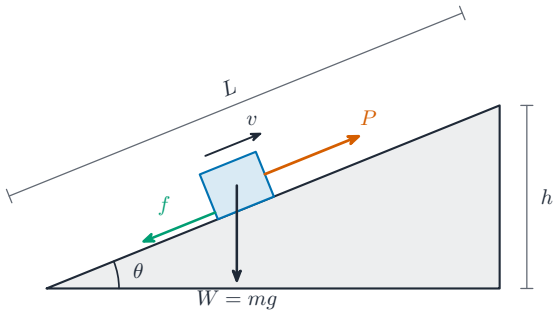


Positive work ($W = +Fs$): force along the motion (top). Negative work ($W = -Fs$): force opposite to the motion, e.g. friction (bottom)

For an object moving up a slope at angle α to the **horizontal** 水平, the work done against gravity in rising a height h is mgh, while the work done by a horizontal push over the slope length L uses $\cos\alpha$.

Two multiple-choice tests of the definition: no work is done on an object that slides at constant velocity along a frictionless surface (no force along the motion), or on one that is simply held still (no distance moved); work is done when a force lifts it. And when a box is pushed at constant velocity across a rough floor, the work done by the push is $F \times d$ for the whole distance, however the path is split into stages.

Worked example. A block of mass 4.9 kg is pushed at constant velocity up a rough slope of length 8.0 m that rises 1.5 m, by a force of 12 N acting along the slope. Find the work done against friction.



A box pushed up a rough slope at constant velocity: the work done by the push pays for the gain in GPE, which needs only the vertical rise h, and for the work against friction along the whole length L

Constant velocity means no change in kinetic energy, so the work done by the push is shared between the gain in gravitational potential energy and the work done against friction. Work by the push: $12 \times 8.0 = 96$ J. Gain in GPE, using the vertical rise: $4.9 \times 9.81 \times 1.5 = 72$ J. So the work done against friction is $96 - 72 = 24$ J, and the friction force is $24/8.0 = 3.0$ N. Using the slope length in mgh is the commonest error on this question.

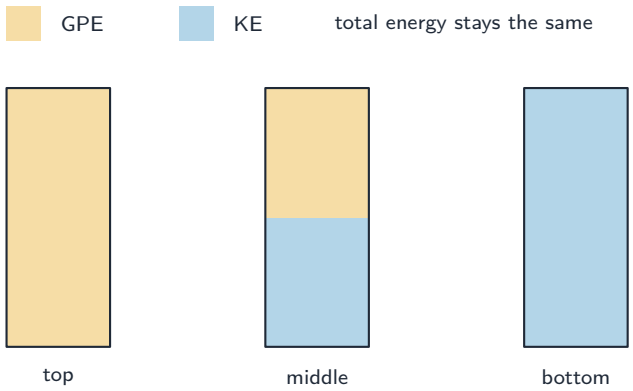
Conservation of energy

Energy is **never made or destroyed** – it only changes from one form to another, or moves from one object to another. In a **closed system** 封闭系统 with no energy transferred across its boundary, the total energy stays constant. This is **conservation of energy** 能量守恒, stated for the marks as: **energy cannot be created or destroyed; it can only be transferred from one form to another** (so the total energy is constant when no energy enters or leaves the system).

When you write an energy equation, list every form the energy starts as and ends as. Common forms in this syllabus: kinetic, gravitational potential, **elastic potential energy** 弹性势能, **electrical** 电的 energy, thermal, sound, **chemical energy** 化学能.

A ball rolling down a **frictionless** 无摩擦 **ramp** 斜坡 turns **gravitational potential energy** 重力势能 into **kinetic energy** 动能: $mgh = \frac{1}{2}mv^2$, so $v = \sqrt{2gh}$. With friction, some of this energy becomes **thermal energy** 热能 of the ramp and the air.

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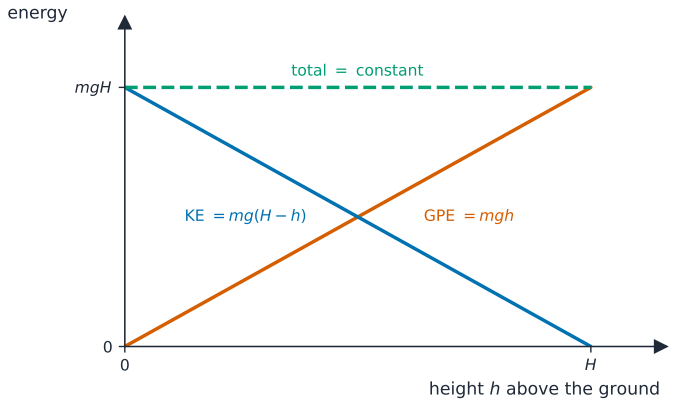
On a frictionless ramp, GPE turns into KE while the total energy stays constant

Worked example. A ball is released from rest at the top of a smooth ramp 1.2 m high. Find its speed at the bottom (take $g = 9.81$ m s^{-2}).

All the gravitational potential energy becomes kinetic energy, so $v = \sqrt{2gh}$ (the mass cancels):

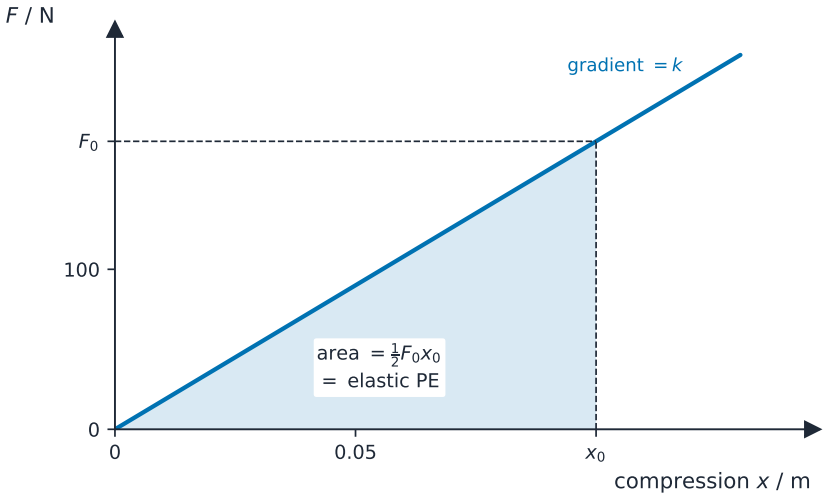
$$v = \sqrt{2 \times 9.81 \times 1.2} \approx 4.9 \text{ m s}^{-1}.$$

The same idea reads a graph. For a ball falling from height H with air resistance negligible, the GPE falls in a straight line with height (mgh), the KE rises in a straight line ($mg(H - h)$), and the two add to the constant mgH at every height.



A falling ball: GPE and KE both change in a straight line with height, and their sum is constant

The energy chain can pass through a spring. A block that hits a spring with kinetic energy 110 J while sliding a little downhill, so that its GPE falls by a further 20 J before the spring stops it, stores $110 + 20 = 130$ J of elastic potential energy at the greatest compression, if nothing is lost to friction. The spring's force–compression graph then gives the compression: the energy stored is the **area** under the line, $\frac{1}{2}F_0x_0$ for a spring that obeys Hooke's law (topic 6), so a maximum force of 2600 N means $x_0 = 2 \times 130/2600 = 0.10$ m. At that instant the resultant force on the block is the spring force minus the component of the weight down the slope, and its acceleration follows from $F = ma$ (topic 3).



The energy stored in a spring is the area under its force–compression graph: $\frac{1}{2}F_0x_0$, which equals $\frac{1}{2}kx_0^2$

Efficiency

The **efficiency** 效率 of a system is

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}} \times 100\%.$$

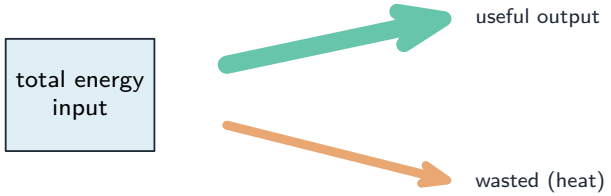
The same idea with power:

$$\text{efficiency} = \frac{\text{useful power output}}{\text{total power input}} \times 100\%.$$

Efficiency is always less than 100% in a real system, because some input energy becomes "useless" forms – usually thermal energy. If a total energy E is supplied and an amount Q is wasted, the useful output is $E - Q$ and the efficiency is $(E - Q)/E$;

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a motor rated at 1.7 kW input with efficiency 53% delivers $0.53 \times 1.7 = 0.90$ kW of useful output.



$$\text{efficiency} = \frac{\text{useful output}}{\text{total input}} \times 100\%$$

Efficiency: only part of the energy input leaves as useful output; the rest is wasted, usually as heat

For an electric motor lifting a load with efficiency η at **voltage** 电压 V and **current** 电流 I, the useful output power is ηVI. From this you can find a force, a lifting speed, or a **tension** 张力.

> **Worked example.** An electric motor lifts a 50 kg load at a steady 0.40 m s^{-1} while drawing 250 W of electrical power. Find its efficiency (take $g = 9.81$ m s^{-2}).

The useful output power is $P = mgv = 50 \times 9.81 \times 0.40 = 196$ W, so

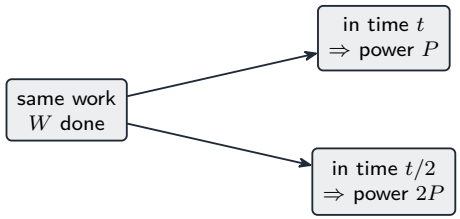
$$\text{efficiency} = \frac{196}{250} \times 100\% \approx 78\%.$$

> **Worked example.** A motor raises a block with a force of 240 N through a vertical distance of 12 m in 60 s. The input power to the motor is 900 W. Find the useful output power and the efficiency.

Work done on the block: $W = Fd = 240 \times 12 = 2880$ J. Useful output power: $P = W/t = 2880/60 = 48$ W. Efficiency: $48/900 \times 100\% = 5.3\%$. Most of the input goes into thermal energy in the motor and its cable. Turned round, a lift motor of useful output 20 kW raising 1500 kg through 20 m takes $t = mgh/P = 1500 \times 9.81 \times 20/20\,000 = 15$ s.

Power

Power 功率 is the rate of doing work, or the rate of transferring energy:



$P = \dfrac{W}{t}$ — less time, more power

The same work done in less time means more power

$$P = \frac{W}{t} = \frac{\Delta E}{\Delta t}.$$

Unit: W = J s^{-1}. Power is a scalar.

■ Power, force and velocity

For an object moving at **velocity** 速度 v with a force F along the direction of motion, in a short time Δt the displacement is $v\,\Delta t$ and the work done is $Fv\,\Delta t$. Dividing by Δt:

$$P = Fv.$$

This is one of the most useful results in mechanics. A "derive $P = Fv$" answer needs exactly those three lines: $P = W/t$, $W = Fs$ for a force along the motion, and $s/t = v$; then use it to solve problems in which a vehicle or a load moves at constant speed.

> **Worked example.** A car travels at a steady 25 m s^{-1} against a total resistive force of 600 N. Find the output power of its engine.

At constant speed the driving force equals the resistive force, so

$$P = Fv = 600 \times 25 = 15\,000 \text{ W} = 15 \text{ kW}.$$

- For a car at constant velocity v on a flat road, the engine power must balance the total resistive force: $P = F_{\text{resist}} \cdot v$. If the **drag** 阻力 grows with v^2, doubling the speed multiplies that drag force by four and the power needed to balance it by **eight**: $P = Fv \propto v^3$. This assumes the stated drag is the resistive force being balanced.
- For lifting a **weight** 重力 mg straight up at constant speed v, the useful output power is $P = mg \cdot v$.
- For an aircraft hovering at a fixed height, the lift force equals the weight, and a large power is needed because air must be pushed downwards all the time.

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- A crane raising 600 kg at a steady 12 m per minute lifts at $v = 0.20$ m s^{-1}, so its useful output power is $mgv = 600 \times 9.81 \times 0.20 = 1.2$ kW. Convert the speed to m s^{-1} first.
- A sailboat pushed by a constant wind force at constant velocity must also feel an equal resistive force from the water: constant velocity means zero resultant force, so the wind's power Fv is all going into work against the water.



A hydroelectric station is rated in megawatts: the energy of the stored water, divided by the time it takes to release it

Gravitational potential energy

In a **uniform** gravitational field (close to a planet's surface), the change in gravitational potential energy of **mass** 质量 m rising or falling through a height Δh is

$$\Delta E_{\text{P}} = mg\Delta h.$$

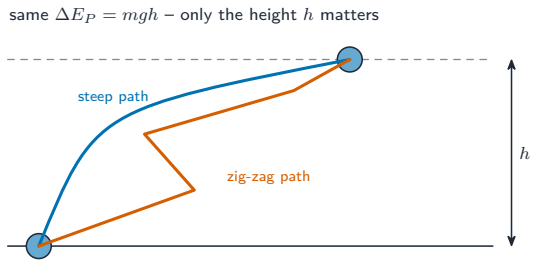
■ Where it comes from

The work done **against** gravity to raise a mass m slowly (no change in kinetic energy) through height Δh equals the gravitational potential energy gained:

- the gravitational force on the mass is mg downwards,
- the force needed to lift it slowly is mg upwards,
- the work done by this force is $W = F \cdot s = mg \cdot \Delta h$,
- this work becomes ΔE_{P}.

So $\Delta E_{\text{P}} = mg\Delta h$. To use it you need m and Δh (and g). You do not need speed or time. The two-mark derivation wants the same four lines, with the condition stated:

the block is raised **at constant speed**, so the lifting force equals the weight mg, and the work done by that force over the vertical distance Δh is $mg\Delta h$, which is the gain in gravitational potential energy. A 100 g object falling 10 m loses $0.100 \times 9.81 \times 10 = 9.8$ J, about 10 J; the mass must be in kilograms.



Gravitational PE depends only on the height risen, not the path taken



A coaster train hauled up the lift hill: the work done against gravity is stored as gravitational potential energy

Kinetic energy

The kinetic energy of an object of mass m moving at speed v is

$$E_{\mathrm{K}} = \tfrac{1}{2}mv^2.$$

Where it comes from

Apply a resultant force F to a mass m that starts at rest. It speeds up evenly from 0 to v over a displacement s. From $v^2 = u^2 + 2as$ with $u = 0$,

$$s = \frac{v^2}{2a}.$$

The work done on the mass is

$$W = F \cdot s = ma \cdot \frac{v^2}{2a} = \tfrac{1}{2}mv^2.$$

All this work becomes kinetic energy, so $E_{\mathrm{K}} = \tfrac{1}{2}mv^2$. In the exam, state the assumptions as you go: the object starts from rest, the force is constant, so the acceleration is uniform, and $F = ma$ is Newton's second law.

Because $E_{\mathrm{K}} \propto v^2$, quadrupling the speed multiplies the kinetic energy by sixteen: an object with 1500 J at 10 m s^{-1} has 24 000 J at 40 m s^{-1}. The same square is why a stopping force is found from energy: a ball of mass 1.2 kg moving at 3.0 m s^{-1} that is stopped by a cushion over 0.020 m loses $\tfrac{1}{2} \times 1.2 \times 3.0^2 = 5.4$ J, so the average force on it is $5.4/0.020 = 270$ N: the work done by the cushion equals the kinetic energy lost.

Kinetic energy and momentum

Combining $p = mv$ and $E_{\mathrm{K}} = \tfrac{1}{2}mv^2$:

$$E_{\mathrm{K}} = \frac{p^2}{2m}.$$

This is handy when the **momentum** 动量 is given but not the velocity. For a momentum change from p_1 to p_2 at constant mass, the change in kinetic energy is $(p_2^2 - p_1^2)/(2m)$.

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A pole vaulter: kinetic energy from the run-up is stored in the bent pole, then given back as height

Using energy methods

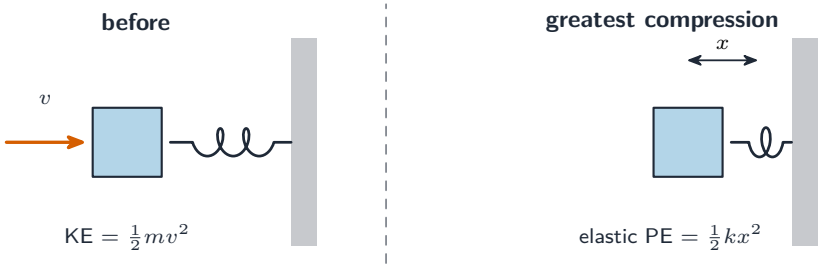
A useful plan for problems that mix forces and energy:

1. **Find the start and end states.** Write the kinetic and potential energies in each.
2. **List any work done by outside forces** (friction, a push). Friction usually takes energy out; a push can add it.
3. **Conservation of energy**: $E_{\text{start}} + W_{\text{in}} = E_{\text{end}} + W_{\text{lost as heat etc.}}$.

Examples:

- A box pushed at **constant velocity** up a ramp of length L rising by h: E_{K} does not change, so the work done by the push goes into ΔE_{P} plus the work done against friction.
- A block sliding into a **spring** 弹簧 with kinetic energy E_{K} on a frictionless surface: at greatest **compression** 压缩 x, all the kinetic energy has become elastic potential energy $\frac{1}{2}kx^2$ (where k is the **spring constant** 劲度系数).
- A ball dropped from height h_1 that bounces to height h_2: the ratio h_2/h_1 is the fraction of mechanical energy kept, $h_2/h_1 = (v_{\text{up}}/v_{\text{down}})^2$.
- A **projectile** 抛体 thrown to the same height at different angles: the final speed is the same (only the height matters); use components to get its direction.
- A **bungee jumper** 蹦极者: from the platform to the point where the cord goes taut, GPE becomes KE; from there the cord stretches and takes energy as elastic potential energy, so the KE reaches its maximum where the cord's pull first equals the weight, then falls to zero at the lowest point, where GPE lost = elastic PE stored (plus any thermal energy).

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A block's kinetic energy becomes elastic potential energy as it compresses the spring

Power when the force is at an angle

Only the component parallel to the velocity transfers energy: $P = Fv\cos\theta$, where θ is the angle between the force and velocity. It follows from $W = Fs\cos\theta$ and $P = W/t$.

> **Worked example.** A 120 N pull acts at 60° to the velocity of a trolley moving at 2.0 m/s. $P = Fv\cos\theta = 120 \times 2.0 \times \cos 60° = 120$ W. The perpendicular component does no work.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
work done by a force	the product of the force and the distance moved in the direction of the force
principle of conservation of energy	energy cannot be created or destroyed, only transferred from one form to another, so the total energy is constant when no energy crosses the system boundary
efficiency	the ratio of useful energy (or power) output from a system to the total energy (or power) input, usually as a percentage
power	the work done per unit time, or the rate at which energy is transferred
gravitational potential energy	the energy an object has because of its position in a gravitational field
kinetic energy	the energy an object has because of its motion
elastic potential energy	the energy stored in an object that has been stretched or compressed

Exam tips

- **Work = force × distance moved in the direction of the force** – use $Fs\cos\theta$ when they are at an angle.
- Use **conservation of energy**: loss in GPE = gain in KE (+ work done against resistance).
- **Power = work / time** = Fv; efficiency = useful output ÷ total input.
- GPE uses the **vertical** height gained, not the distance along a slope.

Common mistakes

- Giving the change in height as the answer to a change-in-energy question. Finish the calculation: $\Delta E_{\mathrm{P}} = mg\Delta h$.
- Using the distance along a slope as the height in $mg\Delta h$, or using mass where weight is needed. Use the vertical height gained, and check whether the quantity wanted is m in kg or W in N.
- Putting the resistive force into $P = Fv$ for a car that is accelerating. F is the driving force; only at constant speed does it equal the resistive force.
- Efficiency above 100%, or efficiency as output divided by wasted. It is useful output divided by total input, and the total input is always the larger number.
- Reading a force–extension graph's gradient when the question wants the energy. Energy stored is the area under the line, not its slope.

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Deformation of solids

A-Level Physics

Forces that cause deformation

When a force acts on a solid along its length, the object changes shape (**deformation** 形变). Two cases (treated as one-dimensional here):

- a **tensile** 拉伸 force stretches the object – it makes an **extension** 伸长量 x,
- a **compressive** force squeezes the object – it makes a **compression** 压缩, treated as a negative extension.

The applied force is the **load** 负载. The change from the natural length is the extension (or compression).

Both are measured from the natural, unstretched length, never from the loaded length: on a diagram of a spring under a box, the distance the spring has shortened is its compression. A spring's own mass is usually said to be **negligible** 可忽略的, so the only forces on it are the load and the tension it provides.

Hooke's law and the spring constant



A spring obeys Hooke's law: extension is proportional to the force applied.

For many materials at small extensions, **the extension is proportional to the load** – this is **Hooke's law** 胡克定律. The constant that links them is the **spring constant** 劲度系数 k:

$$F = kx \qquad \Longleftrightarrow \qquad k = \frac{F}{x}.$$

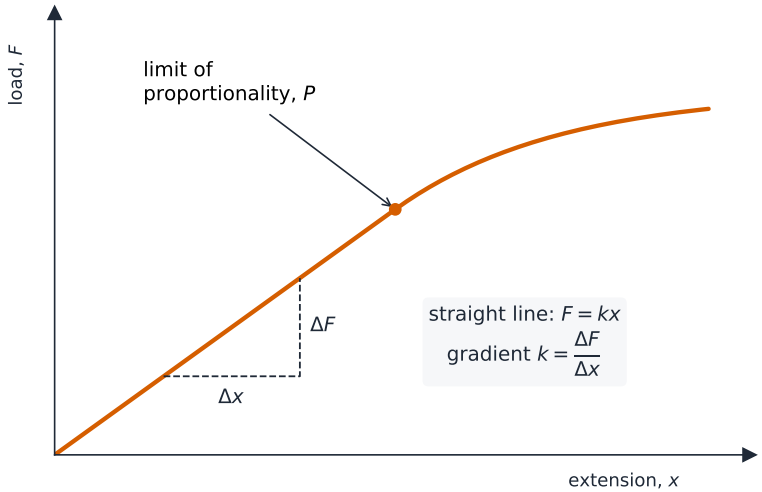
Unit of k: N m^{-1}.

> **Worked example.** A spring stretches by 4.0 cm when a 2.0 N load is hung from it. Find its spring constant.

Converting the extension to metres (4.0 cm = 0.040 m):

$$k = \frac{F}{x} = \frac{2.0}{0.040} = 50 \text{ N m}^{-1}.$$

For the one-mark "state Hooke's law", write it in words: **the extension is directly proportional to the applied force (load), provided the limit of proportionality is not exceeded.** The spring constant is the **force per unit extension**; the multiple-choice distractors are "extension per unit force" (that is $1/k$) and $\frac{1}{2}Fx$ (the energy stored).



A load–extension graph: straight up to the limit of proportionality P, then it curves

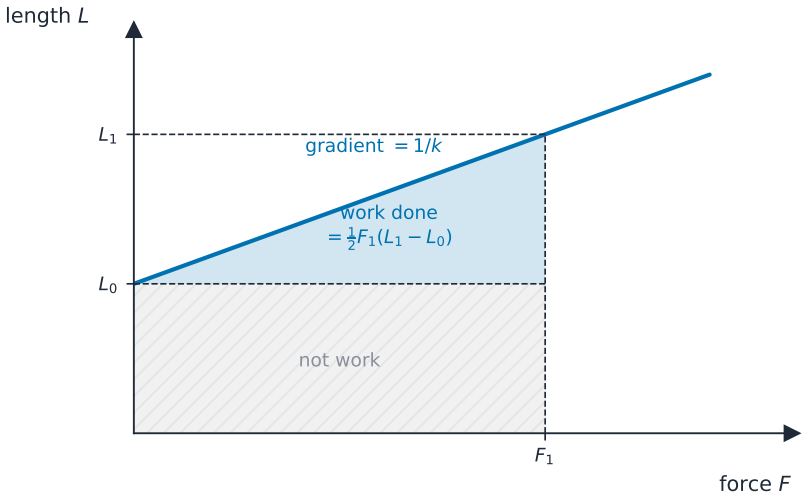
Reading a graph:

- A force–extension (F against x) graph has gradient k in the Hooke's-law region.
- An extension–force (x against F) graph has gradient $1/k$ in the Hooke's-law region.

A common trap: if a graph plots **length** L against force, you can still find the spring constant from the gradient (since $L = L_0 + F/k$, the gradient is $1/k$ – read it off carefully).

> **Worked example.** A spring has length 90 mm under a force of 0.80 N and 115 mm under 1.30 N. Find the spring constant and the unstretched length.

The extra force 0.50 N produces the extra extension 25 mm = 0.025 m, so $k = 0.50/0.025 = 20$ N m^{-1}. At 0.80 N the extension is $x = F/k = 0.80/20 = 0.040$ m, so the unstretched length is $90 - 40 = 50$ mm. On a graph of length against force the intercept on the length axis is L_0, the gradient is $1/k$, and the work done in stretching the spring is the **triangle between the line and the level** $L = L_0$, not the whole area down to the force axis: the rectangle below L_0 is the unstretched length multiplied by the force, which means nothing.



Length against force: the intercept is L_0, the gradient is $1/k$, and only the triangle above L_0 is the work done

Limit of proportionality

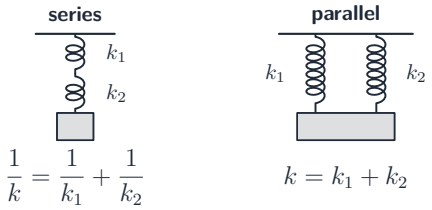
Hooke's law only holds up to the **limit of proportionality** 比例极限. Past this point the F against x line curves and is no longer straight. The material may still be **elastic** 弹性 (it returns to its first length when you remove the load) a little further, then it becomes **plastic** 塑性.

Springs in series and parallel

You may need to combine spring constants:

- **Series** 串联 (one spring hangs from another): the same load passes through both, the total extension is the sum, so $\dfrac{1}{k_{\text{total}}} = \dfrac{1}{k_1} + \dfrac{1}{k_2}$.
- **Parallel** 并联 (two springs side by side holding the same load): each takes half the load (if they are identical), the extensions are equal, so $k_{\text{total}} = k_1 + k_2$.

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Combining spring constants: series gives a softer spring, parallel a stiffer one

Worked example. Springs of spring constant 6.0 N cm^{-1} and 4.0 N cm^{-1} are joined end to end and a load of 80 N hangs from them. Find the total extension.

Each spring carries the full 80 N, so the extensions are $80/6.0 = 13.3$ cm and $80/4.0 = 20$ cm: a total of 33 cm (the same as $k = 2.4$ N cm^{-1} from $1/k = 1/6.0 + 1/4.0$). Three identical springs give the largest combined constant when all three are in parallel ($3k$) and the smallest when all are in series ($k/3$). When a load is shared by two identical springs in parallel, each carries **half the force** and so stores a **quarter of the energy** of one spring holding the whole load; the pair together stores half as much as the single spring did. A spring constant in N cm^{-1} is 100 times smaller than the same constant in N m^{-1}; give the unit the question asks for.

Stress, strain and the Young modulus

For a wire of uniform cross-section under a tensile load:

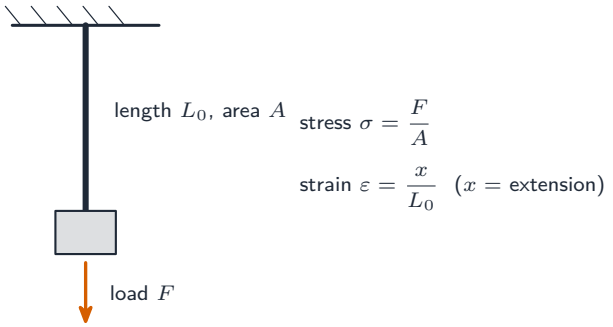
- **Stress** 应力 $\sigma = \dfrac{F}{A}$, where F is the load and A is the **cross-sectional area** 横截面积. Unit: Pa.
- **Strain** 应变 $\varepsilon = \dfrac{x}{L_0}$, where x is the extension and L_0 is the original length. Strain has no unit (it is a ratio of lengths).

For the one-mark definitions: **stress is the force per unit cross-sectional area** (the force acting normally to the area); **strain is the extension per unit original length**. The unit of stress, Pa = N m^{-2}, is kg m^{-1} s^{-2} in base units. The area of a round wire comes from its diameter d: $A = \pi d^2/4 = \pi r^2$, and using d in place of r makes the area four times too big. Convert mm^2 to m^2 with 10^{-6}.

Worked example. A tensile force of 18 N acts on a wire of cross-sectional area 3.2 mm^2. Find the stress.

$\sigma = F/A = 18/(3.2\times10^{-6}) = 5.6\times10^6$ Pa. The same force acts along the whole wire, so a bolt whose diameter is $2d$ at one end and d at the other has four times the stress at the narrow end ($\sigma \propto 1/d^2$), and a wire of three times the radius carries one ninth of the stress.

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Stress is the load per cross-sectional area; strain is the extension per original length

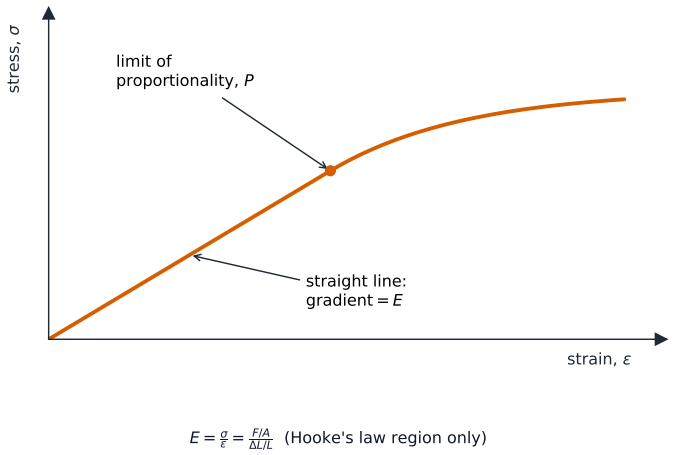
The **Young modulus** 杨氏模量 is the ratio of stress to strain in the Hooke's-law region:

$$E = \frac{\sigma}{\varepsilon} = \frac{F/A}{x/L_0} = \frac{FL_0}{Ax}.$$

Unit: Pa (about 10^{11} for metals; e.g. steel $\approx 2.0\times10^{11}$ Pa).

Worked example. A steel wire of length 2.0 m and cross-sectional area 1.5×10^{-7} m^2 stretches by 1.0 mm under a load of 15 N. Find the Young modulus.

$$E = \frac{FL_0}{Ax} = \frac{15\times2.0}{(1.5\times10^{-7})(1.0\times10^{-3})} = 2.0\times10^{11}\text{ Pa}.$$



A stress–strain graph, straight up to the limit of proportionality P

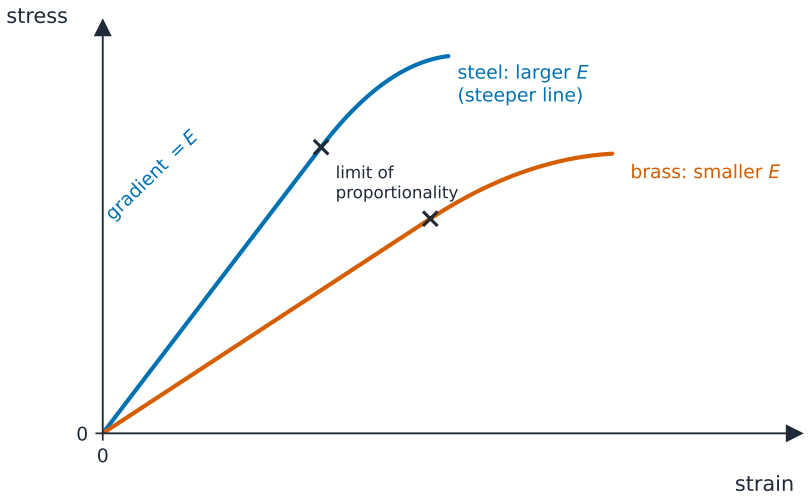
The Young modulus is a property of the **material** – it does not depend on the wire's shape or size. The spring constant k depends on both the material and the size: $k = EA/L_0$.

For the one-mark definition: **the Young modulus is the ratio of stress to strain** (for a material deformed within its limit of proportionality). Because it belongs to the material, a thicker wire of the same metal has the **same** Young modulus: it stretches less under the same load because its area is larger, not because the metal is stiffer. A "show that $k = EA/L_0$" question wants two lines: $k = F/x$ and $E = FL_0/(Ax)$, so $E = kL_0/A$. Two things follow: a wire's spring constant stays **constant** while Hooke's law holds, whatever the force, and a wire of the same metal with twice the diameter has four times the spring constant.

> **Worked example.** A copper wire of length 1.7 m and diameter 0.64 mm has Young modulus 1.2×10^{11} Pa. Find its spring constant.

$A = \pi(0.32 \times 10^{-3})^2 = 3.2 \times 10^{-7}$ m^2, so $k = EA/L_0 = 1.2 \times 10^{11} \times 3.2 \times 10^{-7}/1.7 = 2.3 \times 10^4$ N m^{-1}.

On a stress–strain graph the gradient of the straight part is E, so of two materials drawn on the same axes the **steeper** line is the stiffer material, and a wire with double the Young modulus is drawn as a line through the origin with twice the gradient. On a force–extension graph the gradient is EA/L_0 and the Young modulus is the gradient multiplied by L_0/A; only a stress–strain graph has a gradient equal to E itself.



Two metals on one stress–strain graph: the steeper line has the larger Young modulus

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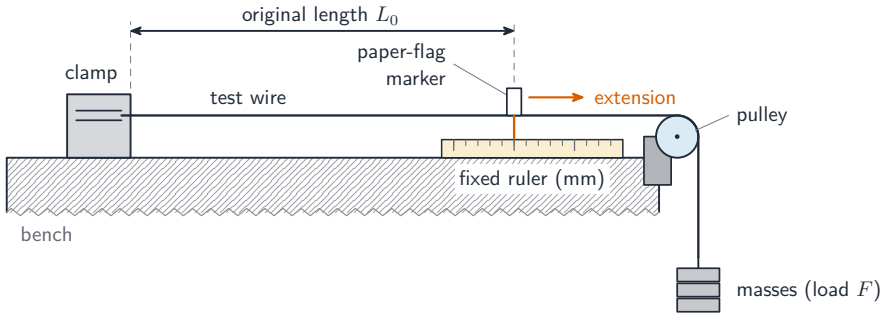
■ Experiment to find the Young modulus of a metal wire

A standard setup:

1. Clamp one end of a long, thin wire to a fixed support. Pass the wire over a **pulley** 滑轮 at the edge of the bench so it hangs straight down.
2. Measure the original length L_0 between the clamp and a marker near the pulley, using a metre rule.
3. Measure the **diameter** 直径 d of the wire at several places with a **micrometer** 螺旋测微器 and take the average. Work out $A = \pi d^2/4$.
4. Hang weights one at a time. Record the load F and the extension x (how far the marker moves against a fixed scale).
5. Plot F against x. In the straight region the gradient is EA/L_0, so $E =$ gradient $\times$ L_0/A.

Why a **long, thin** wire? To make the extension big enough to measure well. Why repeat readings and measure d at several places? To reduce **random error** 随机误差 and check the wire is uniform.

A "describe an experiment" answer earns its marks for the **quantities measured and how** (L_0 with a metre rule; d with a micrometer at several points; the load from the masses or a newton meter; the extension from a marker read against a fixed scale, or a vernier scale), the **graph** (F against x, gradient EA/L_0, or stress against strain, gradient E) and the **precautions**: safety goggles in case the wire snaps, a small load first to straighten kinks, and readings taken on unloading as well as loading to check the wire stayed elastic.



Apparatus for measuring the Young modulus of a wire

Elastic and plastic behaviour

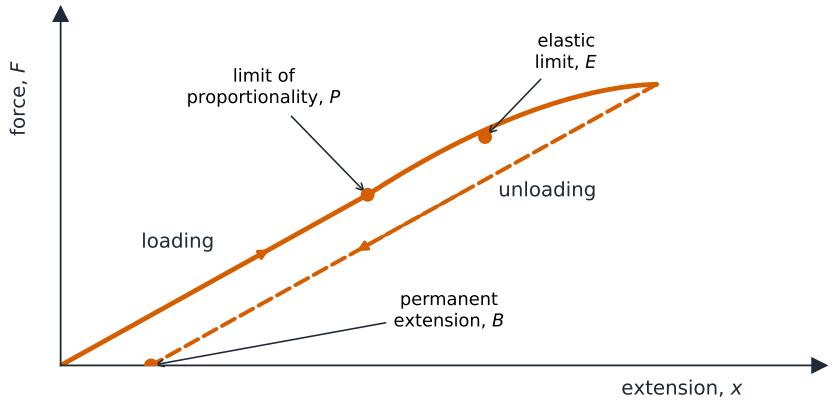
As the load grows:

1. **Elastic and straight (Hooke obeyed)** – up to the limit of proportionality. Removing the load returns the object to its first length.

2. **Elastic but curved** – between the limit of proportionality and the **elastic limit** 弹性极限. The extension is no longer straight in the load, but on unloading the object still returns to its first length.
3. **Plastic** – past the elastic limit. On unloading, the object does **not** return to its first length; a permanent extension stays.

Hooke's law only holds in the straight, elastic region.

In words: **elastic deformation** means the object returns to its original length (or shape) when the load is removed; **plastic deformation** means it does not, and a permanent extension remains; the **elastic limit** is the maximum load (or extension) for which the deformation is still elastic; the **limit of proportionality** is the point beyond which the extension is no longer proportional to the load. On a graph the limit of proportionality is where the line stops being straight. The elastic limit lies a little beyond it and cannot be read from a loading line alone. So in a "which statement must be correct" item, the end of the straight part *is* the limit of proportionality, but whether a later point is the elastic limit or the breaking point cannot be told from the shape of the loading line.



Hooke up to limit of proportionality · plastic beyond yield

Force–extension past the elastic limit: P and E marked, with a permanent extension B left after unloading

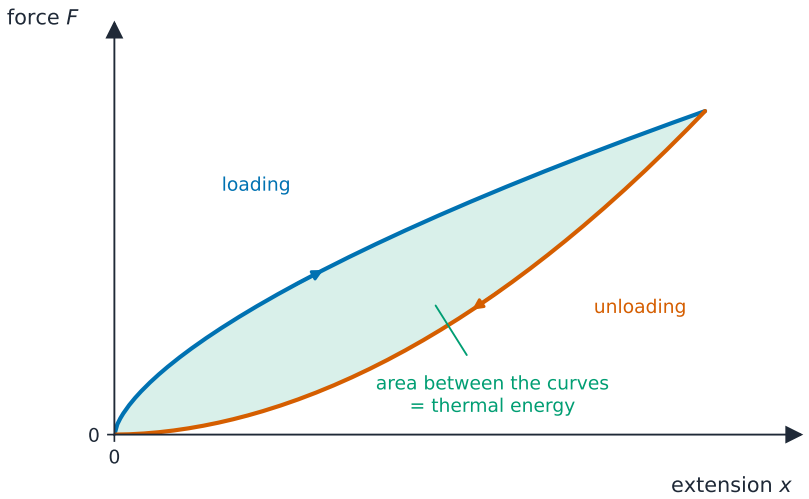


A modern universal (tensile) testing machine stretches a sample and records the force and extension

On a force–extension graph for a material taken into the plastic region and then unloaded, the loading line and the unloading line are different. The unloading line is parallel to the first Hooke line but shifted to the right (the permanent extension left when the load reaches zero). The area between the loading and unloading lines is the energy turned into **thermal energy** 热能 in the material.

A **rubber band** 橡皮筋 also has different loading and unloading curves, but it returns to its original length: its deformation is elastic. The area between the two curves is again energy dissipated as thermal energy (**elastic hysteresis** 弹性滞后), and the energy recovered on unloading is the area under the lower curve. The two-mark "explain why the work done in stretching the wire is not equal to the energy recovered when the force is removed" answer says that the wire was taken past its elastic limit, so part of the deformation is plastic and a permanent extension remains, and that part of the work done became thermal energy in the wire.

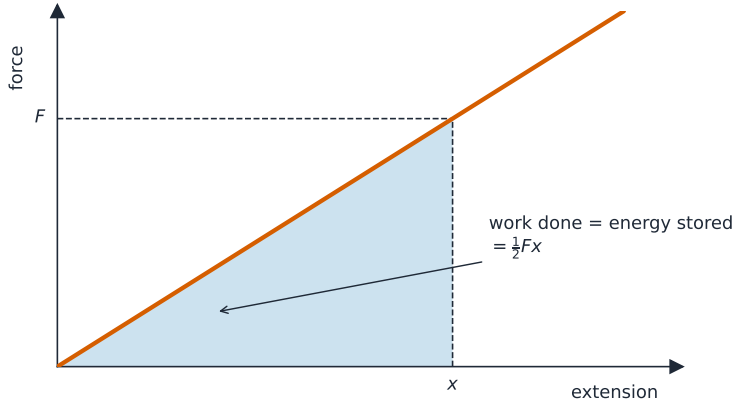
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A rubber band returns to its original length, but the area between loading and unloading is energy lost as thermal energy

Energy stored in a stretched material

The **work done** in stretching a material from 0 to extension x, as the load grows from 0 to F, is the **area under the force–extension graph**.



$E = \frac{1}{2}Fx = \frac{1}{2}kx^2$ (area under F–x graph)

The work done stretching a material is the area under the force–extension graph

Why the area: work done is force multiplied by the distance moved, but here the force grows as the material stretches, so the work is the sum of $F\,\Delta x$ over many small extensions, which is the area under the line. Within the limit of proportionality the force rises uniformly from 0 to F, so the average force is $\frac{1}{2}F$ and the work is $\frac{1}{2}Fx$. On a force–extension graph the gradient is the spring constant.

Hooke's-law material

When Hooke's law holds, the F against x graph is a straight line through the origin. The area under it from 0 to x is a triangle:

$$E_\mathrm{P} = \tfrac{1}{2}Fx = \tfrac{1}{2}kx^2.$$

This is the **elastic potential energy** 弹性势能 stored in a spring or wire stretched within its limit of proportionality. An equal form:

$$E_\mathrm{P} = \frac{F^2}{2k}.$$

> **Worked example.** A spring of spring constant 50 N m^{-1} is stretched by 0.20 m, within its limit of proportionality. Find the elastic potential energy stored.
>
> $$E_\mathrm{P} = \tfrac{1}{2}kx^2 = \tfrac{1}{2} \times 50 \times 0.20^2 = 1.0 \text{ J}.$$

> **Worked example.** A wire of spring constant 2.0×10^4 N m^{-1} is already extended by 2.0 mm. Find the work done to increase its extension to 3.0 mm.

The stored energy rises from $\frac{1}{2}kx_1^2$ to $\frac{1}{2}kx_2^2$:

$$\begin{aligned} W &= \tfrac{1}{2} \times 2.0 \times 10^4 \\ &\quad \times \left[(3.0 \times 10^{-3})^2 - (2.0 \times 10^{-3})^2\right] \\ &= 0.050 \text{ J} \end{aligned}$$

This is the trapezium under the line between the two extensions, not $\frac{1}{2}k(x_2 - x_1)^2$. Because $E_\mathrm{P} \propto x^2$ at fixed k, doubling the extension of a wire stores four times the energy (0.65 J becomes 2.6 J), and a stored energy gives the extension as $x = \sqrt{2E_\mathrm{P}/k}$: a spring of $k = 400$ N m^{-1} storing 0.32 J is compressed by $\sqrt{2 \times 0.32/400} = 0.040$ m. For two wires joined end to end the tension is the same in both, so the total energy stored is $\frac{1}{2}Fx_1 + \frac{1}{2}Fx_2$ with each wire's own extension.

Non-Hooke material

For a graph that is not a straight line (a stretched rubber band, or a spring past its limit of proportionality), find the area by counting grid squares or by using **trapezia** 梯形. The same idea holds: **the area under the force–extension graph is the work done on the material**. To estimate the work done up to the breaking point, count the squares under the whole curve (part squares as halves) and multiply by the

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energy one square represents, the force step multiplied by the extension step; saying that the area was found by counting squares is the "explain your reasoning" mark.

Comparing stored energy

A common multiple-choice case: two materials are stretched by the same force, or by the same extension. Using $E_P = \frac{1}{2}Fx$:

- same F, smaller k (less stiff) $\rightarrow$ larger x $\rightarrow$ more energy stored.
- same x, larger k (stiffer) $\rightarrow$ larger F $\rightarrow$ more energy stored.

A sketch of E_P against extension or compression is a curve through the origin that gets steeper ($E_P \propto x^2$), not a straight line. Of two wires of the same length and area under the same load, the one with the smaller Young modulus extends more and so stores more energy.

When a stretched spring is released onto a mass, the elastic potential energy becomes **kinetic energy** 动能 (and gravitational potential energy if the mass rises). Set $\frac{1}{2}kx^2$ equal to $\frac{1}{2}mv^2$ (plus any mgh) to find the speed or height.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
Hooke's law	the extension is directly proportional to the applied force, provided the limit of proportionality is not exceeded
spring constant	the force per unit extension
limit of proportionality	the point beyond which the extension is no longer proportional to the applied force
elastic limit	the maximum force (or extension) for which the material returns to its original length when the force is removed
elastic deformation	the material returns to its original length (shape) when the force is removed
plastic deformation	the material does not return to its original length when the force is removed; a permanent extension remains
stress	the force per unit cross-sectional area
strain	the extension per unit original length
Young modulus	the ratio of stress to strain (within the limit of proportionality)

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Term	Definition
elastic potential energy	the energy stored in an object because it has been stretched or compressed

Exam tips

- **Hooke's law ($F = kx$)** holds only up to the limit of proportionality.
- **Stress $= F/A$, strain $= x/L$, Young modulus = stress/strain** (gradient of the straight part of the stress-strain graph) – watch the units (Pa).
- Energy stored = **area under the force-extension graph** $= \frac{1}{2}Fx$ up to the limit of proportionality (where the line is straight).
- Distinguish **elastic** (returns to shape) from **plastic** (permanent) deformation.

Common mistakes

- Using the diameter as the radius in $A = \pi r^2$, or leaving an area in mm^2. Halve the diameter first; 1 mm^2 = 10^{-6} m^2.
- Reading the work done off a length–force graph as the whole area down to the axis. Only the triangle above L_0 is work.
- Treating stored energy as proportional to extension. It goes as x^2: double the extension, four times the energy; and the work done between two extensions is the difference of two $\frac{1}{2}kx^2$ values.
- Stating Hooke's law without its condition. "Provided the limit of proportionality is not exceeded" is part of the law.
- Swapping the limit of proportionality (the end of the straight line) and the elastic limit (the end of elastic behaviour, a little beyond it).
- Saying a thicker wire has a larger Young modulus. The modulus is the material's; the thicker wire has a larger spring constant.

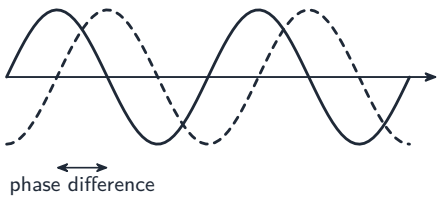
Waves

A-Level Physics

Progressive waves



Ripples spreading on water are progressive waves that carry energy outward.



Two waves of the same frequency, shifted by a phase difference

A **wave** 波 carries **energy** 能量 from one place to another without moving matter overall. The particles of the **medium** 介质 **oscillate** 振动 about fixed rest positions; only the disturbance (and its energy) **propagates** 传播. Examples: a **transverse wave** 横波 on a rope, a **longitudinal wave** 纵波 on a slinky spring, ripples on water, and sound in air. A wave that travels and carries energy is a **progressive wave** 行波.

Wave motion 波动 is a series of oscillations of the particles of a medium, each passing the disturbance on to the next: shake one end of a rope, push one end of a spring, or touch the surface of the water in a **ripple tank** 水波槽, and the oscillation travels away from where it started. For the one-mark definition, a progressive wave transfers energy from one place to another without transferring matter; the particles only oscillate about their rest positions.

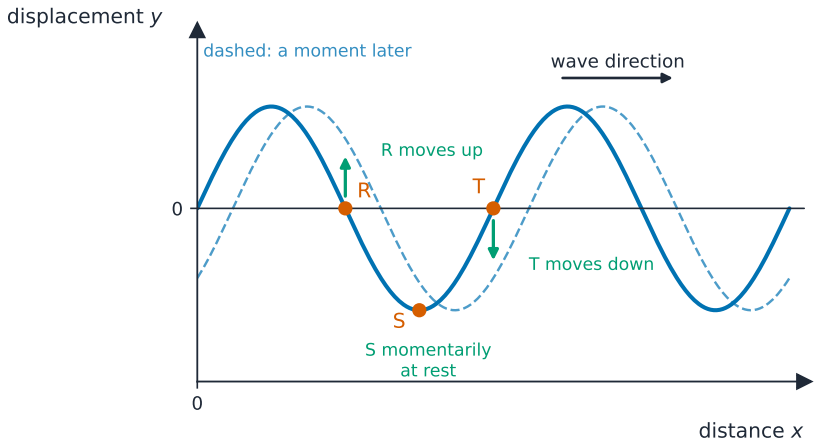
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Key terms

- **displacement** 位移 y – how far a particle has moved from its rest position at a moment. A **vector** 矢量.
- **amplitude** 振幅 A – the largest displacement from the rest position.
- **wavelength** 波长 λ – the shortest distance along the wave between two points that move **in phase** 同相 (for example, two next-door **crests** 波峰).
- **period** 周期 T – the time for one full oscillation of a particle.
- **frequency** 频率 f – the number of full oscillations per second; $f = 1/T$. Unit: **hertz** 赫兹, Hz.
- **speed** 速率 v – how fast a crest travels along the medium.
- **phase difference** 相位差 – the fraction of a cycle by which one oscillation leads or lags another. Given in **radians** 弧度 (a full cycle is 2π) or degrees (a full cycle is 360°).

Two points one wavelength apart are in phase (phase difference 0 or 2π). Two points half a wavelength apart are exactly out of phase (phase difference π).

To find the phase difference between two points from a displacement–distance graph, divide their separation by the wavelength and multiply by 360° (or 2π): two points 0.50 m apart on a wave of wavelength 2.0 m ($v = 600$ m s^{-1}, $f = 300$ Hz) differ in phase by $0.25 \times 360° = 90°$. Only the fraction of a cycle matters, so 450° is the same as 90°. From two displacement–time graphs, read the time shift between the peaks as a fraction of the period. Which way is a point moving at the instant a graph shows? The whole profile moves along, so each point takes up the displacement that its neighbour on the side the wave comes from has now; a point at a crest or a trough is momentarily at rest.



Which way each point moves: towards the displacement its neighbour on the incoming side has now; a crest or trough is momentarily at rest

Worked example. Points R and T on a string are 0.62 cm apart and a quarter of a cycle out of phase. The wave speed is 0.27 m s^{-1}. Find the frequency.

A quarter of a cycle is a quarter of a wavelength, so $\lambda = 4 \times 0.62 = 2.5$ cm and $f = v/\lambda = 0.27/0.025 = 11$ Hz.

The wave equation

In one period T, the wave moves forward by one wavelength λ. So speed = distance/time = $\lambda/T = \lambda f$:

$$v = f\lambda.$$

This comes straight from the definitions of speed, frequency and wavelength, and works for every progressive wave. For the two-mark derivation write both steps: in one period T the wave travels one wavelength λ, so $v = \lambda/T$; and $f = 1/T$, so $v = f\lambda$. Because v is fixed by the medium, a higher frequency means a shorter wavelength. A period question is the same equation the other way round: light of wavelength 460 nm has $f = c/\lambda = 6.5 \times 10^{14}$ Hz and $T = 1/f = 1.5 \times 10^{-15}$ s.

Reading a CRO trace

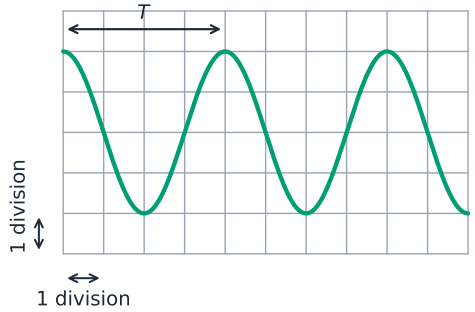
A **cathode-ray oscilloscope** 示波器 (CRO) draws a voltage signal – for sound, the output of a microphone – against time. Two controls matter:

- **time-base** 时基 (seconds per division across): turns horizontal distance on the screen into time. Read the period T as the distance between two next-door peaks, then $f = 1/T$.
- **y-gain** 垂直增益 (volts per division up): turns vertical distance into voltage. The amplitude in volts is the peak height from the centre line.

If the time-base is 5 ms/div and one full cycle takes 4 divisions, then $T = 4 \times 5$ ms = 20 ms and $f = 50$ Hz.

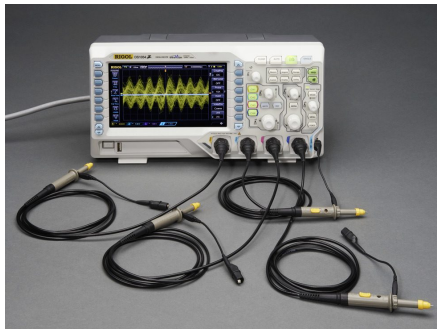
The time-base is the time represented by one division (or one centimetre) across the screen, in s div^{-1}. To set it for a given signal, work backwards: a 2000 Hz sound has $T = 0.50$ ms, so for one cycle to span 2.5 cm the time-base must be $0.50/2.5 = 0.20$ ms cm^{-1}. For the amplitude, multiply the peak height in divisions by the y-gain: 2.0 divisions at 3.5 mV cm^{-1} is 7.0 mV. When the intensity of the sound is reduced to a quarter at the same frequency, the trace keeps its period and its peaks halve in height, because $I \propto A^2$.

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period = n × timebase · peak voltage = n × Y-gain

Reading the period T from a CRO trace using the grid and time-base



A real oscilloscope: the grid lets you read off the period and the amplitude

Intensity of a wave

A wave carries energy. The **intensity** 强度 at a point is the **power** 功率 passing through unit area at right angles to the direction of travel:

$$I = \frac{P}{A}.$$

Unit: W m^{-2}.

Intensity is proportional to the square of the amplitude:

$$I \propto A^2.$$

For a **point source** 点源 sending out energy equally in all directions, the **wavefronts** 波前 are spheres; the surface area at distance r is $4\pi r^2$, so

$$I = \frac{P}{4\pi r^2}, \qquad I \propto \frac{1}{r^2}.$$

Doubling the distance cuts the intensity to a quarter, which means the amplitude is halved (since $I \propto A^2$).

> **Worked example.** A lamp emits 60 W of light equally in all directions. Find the intensity of the light 2.0 m away.

$$I = \frac{P}{4\pi r^2} = \frac{60}{4\pi(2.0)^2} \approx 1.2 \text{ W m}^{-2}.$$

> **Worked example.** Light of power 750 W falls at right angles on a square solar panel of side 1.2 m. Find the intensity.

$I = P/A = 750/1.2^2 = 520$ W m^{-2}. A smaller panel in the same light receives the same intensity but less power, in proportion to its area. The other way round, a magnifying glass of radius r collects a power $I \times \pi r^2$ and concentrates it on a small spot.

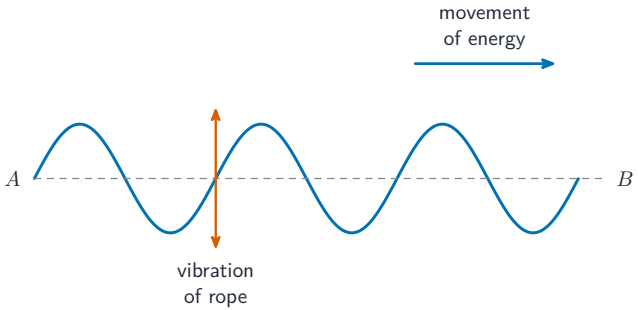
Two rules the multiple-choice questions turn on. Because $I \propto A^2$, the ratio of two intensities is the **square** of the ratio of the amplitudes: waves of amplitude 3.0 cm and 2.0 cm have intensities in the ratio 2.25, and when an amplitude falls to a half the intensity falls to a quarter. For a point source, $I \propto 1/r^2$ means $A \propto 1/r$, so a sketch of A/A_0 against d/x_0 is a curve falling as $1/d$, not a straight line. The one-line statement the scheme wants: intensity is proportional to the amplitude squared.

Transverse and longitudinal waves

Transverse waves

The particles oscillate **perpendicular** 垂直 to the direction the energy travels. A wave on a rope, all electromagnetic waves, and S-waves in the Earth are transverse.

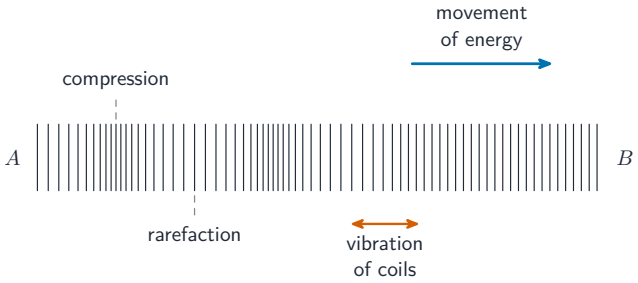
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Transverse wave on a rope

Longitudinal waves

The particles oscillate **parallel** to the direction the energy travels. Sound in any medium, P-waves in the Earth, and the squashes on a slinky are longitudinal. The wave is made of **compressions** 压缩 (higher pressure, particles close together) and **rarefactions** 稀疏 (lower pressure, particles spread out).



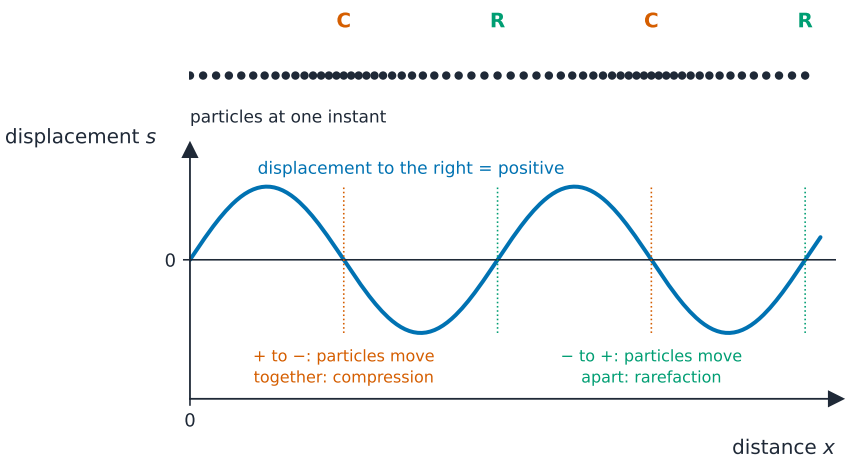
Longitudinal wave on a slinky spring

Graphs of waves

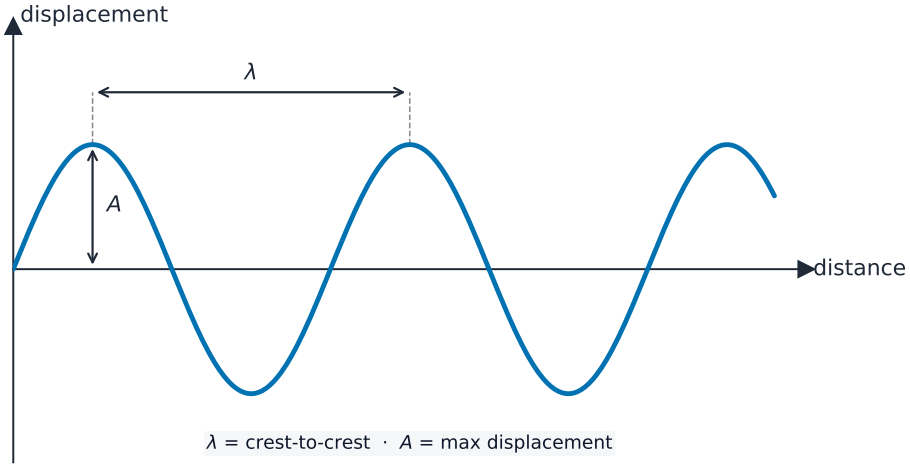
A graph of **particle displacement against position** at one moment looks like a **sine curve** 正弦曲线 for both kinds of wave. The difference: for a transverse wave the displacement axis is the real sideways displacement; for a longitudinal wave it is the small back-and-forth displacement along the direction of travel (positive one way, negative the other).

For a longitudinal wave take displacement to the **right** as positive. Where the graph crosses zero going from positive to negative, the particles on either side have moved **towards** that point, so it is a compression; where it crosses from negative to positive they have moved apart, a rarefaction. Two neighbouring compressions are one wave-

length apart, and a compression and the next rarefaction are half a wavelength apart. The direction of motion of a particle follows the same rule as for a transverse wave: it moves towards the displacement of its neighbour on the side the wave comes from.

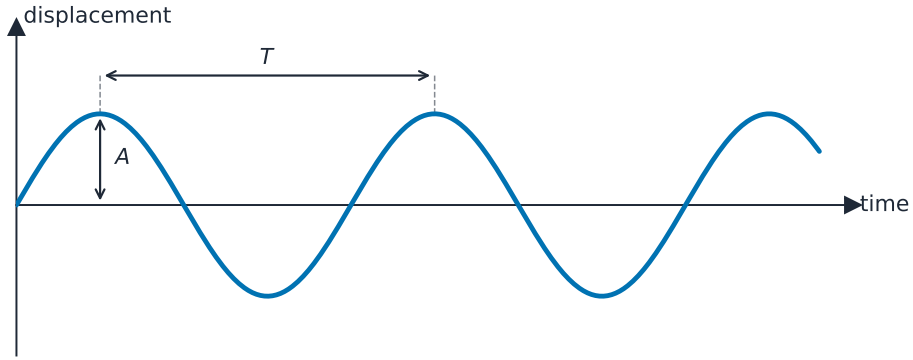


The same longitudinal wave as particles and as a graph: a compression where the displacement changes from positive to negative, a rarefaction where it changes from negative to positive



A displacement–distance graph shows the wave's amplitude and wavelength

A graph of **particle displacement against time** at one point in space is also a sine curve for both kinds. Read the period T from this graph.



A displacement–time graph shows the wave's amplitude and period

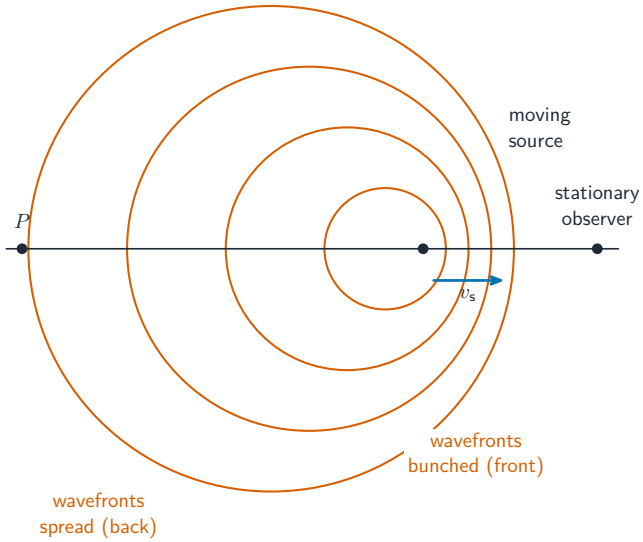
The two-mark comparison, with reference to the direction of energy transfer: in a transverse wave the oscillations are **perpendicular** to the direction of energy transfer; in a longitudinal wave they are **parallel** to it. Both kinds transfer energy without transferring matter, both can be reflected, refracted and diffracted, and both can form stationary waves (topic 8). Only transverse waves can be polarised, and only electromagnetic waves, which are all transverse, can travel through a vacuum; sound needs a medium. Reading the two graphs together: the displacement–distance graph gives λ, the displacement–time graph gives T, and $v = \lambda/T$.

Doppler effect (moving source, stationary observer)

When the **source** 波源 of a sound moves relative to a **stationary** 静止 **observer** 观察者, the heard frequency is different from the source frequency. This is the **Doppler effect** 多普勒效应.

- source moving towards the observer: the wavefronts in front are squashed, so the wavelength is shorter and the heard frequency is **higher**.
- source moving away from the observer: the wavefronts behind are spread out, so the wavelength is longer and the heard frequency is **lower**.

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A moving source squashes the wavefronts ahead of it, raising the observed frequency

The formula (source moving at speed v_s along the line to the observer; wave speed v, source frequency f_s, heard frequency f_o):

$$f_o = \frac{v \cdot f_s}{v \pm v_s}.$$

Choose the sign to match the physics:

- minus sign on the bottom when the source moves **towards** the observer ($f_o > f_s$),
- plus sign when the source moves **away** ($f_o < f_s$).

You only need the case of a stationary observer.

Worked example. A car horn at $f_s = 800$ Hz moves at 30 m s^{-1} towards a still listener. Speed of sound $v = 340$ m s^{-1}:

$$f_o = \frac{340 \times 800}{340 - 30} = \frac{272\,000}{310} \approx 877 \text{ Hz}.$$

Moving away at the same speed, the listener hears $340 \times 800/(340 + 30) = 735$ Hz: the frequency is constant while the car approaches, drops as it passes, and is constant again, lower, as it recedes. A sketch of observed frequency against time for a source passing at constant speed is two level lines joined by a fall. Working backwards, a source of 1200 Hz heard as 960 Hz is moving away ($f_o < f_s$): $960 = 1200 \times 340/(340 + v_s)$ gives $v_s = 85$ m s^{-1}. A buzzer swung in a horizontal circle at 25 m s^{-1} while emitting 846 Hz is heard between $846 \times 330/355 = 786$ Hz (moving directly away) and $846 \times 330/305 = 915$ Hz (moving directly towards), once each per revolution. If the period of a CRO trace of the sound rises continuously, the observed frequency is falling: the source is moving away with increasing speed. The Doppler effect happens for all waves, light included (topic 25); only the sound formula is examined here.

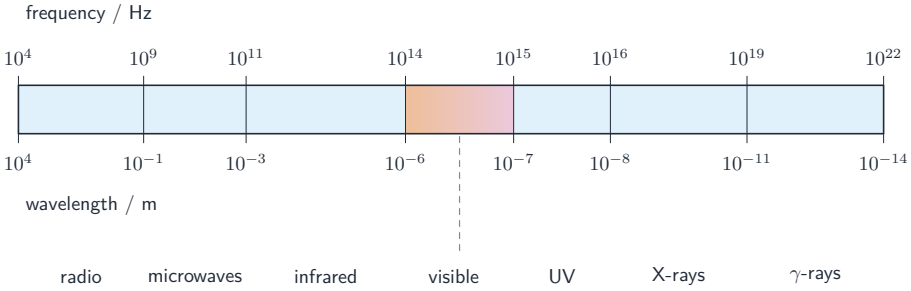
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Electromagnetic spectrum

All **electromagnetic waves** 电磁波 (EM waves) are **transverse** and travel in a **vacuum** 真空 at the same speed:

$$c = 3.00 \times 10^8 \text{ m s}^{-1}.$$

The **electromagnetic spectrum** 电磁波谱 includes radio waves, **microwaves** 微波, **infrared** 红外线, visible light, **ultraviolet** 紫外线, **X-rays** X 射线 and **γ-rays** γ 射线.



The electromagnetic spectrum

Approximate wavelength ranges in free space (learn the orders of magnitude):

- radio waves: $> 10^{-1}$ m (up to many km).
- microwaves: 10^{-3} m to 10^{-1} m.
- infrared: $\sim 7 \times 10^{-7}$ m to 10^{-3} m.
- visible light: 400 nm (violet) to 700 nm (red), i.e. 4×10^{-7} m to 7×10^{-7} m.
- ultraviolet: $\sim 10^{-8}$ m to 4×10^{-7} m.
- X-rays: $\sim 10^{-11}$ m to 10^{-8} m.
- γ-rays: $< 10^{-11}$ m.

The boundaries between regions are not sharp. Use $c = f\lambda$ to change between wavelength and frequency. Only light with wavelengths 400–700 nm can be seen.

To identify a region, convert to a wavelength with $\lambda = c/f$ and compare with the ranges: 2.1 cm is a microwave; 138 pm $= 1.4 \times 10^{-10}$ m is an X-ray; 30 THz gives $\lambda = 1.0 \times 10^{-5}$ m, infrared; 3.0×10^{16} Hz gives 1.0×10^{-8} m, ultraviolet. Visible light spans frequencies of about 4.3×10^{14} to 7.5×10^{14} Hz, so a wave of 5.0×10^{14} Hz can

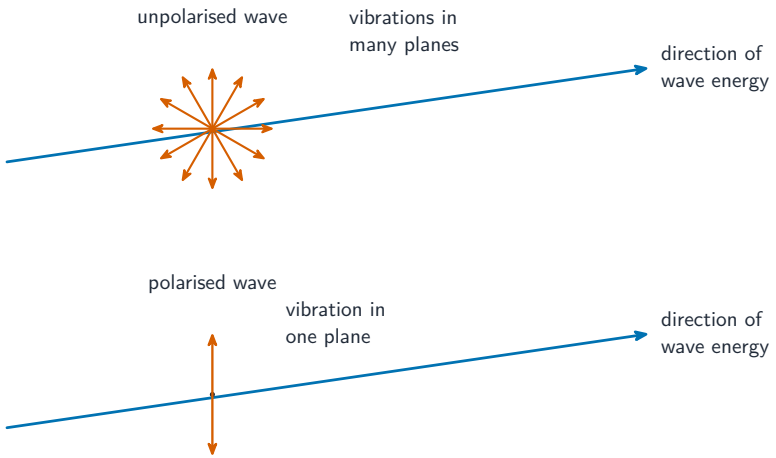
be seen and one of wavelength 5.0×10^{-6} m (infrared) cannot. Red light has a longer wavelength and a lower frequency than green. A list "in order of increasing wavelength" runs γ-rays, X-rays, ultraviolet, visible, infrared, microwaves, radio waves. A pulse of light reflected from a wall 150 m away returns after $2 \times 150/(3.00 \times 10^8) = 1.0$ μs.

Polarisation

Polarisation 偏振 means making a transverse wave oscillate in one plane only.

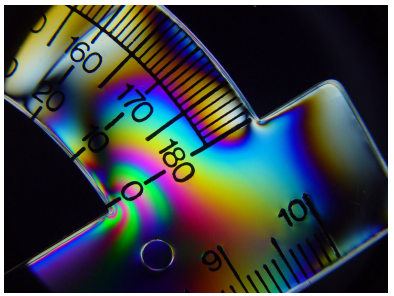
- Only **transverse** waves can be polarised – the oscillation is perpendicular to the direction of travel, so different perpendicular planes are real choices.
- **Longitudinal** waves (sound) cannot be polarised – the oscillation is along the direction of travel, so there is no other plane.

So polarisation is a test: if a wave can be polarised, it must be transverse.



Unpolarised waves vibrate in many planes; a polarised wave vibrates in one plane

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A see-through plastic protractor placed between two crossed polarising filters. Light only reaches your eye because the stressed plastic rotates its plane of polarisation – the colours map where the plastic is squeezed most. With ordinary light it would just look clear

Malus's law

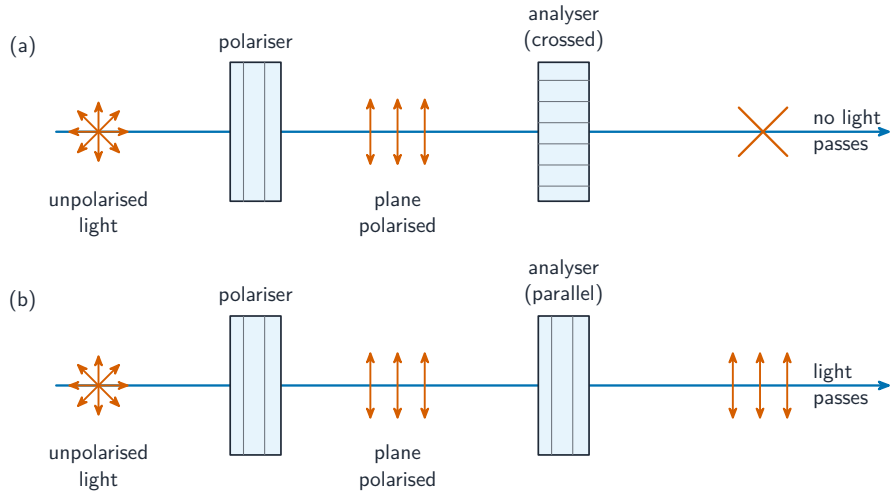
Plane-polarised 平面偏振 light of intensity I_0 passes through a **polarising filter** 偏振片 whose **transmission axis** 透光轴 is at angle θ to the plane of polarisation. The transmitted intensity is given by **Malus's law** 马吕斯定律:

$$I = I_0 \cos^2 \theta.$$

- $\theta = 0°$: filter lined up with the polarisation, $I = I_0$, all passes through.
- $\theta = 90°$: filter at right angles, $I = 0$, all blocked.
- $\theta = 60°$: $I = I_0 \cos^2 60° = I_0 \cdot 0.25 = I_0/4$.

Worked example. Plane-polarised light of intensity 12 W m^{-2} meets a polarising filter whose axis is at 30° to the plane of polarisation. Find the transmitted intensity.

$$I = I_0 \cos^2 \theta = 12 \times \cos^2 30° = 12 \times 0.75 = 9.0 \text{ W m}^{-2}.$$



Crossed filters (a) block the light; parallel filters (b) let it pass

For two filters in a row, use Malus's law twice with the angle between each pair. Be careful with the angle each time – after the first filter the polarisation is along that filter's axis, and the second filter's angle is measured from there.

(You do not need to work out the effect of a polarising filter on an unpolarised wave.)

When a filter is rotated through 360° in front of plane-polarised light, the transmitted intensity varies as $\cos^2\theta$: it is a maximum when the transmission axis is parallel to the plane of polarisation (0° and 180°), zero when perpendicular (90° and 270°), and never negative. Because $I \propto A^2$, the transmitted **amplitude** is $A_0\cos\theta$: at 45° the intensity halves and the amplitude falls to 0.71 of its value. For two filters, the first sets the plane of polarisation along its own axis, so the second filter's angle is measured from that: vertically polarised light through filters at 50° and then 80° to the vertical keeps $\cos^2 50° \times \cos^2 30° = 0.31$ of its intensity. The one-mark definitions: polarisation is the oscillation of a transverse wave in a single plane, which contains the direction of travel; sound cannot be polarised because its oscillations are along the direction of travel, so there is no plane to select.

■ Reading wave motion over time

- A particle oscillating between $-A$ and $+A$ travels a total distance $4A$ **in one complete cycle**. Over n complete cycles, $D = 4nA$; this is particle travel, not wave travel.
- In n periods, the wave travels $s = n\lambda$ and the time is $t = nT = n/f$.
- After $T/4$, a progressive-wave profile has moved $\lambda/4$ in its propagation direction. A stationary-wave profile changes height without moving its nodes: do not shift it sideways.

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- If a polariser starts perpendicular to the incident polarisation and rotates by ϕ, its angle to the polarisation is $90° - \phi$, so $I = I_0\sin^2\phi$.

> **Worked example.** A particle has amplitude 3.0 mm and completes four cycles. $D = 4nA = 4 \times 4 \times 3.0 = 48$ mm. A wave of frequency 25 Hz takes $t = n/f = 4/25 = 0.16$ s for those cycles.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
progressive wave	a wave that transfers energy from one place to another without transferring matter
transverse wave	a wave whose oscillations are perpendicular to the direction of energy transfer
longitudinal wave	a wave whose oscillations are parallel to the direction of energy transfer
displacement	the distance of a particle from its equilibrium (rest) position, in a stated direction
amplitude	the maximum displacement of a particle from its equilibrium position
wavelength	the minimum distance between two points on the wave that are in phase (for example, adjacent crests)
period	the time for one complete oscillation of a particle (or for the wave to travel one wavelength)
frequency	the number of oscillations per unit time (or the number of wavefronts passing a point per unit time)
phase difference	the fraction of a cycle, as an angle, by which one oscillation leads or lags another
intensity	the power per unit area, at right angles to the direction of travel
Doppler effect	the change in the observed frequency of a wave when the source moves relative to the observer
polarisation	the oscillations of a transverse wave are in one plane only, containing the direction of travel

Term	Definition
time-base	the time represented by one division across the screen of a CRO

Exam tips

- Define terms precisely (displacement, amplitude, wavelength, period, frequency) and use $v = f\lambda$.
- Distinguish **transverse** (vibration perpendicular to travel) from **longitudinal** (parallel); only transverse waves can be **polarised**.
- For the **Doppler effect**, the observed frequency **rises** as the source approaches and falls as it recedes.
- Learn the **electromagnetic spectrum** order; all its waves travel at c in a vacuum.

Common mistakes

- A phase difference bigger than one cycle, or degrees where radians were asked. Only the fraction of a cycle matters: 450° is 90°, and 90° is $\pi/2$.
- Reading the wavelength from a displacement–time graph. That graph gives the period; the wavelength comes from the displacement–distance graph.
- Doubling the amplitude and "doubling" the intensity. Intensity goes as the amplitude squared: four times.
- Choosing the Doppler sign by feel. Towards means minus on the bottom (frequency up); away means plus (frequency down). Check that the answer lies on the right side of f_s.
- Using the first filter's angle for the second. After a filter the light is polarised along that filter's axis, and the next angle is measured from there.
- Saying that sound can be polarised. Only transverse waves can.

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Superposition

A-Level Physics

Principle of superposition

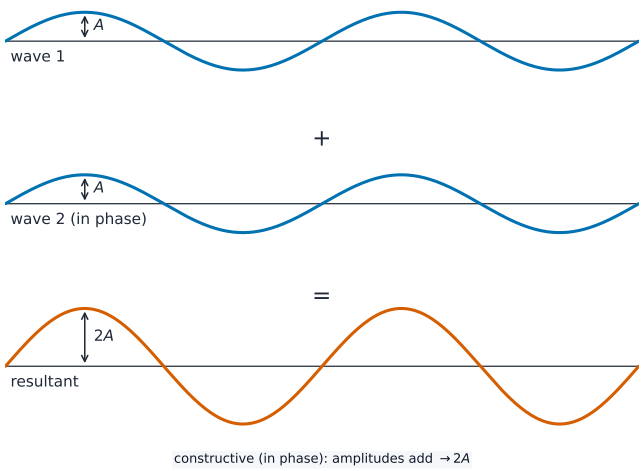
When two or more **waves** 波 overlap at a point, the **displacement** 位移 there is the **vector sum** 矢量和 of the displacements each wave would make on its own. This is the **principle of superposition** 叠加.

The waves pass through each other and come out unchanged. Superposition is the base of everything in this topic.

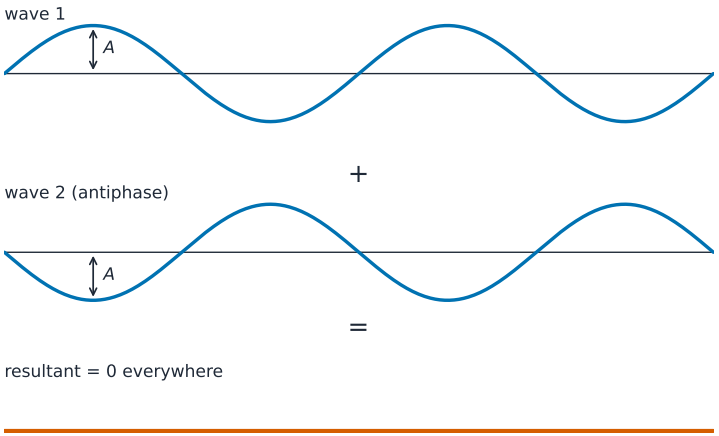
For the two-mark statement: **when two or more waves meet at a point, the resultant displacement is the sum of the displacements of the individual waves.** It is the displacements that add, not the amplitudes or the intensities, and the principle applies whenever waves of the same type overlap; the waves do not have to be coherent or of equal amplitude.

If two waves of **amplitude** 振幅 A_1 and A_2 meet:

- **in phase** 同相 (**crest** 波峰 meets crest): the amplitude is $A_1 + A_2$ (**constructive interference** 相长干涉).
- exactly out of phase (crest meets **trough** 波谷, **phase difference** 相位差 π): the amplitude is $|A_1 - A_2|$ (**destructive interference** 相消干涉).
- any other phase difference ϕ: the amplitude is somewhere between these two.



Two waves arriving in phase add to give double the amplitude (constructive)



destructive (out of phase by $\lambda/2$): cancel if equal A

Two waves arriving exactly out of phase cancel to zero (destructive)

For **intensity** 强度, $I \propto A^2$. Two equal waves meeting in phase give intensity $(2A)^2 = 4A^2$ – **four times** the intensity of one wave alone.

Because $I \propto A^2$, work in amplitudes first. Two waves of intensities I and $4I$ have amplitudes A and $2A$: superposing in phase gives amplitude $3A$ and intensity $9I$, in antiphase amplitude A and intensity I. So two coherent waves of different intensities never cancel completely: the minima have intensity $(A_2 - A_1)^2$, not zero. A wave of amplitude $2A$ meeting one of amplitude $A/2$ travelling the opposite way gives a resultant that varies between $2.5A$ and $1.5A$, a stationary pattern with no true nodes. Doubling the amplitude of one of two equal waves that meet in phase takes the resultant from $2A$ to $3A$, so the intensity rises from $4A^2$ to $9A^2$: 2.25 times.

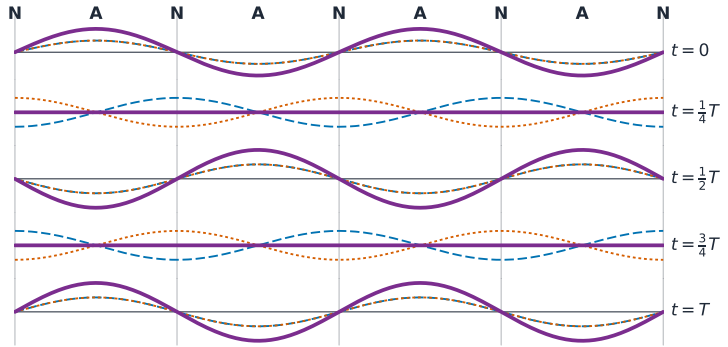
Stationary (standing) waves

When two identical **progressive waves** 行波 travel in **opposite directions** and overlap, they make a **stationary wave** 驻波. Examples: a wave on a string **reflected** 反射 from a fixed end overlapping the incoming wave; sound in an air column reflected from a closed end; microwaves between an emitter and a metal sheet.

Explain how the stationary wave is formed (three or four marks, asked for a string, an air column and microwaves alike): the wave from the source travels to the far end (the wall, the closed end, the metal plate) and is **reflected**; the incident and reflected waves have the same frequency, wavelength and speed and travel in **opposite directions**; they **superpose** where they overlap; at the points where they always meet in phase the displacements add to give the maximum amplitude, an antinode, and at the points where they always meet in antiphase they cancel, a node. For "state the conditions": two waves of the same type, with the same frequency (and wavelength)

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and speed, travelling in opposite directions along the same line; equal amplitudes are needed only for the nodes to have zero amplitude. The metal plate in the microwave experiment is there to reflect the waves back along their own path.



stationary wave: nodes and antinodes from two opposite waves

A stationary wave forms where two opposite waves overlap (N marks a node, A an antinode)

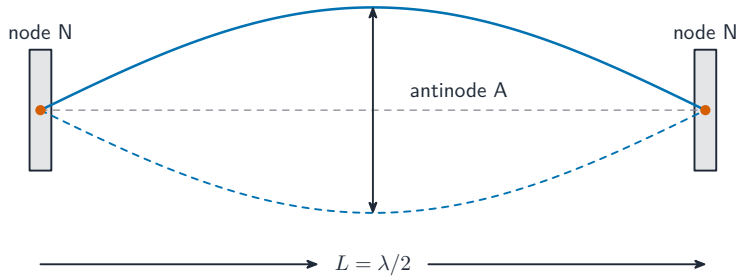
Nodes and antinodes

In a stationary wave:

- **node** 波节 – a point that is always at zero displacement (the two waves always cancel). The distance between next-door nodes is $\lambda/2$.
- **antinode** 波腹 – a point of largest amplitude (the two waves always add). The distance between next-door antinodes is $\lambda/2$.
- a node and the next antinode are $\lambda/4$ apart.

Particles between two nodes oscillate **in phase** with each other, but with different amplitudes (largest at the antinode, zero at the nodes). Particles on opposite sides of a node oscillate in **antiphase** 反相 (phase difference π).

So the phase difference between two points on the same loop (between adjacent nodes) is 0°; between points on neighbouring loops it is 180°; and between points on loops separated by one whole loop it is 0° again. Every point reaches its maximum displacement at the same instant and passes through zero at the same instant. A sketch of the string a quarter of a cycle after the instant of maximum displacement is a straight line along the rest position; half a cycle later it is the mirror image of the first sketch. In one period a particle at an antinode travels four amplitudes, so a particle that moves 72 mm in one and a half periods has an amplitude of 12 mm.



Fundamental mode on a stretched string – one loop, with L equal to half a wavelength

> **Worked example.** A string of length 0.80 m is fixed at both ends and vibrates in its fundamental mode, where the wave speed is 240 m s^{-1}. Find the fundamental frequency.

One loop fits the string, so $\lambda = 2L = 1.60$ m. Then

$$f = \frac{v}{\lambda} = \frac{240}{1.60} = 150 \text{ Hz}.$$

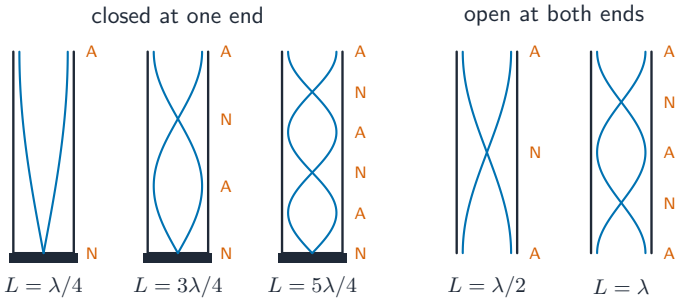
How a stationary wave differs from a progressive wave: it does not carry **energy** 能量 along its length, the pattern does not move along, and the nodes stay fixed; a progressive wave has the same amplitude everywhere and carries energy. The three differences the scheme lists: a progressive wave transfers energy and a stationary wave does not; in a progressive wave every point has the same amplitude, in a stationary wave the amplitude varies from zero at a node to a maximum at an antinode; in a progressive wave neighbouring points differ in phase, in a stationary wave all the points between two nodes are in phase.

■ Measuring wavelength from node spacing

Drive a string with a vibrator at frequency f until a stationary pattern appears. Measure the distance between two well-separated nodes and divide by the number of half-**wavelengths** 波长 between them. Then λ is known, and $v = f\lambda$ gives the wave speed.

For a tube closed at one end and open at the other (a **resonance tube** 共鸣管), the closed end is a displacement node and the open end is a displacement antinode. The **fundamental** 基频 has $L = \lambda/4$; the next resonance is at $L = 3\lambda/4$; and so on. For a tube open at both ends, both ends are antinodes; the fundamental is at $L = \lambda/2$.

The general rule: a closed end is a node and an open end an antinode, and a string fixed at both ends has a node at each end. So a closed pipe fits an odd number of quarter-wavelengths ($L = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots$), while an open pipe and a fixed string fit whole half-wavelengths ($L = \lambda/2, \lambda, 3\lambda/2, \ldots$); a pipe open at both ends with n nodes has $n + 1$ antinodes. Frequencies follow from $f = v/\lambda$: a closed pipe whose lowest note is 820 Hz has $\lambda = 4L$, and a corridor 13.2 m long with a reflecting door at each end resonates lowest at $f = v/2L = 330/26.4 = 12.5$ Hz.



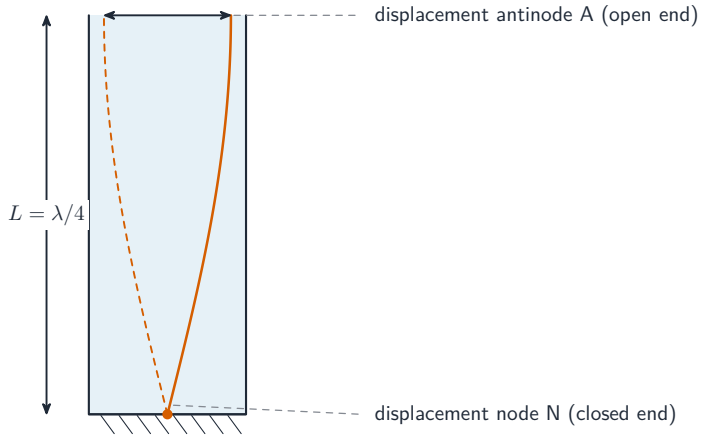
Modes of an air column: a closed end is a node, an open end an antinode

> **Worked example.** A loudspeaker at the open end of a tube 0.51 m long, closed at the other end, sets up a stationary wave with two nodes and two antinodes. Find the wavelength and, with $v = 340$ m s^{-1}, the frequency.

Two nodes and two antinodes is the second mode of a closed pipe, $L = 3\lambda/4$, so $\lambda = 4 \times 0.51/3 = 0.68$ m and $f = 340/0.68 = 500$ Hz. The longest wavelength that can resonate in this tube is $4L = 2.0$ m.

The experiments the syllabus names all measure λ from node or antinode spacing. In the **resonance tube**, a tube is raised out of water while a loudspeaker or tuning fork sounds at its open end; the sound is suddenly loud at the first resonance, when the air column is $\lambda/4$ long, and again at $3\lambda/4$, so the tube moves $\lambda/2$ between the two (a student needs no equipment to detect the resonance: the note becomes loud). In a **dust tube**, fine powder settles into heaps at the displacement nodes, $\lambda/2$ apart. With **microwaves**, a receiver moved along the line between the transmitter and a metal reflecting plate reads a minimum every $\lambda/2$: a receiver that starts at a minimum and passes six more minima in 1.05 m has crossed 3λ, so $\lambda = 0.35$ m and, with $v = 340$ m s^{-1} for the equivalent sound experiment, $f = 970$ Hz. A stationary wave in a microwave oven melts chocolate at the antinodes, $\lambda/2$ apart, so the spot spacing and the oven's frequency (2.45 GHz) give the speed of light from $c = f\lambda$.

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Fundamental mode in a closed pipe – a node at the closed end, an antinode at the open end

Diffraction

Diffraction 衍射 is the spreading of a wave after it passes through a gap or around an **obstacle** 障碍物. All waves diffract – water, sound, light, microwaves.

For the two-mark "state what is meant by diffraction": **the spreading of a wave as it passes through a gap (an aperture) or around the edge of an obstacle**, into the region behind it. Diffraction changes the direction the wave travels in, but not its speed, frequency or wavelength.

The amount of spreading depends on the ratio of **wavelength to gap width**:

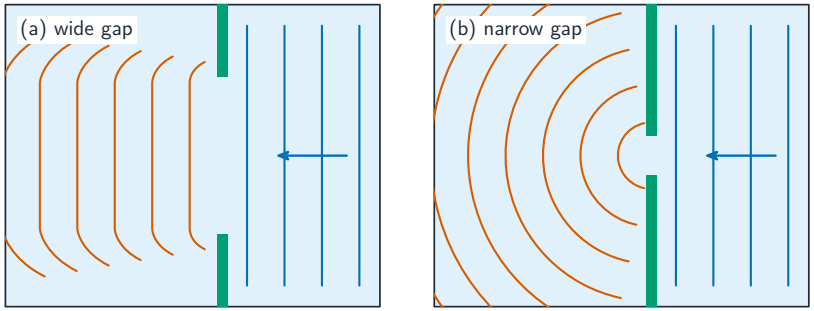
- gap **much wider** than λ: very little spreading; the wave goes nearly straight through.
- gap **about the size of** λ: a lot of spreading; the wave fans out.
- gap **smaller than** λ: very strong spreading; the gap acts almost like a point source.

Show this with water waves in a **ripple tank** 水波槽: straight waves meet a barrier with a gap, and the waves curve more as the gap is made narrower. The same idea is why you can hear someone around a corner (speech has λ near 1 m, close to the gap size) but cannot see them (visible light has $\lambda \sim 500$ nm, far smaller than the gap).

For the strongest spreading the gap should be about one wavelength wide. So to increase the diffraction of a given wave, make the gap narrower; for a given gap, use a longer wavelength, which means a lower frequency. Radio waves of wavelength 1.5 km diffract around a mountain and reach an aerial behind it; microwaves of 1.5 cm do not. Sound of 0.44 kHz in air has $\lambda = 330/440 = 0.75$ m, so features of about 0.75 m diffract it most, and of the sounds passing through a doorway 0.80 m wide the low

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frequencies spread out most. Making the gap many wavelengths wide, or raising the frequency, reduces the spreading.



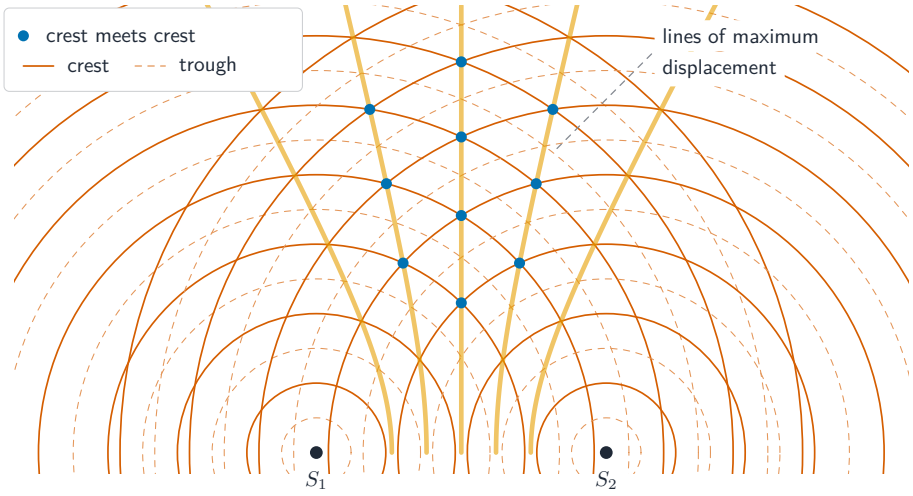
Diffraction in a ripple tank – a wide gap (a) spreads the waves little, a narrow gap (b) much more

Interference

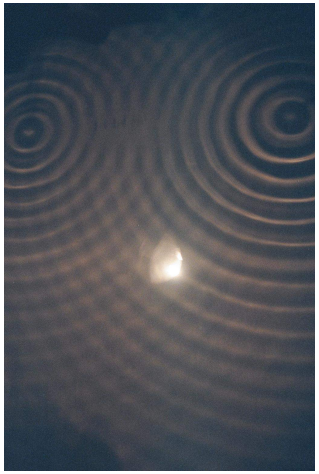


The shifting colours on a soap bubble come from the interference of light.

Interference 干涉 is the superposition of two **coherent** 相干 waves to give a steady pattern of high-amplitude regions (constructive) and low-amplitude regions (destructive).



Two coherent sources give lines of maximum displacement where crests meet crests



The same effect in a real ripple tank – two coherent sources give a steady interference pattern

Coherence

Two sources are **coherent** when they emit waves with a **constant phase difference** (which also needs the same frequency). Two separate lamps are not coherent – their phase changes randomly, so any pattern flickers too fast to see and you get only an average.

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The one-mark definition: coherent waves have a **constant phase difference**, which requires the same frequency. They need not be in phase with each other: two coherent sources emitting 180° apart give a pattern whose central line is a minimum. Two separate lasers, or a lamp and a laser, are not coherent even when their frequencies happen to match, so no steady pattern forms.

To make coherent light from one source, pass it through two **slits** 狭缝 in a **double-slit** 双缝 setup. Both slits are lit by the same wavefront, so the two beams keep a fixed phase relationship.

Conditions for a clear pattern

To see two-source fringes you need:

1. two **coherent** sources (constant phase difference).
2. roughly equal amplitudes (or the dark regions are not very dark).
3. the waves overlap where you look.
4. for light (a transverse wave), the same plane of **polarisation** 偏振.

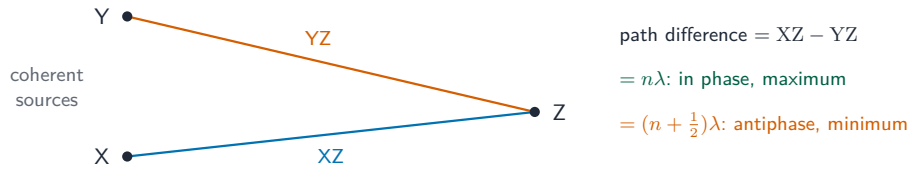
In practice, for light: a single slit (or a laser) makes the two slits coherent; the slits are narrow, so each diffracts the light into the region where the two beams overlap; and the slits are close together with the screen far away, so that the fringes are wide enough to see.

Path difference

For two coherent sources, what happens at a point depends on the **path difference** 路程差 Δx between the two waves arriving there:

- constructive: $\Delta x = n\lambda$ (for whole numbers $n = 0, 1, 2, \ldots$).
- destructive: $\Delta x = (n + \frac{1}{2})\lambda$.

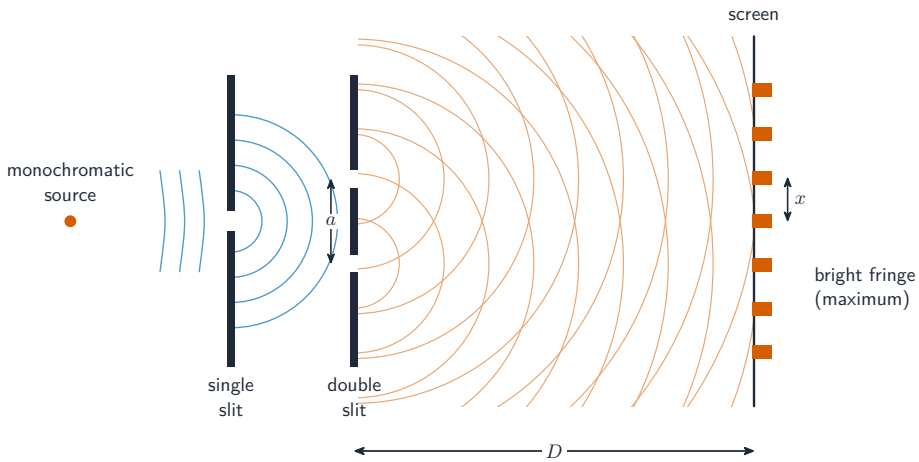
The path difference fixes the **phase difference**: one wavelength of path is 360°. Waves that have travelled 100 cm and 80 cm from two in-phase sources of wavelength 8.0 cm arrive with a path difference of 2.5λ, a phase difference of 180°, and cancel; microwaves of wavelength 4 cm whose paths differ by 6 cm (1.5λ) give a minimum, of zero intensity only if the two amplitudes are equal. Lowering both source frequencies equally lengthens the wavelength, so the same path difference is a smaller number of wavelengths and the point is no longer a minimum.



What happens at Z depends on the path difference XZ − YZ, measured in wavelengths

■ Double-slit (Young's) experiment

For two slits a distance a apart, with a screen a distance D away (assume $D \gg a$), light of wavelength λ makes fringes on the screen.



Young's double-slit experiment – the single slit makes the two slits coherent sources

The **fringe spacing** 条纹间距 x (one **fringe** 条纹 to the next) is

$$\lambda = \frac{ax}{D}, \qquad x = \frac{\lambda D}{a}.$$

Bright fringes (**maximum** 极大) are where the path difference is a whole number of λ; dark fringes (**minimum** 极小) where it is $(n+\frac{1}{2})\lambda$. The fringes are equally spaced.

To make the fringe spacing smaller: increase a (slits further apart), reduce D (screen closer), or use a shorter λ (bluer light).

Explain how the pattern of bright and dark fringes is formed (three marks): the light **diffracts** at each slit; the two diffracted beams **overlap** and superpose; where the path difference from the two slits is a whole number of wavelengths the waves arrive in phase and interfere constructively, giving a bright fringe, and where it is an odd number of half-wavelengths they arrive in antiphase and interfere destructively, giving a dark fringe. A brighter source makes the bright fringes brighter but does not change their spacing; changing to blue light makes the spacing smaller, so to keep the same spacing the slits must be moved closer together; making each slit narrower increases the diffraction, so fringes appear across a wider region, with the spacing unchanged.

> **Worked example.** In a double-slit experiment the slits are 0.50 mm apart and lit by light of wavelength 600 nm. The screen is 2.0 m away. Find the fringe spacing.

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$$x = \frac{\lambda D}{a} = \frac{(600\times10^{-9})(2.0)}{0.50\times10^{-3}} = 2.4\times10^{-3}\ \text{m} = 2.4\ \text{mm}.$$

> **Worked example.** Red light of wavelength 680 nm falls on slits 0.16 mm apart. The distance between the centres of the first and ninth dark fringes is 3.2 cm. Find the distance D to the screen.

Eight fringe spacings make 3.2 cm, so $x = 4.0$ mm and $D = ax/\lambda = (0.16\times10^{-3})(4.0\times10^{-3})/(680\times10^{-9}) = 0.94$ m. On a screen 5.0 cm wide centred on the pattern, with $x = 2.4$ mm, ten bright fringes fit on each side of the central one: 21 in all. A graph of x against a is a curve falling as $1/a$; a graph of x against λ (or against D) is a straight line through the origin with gradient D/a (or λ/a), from which a can be found.

Diffraction grating

A **diffraction grating** 衍射光栅 has many equally spaced slits – often hundreds or thousands per millimetre. Each slit is a coherent source. A maximum is seen at angle θ from the **normal** 法线 to the grating when

$$d\sin\theta = n\lambda,$$

where d is the **slit spacing** 缝间距 (centre to centre), $n = 0, \pm1, \pm2, \ldots$ is the **order** 级次, and λ is the wavelength.

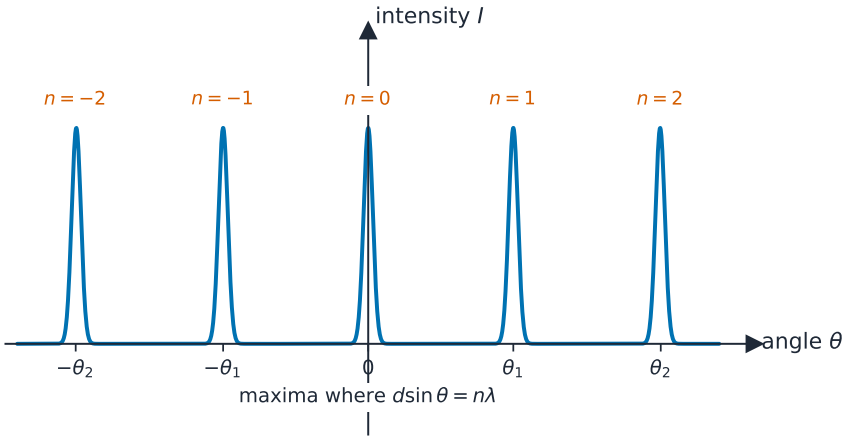
> **Worked example.** A diffraction grating has 500 lines per mm. Light of wavelength 600 nm is shone normally on it. Find the angle of the first-order ($n = 1$) maximum.

The slit spacing is $d = \dfrac{1}{500}$ mm $= 2.0\times10^{-6}$ m, so

$$\sin\theta = \frac{n\lambda}{d} = \frac{600\times10^{-9}}{2.0\times10^{-6}} = 0.30 \quad\Rightarrow\quad \theta \approx 17°.$$

Compared with the double slit, a grating gives much **sharper** maxima, because more slits add together – every other direction is cancelled by many slits.

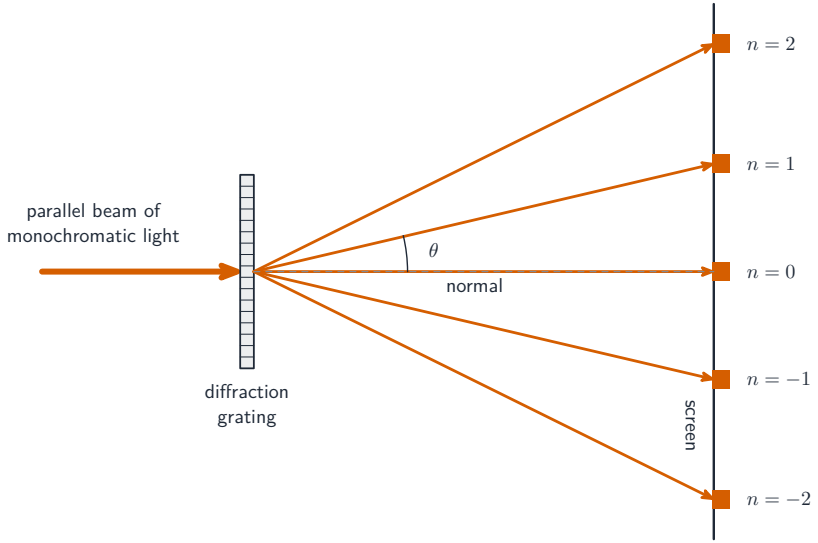
To **describe the diffraction at the grating**: the light spreads out (diffracts) at every slit, and the waves from all the slits superpose; in the directions where the path difference between neighbouring slits is a whole number of wavelengths they are all in phase, so sharp maxima form there and almost nothing in between. A graph of intensity against angle is a set of narrow peaks at $\theta = 0$ and at $\pm\theta_1, \pm\theta_2, \ldots$, where $\sin\theta_n = n\lambda/d$: equally spaced in $\sin\theta$, so slightly further apart in θ at the higher orders.



Intensity against angle for a grating: sharp maxima where $d\sin\theta = n\lambda$

Worked example. Light of wavelength 680 nm is incident normally on a grating with 450 lines per mm. Find the angle between the two second-order maxima.

$d = 1/450$ mm $= 2.22\times10^{-6}$ m, so $\sin\theta_2 = 2\times680\times10^{-9}/(2.22\times10^{-6}) = 0.612$ and $\theta_2 = 37.7°$; the two second-order beams are $2\theta_2 = 75°$ apart.



A diffraction grating splits monochromatic light into sharp maxima on a screen

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Slit spacing from "lines per mm"

If a grating has N lines per millimetre, then $d = 1/N$ millimetres $= 10^{-3}/N$ metres. For 450 lines per mm, $d = 1/450$ mm ≈ 2.22 μm.

Highest order

For a given grating and wavelength, $\sin\theta = n\lambda/d$ cannot be more than 1, so the highest order seen is

$$n_{\max} = \left\lfloor \frac{d}{\lambda} \right\rfloor .$$

If $d/\lambda = 3.27$, orders up to $n = 3$ exist; $n = 4$ would need $\sin\theta > 1$ and is not seen.

So the total number of maxima on a wide screen is $2n_{\max}+1$: for 700 nm light and 400 lines per mm, $d/\lambda = 3.57$, so $n_{\max} = 3$ and seven beams are seen. A shorter wavelength gives more orders at smaller angles. With white light every order except the zero order is a spectrum, violet nearest the centre and red furthest out, because θ grows with λ; the zero order stays white. Two wavelengths give a maximum at the same angle when $n_1\lambda_1 = n_2\lambda_2$: the third order of 400 nm coincides with the second order of 600 nm.

Finding λ with a grating

Shine parallel light of unknown wavelength straight at the grating. Measure the angle θ_1 of the first-order maximum from the centre. Then $\lambda = d\sin\theta_1$. Repeating for higher orders and averaging reduces error.

Measure the angle between the first-order maxima on the two sides and halve it, which cancels any error in setting the zero; higher orders give larger angles and so a smaller percentage uncertainty; and plotting $\sin\theta$ against n for several orders gives a straight line through the origin of gradient λ/d, so $\lambda = Gd$ (or, for a known wavelength, $d = \lambda/G$). Two things must be right: θ is measured from the **normal** to the grating, not from its surface, and d is the distance between adjacent lines, so 400 lines per mm means $d = 2.5$ μm, never 400.

source phase and path phase

Coherent sources have a **fixed** phase difference; they need not emit in phase. The phase difference at an observation point combines the initial source phase with the phase change due to the path difference. Use a consistent sign convention and reduce the result modulo 360°.

For a lagging second source, $\delta_{\text{arrival}} = \delta_{\text{source}} + 360°(r_2 - r_1)/\lambda$. A multiple of 360° gives a maximum; 180° modulo 360° gives a minimum (zero only for equal amplitudes).

Worked example. Sources emit 180° apart. An extra path of $\lambda/2$ adds 180°, giving 360° at arrival: a **maximum**, not a minimum. In a wavefront diagram, a crossing of two crests is a maximum; a crest meeting a trough is a minimum.

■ Overlapping orders and a screen measurement

Two wavelengths give maxima in the same direction when $n_1\lambda_1 = n_2\lambda_2$. For example, second-order 600 nm and third-order 400 nm overlap because $n_1\lambda_1 = 2\times600 = 1200$ nm and $n_2\lambda_2 = 3 \times 400 = 1200$ nm.

If a grating order is measured on a screen a distance L along the normal, its displacement y gives $\theta = \tan^{-1}(y/L)$. Then apply $d\sin\theta = n\lambda$. This geometry differs from the small-angle double-slit fringe-spacing relation.

■ Demonstrating sound interference

Drive two loudspeakers from the same signal generator so their waves have the same frequency and a fixed source phase difference. Move a microphone, connected to an oscilloscope or sound-level display, across their overlapping sound field. Alternating large and small signal amplitudes show constructive and destructive interference. At a large-amplitude position the arrival phases match; at a small-amplitude position they oppose. Changing the microphone position changes the path difference. Unrelated speakers at the same nominal frequency need not maintain coherence.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
principle of superposition	when two or more waves meet at a point, the resultant displacement is the sum of the displacements of the individual waves
stationary wave	the pattern formed when two progressive waves of the same frequency and speed travel in opposite directions and superpose, with nodes and antinodes that do not move
node	a point on a stationary wave where the displacement is always zero
antinode	a point on a stationary wave where the amplitude is a maximum
diffraction	the spreading of a wave as it passes through a gap or around the edge of an obstacle
interference	the superposition of waves from coherent sources, giving a steady pattern of maxima and minima
coherence	waves that have a constant phase difference (and so the same frequency)
path difference	the difference between the distances travelled by two waves from their sources to a point
fringe spacing	the distance between the centres of two adjacent bright (or dark) fringes
order of a maximum	the whole number n in $d\sin\theta = n\lambda$, the number of wavelengths of path difference between adjacent slits

Exam tips

- Two-source interference: **constructive** when path difference $= n\lambda$, **destructive** when $= (n + \frac{1}{2})\lambda$; the sources must be **coherent**.
- Double slit: $\lambda = ax/D$; diffraction grating: $d\sin\theta = n\lambda$ – know every symbol.
- On a **stationary wave** mark nodes and antinodes; adjacent nodes are $\lambda/2$ apart; it stores energy but does not transfer it.
- A stationary wave needs two waves of the **same frequency travelling in opposite directions**.

■ Common mistakes

- Adding amplitudes or intensities in the principle of superposition. Displacements add; the intensity then follows from the resultant amplitude squared.
- "Adjacent nodes are one wavelength apart." Half a wavelength; a node to the next antinode is a quarter.
- Using $d =$ lines per millimetre in $d\sin\theta = n\lambda$. Invert: $d = 10^{-3}/N$ metres.
- Measuring θ from the grating surface, or forgetting that the angle between the two first-order beams is $2\theta_1$.
- Writing "in phase" for coherent. Coherent means a constant phase difference; the sources may be permanently out of step.
- Saying the fringe spacing changes when the source is made brighter, or that narrower slits change the spacing. Brightness and slit width change the contrast and the number of visible fringes, not $x = \lambda D/a$.

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Electricity

A-Level Physics

Electric current

An **electric current** 电流 is a flow of **charge carriers** 载流子. In a metal the carriers are negative conduction **electrons** 电子; in an **electrolyte** 电解质 they are positive and negative **ions** 离子; in a **semiconductor** 半导体 they may be electrons or "**holes**" 空穴. The **conventional current** 常规电流 direction is the way **positive** charge would flow – opposite to the real flow of electrons in a wire.

Charge

Charge is **quantised** 量子化: the smallest free unit of charge is the **elementary charge** 基本电荷

$$e = 1.60 \times 10^{-19}\ \text{C}.$$

Every free charge in this syllabus is a whole-number multiple of e. The unit of charge is the **coulomb** 库仑, C.

So a particle can carry 4.8×10^{-19} C (three electrons' worth) or -2.4×10^{-18} C (fifteen), but never 1.1×10^{-19} C or 6.4×10^{-20} C; a set of measurements that gives a charge carrier 2.5×10^{-19} C cannot be right, because that is not a multiple of e. For the one-mark definitions: an **electric current is a flow of charge carriers**; the **coulomb** is the charge that passes a point in one second when the current is one ampere (an ampere multiplied by a second); a **charge carrier** can be an electron, a proton, an ion or an α-particle, but never a neutron.

Current as the rate of flow of charge

If charge Q passes a point in time t, the current is

$$I = \frac{Q}{t}, \qquad Q = It.$$

Unit of current: ampere, A ($= \text{C s}^{-1}$). For a changing current, the charge that has flowed in a time is the **area under** an I–t graph.

> **Worked example.** A current of 0.50 A flows for 2.0 minutes. Find the charge that passes, and how many electrons this represents. ($e = 1.60 \times 10^{-19}$ C.)

$$Q = It = 0.50 \times 120 = 60\ \text{C}, \qquad N = \frac{Q}{e} = \frac{60}{1.60 \times 10^{-19}} \approx 3.8 \times 10^{20}.$$

The same two lines answer "how many electrons pass in 10 hours at 4.0 mA" (convert the time to seconds: $N = It/e = 4.0 \times 10^{-3} \times 36\,000/(1.60 \times 10^{-19}) = 9.0 \times 10^{20}$) and,

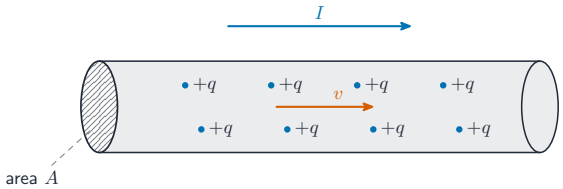
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the other way round, "the average current when 6.0×10^{23} electrons pass in 24 hours" ($I = Ne/t = 1.1$ A). A beam of α-particles carries a current too: each carries $2e$, so a beam current of 6.9×10^{-9} A is $6.9 \times 10^{-9}/(2 \times 1.60 \times 10^{-19}) = 2.2 \times 10^{10}$ particles per second. A lightning strike that transfers 1×10^{20} electrons in 30 μs is a current of $Ne/t = 5.3 \times 10^{5}$ A.

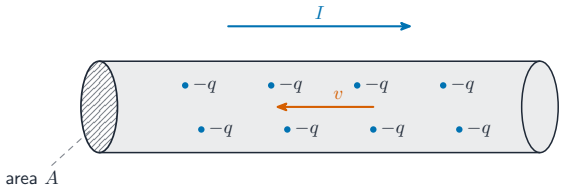
Drift velocity equation

For a uniform conductor of cross-section area A, with n charge carriers per unit volume (the **number density** 数密度), each carrying charge q, moving with average **drift velocity** 漂移速度 v:

$$I = Anvq.$$



(a) positive carriers drift with I



(b) electrons drift against I

Charge carriers drifting inside a conductor: positive carriers drift with I, electrons against it

> **Worked example.** A copper wire of cross-sectional area 1.0×10^{-6} m^2 carries a current of 5.0 A. Copper has $n = 8.5 \times 10^{28}$ free electrons per m^3. Find the drift velocity. ($e = 1.60 \times 10^{-19}$ C.)

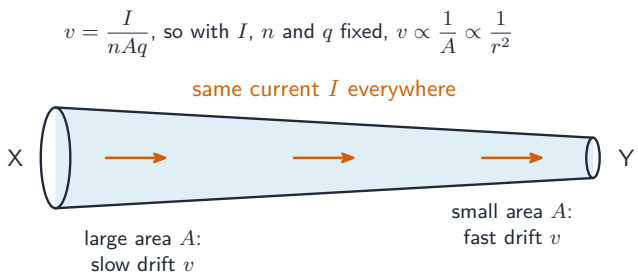
Rearranging $I = Anvq$ gives $v = \dfrac{I}{Anq}$:

$$v = \frac{5.0}{(1.0 \times 10^{-6})(8.5 \times 10^{28})(1.60 \times 10^{-19})} \approx 3.7 \times 10^{-4}\ \text{m s}^{-1}.$$

The electrons drift very slowly – less than a millimetre per second.

Two "show that" steps that often precede this calculation. The **number density** of a metal with one free electron per atom is $n = \dfrac{\text{density} \times N_\text{A}}{\text{molar mass}}$: for copper, $8.9 \times 10^3 \times 6.02 \times 10^{23} / 0.0635 = 8.5 \times 10^{28}$ m^{-3}. And the **time** for an electron to drift the length of a wire is $t = L/v$: at 3.7×10^{-4} m s^{-1} a 2.0 m wire takes 5 400 s, an hour and a half, even though the current is established almost instantly. Between them, the equation $I = Anvq$ is used as a ratio far more often than as a calculation:

- **two wires in series** carry the same current; if wire Y has twice the diameter of X (four times the area) and the same metal, its electrons drift at a quarter of the speed. Two wires of the same length and diameter but different metals have drift speeds in the inverse ratio of their number densities.
- **a wire that narrows** (a wedge, a tapered rod, a cable of thick and thin strands) carries the same current at every cross-section, so the drift speed rises where the area falls: $v \propto 1/A \propto 1/r^2$. A sketch of v against distance along a tapering wire is a curve that rises more and more steeply towards the narrow end, not a straight line.
- the p.d. across a uniform wire is proportional to its length at fixed current, because $V = IR = I\rho L/A$ with I, ρ and A constant.



The same current everywhere, so the electrons drift faster where the wire is thinner

Use this to compare currents:

- a thinner wire (smaller A) at the same I needs a faster drift v.
- a semiconductor has far fewer free carriers than a metal (smaller n), so for the same I the drift velocity is much larger.
- in **series** 串联 components, I is the same everywhere, so if A stays the same but the material changes, nv changes the other way.

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Potential difference

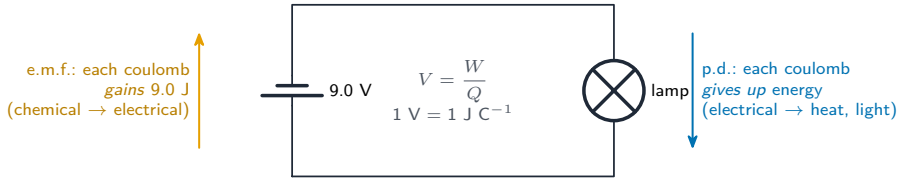
The **potential difference** 电势差 (p.d.) across a component is the **energy** 能量 transferred per unit charge as that charge passes through it:

$$V = \frac{W}{Q}.$$

Unit: **volt** 伏特, V (= J C^{-1}).

If 1 J of electrical energy changes into other forms (thermal, light, kinetic, …) when 1 C of charge passes through a component, the p.d. across it is 1 V.

The **electromotive force** 电动势 (e.m.f.) of a source is the energy given **per unit charge by the source**. The formula is the same as for p.d.; the difference is direction: e.m.f. is energy given **to** the charge by the source; p.d. is energy given **up** by the charge to the component.



e.m.f. is energy given to each coulomb; p.d. is energy each coulomb gives up.

The definitions the examiner accepts: the **potential difference** across a component is the energy transferred (from electrical to other forms) per unit charge passing through it; the **e.m.f.** of a source is the energy transferred from other forms (chemical, in a cell) to electrical energy per unit charge in driving the charge round a complete circuit; one **volt** is one joule per coulomb. A battery "marked 9.0 V" therefore gives each coulomb 9.0 J of electrical energy for the whole circuit, and the product of charge and p.d. is the energy transferred.

Electrical power



High-voltage power lines carry electrical energy across the country.

Combining $V = W/Q$ and $I = Q/t$:

$$P = \frac{W}{t} = VI.$$

Using Ohm's law $V = IR$:

$$P = VI = I^2R = \frac{V^2}{R}.$$

Pick the form with the quantities you know. Examples:

- two heaters of equal resistance – the one with the larger current gives more **power** 功率 ($P = I^2R$).
- two resistors in **parallel** 并联 across the same **voltage** 电压 – the one with smaller R gives more power ($P = V^2/R$).
- a kettle marked "2.4 kW, 240 V" draws $I = P/V = 10$ A and has resistance $R = V^2/P = 24\ \Omega$.

Energy transferred in time t is $E = Pt$.

> **Worked example.** Two lamps, P rated 250 V, 50 W and Q rated 250 V, 200 W, are connected in series to a 250 V supply. Which is brighter?

Their resistances at the rated p.d. are $R = V^2/P$: 1250 Ω for P and 313 Ω for Q. In series they carry the same current, so $P = I^2R$ makes the larger resistance, lamp P, dissipate more power: the "weaker" lamp is the brighter one. In **parallel** the p.d. is the same across both and $P = V^2/R$ favours the smaller resistance: Q. "State and explain which resistor dissipates more power" is answered by naming the quantity the two share (current in series, p.d. in parallel) and the form of the power equation that uses it.

> **Worked example.** A supply delivers 2.4 kW at 240 V to a kettle through two cables of resistance 0.10 Ω each. Find the power lost in the cables.

The current is $I = P/V = 10$ A, so the cables dissipate $I^2R = 10^2 \times 0.20 = 20$ W, and the kettle receives 2380 W. The **efficiency** of a circuit that exists to power one component is that component's power divided by the total power from the supply: with 6.0 V across a 0.86 Ω series resistor and 4.5 V across the component at the same current, the efficiency is $4.5/(6.0 + 4.5) = 43\%$. A thermistor across a fixed p.d. dissipates $P = V^2/R$, so as its temperature rises and R falls, the power rises. A kettle draws about 10 A from a 250 V supply; a mobile phone charger a fraction of an ampere.

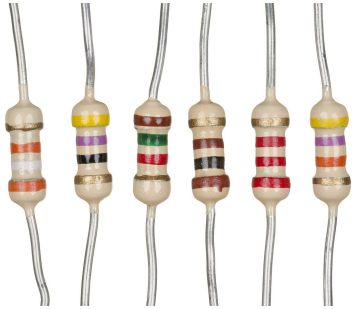
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Resistance and Ohm's law

The **resistance** 电阻 R of a component is

$$R = \frac{V}{I}.$$

Unit: **ohm** 欧姆, Ω ($= \text{V A}^{-1}$). Resistance depends on the conditions (such as temperature) when it is measured.



Real fixed resistors – the coloured bands code the resistance in ohms

Ohm's law

A conductor obeys **Ohm's law** 欧姆定律 when the current through it is **proportional to the p.d.** across it, as long as the conditions (especially temperature) stay constant. For such a conductor R is constant and the I–V graph is a straight line through the origin.

Ohm's law is an experimental result, not a definition. The definition $R = V/I$ works for **any** component; only ohmic ones have constant R.

For the marks: **resistance** is the ratio of the potential difference across a component to the current in it; the **ohm** is the resistance of a component in which a p.d. of one volt produces a current of one ampere (a volt per ampere); **Ohm's law** states that the current in a metallic conductor is directly proportional to the potential difference across it, provided that its temperature (and other physical conditions) remains constant. Of several I–V graphs, only a straight line through the origin obeys Ohm's law; a straight line that misses the origin, or any curve, does not.

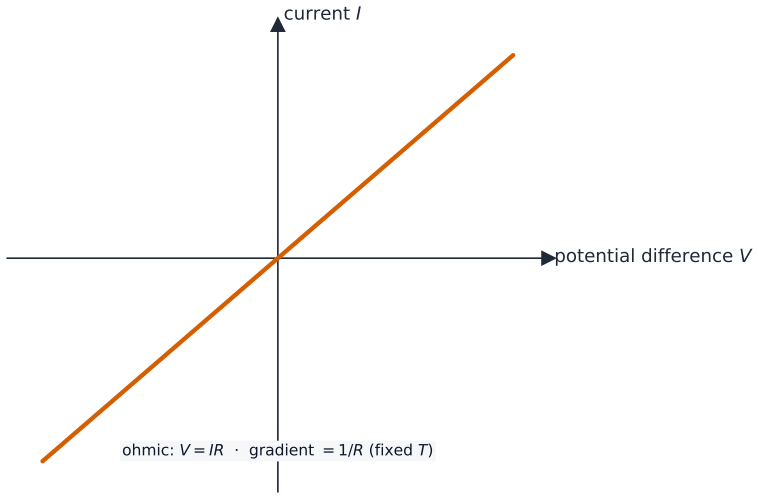
I–V characteristics

You should be able to sketch these:

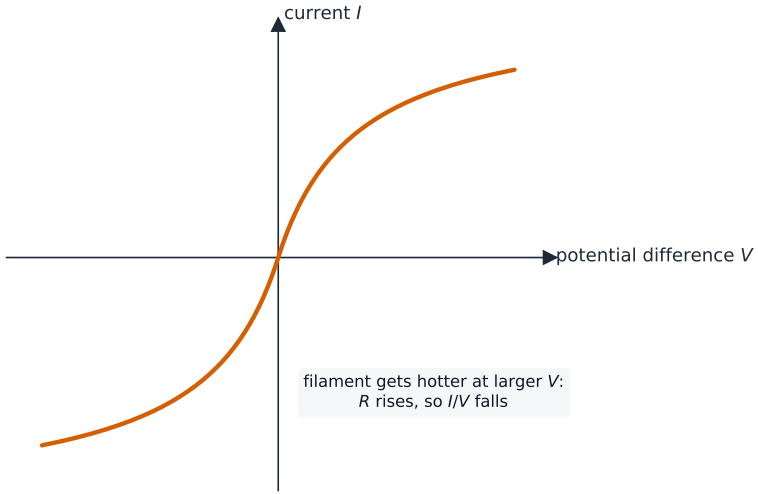
- **metal wire at constant temperature** – a straight line through the origin (constant R). Reversing the p.d. drives the current the other way, giving a straight line in both directions.

- **filament lamp** 灯丝灯泡 – through the origin, steep at first, then flatter as V (and I) grow. Reason: more current heats the filament, so its resistance rises and the gradient $1/R$ falls.
- **semiconductor diode** 二极管 – almost no current for negative V or small positive V. Above a "switch-on" voltage (about 0.7 V for silicon), the current rises sharply.

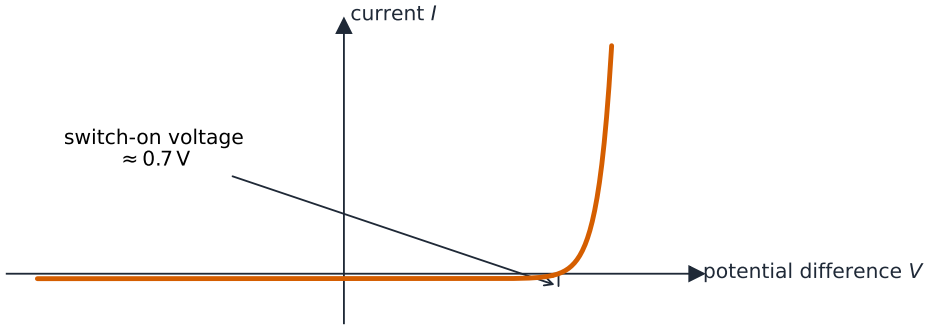
Explain the shape of the filament lamp's line (three marks): as the current increases the filament's temperature rises; the lattice ions vibrate with larger amplitude, so the free electrons collide with them more often; each collision takes energy from the electrons, so the resistance increases; on the graph the ratio V/I grows and the gradient falls. Run backwards for "the current decreases": the temperature falls, so the resistance falls. Two things to read off any characteristic. The resistance at a point is V/I for that point, never the gradient of a curve, so a diode's resistance is very large (infinite, in practice) up to the switch-on p.d. and then falls steeply as V rises further, and of four components at the same p.d. the one with the smallest current has the greatest resistance. And when a lamp (or a diode) is in series with a resistor, the two carry the same current and their p.d.s add: read the current from the component's curve at its own p.d., then use $V = IR$ for the resistor, and the supply p.d. is the sum.



I–V characteristic of an ohmic conductor (metal wire at constant temperature)



I–V characteristic of a filament lamp



Si diode: conducts above ≈ 0.7 V · reverse blocks

I–V characteristic of a semiconductor diode

Resistivity

For a uniform conductor of length L and cross-section area A,

$$R = \frac{\rho L}{A}.$$

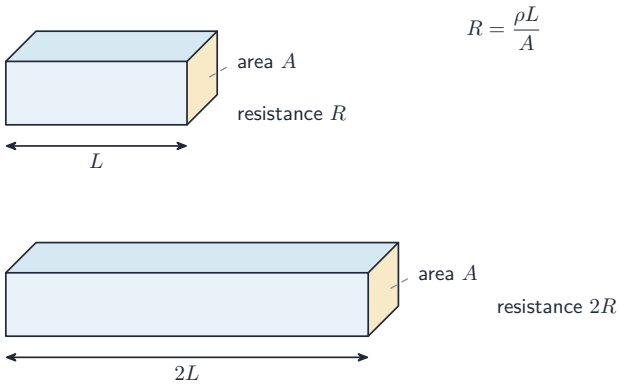
ρ is the **resistivity** 电阻率, a property of the material, with unit Ω m. Doubling the length doubles R; doubling the area halves it; halving the diameter quarters the area and so makes R four times bigger.

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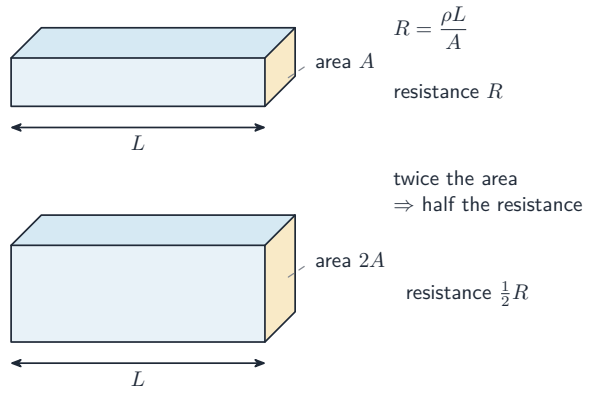
Worked example. A copper wire of length 2.0 m and cross-sectional area 1.7×10^{-7} m^2 has resistivity 1.7×10^{-8} Ω m. Find its resistance.

$$R = \frac{\rho L}{A} = \frac{(1.7 \times 10^{-8})(2.0)}{1.7 \times 10^{-7}} = 0.20\ \Omega.$$

Typical values: copper at room temperature $\rho \sim 1.7 \times 10^{-8}$ Ω m; an **insulator** 绝缘体 $\rho \sim 10^{15}$ Ω m or more.



A longer conductor has more resistance – doubling L doubles R

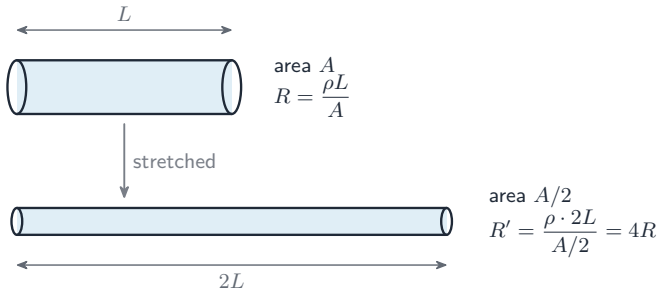


A wider conductor has less resistance – doubling A halves R

The resistivity of a metal **rises with temperature** (more **lattice vibration** 晶格振动 **scatters** 散射 the electrons), which is why the filament lamp's I–V line curves.

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Most resistivity questions are ratios. **Stretching** a wire keeps its volume ($A \times L$) constant, so if the length becomes k times longer the area becomes k times smaller and $R = \rho L/A$ becomes k^2 times larger: a wire three times as long (same mass, same metal) has nine times the resistance, and a wire whose diameter falls to 0.940 of its value has its area multiplied by 0.884, its length divided by 0.884, and its resistance multiplied by $1/0.884^2 = 1.28$.



Stretched at constant volume: twice the length, half the area, four times the resistance

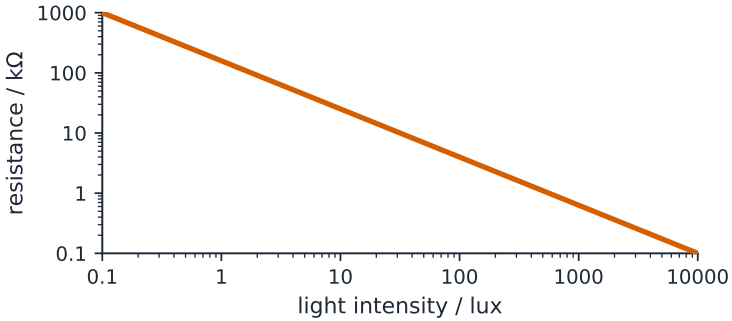
Worked example. A copper lightning rod of resistance 9.6 Ω and length 20 m has resistivity 1.7×10^{-8} Ω m. Find its radius.

$A = \rho L/R = 1.7 \times 10^{-8} \times 20/9.6 = 3.5 \times 10^{-8}$ m^2, and $r = \sqrt{A/\pi} = 1.1 \times 10^{-4}$ m. Doubling the radius (same length) quarters the resistance. The other standard ratios: **strands in parallel** – seven identical strands share the current, so the cable's resistance is one seventh of one strand's; two wires of the same resistance where one metal has twice the resistivity need the second wire to have twice the area (a diameter $\sqrt{2}$ times larger) for the same length; a wire's resistance per unit length, 0.92 Ω m^{-1}, multiplied by its area, 5.3×10^{-7} m^2, is its resistivity, 4.9×10^{-7} Ω m; and a cylinder of conducting putty 60 mm long and 20 mm across, or a cube of side a ($R = \rho a/a^2 = \rho/a$), uses the same $R = \rho L/A$ with the shape's own length and end area. When the wire is held under tension, $R_0 = \rho L/A$ still gives its resistance, and a stretch that lengthens it and thins it raises R for both reasons. Because $\rho = RA/L = R\pi d^2/(4L)$, the percentage uncertainty in a measured resistivity is the sum of the percentage uncertainties in R and L plus **twice** that in d.

Light-dependent resistor (LDR)

A **light-dependent resistor** 光敏电阻 (LDR) is a semiconductor whose **resistance falls as the light intensity rises**. In bright light R may be a few hundred Ω; in the dark it can be in the megaohms. LDRs are used in light-sensing circuits (street lamps,

camera light meters). Here the **light intensity** 光强 controls the resistance.

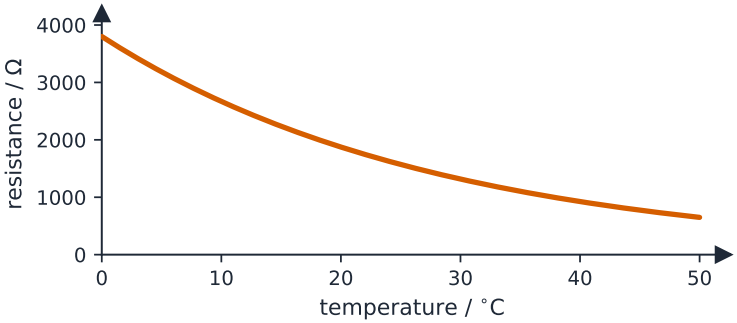


LDR: more light ⇒ lower R

Resistance of an LDR decreases as light intensity increases

Thermistor

In this syllabus a **thermistor** 热敏电阻 has a **negative temperature coefficient** 负温度系数: its **resistance falls as its temperature rises**. This is useful for sensing temperature – put it in a **potential divider** 分压器 and the output voltage changes with temperature.



NTC: R falls as T rises

Resistance of a thermistor falls as temperature rises

This is the opposite of a metal: in a semiconductor, more thermal energy frees more charge carriers, and this matters more than the extra scattering.

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A sketch of a thermistor's resistance against temperature starts at R_0 at 0 °C and falls along a curve that flattens: the fall is not linear, which is the disadvantage of a thermistor as a thermometer (its scale is not uniform, so it needs calibrating). When the light on an LDR is increased, or a thermistor is warmed, its resistance falls, the current in its circuit rises, and the p.d. across it (in series with a fixed resistor) falls while the p.d. across the resistor rises; a fixed resistor and a metal wire at constant temperature keep their resistance, and a filament lamp's rises with current.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
electric current	a flow of charge carriers
coulomb	the charge passing a point in one second when the current is one ampere
potential difference	the energy transferred from electrical to other forms per unit charge passing through a component
electromotive force (e.m.f.)	the energy transferred from other forms to electrical energy per unit charge, by a source driving charge round a complete circuit
volt	one joule per coulomb
resistance	the ratio of the potential difference across a component to the current in it
ohm	the resistance of a component in which a potential difference of one volt produces a current of one ampere
Ohm's law	the current in a metallic conductor is directly proportional to the potential difference across it, provided its temperature remains constant
resistivity	the constant ρ in $R = \rho L/A$, a property of the material with unit Ω m
number density	the number of charge carriers per unit volume of the conductor

Exam tips

- Use $I = Q/t$, $V = W/Q$ (energy per unit charge) and $P = VI = I^2R = V^2/R$.
- **Ohm's law ($V = IR$)** applies only to an ohmic conductor at constant temperature – a filament lamp is non-ohmic.
- Sketch and interpret the **I-V characteristics** of a resistor, filament lamp and diode.
- An **LDR's** resistance falls with light; a **thermistor's** falls as temperature rises.

Common mistakes

- Taking the resistance from the gradient of a curved I–V graph. Resistance is V/I at the point; only for a straight line through the origin is it the reciprocal of the gradient.
- Leaving a time in minutes or hours in $Q = It$. Convert to seconds first.
- Saying the filament lamp's resistance rises "because of the voltage". The chain is current, temperature, ion vibration, more collisions, more resistance.
- Forgetting that a stretched wire changes in two ways. At constant volume the area falls as the length rises, so $R \propto L^2$.
- Using the diameter as the radius in $A = \pi r^2$, or leaving mm^2 unconverted.
- Comparing powers with the wrong form. Series components share the current, so use I^2R; parallel components share the p.d., so use V^2/R.

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D.C. circuits

A-Level Physics

Practical circuits

e.m.f. and p.d.

The **electromotive force** 电动势 (e.m.f.) ε of a source is the **energy** 能量 given to each unit of charge by the source as it drives the charge around a full circuit. Unit: volt.

The **potential difference** 电势差 (p.d.) across a component is the energy changed from electrical to other forms by each unit of charge as it passes through that component.

Both are in volts; they differ in energy direction:

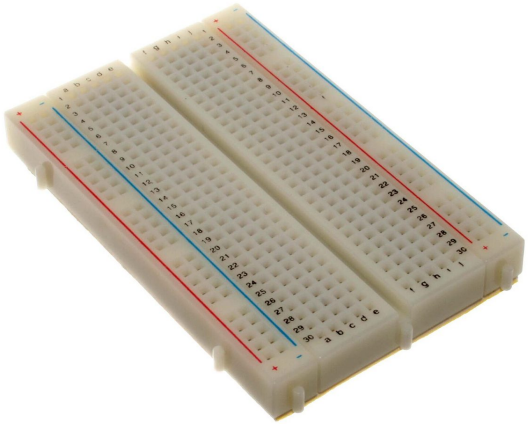
- e.m.f. – energy put **into** the circuit by the source (chemical → electrical in a battery, mechanical → electrical in a generator).
- p.d. – energy taken **out** of the electrical form (electrical → thermal in a resistor, → light in a lamp, → kinetic in a motor).

The examiner's wording is fixed. e.m.f. is "the energy transferred **per unit charge** by the source in driving charge round a **complete circuit**"; p.d. is "the energy transferred per unit charge from electrical to other forms". Both are energy per charge, so the volt is a joule per coulomb (1 V = 1 J C^{-1}). Statements that are **always** true of a source: the e.m.f. is the terminal p.d. when **no current** flows; the total energy it gives to a charge Q is εQ. The name is a trap: an electromotive "force" is not a force and is not measured in newtons.

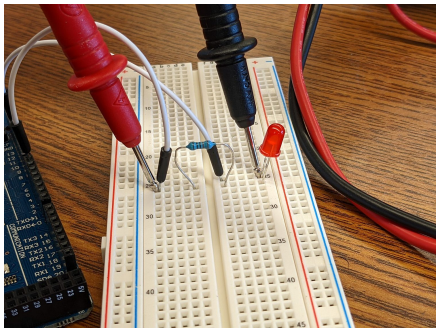
> **Worked example.** A cell of e.m.f. ε and internal resistance r drives a charge Q round a circuit whose terminal p.d. is V. Compare the energy transferred by the cell with the energy dissipated outside it.

The cell transfers εQ in total; the external circuit receives VQ. Since $V = \varepsilon - Ir < \varepsilon$ whenever a current flows, $VQ < \varepsilon Q$ – the difference $(\varepsilon - V)Q = IrQ$ is the energy turned to heat inside the cell.

In the lab you often build a circuit on a **breadboard** 面包板 (a board with rows of holes that connect components without soldering) and measure currents and p.d.s with a **multimeter** 万用表.



The bare board: the two long rails carry power, and each short row of holes is joined underneath



A real circuit on a breadboard, being measured with a multimeter

Internal resistance

A real source has some **internal resistance** 内阻 r – usually the resistance of the **electrolyte** 电解质 in a **cell** 电池, or the wire windings in a generator. When current I flows, an internal p.d. of Ir is "lost" inside the source, so the **terminal p.d.** 端电压 across the outside circuit is

$$V_{\text{terminal}} = \varepsilon - Ir.$$

So:

- **no current** (**open circuit** 开路, $I = 0$): the terminal p.d. equals the e.m.f.
- **larger current**: the terminal p.d. falls.

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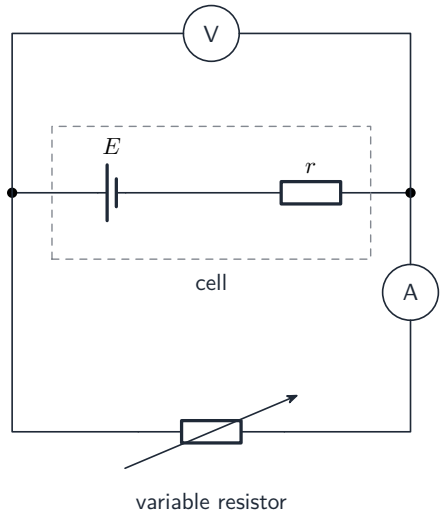
- **short circuit** 短路 ($R_{\text{external}} \to 0$): $I = \varepsilon/r$, a large current, with all the energy turned to heat inside the source.

To measure r, change the outside resistance and plot V_{terminal} against I: the line has y-intercept ε and gradient $-r$.

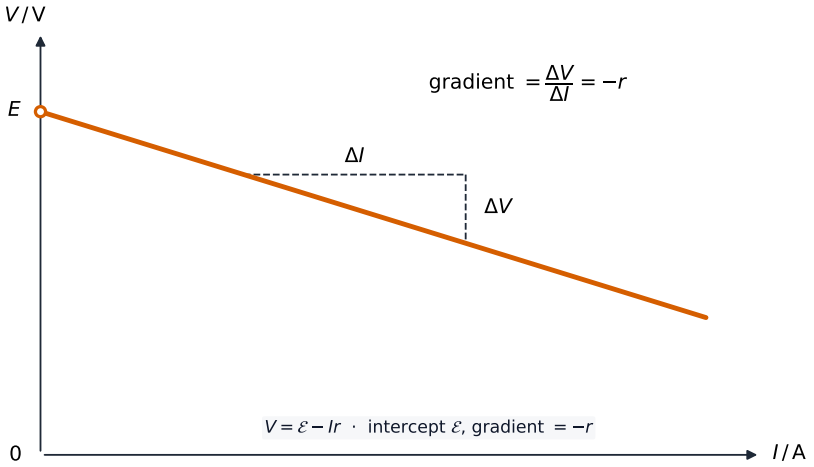
> **Worked example.** A cell of e.m.f. 1.5 V and internal resistance 0.50 Ω is connected to a 2.5 Ω resistor. Find the current and the terminal p.d.

The e.m.f. drives the current through both resistances: $I = \dfrac{\varepsilon}{R+r} = \dfrac{1.5}{2.5+0.50} = 0.50$ A. Then

$$V_{\text{terminal}} = \varepsilon - Ir = 1.5 - 0.50 \times 0.50 = 1.25 \text{ V}.$$



Circuit for measuring the e.m.f. and internal resistance of a cell



Terminal p.d. against current – the intercept is the e.m.f. and the gradient is minus the internal resistance

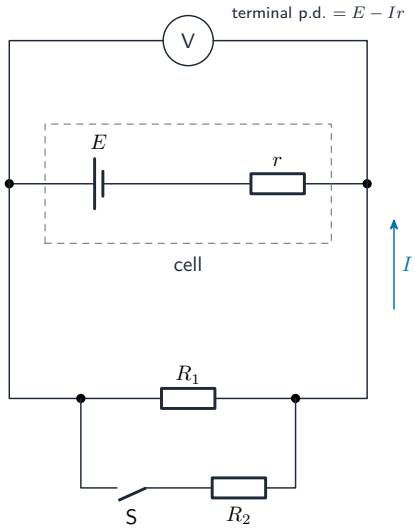
The **power** 功率 given to the outside load is $P_{\text{ext}} = (\varepsilon - Ir)I$; the power lost inside is $P_{\text{int}} = I^2 r$; the total power from the source is εI.

The **efficiency** 效率 of the source is $\dfrac{P_{\text{ext}}}{\varepsilon I} = \dfrac{VI}{\varepsilon I} = \dfrac{V}{\varepsilon} = \dfrac{R}{R+r}$: a large load resistance wastes little energy inside the source.

> **Worked example.** A cell of e.m.f. 2.0 V and internal resistance 0.40 Ω drives a current of 1.5 A through a wire. Show that the terminal p.d. is 1.4 V and find the percentage efficiency with which the cell supplies power to the wire.

$V = \varepsilon - Ir = 2.0 - 1.5 \times 0.40 = 2.0 - 0.60 = 1.40$ V. Power to the wire $= VI = 1.4 \times 1.5 = 2.1$ W; total power from the cell $= \varepsilon I = 2.0 \times 1.5 = 3.0$ W. Efficiency $= 2.1/3.0 = 0.70$, i.e. 70%. In a "show that" question, write every step and the unrounded value (1.40 V) before the value you were given.

When the outside circuit changes, argue through the current. Closing a switch that adds a second resistor **in parallel** lowers the total external resistance, so the current in the cell rises, the internal "lost volts" Ir rise, and the terminal p.d. $\varepsilon - Ir$ falls; the p.d. across the original resistor (which is the terminal p.d.) therefore falls, and so does its current, even though the cell's current rose. This chain – resistance → current in the cell → Ir → terminal p.d. – is the model answer for almost every "state and explain the effect" question in this topic.



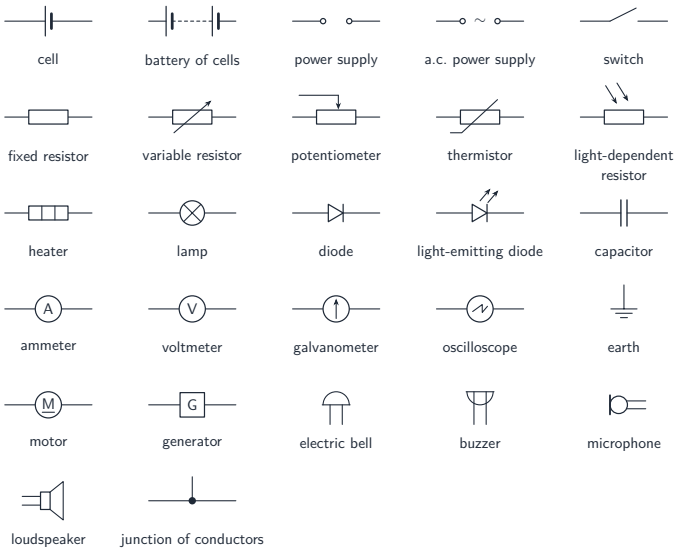
Closing S adds a parallel branch: the current in the cell rises, so Ir rises and the terminal p.d. read by the voltmeter falls

Cells joined together. In series, e.m.f.s add and internal resistances add: three cells of e.m.f. ε and internal resistance r give 3ε and $3r$. A cell connected the **wrong way round** subtracts its e.m.f. (ten 1.5 V cells with one reversed give $8 \times 1.5 = 12$ V). Identical cells in parallel give the same e.m.f. ε with a smaller internal resistance r/N, so they can supply a larger current.

Circuit symbols

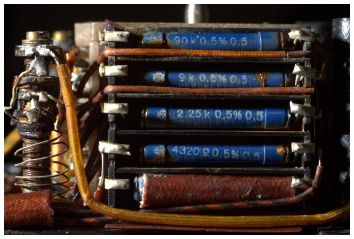
You must recognise and draw the standard symbols in the syllabus: cell, battery, switch, resistor, variable resistor, **ammeter** 电流表, **voltmeter** 电压表, lamp, **diode** (and **LED** 发光二极管), **capacitor** 电容器, inductor, thermistor, light-dependent resistor, **fuse** 保险丝, earth, junction. An ideal ammeter has zero **resistance** 电阻 and goes in **series** 串联. An ideal voltmeter has infinite resistance and goes in **parallel** 并联.

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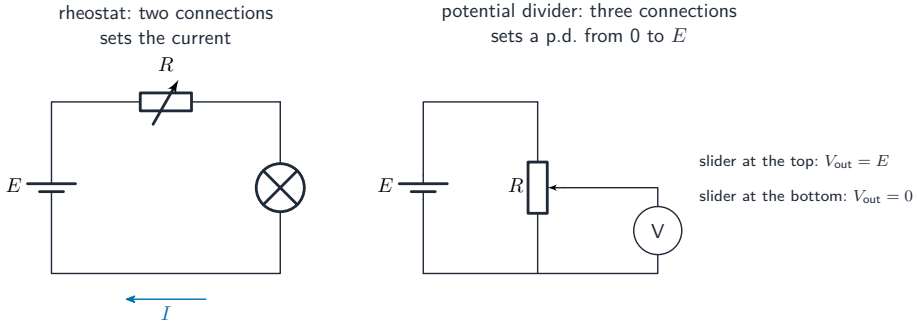
The standard circuit symbols you need to recognise and draw

Drawing a circuit diagram. Marks are lost for symbols, not physics. Use ruler-straight lines and the standard symbols; an ammeter goes **in series** with the component whose current it measures and a voltmeter **in parallel** with (across) the component whose p.d. it measures; a cell with internal resistance is drawn as an ideal cell in series with a resistor r; and a variable resistor may be used in two ways, shown below. A "complete the circuit diagram" question usually wants exactly this measuring arrangement: source, ammeter and variable resistor in one series loop, voltmeter across the component under test.



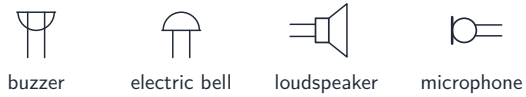
A digital multimeter across a resistor bank: the voltmeter goes in parallel, the ammeter in series

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A variable resistor as a rheostat (two connections, sets the current) and as a potential divider (three connections, sets a p.d. from 0 to E)

■ Audio-device symbols



Audio circuit symbols: buzzer, bell, loudspeaker and microphone

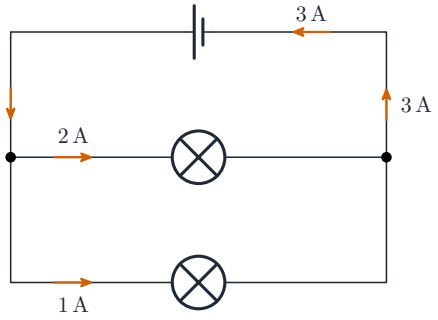
A **buzzer** and an **electric bell** turn electrical energy into sound; a **loudspeaker** turns a varying electrical signal into sound; a **microphone** turns sound into a varying electrical signal. Learn their distinct outlines and connections. The cup-shaped symbol with a flat top and two leads is the buzzer used in the AS examination; a device's function alone does not identify the symbol.

Kirchhoff's laws

■ First law (junction rule)

At any **junction** 节点, **the total current flowing in equals the total current flowing out**. This follows from **conservation of charge** 电荷守恒 – charge cannot build up at a point in a steady circuit, so charge in per second equals charge out per second.

For a junction with three wires: $I_1 = I_2 + I_3$ if currents 2 and 3 flow out and current 1 flows in.



Current divides at a junction in a parallel circuit (3 A in equals 2 A plus 1 A)

Two ways the examiner asks it: "state the law" (one mark: the sum of the currents **into** a junction equals the sum of the currents **out of** it) and "state the conservation law behind it" (charge). Do not answer "energy" for the first law or "charge" for the second – the pairing is tested in almost every Paper 1.

Second law (loop rule)

Around any **closed loop** 回路, **the total e.m.f. equals the total p.d. across the components in that loop**. This follows from **conservation of energy** 能量守恒: as a unit of charge goes once round a loop, the energy it gains from sources equals the energy it gives up to components.

Pick a direction round the loop. Take an e.m.f. as positive when the loop direction goes from − to + of the source, and a p.d. as positive when the loop direction is the conventional current direction through the resistor.

In symbols, round any closed loop $\sum \varepsilon = \sum IR$. A source you pass from + to − counts as a **negative** e.m.f. (it is being charged, or opposes the other source), and a resistor you pass **against** its current counts as a negative p.d.

> **Worked example.** Two batteries are in one loop with their e.m.f.s opposed: 12.0 V with internal resistance 1.0 Ω, and 8.0 V with internal resistance 0.50 Ω. Find the current.

Going round the loop, the net e.m.f. is $12.0 - 8.0 = 4.0$ V and the total resistance is $1.0 + 0.50 = 1.5\ \Omega$ (the internal resistances are in series), so $I = 4.0/1.5 = 2.7$ A. The current flows in the direction driven by the larger e.m.f.

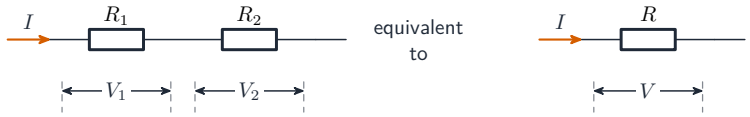
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Combining resistors

Resistors in series. *Derivation using Kirchhoff's laws:* there is no junction between the resistors, so by the **first law** the same current I passes through each. By the **second law** the e.m.f. round the loop equals the sum of the p.d.s:

$$\varepsilon = IR_1 + IR_2 + \ldots = I(R_1 + R_2 + \ldots),$$

so $R_{\text{series}} = R_1 + R_2 + \ldots$.

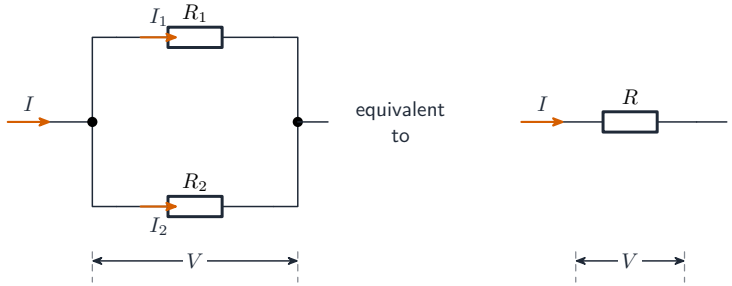


Two resistors in series and their single equivalent resistor

Resistors in parallel. *Derivation:* by the **second law**, each resistor forms its own loop with the source, so each has the same p.d. V across it. By the **first law** the current entering the junction equals the sum of the branch currents:

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \ldots = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots\right),$$

so $\dfrac{1}{R_{\text{parallel}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \ldots$.



Two resistors in parallel and their single equivalent resistor

Two equal resistors R in parallel give $R/2$; N equal ones give R/N. A parallel combination is **always smaller** than any of its resistors; a series combination is always larger.

> **Worked example.** A 4.0 Ω resistor and a 12 Ω resistor are connected in parallel. Find their combined resistance.

$$\frac{1}{R} = \frac{1}{4.0} + \frac{1}{12} = \frac{3}{12} + \frac{1}{12} = \frac{4}{12} = \frac{1}{3} \quad \Rightarrow \quad R = 3.0\ \Omega.$$

■ Resistor networks and power

Reduce a mixed network **one step at a time**: replace each series chain by its sum and each parallel pair by $\dfrac{R_1 R_2}{R_1 + R_2}$ (the "product over sum" form, valid for two resistors only), redraw, and repeat until one resistor is left. Two rules decide **which resistor dissipates the most power** without any arithmetic: components carrying the **same current** dissipate more in the **larger** resistance ($P = I^2R$); components with the **same p.d.** dissipate more in the **smaller** resistance ($P = V^2/R$). In a series–parallel mix the single resistor that carries the **whole** current usually dissipates the most, because a parallel branch carries only a share of it.

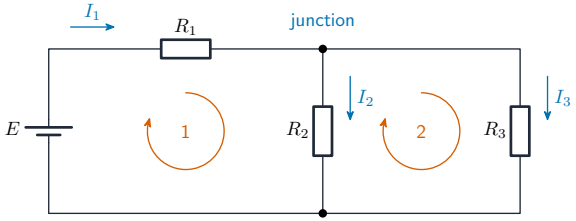
> **Worked example.** Three resistors, each of resistance R, are connected with two in series and that pair in parallel with the third. The total resistance between the ends is 8.0 Ω. Find R.

The series pair is $2R$; in parallel with R: $R_\mathrm{T} = \dfrac{2R \times R}{2R + R} = \dfrac{2R}{3}$. So $\dfrac{2R}{3} = 8.0$, giving $R = 12\ \Omega$. Check: the single resistor R carries twice the current of the pair (same p.d., half the resistance), so it dissipates the most power.

■ Solving a circuit

1. **Label** every current with a symbol and a chosen direction.
2. Use **Kirchhoff's first law** 基尔霍夫第一定律 at each junction to link the currents.
3. Use **Kirchhoff's second law** 基尔霍夫第二定律 around each loop to get equations in the p.d.s.
4. **Use** $V = IR$ for each resistor.
5. Solve the equations together.

For symmetric resistor networks, use the symmetry to spot branches with equal currents – the branch with the most current gives the most power ($P = I^2R$).



The standard two-loop circuit: one junction equation and two loop equations fix all three currents

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> **Worked example.** In the circuit above $\varepsilon = 12$ V (negligible internal resistance), $R_1 = 2.0\ \Omega$, $R_2 = 6.0\ \Omega$ and $R_3 = 3.0\ \Omega$. Find the three currents.

Kirchhoff's method. Junction: $I_1 = I_2 + I_3$. Loop 1 (through ε, R_1, R_2): $12 = 2.0I_1 + 6.0I_2$. Loop 2 (through R_2 and R_3, no source): $0 = 6.0I_2 - 3.0I_3$, so $I_3 = 2I_2$. Then $I_1 = 3I_2$ and $12 = 6.0I_2 + 6.0I_2$, giving $I_2 = 1.0$ A, $I_3 = 2.0$ A and $I_1 = 3.0$ A.

Reduction check. 6.0 Ω and 3.0 Ω in parallel give 2.0 Ω; with R_1 the total is 4.0 Ω, so $I_1 = 12/4.0 = 3.0$ A and the p.d. across the parallel pair is $3.0 \times 2.0 = 6.0$ V, giving $I_2 = 6.0/6.0 = 1.0$ A and $I_3 = 6.0/3.0 = 2.0$ A. Both methods must agree; a negative answer for a current simply means you guessed its direction the wrong way.

Potential dividers

A **potential divider** 分压器 is two (or more) resistors in series across a source. The p.d. across each resistor is in **direct proportion** to its resistance:

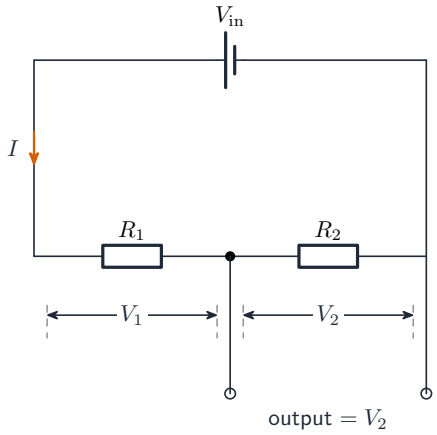
$$V_1 = V_\mathrm{in} \cdot \frac{R_1}{R_1 + R_2}, \qquad V_2 = V_\mathrm{in} \cdot \frac{R_2}{R_1 + R_2}.$$

The output (tapped between R_1 and R_2) can be set to any **voltage** 电压 between 0 and V_in by choosing the resistances. A **potentiometer** 电位差计 used with all three of its connections (a slider on a uniform-resistance track) gives a smoothly variable divider; the same component with only two connections is a **rheostat** 变阻器, which just changes the current.

> **Worked example.** A 6.0 V supply is connected across a 2.0 kΩ resistor in series with a 4.0 kΩ resistor. Find the output voltage tapped across the 4.0 kΩ resistor.

$$V_2 = V_\mathrm{in} \cdot \frac{R_2}{R_1 + R_2} = 6.0 \times \frac{4.0}{2.0 + 4.0} = 4.0\ \mathrm{V}.$$

The same answer comes from the **current-first** route the mark scheme often lays out: $I = V_\mathrm{in}/(R_1 + R_2) = 6.0/6000 = 1.0$ mA, then $V_2 = IR_2 = 1.0 \times 10^{-3} \times 4000 = 4.0$ V. Two consequences worth remembering: the **larger** resistance takes the **larger** share of the p.d.; and the ratio formula holds only while **no current is drawn from the output** – a load (or a low-resistance voltmeter) in parallel with R_2 lowers the effective resistance of R_2 and so lowers V_2.



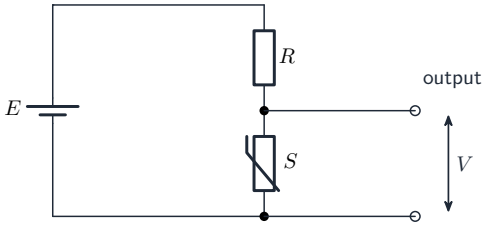
A potential divider – the p.d. splits between R1 and R2 in proportion to their resistances

■ Sensor circuits

Replace one fixed resistor with a **sensor** 传感器 whose resistance changes with a physical quantity:

- **thermistor** 热敏电阻 (NTC): R falls as temperature rises. In a divider, the output voltage changes with temperature in a fixed direction.
- **light-dependent resistor** 光敏电阻 (LDR): R falls as **light intensity** 光强 rises, giving a brightness-dependent output.

Connect the output to a **transistor** 晶体管 base or a **comparator** 比较器 to switch a load on or off when the temperature or light passes a **threshold** 阈值.



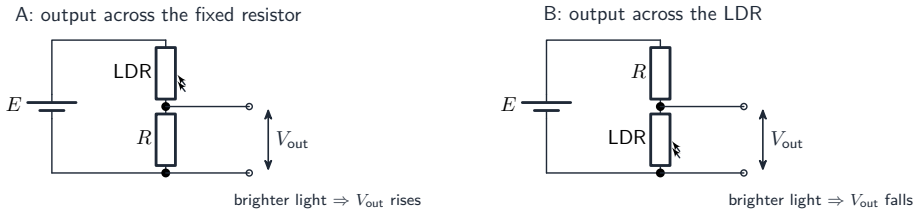
A thermistor in a potential divider gives an output voltage that changes with temperature

Which way does the output go? The output can be taken across the **sensor** or across the **fixed resistor**, and the two choices respond in opposite directions. Argue through the current, in this order, and you have the full three-mark answer:

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1. the light gets brighter (or the temperature rises), so the resistance of the LDR (or thermistor) **falls**;
2. the total resistance of the series circuit falls, so the **current rises** ($I = \varepsilon/R_{\text{total}}$);
3. the p.d. across the **fixed resistor** $= IR$ **rises**, so the p.d. across the **sensor** $= \varepsilon - IR$ **falls**.

So an output across the fixed resistor **rises** with light or temperature; an output across the sensor **falls**. If the source has internal resistance, add one more link: the larger current also increases Ir, so the terminal p.d. falls slightly.



Where you take the output decides the direction of the change: across the fixed resistor it rises with light, across the LDR it falls

Worked example. A battery of e.m.f. 9.0 V and negligible internal resistance is connected in series with an LDR and a 1200 Ω resistor. In the light the LDR has a resistance of 1800 Ω. Calculate the p.d. across the LDR, and state and explain what happens to it when the light intensity decreases.

$V_{\text{LDR}} = 9.0 \times \dfrac{1800}{1800 + 1200} = 5.4$ V. Less light → the resistance of the LDR increases → the total resistance increases and the current **decreases** → the p.d. across the 1200 Ω resistor (IR) decreases → the p.d. across the LDR ($9.0 - IR$) **increases**.

Worked example. A thermistor in series with a 5800 Ω resistor across a 6.0 V supply gives a p.d. of 2.9 V across the resistor. Find the resistance of the thermistor.

Current $= 2.9/5800 = 5.0 \times 10^{-4}$ A. The thermistor takes the remaining $6.0 - 2.9 = 3.1$ V, so $R = 3.1/(5.0 \times 10^{-4}) = 6200$ Ω (or, by ratio, $R = 5800 \times 3.1/2.9$).

■ Potentiometer and the null method

A **potentiometer** is a uniform resistance wire of length L_0 with a sliding contact (**jockey** 滑动触头). The resistance per unit length is uniform, so the p.d. from one end to the jockey is proportional to the length:

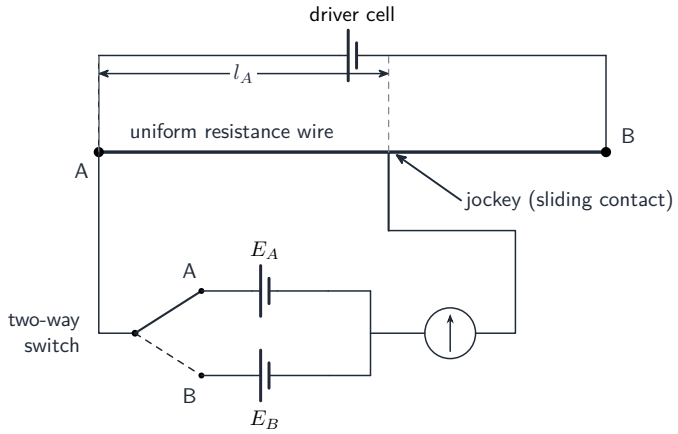
$$V_x = V_{\text{full}} \cdot \frac{x}{L_0}.$$

To **compare two e.m.f.s** (an unknown cell against a standard cell), connect each

in turn with the jockey through a **galvanometer** 检流计. Slide the jockey until the galvanometer reads zero (a **null** – no current flows through the cell being measured, because the potentiometer's voltage there exactly opposes the cell's e.m.f.). The **balance length** 平衡长度 – the wire length from the end to the jockey at balance – is proportional to the e.m.f. being measured, so the two lengths are in the ratio of the e.m.f.s:

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}.$$

This is a **null method** 零点法: you find the balance (zero current) instead of measuring a current's value. Its advantage is that at balance the unknown cell gives no current, so its internal resistance does not affect the result.



A potentiometer comparing two cell e.m.f.s by the null method

Why the null method is better than a voltmeter. At balance no current is drawn from the cell being measured, so there is no Ir drop inside it: the balance point measures the **e.m.f.**, not the terminal p.d. A voltmeter always draws some current, so it reads slightly less than the e.m.f.

Reading the balance point. The p.d. per unit length of the wire is fixed by the **driver cell** 驱动电池 and the resistance of the wire. Anything that makes the p.d. being balanced **larger** moves the balance point **further** along the wire; anything that makes the p.d. per unit length larger (a driver cell of larger e.m.f., or a wire that takes a larger share of the driver's p.d.) makes the balance length **shorter**. Replacing the wire by one of the same length but **greater diameter** lowers its resistance ($R = \rho L/A$): if the driver cell has negligible internal resistance and nothing else is in series, the p.d. across the wire is still the full e.m.f. and the balance length does not move; if the driver has internal resistance or a series resistor, the wire's share of the e.m.f. falls, the p.d. per metre falls, and the balance length grows. Say which case you are in.

> **Worked example.** A potentiometer wire XY of length 2.0 m carries a p.d. of 1.50 V. A cell of e.m.f. E is balanced when the jockey is 1.6 m from X. Find E, and state what happens to the balance length if the driver cell is replaced by one of larger e.m.f.

$E = 1.50 \times \dfrac{1.6}{2.0} = 1.2$ V. A larger driver e.m.f. gives a larger p.d. per metre, so the same 1.2 V is reached at a **shorter** length: the jockey must move towards X.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
electromotive force (e.m.f.)	the energy transferred per unit charge by a source in driving charge round a complete circuit
potential difference (p.d.)	the energy transferred per unit charge from electrical energy to other forms of energy
internal resistance	the resistance to current inside a source of e.m.f., which causes a p.d. Ir across the source when a current flows
terminal p.d.	the p.d. across the terminals of a source, equal to e.m.f. $-Ir$
Kirchhoff's first law	the sum of the currents into a junction is equal to the sum of the currents out of the junction (conservation of charge)
Kirchhoff's second law	the sum of the e.m.f.s round a closed loop is equal to the sum of the p.d.s round the loop (conservation of energy)
potential divider	two or more resistors in series across a supply, giving a p.d. across one of them that is a fraction of the supply p.d.
potentiometer	a uniform resistance wire with a sliding contact, used to compare p.d.s by finding the length at which a galvanometer reads zero

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Term	Definition
null method	a measurement made by adjusting a circuit until a meter reads zero, so no current is drawn from the component being measured
balance length	the length of potentiometer wire between one end and the sliding contact when the galvanometer reads zero

Exam tips

- Apply **Kirchhoff's laws**: current into a junction = current out (charge conserved); $\sum$ e.m.f. = $\sum$ p.d. round a loop (energy conserved). Name the law you are using when a question says "use Kirchhoff's laws".
- Combine resistors: **series** $R = R_1 + R_2$; parallel $1/R = 1/R_1 + 1/R_2$. A parallel combination is always smaller than its smallest resistor.
- A **potential divider** splits voltage in the ratio of the resistances; the larger resistance takes the larger p.d.
- Include **internal resistance**: e.m.f. $= I(R + r)$ – the "lost volts" are Ir. When a circuit changes, argue resistance $\rightarrow$ current in the cell $\rightarrow$ Ir $\rightarrow$ terminal p.d.
- "Negligible internal resistance" means the terminal p.d. **is** the e.m.f., whatever the current. Look for the phrase before you start.
- Read a V–I graph of a source as $V = \varepsilon - Ir$: intercept ε, gradient $-r$, and the current at $V = 0$ is the maximum (short-circuit) current ε/r.
- In Paper 5, rearrange the circuit relation into $y = mx + c$ before plotting: a cell of e.m.f. ε and internal resistance r feeding n equal resistors R in parallel obeys $\varepsilon = I\left(\dfrac{R}{n} + r\right)$, so $\dfrac{1}{I} = \dfrac{R}{\varepsilon}\cdot\dfrac{1}{n} + \dfrac{r}{\varepsilon}$ – a graph of $1/I$ against $1/n$ has gradient R/ε and intercept r/ε.

Common mistakes

- Writing $V = IR$ with the e.m.f. and the external resistance only, forgetting r. Use $\varepsilon = I(R + r)$.
- Saying e.m.f. is "the force that pushes the charge". It is energy per unit charge; the volt is a joule per coulomb.
- Pairing the laws with the wrong conservation law. First law – charge; second law – energy.
- Using "product over sum" for three parallel resistors. It works for two only; otherwise add the reciprocals.

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- Explaining a sensor circuit by "the resistance changes so the voltage changes". The marks are for the chain: resistance $\rightarrow$ total resistance $\rightarrow$ current $\rightarrow$ IR across the fixed resistor $\rightarrow$ the rest across the sensor.
- Claiming a potentiometer at balance "draws no current from the driver cell". It draws none from the **cell being measured**; the driver cell always supplies the wire current.
- Rounding a "show that" value before the last line – write 1.40 V, then say it is 1.4 V.

Particle physics

A-Level Physics

The nuclear atom

■ Geiger–Marsden α-particle scattering

Alpha particles α 粒子 fired at a thin gold foil were seen to:

- mostly pass straight through, with very little **deflection** 偏转,
- sometimes deflect through small angles,
- rarely (about 1 in 8000) deflect through angles greater than 90°.

From this Rutherford worked out:

- the atom is **mostly empty space** (most α-particles pass straight through),
- there is a tiny, dense, positively charged **nucleus** 原子核 at the centre (the rare large deflections need a concentrated charge to push the α away),
- almost all of the atom's mass is in this nucleus.

Order of magnitude: atom diameter $\sim 10^{-10}$ m, nucleus diameter $\sim 10^{-15}$ m – the nucleus is about 10^5 times smaller than the atom.

The three results and what each proves – the mark scheme pairs them exactly like this:

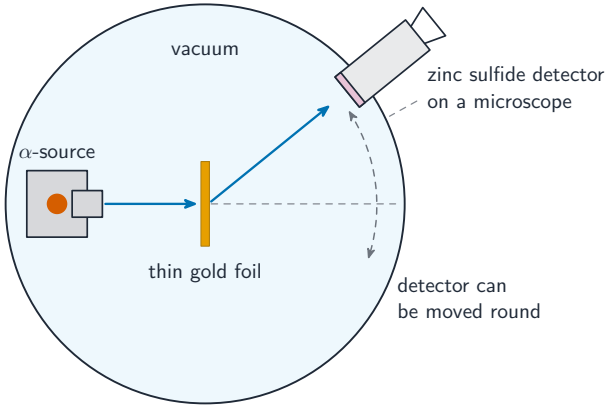
Observation	Conclusion
the vast majority pass straight through with little or no deflection	most of the atom is empty space
a small number are deflected through small angles	there is a concentrated positive charge (the nucleus) that repels the positive α-particles
a very small minority (about 1 in 8000) are deflected through more than 90°, some straight back	the nucleus is very small compared with the atom, and contains almost all of the atom's mass

Three details of the experiment are also asked: the foil is **thin** so that each α-particle meets at most one nucleus (and is not absorbed); the chamber is a **vacuum** so the α-particles are not stopped or scattered by air; gold is used because it can be beaten into a very thin sheet and has a heavy, highly charged nucleus.

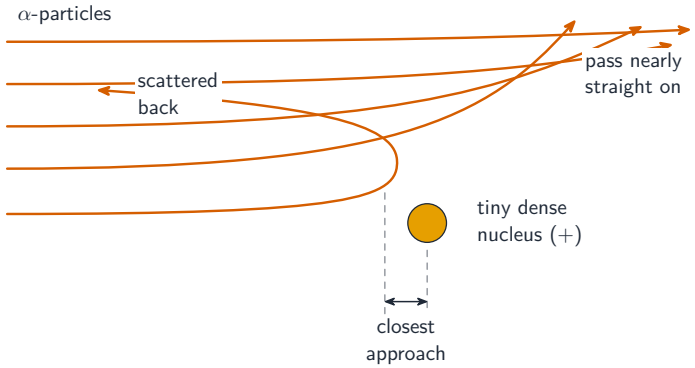
> **Worked example.** Explain why a very small minority of the α-particles are scattered through angles greater than 90°.

Only an α-particle that approaches a nucleus almost **head-on** is turned back. The nucleus and the α-particle are both positive, so the **electrostatic** 静电 repulsion is very large at small separation, and the nucleus is much more massive than the α-particle, so it is the α-particle that is turned round. Because the nucleus is tiny, very few α-particles get that close – hence the small minority. As the α-particle approaches, its kinetic energy is converted to electric potential energy; at the point of closest approach it is momentarily at rest and all of its kinetic energy has become potential energy.

The deflection depends on how close the path passes to the nucleus: the smaller the distance, the larger the angle, and the paths are symmetrical about the line through the nucleus.



The α-scattering experiment – α-particles strike a thin gold foil in a vacuum



Most α-particles pass nearly straight through; a few are deflected sharply by the tiny nucleus

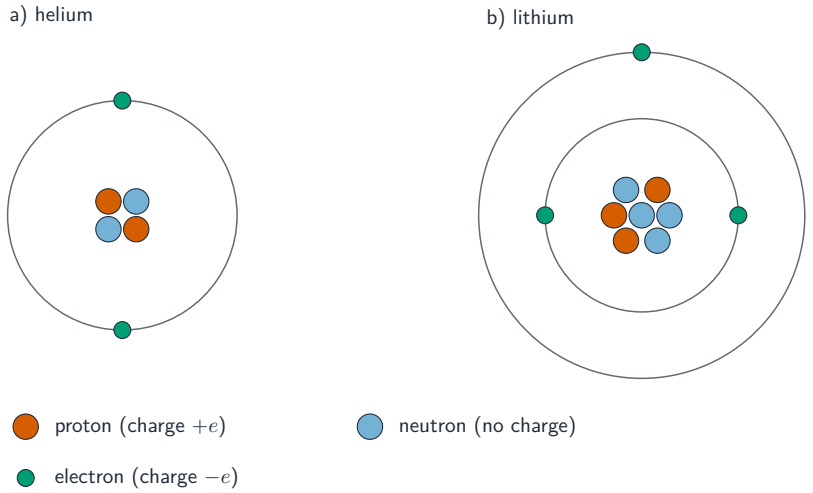
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Simple nuclear model

An atom has:

- a central nucleus of **protons** 质子 (positive, charge $+e$) and **neutrons** 中子 (no charge),
- **electrons** 电子 (charge $-e$) around the nucleus.

The proton and neutron have almost the same mass (≈ 1 u); the electron is about $\frac{1}{1836}$ of the proton's mass.



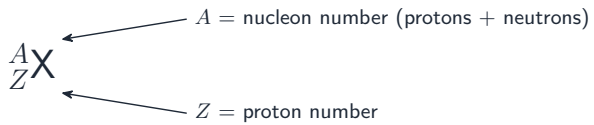
Simple models of a helium atom and a lithium atom (not to scale)

Worked example. Describe the structure of an atom of uranium-238, $^{238}_{92}\mathrm{U}$.

A nucleus containing 92 protons and $238 - 92 = 146$ neutrons, with 92 electrons in orbit around it. "Orbital electrons" is the syllabus phrase – the electrons are outside the nucleus, and the neutral atom has as many electrons as protons.

Notation and key numbers

For a **nuclide** 核素 written $^{A}_{Z}\mathrm{X}$:



Nuclide notation: nucleon number on top, proton number below

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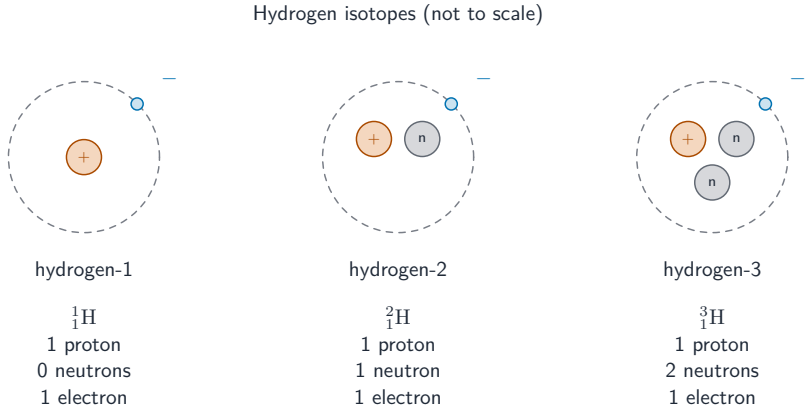
- **proton number** 质子数 Z (also the atomic number): the number of protons. It fixes the element.
- **nucleon number** 核子数 A (also the mass number): the total number of **nucleons** 核子 (protons + neutrons).
- number of neutrons $N = A - Z$.

A neutral atom has the same number of electrons as protons.

Isotopes

Isotopes 同位素 are atoms of the same element (same Z) with different numbers of neutrons (different A). They behave the same chemically but differently in the nucleus. Example: $^{12}_{6}\mathrm{C}$ and $^{14}_{6}\mathrm{C}$ are isotopes of carbon.

The examiner's wording: isotopes are nuclei (or atoms) with the **same number of protons** but **different numbers of neutrons** – equivalently the same proton number Z and different nucleon numbers A. Hydrogen has three:



The three isotopes of hydrogen: the same one proton, different numbers of neutrons

Worked example. Tritium, $^{3}_{1}\mathrm{H}$, is an isotope of hydrogen. State the numbers of protons, neutrons and electrons in a neutral tritium atom, and give the quark composition of its nucleus.

$Z = 1$ proton; $A - Z = 3 - 1 = 2$ neutrons; 1 electron (neutral, so electrons = protons). The nucleus is one proton (uud) and two neutrons (udd, udd): **4 up quarks and 5 down quarks**. Charge check: $4 \times \frac{2}{3} - 5 \times \frac{1}{3} = +1$, the proton number. A "labelled diagram" of the atom is the nucleus (1 p, 2 n) with one electron orbiting outside it.

■ Conservation laws in nuclear processes

In any nuclear process:

- nucleon number A is conserved (total A before = total A after),
- **charge** is conserved (this is **conservation of charge** 电荷守恒).

These two rules let you balance decay and reaction equations.

■ Unified atomic mass unit

The **unified atomic mass unit** 统一原子质量单位, symbol u, is set so that an atom of ${}^{12}_{6}\mathrm{C}$ has mass exactly 12 u. Numerically,

$$1\ \mathrm{u} = 1.661 \times 10^{-27}\ \mathrm{kg}.$$

A proton has mass ≈ 1.007 u; a neutron ≈ 1.009 u; an electron $\approx 5.5 \times 10^{-4}$ u.

> **Worked example.** A nucleus X has 14 nucleons and p protons. Its charge-to-mass ratio is 4.1×10^{7} C kg^{-1}. Find p and identify X.

Charge $= pe$; mass ≈ 14 u. So $\dfrac{pe}{14\mathrm{u}} = 4.1{\times}10^{7}$, giving $p = \dfrac{4.1 \times 10^{7} \times 14 \times 1.66 \times 10^{-27}}{1.60 \times 10^{-19}} = 5.96 \approx 6$. Six protons and 14 nucleons: X is carbon-14, ${}^{14}_{6}\mathrm{C}$. Keep every figure until the end, then round to the nearest whole number of protons.

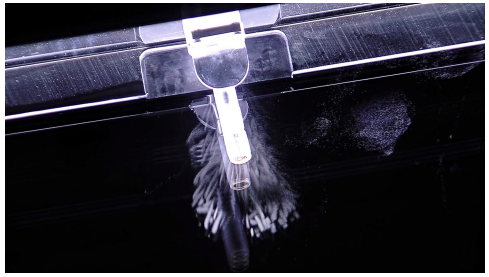
Radioactive emissions

An unstable nucleus rearranges itself and gives out one of three kinds of **radiation** 辐射. This is **radioactive** 放射性 decay. Each kind has its own properties.

■ α-radiation

- **Made of**: a helium-4 nucleus, ${}^{4}_{2}\alpha$ (two protons + two neutrons).
- **Mass**: ≈ 4 u.
- **Charge**: $+2e$.
- **Range in air**: a few cm. Stopped by a sheet of paper.
- **Ionising power**: strong – it is good at **ionising** 电离.
- **Energy spectrum**: **discrete** 分立 (one decay gives α-particles at one or a few sharp energies).

A **cloud chamber** 云室 makes the tracks visible: each α-particle leaves a short, straight, thick trail of tiny droplets as it ionises the air. The short equal lengths show the α-particles all carry about the same energy.



Alpha-particle tracks in a cloud chamber, fanning out from an americium-241 source

■ β-radiation

Two types of **beta particle** β 粒子:

- β^{-}: a fast electron, given out when a neutron turns into a proton.
- β^{+}: a **positron** 正电子 (the electron's antiparticle), given out when a proton in a proton-rich nucleus turns into a neutron.

Properties (both types):

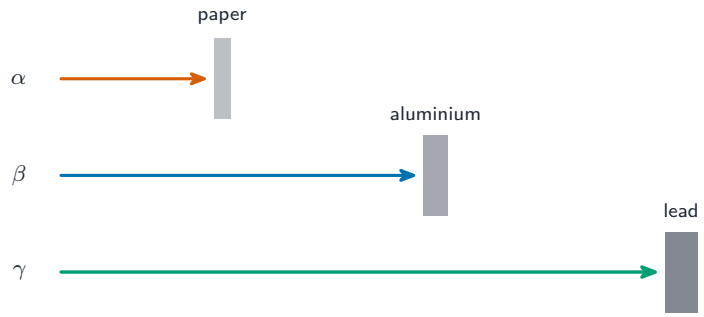
- **Mass**: $\approx 1/1836$ u (much less than α).
- **Charge**: $-e$ for β^{-}, $+e$ for β^{+}.
- **Range in air**: about 1 m. Stopped by a few mm of aluminium.
- **Energy spectrum**: **continuous** 连续 up to a maximum (see below).

■ γ-radiation

- **Made of**: a high-energy **photon** 光子 – part of the **electromagnetic spectrum** 电磁波谱.
- **Mass**: zero (rest mass).
- **Charge**: zero.
- **Range in air**: large (follows the inverse-square law). Strongly **attenuated** 衰减 by several cm of lead or about a metre of concrete.
- **Ionising power**: weakest.
- **Energy spectrum**: discrete (a **gamma ray** γ 射线 is given out as the nucleus drops between two nuclear energy levels).

A nucleus often gives out a γ-photon as a "tidy-up" step after an α or β decay leaves the **daughter nucleus** 子核 in an **excited state** 激发态.

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Penetrating power: α is stopped by paper, β by aluminium, γ only attenuated by lead

Mass and charge, side by side – a table question asks for exactly these, in units of u and e:

Particle	Mass	Charge
proton	1 u (1.007 u)	$+e$
neutron	1 u (1.009 u)	0
electron, β^-	$\frac{1}{1836}$ u (5.5×10^{-4} u)	$-e$
positron, β^+	$\frac{1}{1836}$ u	$+e$
α-particle	4 u	$+2e$
γ-photon	0	0
neutrino, antineutrino	almost zero	0

Worked example. Compare an α-particle with a β^+ particle in terms of their masses and charges (3 marks).

Both are positively charged, but the α-particle's charge ($+2e$) is twice that of the β^+ ($+e$). The α-particle's mass (4 u) is about 7300 times the mass of the β^+ (5.5×10^{-4} u). Give a ratio, not just "heavier": *twice the charge* and *about 7000 times the mass* are the marking points.

Because they carry charge, α- and β-particles are deflected by electric and magnetic fields – in opposite directions for opposite signs, the light β far more than the heavy α – while γ-rays, being uncharged, pass straight through.

Antiparticles, neutrinos and antineutrinos

Every particle has an **antiparticle** 反粒子 with the **same mass** but **opposite charge**. The positron is the antiparticle of the electron.

In β-decay, a third particle is always given out as well:

- β^- decay: an **antineutrino** 反中微子 $\bar{\nu}_e$.
- β^+ decay: a **neutrino** 中微子 ν_e.

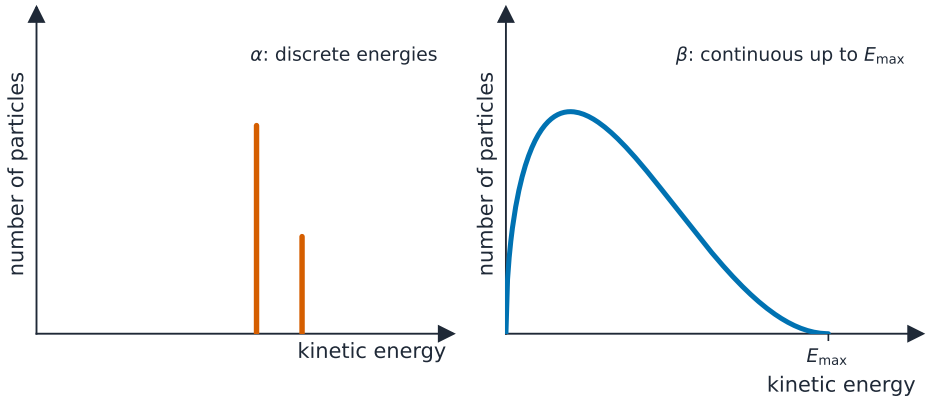
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Neutrinos and antineutrinos have zero charge, very small mass, and barely interact – they are very hard to detect, but they must be there to balance energy, **momentum** 动量 and other conserved quantities in β-decay.

Why β has a continuous spectrum (and α does not)

In α-decay the **energy** 能量 released is shared between just two particles (the daughter nucleus and the α). Conservation of momentum and energy then fixes the α's energy to one value (discrete).

In β-decay the energy is shared between **three** particles (the daughter nucleus, the β, and the (anti)neutrino). The β can take any share from zero up to a maximum, so its energy spectrum is continuous.



α-particles from one decay have fixed (discrete) energies; β-particles share the energy with an (anti)neutrino, so their spectrum is continuous up to a maximum

Worked example. The energy spectrum of the β^- particles from a source is continuous, from zero up to a maximum. Explain why (3 marks).

Each decay releases a fixed amount of energy. That energy is shared between the β^- particle, the **antineutrino** and the recoiling daughter nucleus. The antineutrino can take any share, so the β^- particle is left with any energy from zero up to the maximum – the maximum being when the antineutrino carries away almost none. (For an α-particle there are only two bodies, so momentum conservation fixes the share and the energy is discrete.)

Writing decay equations

A general α-decay:

$$^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\alpha$$

(check: $A = (A-4)+4$, $Z = (Z-2)+2$.)
A general β^- decay:

$$^{A}_{Z}\mathrm{X} \to {}^{A}_{Z+1}\mathrm{Y} + {}^{0}_{-1}\beta + \bar{\nu}_\mathrm{e}.$$

A general β^+ decay:

$$^{A}_{Z}\mathrm{X} \to {}^{A}_{Z-1}\mathrm{Y} + {}^{0}_{+1}\beta + \nu_\mathrm{e}.$$

Worked example. Uranium-238, $^{238}_{92}\mathrm{U}$, decays by α-emission; carbon-14, $^{14}_{6}\mathrm{C}$, decays by β^--emission. Find each daughter nuclide.

α-decay lowers A by 4 and Z by 2; β^--decay leaves A unchanged and raises Z by 1:

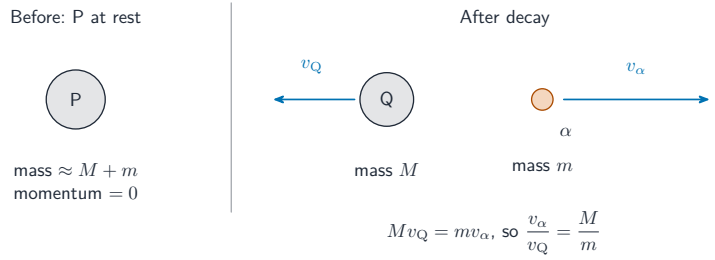
$$^{238}_{92}\mathrm{U} \to {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\alpha, \qquad {}^{14}_{6}\mathrm{C} \to {}^{14}_{7}\mathrm{N} + {}^{0}_{-1}\beta + \bar{\nu}_\mathrm{e}.$$

Determine quantitatively the changes. α-emission: A decreases by 4, Z decreases by 2. β^--emission: A unchanged, Z increases by 1. β^+-emission: A unchanged, Z decreases by 1. γ-emission: no change in either. A decay chain that ends at a known nuclide is solved from these: the number of α-decays is $(A_\mathrm{start} - A_\mathrm{end})/4$, and then the number of β^--decays makes Z come out right.

Worked example. Thorium-230, $^{230}_{90}\mathrm{Th}$, decays in stages, by α- and β^--emission, to lead-206, $^{206}_{82}\mathrm{Pb}$. Find the number of each kind of decay.

α-decays: $(230-206)/4 = 6$. Six α-decays alone would lower Z by 12, to 78; the final Z is 82, so there are $82-78 = 4$ β^--decays.

A nucleus at rest recoils. When a stationary nucleus emits an α-particle, momentum is conserved: the daughter nucleus and the α-particle move off in **opposite directions** with momenta of **equal magnitude**. Since $p = mv$, the lighter α-particle moves much faster, and since $E_\mathrm{k} = p^2/2m$ it also takes most of the kinetic energy – the shares are in inverse proportion to the masses.



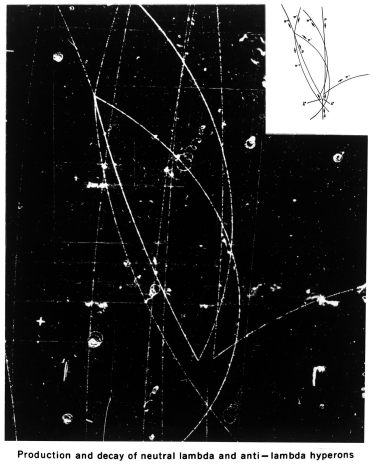
A nucleus at rest decays: the daughter recoils with momentum equal and opposite to the α-particle's

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Worked example. A stationary nucleus P of mass 243 u emits an α-particle of mass 4 u at 1.5×10^7 m s^{-1}. Find the speed of the daughter nucleus Q and the ratio of the kinetic energy of the α-particle to that of Q.

Q has mass $243-4 = 239$ u. Momentum: $239 v_\mathrm{Q} = 4\times1.5\times10^7$, so $v_\mathrm{Q} = 2.5\times10^5$ m s^{-1}, in the opposite direction to the α-particle. Kinetic energies: $\dfrac{E_\alpha}{E_\mathrm{Q}} = \dfrac{p^2/2m_\alpha}{p^2/2m_\mathrm{Q}} = \dfrac{m_\mathrm{Q}}{m_\alpha} = \dfrac{239}{4} \approx 60$. The α-particle takes about 98% of the energy released.

Fundamental particles



A bubble chamber reveals the curved tracks of charged particles.



Where the particles come from: a linear accelerator uses electric fields to drive charged particles fast enough to probe the nucleus

Some particles are **fundamental particles** 基本粒子 (point-like, with no smaller parts as far as we know); others are built from fundamental ones.

The one-mark definition: *a fundamental particle is one that cannot be broken down into smaller particles* (it has no internal structure). Electrons, neutrinos and quarks are fundamental; protons and neutrons are not.

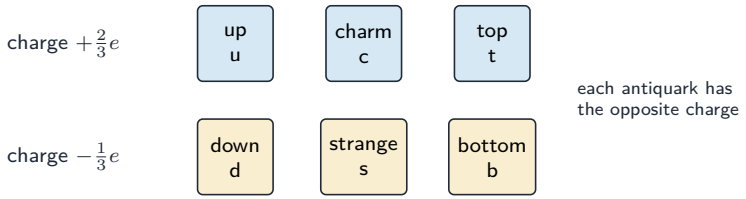
Quarks

A **quark** 夸克 is a **fundamental** 基本的 particle. There are six **flavours** 味:

- **up** (u), charge $+\frac{2}{3}e$,
- **down** (d), charge $-\frac{1}{3}e$,
- **charm** (c), charge $+\frac{2}{3}e$,
- **strange** (s), charge $-\frac{1}{3}e$,
- **top** (t), charge $+\frac{2}{3}e$,
- **bottom** (b), charge $-\frac{1}{3}e$.

Each quark has an **antiquark** 反夸克 with the same size of charge but the opposite sign: $\bar{u}$ (charge $-\frac{2}{3}e$), $\bar{d}$ (charge $+\frac{1}{3}e$). No other quark property is tested.

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The six quarks: up/charm/top carry $+\frac{2}{3}e$, down/strange/bottom carry $-\frac{1}{3}e$

Written as the table a question asks you to complete:

Quark	Charge	Antiquark	Charge
up (u), charm (c), top (t)	$+\frac{2}{3}e$	$\bar{u}$, $\bar{c}$, $\bar{t}$	$-\frac{2}{3}e$
down (d), strange (s), bottom (b)	$-\frac{1}{3}e$	$\bar{d}$, $\bar{s}$, $\bar{b}$	$+\frac{1}{3}e$

> **Worked example.** By reference to quark composition, show that the charge of a proton is $+1.6 \times 10^{-19}$ C.

A proton is uud: charge $= \frac{2}{3}e + \frac{2}{3}e - \frac{1}{3}e = +e = +1.6 \times 10^{-19}$ C. Write the three fractions and their sum – the mark is for the arithmetic, not the answer.

Hadrons: baryons and mesons

Particles built from quarks are **hadrons** 强子. Two types:

- **baryons** 重子 – three quarks. Examples: proton (u u d), neutron (u d d). Charge check: $\frac{2}{3} + \frac{2}{3} - \frac{1}{3} = +1$ for the proton; $\frac{2}{3} - \frac{1}{3} - \frac{1}{3} = 0$ for the neutron.
- **mesons** 介子 – one quark and one antiquark (for example π^+ is $u\bar{d}$).

Protons and neutrons are **not** fundamental – they are baryons made of quarks.

"Compare baryons and mesons in terms of their constituent particles" (2 marks): both are hadrons made of quarks; a baryon is three quarks (or three antiquarks, for an antibaryon), a meson is one quark and one antiquark. Any charge you are given must come out of the quark charges:

> **Worked example.** (a) A meson has charge $-1e$. Give a possible quark composition. (b) A meson Q has charge 0. Give a possible composition. (c) A baryon is made of three quarks of different flavours, u, d and s. Find its charge. (d) Give the quark composition of an antineutron and its charge.

(a) $d\bar{u}$: $-\frac{1}{3} - \frac{2}{3} = -1$ ($s\bar{u}$ also works). (b) $u\bar{u}$: $+\frac{2}{3} - \frac{2}{3} = 0$ (or $d\bar{d}$). (c) uds: $\frac{2}{3} - \frac{1}{3} - \frac{1}{3} = 0$. (d) The antineutron is the antiparticle of udd, so it is $\bar{u}\bar{d}\bar{d}$: $-\frac{2}{3} + \frac{1}{3} + \frac{1}{3} = 0$ – the same mass and charge as the neutron, since both have charge zero.

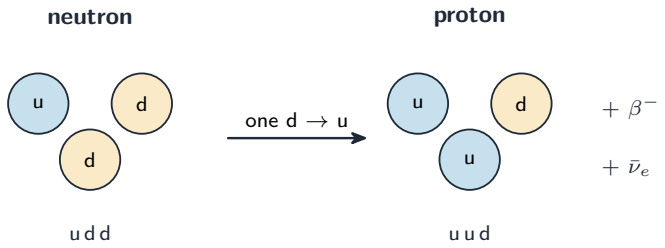
Quark changes in β-decay

In β^- decay a neutron turns into a proton; in quark terms, **one down quark turns into an up quark**:

$$\mathrm{d} \to \mathrm{u} + \beta^- + \bar{\nu}_\mathrm{e}.$$

In β^+ decay a proton turns into a neutron; **one up quark turns into a down quark**:

$$\mathrm{u} \to \mathrm{d} + \beta^+ + \nu_\mathrm{e}.$$



Beta-minus decay: one down quark becomes an up quark, turning a neutron into a proton

"Describe β^+ decay in terms of the fundamental particles involved" (2 marks): an up quark in a proton changes into a down quark, so the proton becomes a neutron, and a positron and an electron neutrino are emitted – $\mathrm{u} \to \mathrm{d} + \beta^+ + \nu_\mathrm{e}$. For β^- decay swap the roles: a down quark becomes an up quark, emitting an electron and an electron antineutrino. Name the (anti)neutrino: it is the "other lepton" the question asks for, and it is what makes the β energy continuous.

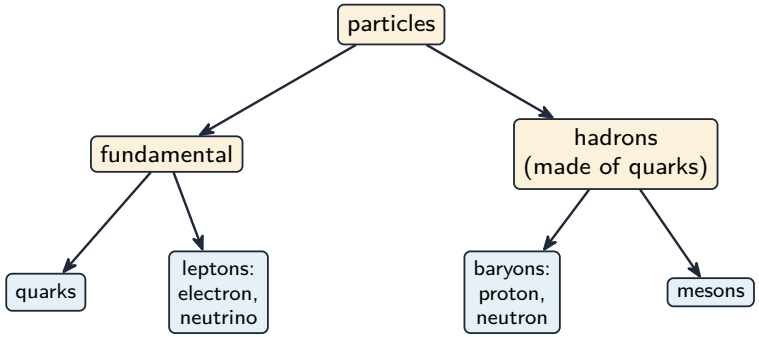
Leptons

Leptons 轻子 are fundamental particles that are not made of quarks. The leptons you need are the **electron** and the **electron neutrino**, with their antiparticles the **positron** and the **electron antineutrino**. (Heavier leptons – the muon and tau – exist but are not tested.) Asked for "two different leptons", give *electron* and *neutrino*.

Classifying particles

To answer "which are fundamental?": **quarks** and **leptons** (electrons, positrons, neutrinos, antineutrinos) are fundamental; protons, neutrons, baryons, mesons and hadrons are **not** – they are built from quarks.

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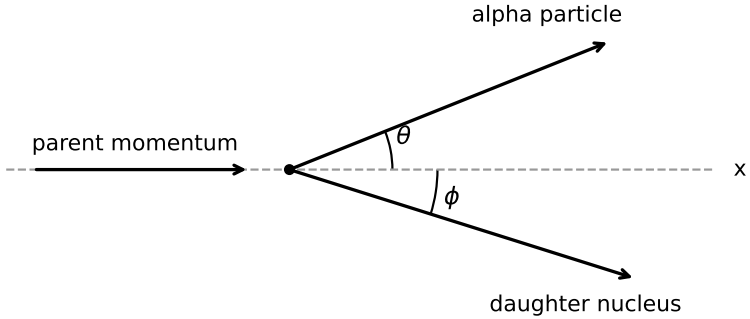
Fundamental particles (quarks, leptons) versus hadrons (baryons, mesons)

Worked example. In the list – antineutrino, β^+ particle, neutron, positron, proton – underline the hadrons.

Neutron and proton (three quarks each). The antineutrino and the positron are leptons; the β^+ particle *is* a positron. The same list asked as "which are not fundamental?" has the same answer: neutron and proton.

Antimatter. Every particle has an antiparticle of the same mass and opposite charge; a positron and an electron are alike in mass (and in the size of their charge) and differ in the sign of their charge. An **antihydrogen** atom is an antiproton (charge $-e$, made of $\bar{\mathrm{u}}\bar{\mathrm{u}}\bar{\mathrm{d}}$) with a positron in orbit.

A moving nucleus decays



Momentum components when a moving nucleus decays

Use conservation of momentum **along each axis separately**. Choose x along the parent's initial velocity, so the initial transverse momentum is zero. If the products

move on opposite sides of x, their transverse momenta cancel: $m_\alpha v_\alpha \sin\theta = m_d v_d \sin\phi$. Along x, their momenta add to the parent's initial momentum.

> **Worked example.** Products have momenta 2.0×10^{-20} and 1.0×10^{-20} kg m/s. The daughter moves 30° below x. Transverse conservation gives $\sin\theta = p_d \sin\phi / p_\alpha = (1.0\times10^{-20}\sin 30°)/(2.0\times10^{-20}) = 0.25$, so the alpha particle moves at 14.5° above x. The initial total momentum is not zero; only its transverse component is zero.

■ Count quarks in a whole nucleus

A nucleus ${}^{A}_{Z}X$ has Z protons (uud) and $A-Z$ neutrons (udd). Thus $N_u = 2Z + (A-Z) = A+Z$ and $N_d = Z + 2(A-Z) = 2A - Z$. Check $N_u + N_d = 3A$ and $q = (2N_u - N_d)e/3 = Ze$.

> **Worked example.** Carbon-14 has six protons and eight neutrons: $N_u = A + Z = 14 + 6 = 20$ and $N_d = 2A - Z = 28 - 6 = 22$. In beta-minus decay one down quark becomes up, so the daughter has 21 up and 21 down quarks.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
proton number Z	the number of protons in a nucleus
nucleon number A	the total number of protons and neutrons in a nucleus
isotopes	nuclei of the same element (same number of protons) with different numbers of neutrons
nuclide	a nucleus with a particular number of protons and a particular number of neutrons
unified atomic mass unit	one twelfth of the mass of an atom of carbon-12
antiparticle	a particle with the same mass as the corresponding particle but the opposite charge
fundamental particle	a particle that cannot be broken down into smaller particles
hadron	a particle made of quarks: a baryon (three quarks) or a meson (a quark and an antiquark)

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Term	Definition
lepton	a fundamental particle that is not made of quarks, such as the electron or the neutrino
α-particle	a helium-4 nucleus: two protons and two neutrons, charge $+2e$, mass 4 u
β^- / β^+ particle	an electron / a positron emitted from a nucleus when a neutron becomes a proton / a proton becomes a neutron

Exam tips

- Describe the **nuclear atom** (a small, dense, positive nucleus) using the alpha-scattering evidence – pair each observation with its conclusion.
- Compare α, β and γ by **charge, mass, ionising power and penetration**; when asked to compare two particles, give the ratio ("twice the charge", "about 7000 times the mass").
- Use quark composition (proton *uud*, neutron *udd*) and check that **charge and nucleon number balance** in every equation you write.
- In β^- decay a neutron becomes a proton, an electron and an antineutrino; in β^+ decay a proton becomes a neutron, a positron and a neutrino. Name the (anti)neutrino every time.
- A nucleus at rest that decays gives its two products **equal and opposite momentum**, so the lighter one is faster and carries most of the kinetic energy.
- For a decay chain, count the α-decays from the change in A first, then fix Z with β-decays.

■ Common mistakes

- Concluding "the nucleus is positive" from the *straight-through* result. That result shows empty space; the large-angle deflections show the concentrated positive charge.
- Defining isotopes by "different mass" only. The mark needs *same number of protons, different number of neutrons.*
- Writing ${}^{0}_{-1}\beta$ for β^+ decay. A positron is ${}^{0}_{+1}\beta$, and Z goes *down* by one.
- Forgetting the (anti)neutrino in a β-decay equation, or putting a neutrino with β^-. β^- goes with the antineutrino, β^+ with the neutrino.
- Saying protons or neutrons are fundamental. Only quarks and leptons are.
- Giving a meson as "two quarks". It is a quark **and an antiquark**.
- Treating an antiquark's charge as the same sign as the quark's. $\bar{\text{u}}$ is $-\frac{2}{3}e$, $\bar{\text{d}}$ is $+\frac{1}{3}e$.

Motion in a circle

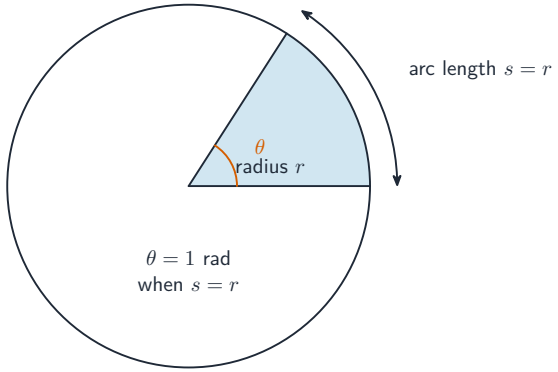
A-Level Physics

Angles in radians

The **radian** 弧度 is the angle made at the centre of a circle by an **arc** 弧 whose length equals the radius. For an arc of length s on a circle of radius r, the angle in radians is

$$\theta = \frac{s}{r}.$$

Radians have no unit (a ratio of lengths). A full circle has $s = 2\pi r$, so $\theta = 2\pi$ rad. A half-circle is π rad; a quarter is $\pi/2$ rad.



One radian is the angle whose arc length equals the radius

To convert: 1 rad $= 180°/\pi \approx 57.3°$. Set your calculator to **radians** for this topic; "degree" mode will give wrong answers.

The one-mark definition. *The radian is the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.* Give it in words – $\theta = s/r$ on its own is not a definition – and keep $s = r\theta$ ready for turning a distance along the arc into an angle, or back.

Uniform circular motion: angular speed

An object moves in a circle of radius r at constant speed v. Define:

- **angular displacement** 角位移 θ – the angle (in radians) turned through by the radius from a chosen start line.
- **angular speed** 角速度 ω – the rate of change of angular displacement.



A spinning fairground ride: every rider turns through the same angle each second.

For uniform motion ω is constant and

$$\omega = \frac{\theta}{t}.$$

Unit: rad s^{-1}.

What stays constant and what varies. In uniform circular motion the **speed**, the angular speed, the period and the *magnitude* of the acceleration are constant; the **velocity**, the **displacement** from the centre and the **acceleration** all change continuously, because their *direction* changes. Asked to "state two quantities that vary", give two of velocity, acceleration, displacement and momentum – and say it is the direction that varies.

Period and frequency

If the object goes once round (2π rad, one **revolution** 圈) in time T (the **period** 周期), then

$$\omega = \frac{2\pi}{T} = 2\pi f,$$

where $f = 1/T$ is the **frequency** 频率 of turning (Hz).

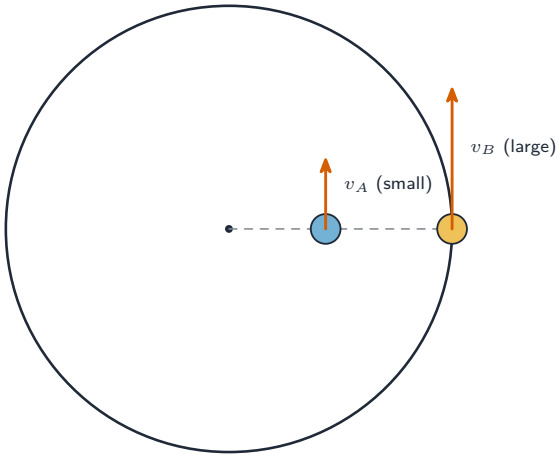
Linear and angular speed

In one period T the object travels a distance $2\pi r$ (the **circumference** 周长) at constant speed, so

$$v = \frac{2\pi r}{T} = r\omega.$$

This links the linear (**tangential** 切向) speed v with the angular speed ω. At a larger radius (for the same angular speed) the linear speed is larger – a child on the edge of a merry-go-round moves faster than one near the centre, even though both go round once in the same time.

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same ω, but $v = r\omega$ —
the outer rider moves faster

Same angular speed, but the rider at the larger radius has the larger linear speed ($v = r\omega$)

Worked example. A fairground ride of radius 4.0 m completes one turn every 8.0 s. Find its angular speed and the linear speed of a rider on the edge.

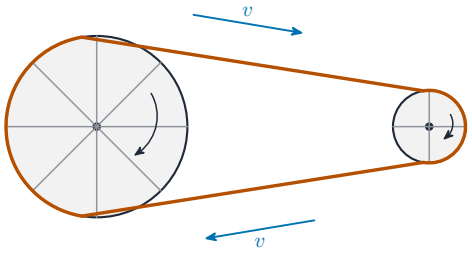
$$\omega = \frac{2\pi}{T} = \frac{2\pi}{8.0} = 0.79 \text{ rad s}^{-1}, \qquad v = r\omega = 4.0 \times 0.79 = 3.1 \text{ m s}^{-1}.$$

Worked example. The minute hand of a clock turns once every hour. A piece of modelling clay on the hand moves a total distance of 0.44 m in 1400 s. Find the angular speed of the hand, the angle the clay turns through in that time, and its distance from the centre of the clock.

$\omega = 2\pi/T = 2\pi/3600 = 1.75 \times 10^{-3}$ rad s^{-1}. Angular displacement $\theta = \omega t = 1.75\times10^{-3}\times1400 = 2.44$ rad. Distance along the arc $s = r\theta$, so $r = s/\theta = 0.44/2.44 = 0.18$ m. Check: $v = s/t = 3.1 \times 10^{-4}$ m s^{-1} and $r\omega = 0.18 \times 1.75 \times 10^{-3} = 3.1 \times 10^{-4}$ m s^{-1} – the same. A common trap in these questions is a radius measured *from the rim*: a lump 1.2 cm in from the edge of a 9.3 cm disc moves in a circle of radius 8.1 cm.

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every link of the chain moves at the same speed v



pedal cog: radius r_1,
angular speed ω_1

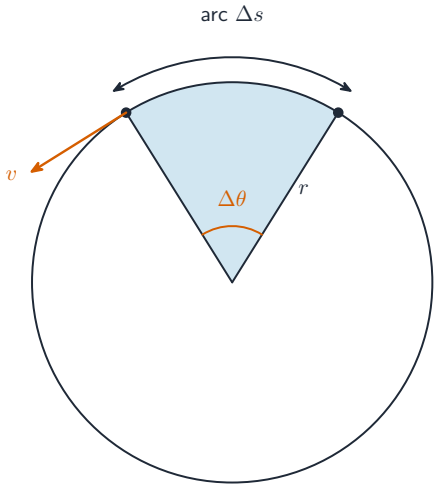
rear cog: radius r_2,
angular speed ω_2

$$v = r_1\omega_1 = r_2\omega_2 \quad \Rightarrow \quad \frac{\omega_2}{\omega_1} = \frac{r_1}{r_2}$$

Two cogs joined by a chain share the chain's speed, not its angular speed: $v = r_1\omega_1 = r_2\omega_2$

Worked example. A bicycle chain passes round a pedal cog of radius 0.095 m and a rear-wheel cog of radius 0.038 m. The pedals turn at 1.2 revolutions per second. Find the speed of the chain and the angular speed of the rear cog. The chain is then moved to a smaller rear cog while the bicycle's speed stays the same; explain, without calculation, what happens to the angular speed of the pedals.

Pedal angular speed $\omega_1 = 2\pi \times 1.2 = 7.5$ rad s^{-1}. Every link of the chain moves at the speed of the rim of the pedal cog: $v = r_1\omega_1 = 0.095 \times 7.5 = 0.72$ m s^{-1}. The rear cog's rim moves at the same speed, so $\omega_2 = v/r_2 = 0.72/0.038 = 19$ rad s^{-1}. With the bicycle's speed unchanged, the rear *wheel* keeps the same angular speed; the smaller cog's rim therefore moves more slowly ($v = r\omega$ with a smaller r), so the chain moves more slowly, and the pedal cog, driven by the same chain, turns at a **lower** angular speed.



As the radius turns through $\Delta\theta$ the object moves an arc Δs at speed v

Centripetal acceleration

An object moving in a circle at constant speed still has a changing **velocity** 速度 – its direction keeps changing, even though its size stays the same. A changing velocity needs an **acceleration** 加速度. This acceleration points **towards the centre** and is the **centripetal acceleration** 向心加速度.

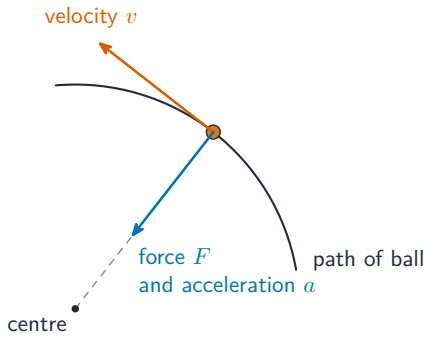
The two-mark description. *In uniform circular motion the speed is constant but the velocity is continuously changing, because its direction changes; the acceleration is constant in magnitude and is always directed towards the centre of the circle, perpendicular to the velocity.* "State what is meant by centripetal acceleration": *the acceleration of an object moving along a circular path, directed towards the centre of the circle.*

■ Size

$$a = \frac{v^2}{r} = r\omega^2.$$

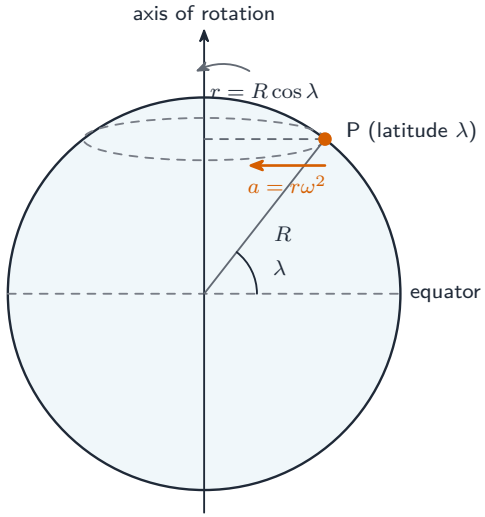
The two forms are equal because $v = r\omega$. Pick the one with the quantities you have.

The centripetal acceleration is **perpendicular** 垂直 to the velocity at every instant – never along the direction of motion. (If part of it were along the motion, the speed would change.) Unit: m s^{-2}.



The velocity points along the tangent; the force and acceleration point to the centre

Why the speed does not change. The centripetal force is perpendicular to the displacement at every instant, so it does **no work** on the object; its kinetic energy, and so its speed, stays constant. Only the direction of the velocity changes – which is exactly what an acceleration perpendicular to the velocity does.



P goes round the small circle once a day: $\omega = 2\pi/(24 \times 3600)$, $v = r\omega$, $a = r\omega^2$, directed towards the axis, not the centre

A point on the rotating Earth moves in a circle about the axis, of radius $R\cos\lambda$, not about the Earth's centre

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Worked example. The Earth is a sphere of radius 6.37×10^6 m rotating once in 24 hours. Cambridge is at **latitude** 纬度 52.2° north. Find the radius of the circle in which Cambridge moves, its speed, and its centripetal acceleration. A student of mass 58.6 kg stands on bathroom scales there; state and explain the effect of the rotation on the reading.

The circle is about the *axis*, so $r = R\cos\lambda = 6.37 \times 10^6 \times \cos 52.2° = 3.90 \times 10^6$ m. $\omega = 2\pi/(24 \times 3600) = 7.27 \times 10^{-5}$ rad s^{-1}, so $v = r\omega = 284$ m s^{-1} and $a = r\omega^2 = 3.90 \times 10^6 \times (7.27 \times 10^{-5})^2 = 2.1 \times 10^{-2}$ m s^{-2}. The student needs a resultant force $ma = 1.2$ N towards the axis, which can only come from the weight exceeding the upward contact force; so the scales read *less* than the weight $mg = 575$ N – by about one newton (the acceleration has a component $a\cos\lambda = 0.013$ m s^{-2} along the vertical, i.e. 0.7 N of the 1.2 N). Small, but the sign and the reason are the marks.

Centripetal force



A Ferris wheel: a centripetal force toward the centre keeps each car moving in a circle.

By Newton's second law, the resultant **force** 力 on a body in circular motion at constant speed is

$$F = ma = \frac{mv^2}{r} = mr\omega^2.$$

This is the **centripetal force** 向心力. It **always points towards the centre** – perpendicular to the velocity.

The centripetal force is not a new kind of force – it is the **net result** of the real forces acting (tension, gravity, friction, electric attraction, normal contact force, …). In a problem, work out which real force(s) provide it.

Worked example. A 0.20 kg ball on a string is whirled in a horizontal circle of radius 0.50 m at 3.0 m s^{-1}. Find the centripetal force (the tension in the string).

$$F = \frac{mv^2}{r} = \frac{0.20 \times 3.0^2}{0.50} = 3.6 \text{ N}.$$

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Where the centripetal force comes from

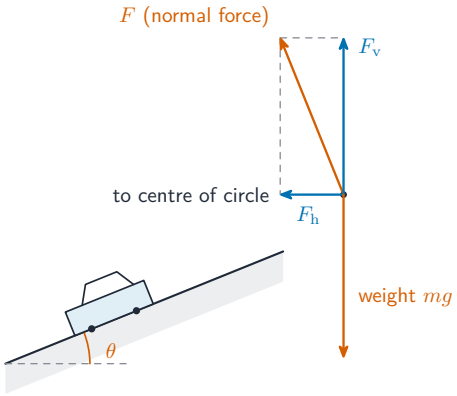
- **Ball on a string** in a horizontal circle: the **tension** 张力 in the string.
- **Car turning a flat corner**: the **friction** 摩擦力 between tyres and road ($F = mv^2/r$). If the car goes too fast, friction is not enough and it skids outwards.

A road is **banked** 倾斜 when its outer edge is raised.

- **Banked corner** 倾斜弯道 (no friction): the horizontal part of the **normal contact force** 支持力; $\tan\theta = v^2/(rg)$ for the angle that needs no friction.
- A planet or **satellite** 卫星 in **orbit** 轨道: the **gravitational attraction** 引力, $GMm/r^2 = mv^2/r$.
- **Electron** 电子 in a circular orbit (Bohr-style model): the **electrostatic** 静电 attraction between the electron and the positive **nucleus** 原子核:

$$\frac{kZe^2}{r^2} = \frac{m_e v^2}{r},$$

where $k = 1/(4\pi\varepsilon_0)$ and Z is the nuclear charge. Solve for v to get the orbital speed; then $T = 2\pi r/v$.



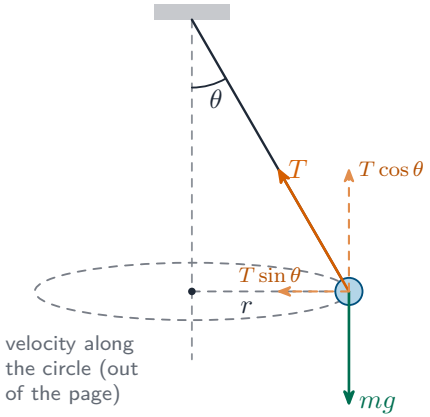
On a banked track the horizontal part of the road's force provides the centripetal force

The tilted force: conical pendulum, cone and swing-ride

A ball on a string swung in a horizontal circle (a **conical pendulum** 圆锥摆), a ball rolling round the inside of a smooth cone, a chair on a fairground swing-ride and a car on a banked track are all the same problem: one force (the tension or the normal contact force) is **tilted** at an angle θ to the vertical. Resolve it. Its **vertical** component balances the weight; its **horizontal** component is the whole centripetal force:

$$F\cos\theta = mg, \qquad F\sin\theta = \frac{mv^2}{r} \qquad \Rightarrow \qquad \tan\theta = \frac{v^2}{rg}.$$

Never add a separate "centripetal force" to the diagram: the two real forces are the weight and the tilted force, and their *resultant* is horizontal, towards the centre.



vertical: $T\cos\theta = mg$

horizontal: $T\sin\theta = \dfrac{mv^2}{r}$

so $\tan\theta = \dfrac{v^2}{rg}$

ball inside a smooth cone: the normal contact force N plays the part of T

Resolve the tilted force: vertical component = mg, horizontal component = mv^2/r

Worked example. A steel ball moves in a horizontal circle of radius 0.12 m on the smooth inside surface of a cone whose surface makes 52° with the horizontal. Name the two forces on the ball, state the direction of their resultant, and find the speed of the ball and the period of its motion.

The forces are the **weight** (vertically down) and the **normal contact force** from the cone's surface (perpendicular to the surface, so at 52° to the vertical); their resultant is horizontal, towards the centre of the circle. Vertically $N\cos 52° = mg$; horizontally $N\sin 52° = mv^2/r$. Dividing: $v^2 = rg\tan 52° = 0.12 \times 9.81 \times 1.28 = 1.51$, so $v = 1.2$ m s^{-1}, and $T = 2\pi r/v = 2\pi \times 0.12/1.23 = 0.61$ s. The mass cancels – the speed does not depend on it.

Worked example. A sphere of mass 0.29 kg hangs from a spring of spring constant 40 N m^{-1} and unstretched length 6.0 cm. It is set moving in a horizontal circle so that the spring makes 30° with the vertical. Find the tension, the radius of the circle and the speed.

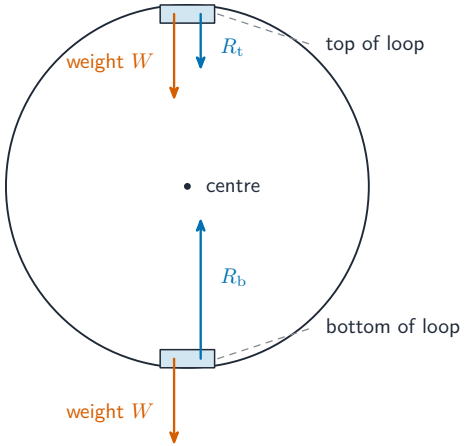
Vertically $T\cos 30° = mg$, so $T = 0.29 \times 9.81/0.866 = 3.3$ N. The extension is $x = T/k = 3.28/40 = 0.082$ m, so the spring's length is $0.060 + 0.082 = 0.142$ m and $r = 0.142\sin 30° = 0.071$ m. Horizontally $T\sin 30° = mv^2/r$: $v^2 = 3.28 \times 0.5 \times 0.071/0.29 = 0.40$, $v = 0.63$ m s^{-1}. Hooke's law supplies the length; the circle supplies the rest.

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Vertical circles

When the circle is upright, the speed is **not** constant (gravity does work) – but at each instant the **net force towards the centre** still equals mv^2/r:

- at the bottom of a loop: tension up, **weight** 重力 down, so $T - mg = mv^2/r$ – the tension is largest here.
- at the top of a loop: tension and weight both point down (towards the centre), so $T + mg = mv^2/r$ – the tension is smallest. For the slowest speed at the top with the string just tight, set $T = 0$: $mg = mv_{\min}^2/r$, giving $v_{\min} = \sqrt{gr}$.



Forces on a person at the top and bottom of a vertical circle

Worked example. A car goes round a vertical loop of radius 2.0 m. Find the minimum speed at the top for the car to keep contact with the track (take $g = 9.81$ m s^{-2}).

At the slowest speed the track force is zero, so gravity alone provides the centripetal force: $mg = mv_{\min}^2/r$, giving $v_{\min} = \sqrt{gr}$:

$$v_{\min} = \sqrt{9.81 \times 2.0} \approx 4.4 \text{ m s}^{-1}.$$

The constant-speed result ($v = r\omega$, ω constant) holds for horizontal circles, or where the force only bends the path (orbits in gravity, charges in a **magnetic field** 磁场).

Circles in fields

> **Worked example.** A helium atom is modelled as a nucleus of charge $+2e$ with two electrons in the same circular orbit of radius 170 pm, always on opposite sides of the nucleus. Find the resultant electric force on one electron and its speed.

Each electron is attracted by the nucleus, distance r away, and repelled by the other electron, distance $2r$ away, along the same line: $F = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{2e^2}{r^2} - \dfrac{e^2}{(2r)^2}\right) = \dfrac{e^2}{4\pi\varepsilon_0 r^2} \times 1.75 = \dfrac{2.31\times10^{-28}}{(1.7\times10^{-10})^2} \times 1.75 = 1.4\times10^{-8}$ N. This is the centripetal force: $v = \sqrt{Fr/m} = \sqrt{1.40\times10^{-8}\times1.7\times10^{-10}/(9.11\times10^{-31})} = 1.6\times10^{6}$ m s^{-1}.

> **Worked example.** A proton (mass 1.67×10^{-27} kg) moving at 2.0×10^{6} m s^{-1} enters a uniform magnetic field of flux density 0.50 T at right angles to the field. Find the radius of its path and the time for one revolution.

The magnetic force Bqv is always perpendicular to the velocity, so it is a centripetal force and the speed is constant: $Bqv = mv^2/r$ gives $r = \dfrac{mv}{Bq} = \dfrac{1.67\times10^{-27}\times2.0\times10^{6}}{0.50\times1.60\times10^{-19}} = 4.2\times10^{-2}$ m, and $T = \dfrac{2\pi r}{v} = \dfrac{2\pi m}{Bq} = 1.3\times10^{-7}$ s – independent of the speed, which is why a faster proton makes a larger circle in the same time. For a satellite the gravitational force plays the same part: $\dfrac{GMm}{r^2} = mr\omega^2$, and the orbital speed and period follow from r alone.

How to structure a circular-motion answer

1. **Find the radius** r and choose v or ω. Use $v = r\omega$ to switch between them.
2. **Find the centripetal acceleration** with $a = v^2/r$ or $r\omega^2$.
3. **List the real forces** and write Newton's second law in the **radial** 径向 direction (towards the centre is positive). Set the net inward force equal to mv^2/r.
4. For **period or frequency**: use $\omega = 2\pi/T$, or $T = 2\pi r/v$.
5. Check the **directions**: centripetal force and acceleration point to the centre; the velocity is along the tangent.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

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Term	Definition
radian	the angle subtended at the centre of a circle by an arc equal in length to the radius
angular displacement	the angle, in radians, through which the radius has turned from its starting position
angular speed	the angle swept out by the radius per unit time (the rate of change of angular displacement)
period	the time taken for one complete revolution
uniform circular motion	motion in a circle at constant speed, with a velocity that changes continuously in direction and an acceleration of constant magnitude directed towards the centre
centripetal acceleration	the acceleration of an object moving in a circular path, directed towards the centre of the circle and perpendicular to the velocity
centripetal force	the resultant force on an object in circular motion, directed towards the centre, of magnitude mv^2/r

Exam tips

- Work in **radians**; angular speed $\omega = 2\pi/T = v/r$, and arc length $s = r\theta$ turns a distance along the circle into an angle.
- **Centripetal acceleration** $a = v^2/r = \omega^2 r$; the net force acts **towards the centre** – it is provided by tension/gravity/friction/a contact force, not an extra force.
- Always state **what provides the centripetal force** in the situation given, and for a tilted force resolve it: vertical component $= mg$, horizontal component $= mv^2/r$.
- Read the radius carefully: a point measured *from the rim* moves in a smaller circle; a point on the rotating Earth moves in a circle about the **axis**, radius $R\cos\lambda$.
- Two cogs on one chain share the chain's **speed**; two points on one rigid wheel share its **angular speed**.

Common mistakes

- Defining the radian as "$180/\pi$ degrees" or as $\theta = s/r$. The mark needs the arc equal in length to the radius.
- Saying the velocity is constant in uniform circular motion. The *speed* is constant; the velocity changes direction, which is why there is an acceleration.
- Drawing a "centripetal force" arrow as an extra force alongside the tension or contact force. It is their resultant, not a third force.

- Using degrees in ωt or $v = r\omega$. Every formula in this topic assumes radians.
- Forgetting that the force does no work: a force perpendicular to the motion changes direction, not speed, so kinetic energy is constant.
- Taking r as the Earth's radius for a point at latitude λ. The circle is about the axis, so $r = R\cos\lambda$.

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Gravitational fields

A-Level Physics

Definition

A **gravitational field** 重力场 is a region where a **mass** 质量 feels a **force** 力 from other masses. The **gravitational field strength** 重力场强度 g at a point is the **gravitational force per unit mass** on a small **test mass** 检验质量 placed there:

$$g = \frac{F}{m}.$$

Unit: N kg^{-1} (the same as m s^{-2} – the acceleration of free fall in the field). g is a **vector** 矢量, pointing the way the force acts – towards the source mass.

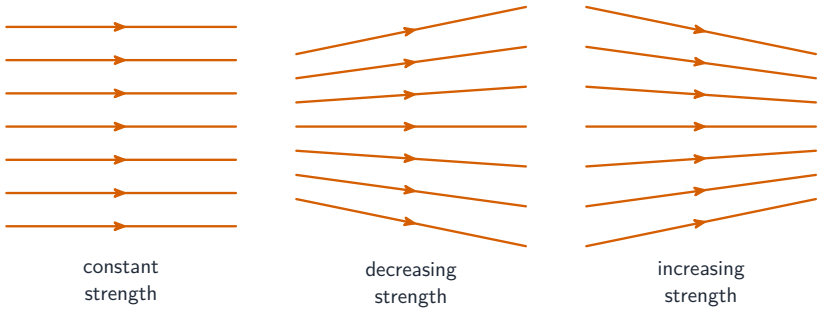
The examiner's wording. A *gravitational field* is a region of space in which a mass experiences a force. *Gravitational field strength* at a point is the gravitational force per unit mass acting on a small test mass placed at the point. The direction of a *field line* at a point is the direction of the force on a (small test) mass placed there. All three are one-mark definitions; "force per unit mass" is the phrase that scores, "the force on 1 kg" is not.

Field lines

A gravitational field is drawn with **field lines** 场线 that point the way the force acts on a test mass:

- around a **point mass** 质点 or a uniform sphere (treated as a point mass from outside), the field lines are **radial** 径向, pointing **inwards**.
- near the Earth's surface over a small area, the field lines are nearly **parallel and equally spaced**, pointing straight down – a **uniform field** 匀强场.

Closer lines mean a stronger field.



Field-line spacing shows the field strength – closer lines mean a stronger field

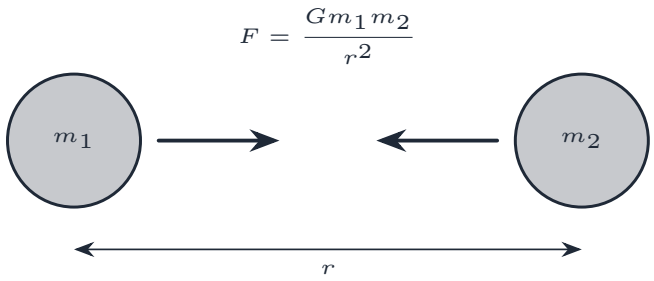
Why g is constant near the surface, in terms of field lines (a two-mark explanation): over a region whose size is small compared with the Earth's radius, the radial field lines are almost parallel and their spacing hardly changes with height, so the field strength – which the spacing represents – is almost the same at the top of the region as at the bottom. Over the whole planet the lines spread out with distance and the field falls.

> **Worked example.** A point P represents a point mass. Draw lines to represent the gravitational field around P, and state what the direction of a line shows.

Straight radial lines, evenly spaced around P, with arrows pointing **inwards** towards P. The direction of a line is the direction of the force on a small test mass placed there – always towards the mass that produces the field, because gravity only attracts.

Newton's law of gravitation

For two point masses m_1, m_2 a distance r apart, the force on each is



Two masses attract along the line joining them

$$F = \frac{Gm_1m_2}{r^2},$$

pulling them together along the line joining them. This is **Newton's law of gravitation** 万有引力定律. The constant $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} is the **universal gravitational constant** 万有引力常量.

In words, as the mark scheme wants it: *the gravitational force between two point masses is proportional to the product of their masses and inversely proportional to the square of their separation* – both proportionalities, and "point masses". Asked to "state the equation and the meaning of any other symbols", give $F = Gm_1m_2/r^2$ with G the gravitational constant and r the distance between the centres.

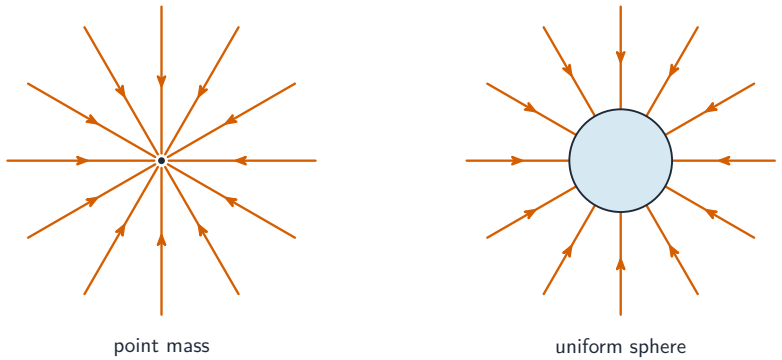
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> **Worked example.** Two isolated uniform spheres, each of mass 1.0×10^3 kg, have their centres 2.0 m apart. Find the gravitational force between them and comment on its size.

$F = \dfrac{6.67 \times 10^{-11} \times 1.0 \times 10^3 \times 1.0 \times 10^3}{2.0^2} = 1.7 \times 10^{-5}$ N. This is about 10^{-9} of each sphere's weight (9.8×10^3 N): gravity between everyday objects is negligible, and only a planet-sized mass produces a noticeable field. "Isolated" in a question means that no other masses need be considered.

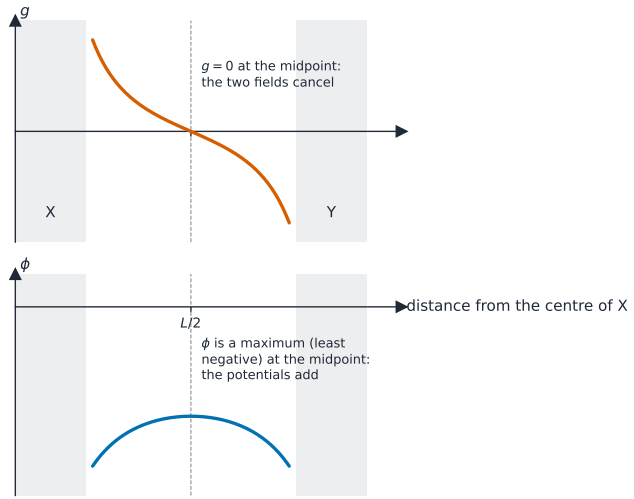
Spheres treated as point masses

For a uniform sphere (such as a planet or star), the field at any point **outside** is the same as that of a point mass equal to the total mass at the centre. So from above the surface, you can treat the Earth as a point mass at its centre. (Points inside a sphere are different, and are not in the syllabus.)



Outside a uniform sphere the field is radial, exactly like a point mass at the centre

Between two masses the fields subtract and the potentials add. Field strength is a vector: on the line joining two spheres the two fields point in opposite directions, so there is a point where they cancel and the resultant field is zero. Potential is a scalar: the two potentials simply add, and both are negative, so the potential is least negative (a maximum) at the point where the field is zero.



Along the line between two equal masses the field strength passes through zero at the midpoint, where the potential has its maximum

Worked example. Two identical isolated uniform spheres X and Y, each of mass M and radius R, have their centres a distance L apart. Point P lies on the line joining the centres. State where on the line the resultant field strength is zero, and find the gravitational potential there.

By symmetry the fields of X and Y are equal and opposite at the **midpoint**, $L/2$ from each centre, so the resultant field is zero there. The potential at P is the sum $\phi = -\dfrac{GM}{L/2} - \dfrac{GM}{L/2} = -\dfrac{4GM}{L}$. A mass released at P would stay there; a mass released anywhere else on the line falls towards the nearer sphere.

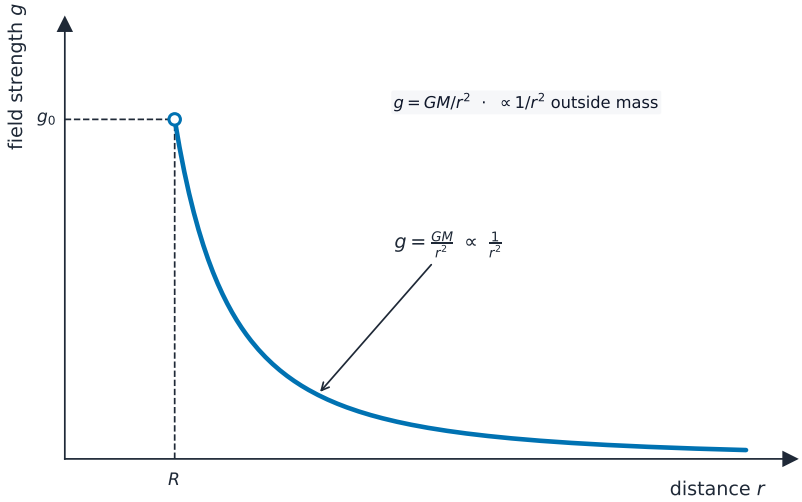
■ Field strength from a point mass

Put the gravitational force on a test mass m at distance r from a point mass M into $g = F/m$:

$$F = \frac{GMm}{r^2}, \qquad g = \frac{GM}{r^2}.$$

So g falls off as $1/r^2$ as you move away from the source.

The two-mark derivation. The force on a test mass m at distance r from a point mass M is $F = GMm/r^2$ (Newton's law). Field strength is force per unit mass, $g = F/m$, so $g = GM/r^2$. Write both lines: the law and the definition are the two marks. Because $g \propto 1/r^2$, halving the distance makes the field **four** times stronger: at distance $x/2$ the field is $4g$, in the same direction, towards M.



Field strength falls off as $1/r^2$ with distance from a point mass

Worked example. Find the gravitational field strength at the Earth's surface. (Earth's mass $M = 6.0 \times 10^{24}$ kg, radius $R = 6.4 \times 10^{6}$ m, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.)

$$g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})}{(6.4 \times 10^{6})^2} \approx 9.8 \text{ N kg}^{-1}.$$

■ Why g is nearly constant near the Earth's surface

The Earth's radius is $R \approx 6.4 \times 10^6$ m. Rising to height h changes the distance from the centre from R to $R + h$. For $h \ll R$ (any building or mountain), $(R + h)/R \approx 1$, so g barely changes – going from 5 m to 10 m high changes r by about one part in a million. In the laboratory, g is effectively constant.

Orbital motion in a gravitational field

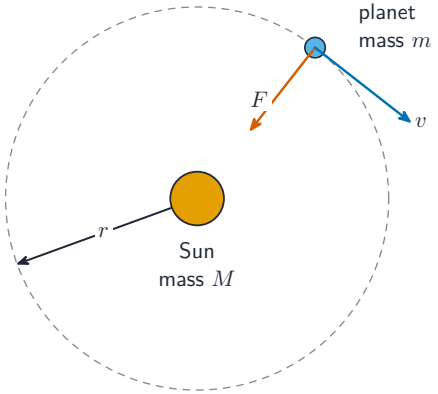
For a **satellite** 卫星 of mass m in a circular **orbit** 轨道 of radius r around a body of mass M, gravity provides the **centripetal force** 向心力:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}.$$

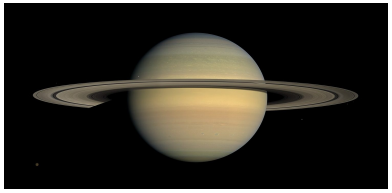
Cancel m (the orbital speed does not depend on the satellite's mass):

$$v = \sqrt{\frac{GM}{r}}.$$

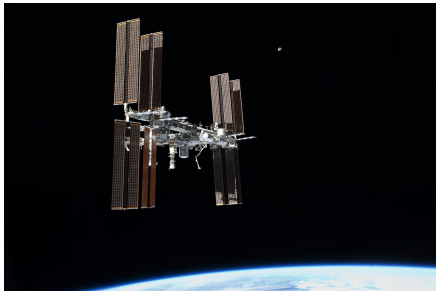
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Gravity provides the centripetal force that keeps a planet in a circular orbit



Saturn, its rings (countless small orbiting pieces) and its moons are all held in orbit by gravity



The International Space Station orbits Earth, held in its path by gravity.

Worked example. A satellite orbits the Earth in a circular orbit of radius $r = 7.0 \times 10^6$ m. Find its orbital speed. (For the Earth, $GM = 4.0 \times 10^{14}$ m^3 s^{-2}.)

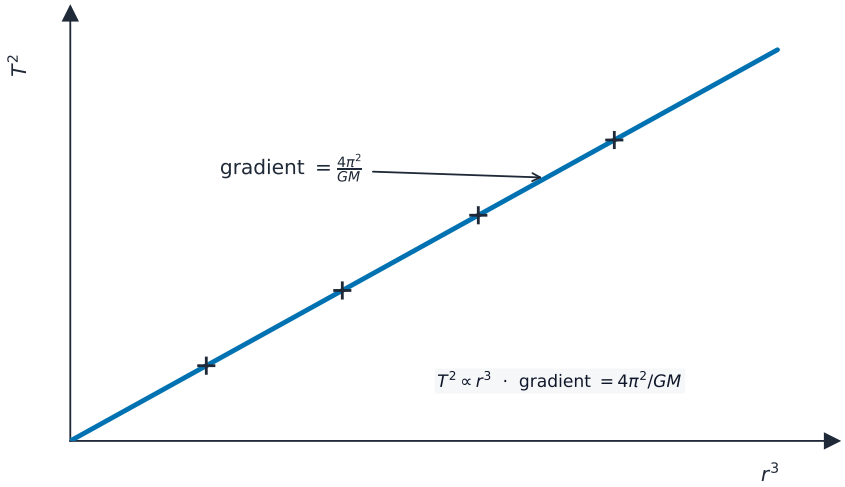
$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{4.0 \times 10^{14}}{7.0 \times 10^6}} \approx 7.6 \times 10^3 \text{ m s}^{-1}.$$

The **period** 周期 follows from $T = 2\pi r/v$:

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$$T = 2\pi\sqrt{\frac{r^3}{GM}}, \quad \text{so} \quad T^2 = \frac{4\pi^2}{GM} \cdot r^3.$$

This is **Kepler's third law** 开普勒第三定律 for circular orbits: $T^2 \propto r^3$. A plot of T^2 against r^3 is a straight line through the origin with gradient $4\pi^2/(GM)$, so orbital data gives the central mass.



Kepler's third law: $T^2 \propto r^3$, a straight line through the origin

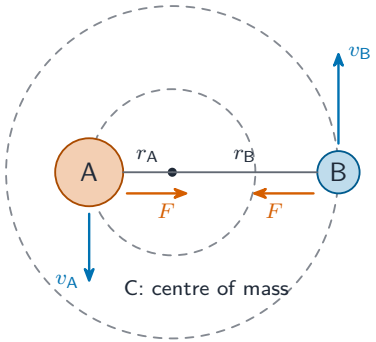
The two-mark derivation, as the exam sets it. For a satellite of mass m in a circular orbit of radius R and period T around a planet of mass M: the gravitational force provides the centripetal force, $\frac{GMm}{R^2} = mR\omega^2$, and $\omega = 2\pi/T$; so $\frac{GM}{R^2} = \frac{4\pi^2 R}{T^2}$, giving $T^2 = \frac{4\pi^2 R^3}{GM}$. For an orbit at height h above a planet of radius R_p the orbital radius is $R_\text{p} + h$, so $T^2 = 4\pi^2(R_\text{p} + h)^3/GM$: a graph of T^2 against $(R_\text{p} + h)^3$ is a straight line through the origin whose gradient gives M.

Worked example. Satellite X, of mass M_s, orbits a planet at a distance $4R$ from its centre; satellite Y, of mass $2M_\text{s}$, orbits at $3R$. Compare their speeds, periods and kinetic energies.

From $v = \sqrt{GM/r}$, $\frac{v_\text{Y}}{v_\text{X}} = \sqrt{\frac{4R}{3R}} = 1.15$: Y moves faster, and the satellite masses do not enter. From $T \propto r^{3/2}$, $\frac{T_\text{Y}}{T_\text{X}} = \left(\frac{3}{4}\right)^{3/2} = 0.65$. Kinetic energy $= \frac{1}{2}mv^2 = \frac{GMm}{2r}$, so $\frac{E_\text{Y}}{E_\text{X}} = \frac{2M_\text{s}/3R}{M_\text{s}/4R} = \frac{8}{3}$. In ratio questions cancel G, M and the satellite mass first; only

the radii and masses that differ survive.

Binary stars. Two stars of masses M_A and M_B a distance d apart orbit their common **centre of mass** 质心, always on opposite sides of it, with the **same period**. The centre of mass divides d in the inverse ratio of the masses, $M_A r_A = M_B r_B$ with $r_A + r_B = d$, and the same gravitational force GM_AM_B/d^2 is the centripetal force on each star.



same period T and angular speed ω

$M_A r_A = M_B r_B, \quad r_A + r_B = d$

$F = \dfrac{GM_AM_B}{d^2} = M_A r_A \omega^2 = M_B r_B \omega^2$

A binary star: both stars circle the centre of mass with the same period, the heavier one on the smaller circle

> **Worked example.** A binary star consists of star A, mass 4.0×10^{30} kg, and star B, mass 2.0×10^{30} kg, with centres 3.3×10^{11} m apart. Find the distance of A from the centre of mass and the period of the orbit.

$M_A r_A = M_B r_B$ with $r_A + r_B = 3.3 \times 10^{11}$ gives $r_A = \dfrac{2.0}{6.0} \times 3.3 \times 10^{11} = 1.1 \times 10^{11}$ m (and $r_B = 2.2 \times 10^{11}$ m). For star A: $\dfrac{GM_AM_B}{d^2} = M_A r_A \omega^2$, so $\omega^2 = \dfrac{GM_B}{d^2 r_A} = \dfrac{6.67 \times 10^{-11} \times 2.0 \times 10^{30}}{(3.3 \times 10^{11})^2 \times 1.1 \times 10^{11}} = 1.1 \times 10^{-14}$ s^{-2}, $\omega = 1.06 \times 10^{-7}$ rad s^{-1} and $T = 2\pi/\omega = 6.0 \times 10^7$ s, about 1.9 years. Using d in the force but r_A in the centripetal term is the whole point of the question.

■ Geostationary orbit

A **geostationary** 地球同步 satellite:

- stays directly above the same point on the Earth (so a fixed dish always points at it),
- has a period of **24 hours** (the same as the Earth's rotation),
- orbits **west to east** (the same way the Earth turns),
- must be directly above the **equator** 赤道.

It must have the same **angular speed** 角速度 as the Earth, in the same direction, in the equatorial plane (or it would drift north–south during the day). From $T = 24$ h and $T^2 = 4\pi^2 r^3/(GM)$, the radius is $r \approx 4.2 \times 10^7$ m (about 3.6×10^7 m above the surface).

> **Worked example.** Calculate the radius of a geostationary orbit around the Earth ($M = 5.98 \times 10^{24}$ kg), and explain why a satellite with the same orbital radius and period may still not be geostationary.

$T = 24 \times 3600 = 8.64 \times 10^4$ s; $r^3 = \dfrac{GMT^2}{4\pi^2} = \dfrac{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times (8.64 \times 10^4)^2}{4\pi^2} = 7.5 \times 10^{22}$ m^3, so $r = 4.2 \times 10^7$ m – a height of about 3.6×10^7 m above the surface. A satellite with this period is geostationary **only** if its orbit is in the plane of the equator and it travels from west to east; with the same period in a tilted orbit (over the poles, say) it returns to the same point each day but is not always above it. Mars turns once in about 25 hours, so a satellite that stays above one point on Mars has a 25-hour period and the same two other features.

Gravitational potential

Gravitational potential 引力势 ϕ at a point is the **work done per unit mass** in bringing a small test mass from **infinity** 无穷远 to that point:

$$\phi = \frac{W}{m}.$$

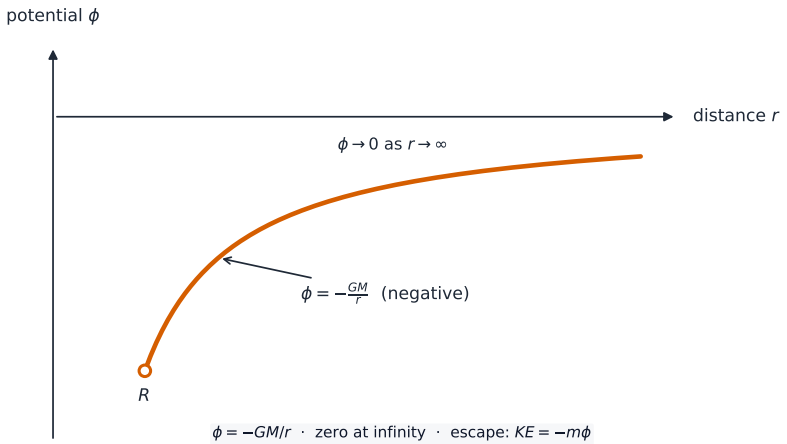
Unit: J kg^{-1}.

The potential is taken as **zero at infinity**. As the test mass falls in towards the source, gravity does the work for you, so ϕ is **negative** everywhere except at infinity. For a point mass M at distance r:

$$\phi = -\frac{GM}{r}.$$

ϕ is a **scalar** 标量. For several masses, add the potentials.

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The gravitational potential well: $\phi = -GM/r$ is negative, rising to zero at infinity

The two-mark definition, and why it is negative. *Gravitational potential at a point is the work done per unit mass in bringing a small test mass from infinity to the point.* Potential is defined as zero at infinity; gravity is attractive, so as the test mass comes in from infinity the field does work *on* it and the mass gives out energy – the work that an external agent must do is negative. Hence the potential at every finite distance is below zero. (Electric potential near a positive charge is positive for the opposite reason: that field repels, so work must be done to bring a positive test charge in.)

Similarities and differences with electric potential. Both are defined as work done per unit mass or per unit positive charge from infinity, both are scalars, both vary as $1/r$ from a point source and both are zero at infinity. The difference: gravitational potential is *always negative*, because gravity only attracts, whereas electric potential can be positive or negative depending on the sign of the charge.

Reading a potential graph. Two graphs appear in questions. On ϕ against r the curve is $-GM/r$: the surface value $\phi_s = -GM/R$ gives the mass of the planet, and the **gradient** at any point gives the field strength there, since $g = -\dfrac{\Delta\phi}{\Delta r}$ – field strength is the negative of the **potential gradient** (this is the "relationship between potential and field strength" a question may ask for). On ϕ against $1/r$ the graph is a straight line through the origin with gradient $-GM$, which is the neater way to extract M.

> **Worked example.** The Moon is an isolated uniform sphere of mass 7.3×10^{22} kg and radius 1.7×10^{6} m. Calculate the gravitational potential at its surface, and the minimum speed with which a particle must leave the surface to escape.

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$\phi = -\dfrac{GM}{R} = -\dfrac{6.67 \times 10^{-11} \times 7.3 \times 10^{22}}{1.7 \times 10^{6}} = -2.9 \times 10^{6}$ J kg^{-1}. To escape, the particle's kinetic energy per unit mass must equal the depth of the potential well: $\frac{1}{2}v^2 = 2.9 \times 10^{6}$, so $v = \sqrt{2 \times 2.9 \times 10^{6}} = 2.4 \times 10^{3}$ m s^{-1}. The minus sign carries the meaning: energy of 2.9 MJ per kilogram must be *supplied* to lift the particle out.

Gravitational potential energy of two point masses

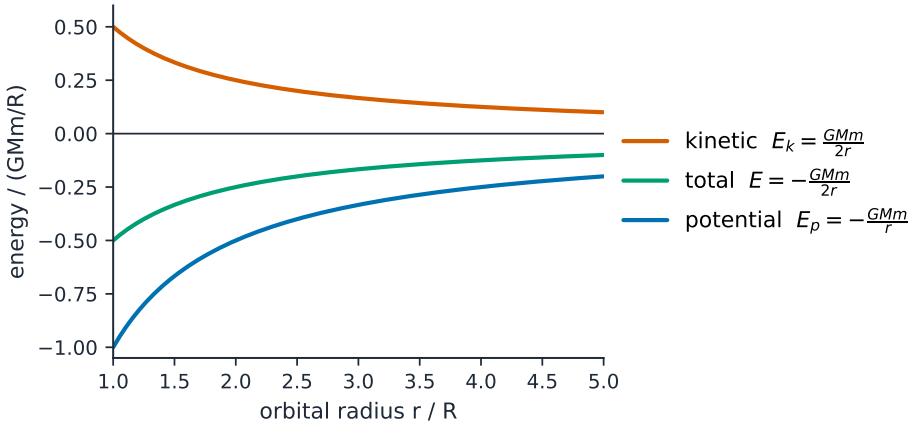
If a test mass m sits where the potential is ϕ, the **gravitational potential energy** 重力势能 of the pair is

$$E_{\mathrm{P}} = m\phi = -\frac{GMm}{r}.$$

Like the potential, E_{P} is **negative** and reaches zero only at infinite separation. Closer masses have more negative potential energy (more tightly bound).

Link with $\Delta E_{\mathrm{P}} = mg\Delta h$

For small height changes near the surface, r barely changes, so $\Delta E_{\mathrm{P}} \approx mg\Delta h$. For large changes (a satellite moving to a higher orbit) use $-GMm/r$ at each radius and take the difference:



A higher orbit has more total energy but less kinetic energy

$$\Delta E_{\mathrm{P}} = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \quad (r_2 > r_1),$$

which is positive (energy must be supplied to raise the satellite).

Worked example. A satellite of mass 1200 kg is moved from a circular orbit of radius 7.0×10^6 m to one of radius 8.0×10^6 m around the Earth ($GM = 4.0 \times 10^{14}$ m^3 s^{-2}). Find the changes in its gravitational potential energy, its kinetic energy and its total energy.

$\Delta E_\mathrm{P} = GMm\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right) = 4.0 \times 10^{14} \times 1200 \times \left(\dfrac{1}{7.0 \times 10^6} - \dfrac{1}{8.0 \times 10^6}\right) = +8.6 \times 10^9$ J. In a circular orbit $E_\mathrm{K} = \frac{1}{2}mv^2 = \dfrac{GMm}{2r}$, so $\Delta E_\mathrm{K} = \dfrac{GMm}{2}\left(\dfrac{1}{r_2} - \dfrac{1}{r_1}\right) = -4.3 \times 10^9$ J: the higher satellite moves more slowly. The total energy $E_\mathrm{K} + E_\mathrm{P} = -\dfrac{GMm}{2r}$ rises by $+4.3 \times 10^9$ J, which is the energy the rocket motor must supply. A satellite's total energy is negative – it is bound – and it becomes less negative as the orbit widens.

■ Escape velocity (from conservation of energy)

To escape from radius r to infinity, an object's **kinetic energy** 动能 must equal the size of its gravitational potential energy:

$$\tfrac{1}{2}mv_\mathrm{esc}^2 = \frac{GMm}{r}, \qquad v_\mathrm{esc} = \sqrt{\frac{2GM}{r}}.$$

At the Earth's surface, the **escape velocity** 逃逸速度 is ≈ 11 km s^{-1}. It does not depend on the object's mass.

Worked example. Find the escape velocity from the Earth's surface. (For the Earth, $GM = 4.0 \times 10^{14}$ m^3 s^{-2}, $R = 6.4 \times 10^6$ m.)

$$v_\mathrm{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(4.0 \times 10^{14})}{6.4 \times 10^6}} \approx 1.1 \times 10^4 \text{ m s}^{-1} \ (= 11 \text{ km s}^{-1}).$$

Worked example. A particle is projected vertically upwards from the surface of an isolated planet of radius R, where the gravitational potential is ϕ_s. Show that it just reaches a distance r from the centre if its launch speed v satisfies $\frac{1}{2}v^2 = -\phi_\mathrm{s}\left(1 - \dfrac{R}{r}\right)$, and deduce the escape speed.

Potential varies as $1/r$, so at distance r it is $\phi_\mathrm{s}R/r$. When the particle stops, its kinetic energy per unit mass has all become potential energy per unit mass: $\frac{1}{2}v^2 = \phi(r) - \phi_\mathrm{s} = \phi_\mathrm{s}\dfrac{R}{r} - \phi_\mathrm{s} = -\phi_\mathrm{s}\left(1 - \dfrac{R}{r}\right)$, which is positive because ϕ_s is negative. Letting $r \to \infty$ gives $\frac{1}{2}v_\mathrm{esc}^2 = -\phi_\mathrm{s} = GM/R$ – the escape-speed result again, now read straight off the potential.

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Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
gravitational field	a region of space in which a mass experiences a force
gravitational field strength	the gravitational force per unit mass acting on a small test mass placed at the point
field line (direction)	the direction of the force on a small test mass placed at that point
Newton's law of gravitation	the gravitational force between two point masses is proportional to the product of their masses and inversely proportional to the square of their separation
gravitational potential	the work done per unit mass in bringing a small test mass from infinity to the point
gravitational potential energy (two point masses)	the work done in bringing the two masses from infinity to their separation, $E_\mathrm{P} = -GMm/r$
geostationary orbit	an orbit with a period of 24 hours, from west to east, directly above the equator, so the satellite stays above the same point on the surface
escape speed	the minimum speed at which an object must leave the surface to reach infinity with zero kinetic energy
centre of mass (binary star)	the point about which both stars orbit, dividing their separation in the inverse ratio of their masses

Exam tips

- **Newton's law of gravitation** $F = GMm/r^2$ (inverse-square); field strength $g = GM/r^2$ – and quote the derivation as two lines, the law and the definition of g.
- Distinguish **gravitational potential** ($\phi = -GM/r$, always negative, zero at infinity) from field strength; g is the negative gradient of the ϕ–r graph.
- For an orbit set gravity = centripetal force to get $T^2 \propto r^3$; a geostationary orbit has $T = 24$ h plus two more features: above the equator, west to east.
- Use the **orbital radius** (centre to centre, $R_\mathrm{p} + h$), never the height, and for a binary star the separation d in the force but the orbit radius r_A in the centripetal term.

- Energy changes come from $-GMm/r$ at each radius; for a circular orbit $E_K = GMm/2r$ and the total energy is $-GMm/2r$.

Common mistakes

- Defining potential as "force per unit mass" or field strength as "work done per unit mass". Swap them and both marks go.
- Saying the potential is negative "because gravity attracts" and stopping there. The mark needs the work argument: zero at infinity, and the field does the work as the mass comes in.
- Using the height above the surface as r in GM/r^2 or $T^2 \propto r^3$. Add the planet's radius first.
- Writing Kepler's law as $T \propto r$ or $T^2 \propto r^2$. It is $T^2 \propto r^3$: a straight line only on T^2 against r^3.
- Making "geostationary" mean only "24-hour period". Without the equatorial plane and the west-to-east direction it is not geostationary.
- Forgetting that a satellite's kinetic energy *falls* when it is raised to a higher orbit, even though energy had to be supplied.

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Temperature

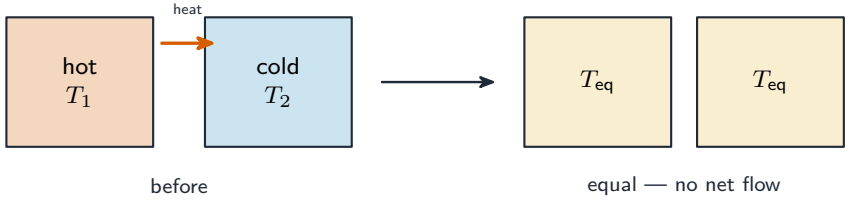
A-Level Physics

Thermal equilibrium

Heat 热量 (thermal energy) flows from a **higher temperature** to a **lower temperature**. When two bodies touch, energy moves until their temperatures are equal – they reach **thermal equilibrium** 热平衡. At equilibrium there is no net flow of energy.

Two regions at the same **temperature** 温度 are in thermal equilibrium with each other – no net energy flows, even though particles still exchange energy.

Temperature decides the **direction** of heat flow. It is not a measure of how much **thermal energy** 热能 a body holds. A small cup of boiling water (100 °C) holds far less energy than a swimming pool at 25 °C, but a piece of metal put in the cup gains energy while one put in the pool loses it.



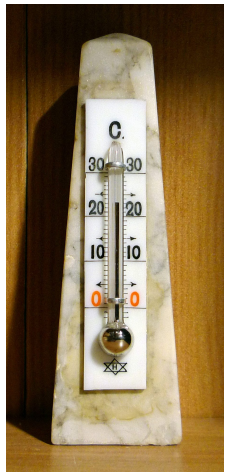
Heat flows from hot to cold until both reach the same temperature – thermal equilibrium

The examiner's wording. *Two objects are in thermal equilibrium when there is no net transfer of thermal energy between them*; that happens when they are at the same temperature. Asked for "the reason why two objects at the same temperature are in thermal equilibrium", the one mark is: *there is no net flow of thermal energy between them* – energy still passes both ways, but equally. Thermal energy is the energy transferred *because of* a temperature difference; temperature is what decides the direction.

> **Worked example.** Two metal cuboids P and Q are in thermal contact and in thermal equilibrium. State what this means, and describe what happens when a hotter cuboid is placed against them.

No net thermal energy passes between P and Q, because they are at the same temperature. A hotter cuboid transfers thermal energy to them (from higher to lower temperature) until all three reach one common temperature; then, and only then, is the whole group in thermal equilibrium.

Measuring temperature



A thermometer measures temperature on a defined scale.

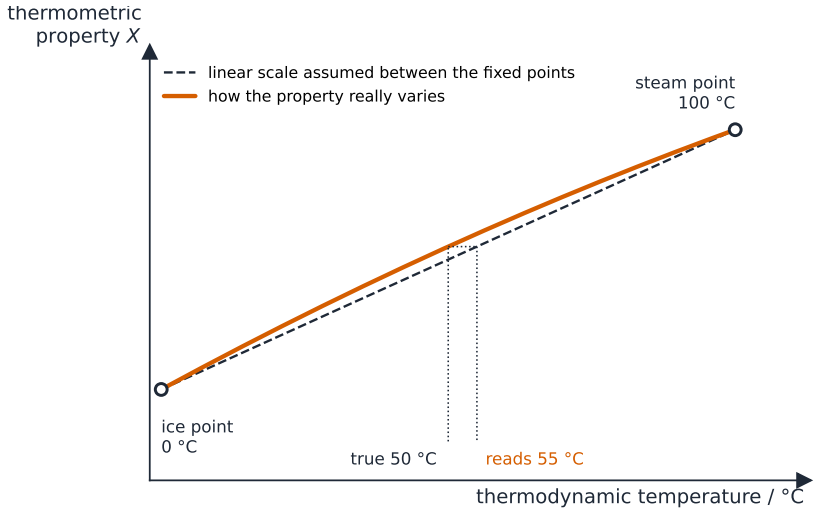
Any physical property that **changes in a repeatable way with temperature** can make a **thermometer** 温度计. Examples:

- **volume of a liquid** – a liquid-in-glass thermometer (mercury or alcohol). As the temperature rises, the liquid expands and rises up a narrow **capillary** 毛细管.
- **volume of a gas at constant pressure** – a gas thermometer. The gas volume rises in step with the absolute temperature.
- **resistance of a metal** – a resistance thermometer. A metal's **resistance** 电阻 rises nearly in step with temperature over a wide range.
- **e.m.f. of a thermocouple** – a **thermocouple** 热电偶 is two different metals joined at two points; the **electromotive force** 电动势 it makes depends on the temperature difference between the joins.

Different thermometers can read slightly differently if the property does not change in a straight line; they agree only at the **calibration** 校准 points.

Fixed points and calibration. A thermometer is calibrated at two **fixed points** 固定点 that are easy to reproduce – the **ice point** (0 °C, pure melting ice) and the **steam point** (100 °C, steam above boiling water at standard pressure). The value of the property is measured at each, and the scale between them is drawn assuming the property changes **linearly** with temperature. That assumption is the weakness: mercury's expansion, a metal's resistance and a thermocouple's e.m.f. all vary slightly differently between the fixed points, so two thermometers that agree at 0 and 100 °C can disagree at 50 °C.

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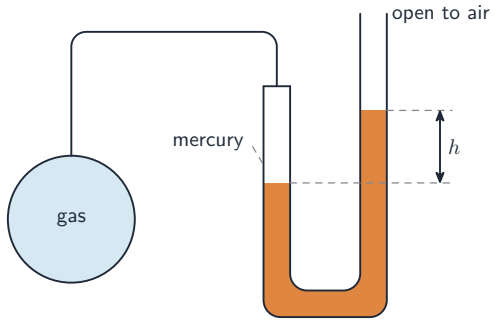
Calibrated at two fixed points and assumed linear in between, a thermometer reads a temperature that depends on how its own property really varies

Why a liquid-in-glass thermometer does not measure thermodynamic temperature (the one-mark reason): its reading depends on the **property of a particular substance** – the expansion of mercury or alcohol – and on the assumption that this expansion is linear between the fixed points. The thermodynamic scale depends on no substance at all.

Why water is a poor thermometric liquid. Its density does not change steadily with temperature: it is greatest at 4 °C, so between 0 and 8 °C two different temperatures give the same density and the reading is ambiguous, and the change per degree is small, so the thermometer is insensitive. Mercury's density falls steadily and almost linearly over the whole range, which is why it was chosen.

> **Worked example.** A platinum resistance thermometer is a coil of platinum wire in a glass tube, connected to a circuit that measures its resistance. Explain how it measures temperature, and give one disadvantage compared with a thermocouple.

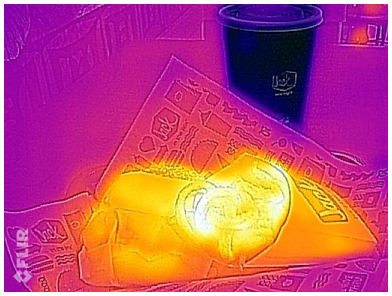
The resistance of the platinum increases with temperature in a known, almost linear way; the resistance is measured at the ice point and the steam point, and any other temperature is read from where the measured resistance falls on the linear scale between them (or from a calibration graph). Disadvantage: the coil, tube and the fluid around them have a large thermal capacity, so the thermometer responds slowly and cannot follow a rapidly changing temperature; it is also bulky and needs a circuit to read it. (Its advantages: accurate, stable and usable over a very wide range.)



A constant-volume gas thermometer – the gas pressure is found from the height difference h

Worked example. In a constant-volume gas thermometer the pressure of the gas is 1.05×10^5 Pa when the bulb is in melting ice and 1.19×10^5 Pa when it is in a warm room. Find the thermodynamic temperature of the room, and explain why this thermometer, unlike a liquid-in-glass one, gives thermodynamic temperature directly.

For a fixed mass of gas at constant volume the pressure is proportional to the thermodynamic temperature, so $\dfrac{T}{273.15} = \dfrac{1.19 \times 10^5}{1.05 \times 10^5}$, giving $T = 310$ K (36 °C). The pressure of a (nearly) ideal gas depends only on its temperature, not on which gas it is, so the reading does not rely on the property of a particular substance – the defining feature of the thermodynamic scale.



A thermal (infrared) camera maps temperature to colour: the hot fries glow bright orange, the cold drink stays dark

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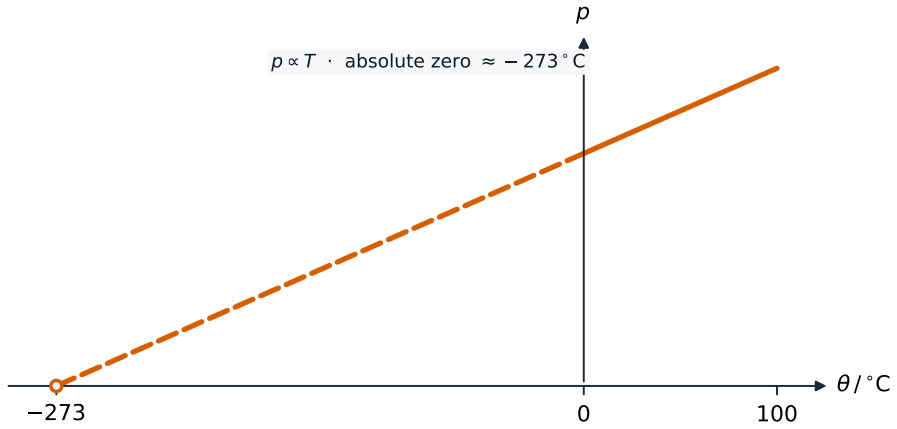
Thermodynamic temperature scale

The **thermodynamic temperature** 热力学温度 (or **absolute temperature** 绝对温度) scale does not depend on any one substance – only on the laws of thermodynamics. Its unit is the **kelvin** 开尔文 (K).

Absolute zero

The lowest possible temperature is **zero kelvin** (0 K), called **absolute zero** 绝对零度. There a system has its least possible **internal energy** 内能 – particles have no random motion to speak of. Nothing can be cooled below this.

As the exam asks it. "State the magnitude and unit of absolute zero on the thermodynamic scale": 0 K (the unit is the kelvin). "State the temperature of absolute zero on the Celsius scale": −273.15 °C (accept −273 °C). "What is absolute zero?": the temperature at which a system has its **minimum internal energy** – the particles have the least kinetic and potential energy they can have – and below which it is impossible to go. The thermodynamic scale is fixed by absolute zero and by the **triple point** 三相点 of water, defined as 273.16 K; it does not depend on the property of any particular substance.



Extrapolating the pressure–temperature line back to zero pressure gives absolute zero, about −273 °C

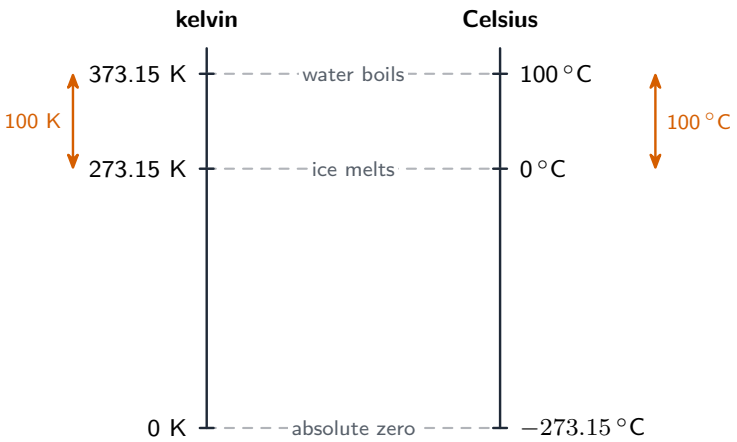
Celsius scale

The **Celsius** 摄氏度 scale θ is shifted from the thermodynamic scale by a fixed amount:

$$T/\mathrm{K} = \theta/^\circ\mathrm{C} + 273.15.$$

So 0 °C = 273.15 K and 100 °C = 373.15 K. A kelvin and a degree Celsius are the **same size**, so a temperature **difference** of 1 K equals 1 °C – but the absolute values

differ by 273.15.



$T/\mathrm{K} = \theta/^\circ\mathrm{C} + 273.15$

The same size of degree, a different zero: add 273.15 to go from °C to K

In gas-law calculations you must always use **absolute** temperatures in kelvin. Using °C gives wrong answers.

Specific heat capacity

The **specific heat capacity** 比热容 c of a substance is the **energy** 能量 needed to raise the temperature of **unit mass** by **one kelvin**:

$$c = \frac{Q}{m\Delta T} \qquad \Longleftrightarrow \qquad Q = mc\Delta T.$$

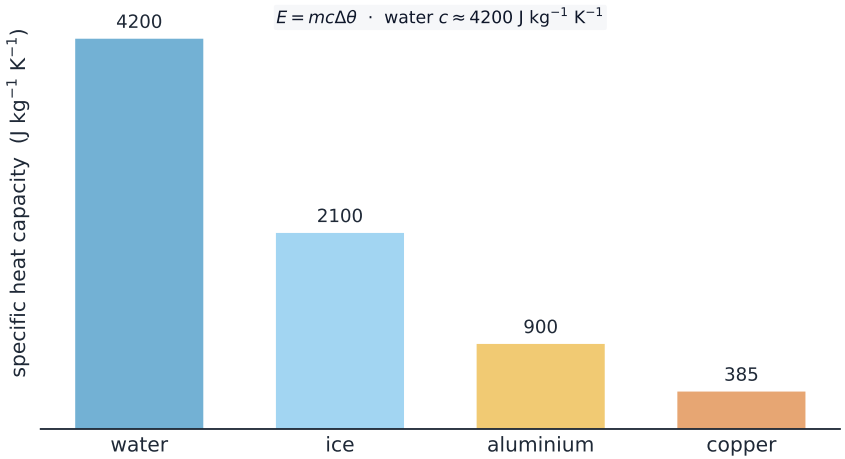
Unit: J kg^{-1} K^{-1}.

The two-mark definition: *specific heat capacity is the energy required per unit mass of the substance to raise its temperature by one kelvin* (or by one degree). Both "per unit mass" and "per unit temperature rise" are needed; "the energy to heat 1 kg by 1 K" scores, "the energy to heat the substance" does not.

Examples:

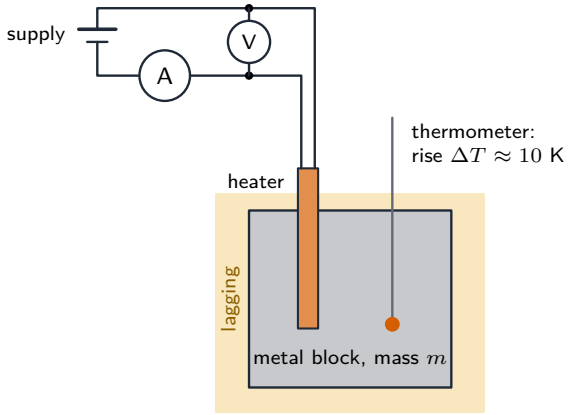
- water: $c \approx 4200$ J kg^{-1} K^{-1} (high – why water is a good coolant and why oceans steady the climate).
- aluminium: $c \approx 900$ J kg^{-1} K^{-1}.
- copper: $c \approx 385$ J kg^{-1} K^{-1}.

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Water's specific heat capacity is far higher than common solids – why it is such a good coolant

To find an unknown c by experiment: supply known energy Q electrically ($Q = VIt$, from the **power** 功率), then measure the temperature rise ΔT of a known **mass** 质量 m. Then $c = Q/(m\Delta T)$. Reduce heat loss with **insulation** 隔热 and use a rise of about 10 K (big enough to measure well, small enough to limit losses).



$Q = VIt, \quad c = \dfrac{Q}{m\,\Delta T}$

The electrical method: measure V, I and t for the energy, and the temperature rise of a known mass

Worked example. How much energy is needed to heat 0.50 kg of water from 20 °C to 100 °C? (Specific heat capacity of water $c = 4200$ J kg^{-1} K^{-1}.)

A temperature difference is the same in K and °C, so $\Delta T = 80$:

$$Q = mc\Delta T = 0.50 \times 4200 \times 80 = 1.68 \times 10^5 \text{ J } (= 168 \text{ kJ}).$$

When two bodies reach thermal equilibrium with no heat lost to the surroundings, the energy gained by the colder one equals the energy lost by the hotter one:

$$m_1 c_1 (T_{\text{eq}} - T_1) = m_2 c_2 (T_2 - T_{\text{eq}}).$$

Worked example. 0.20 kg of water at 80 °C is mixed with 0.30 kg of water at 20 °C, with no heat lost. Find the final temperature.

The heat lost by the hot water equals the heat gained by the cold water (the c of water cancels):

$$0.20\,(80 - T_{\text{eq}}) = 0.30\,(T_{\text{eq}} - 20) \quad \Rightarrow \quad T_{\text{eq}} = 44 \text{ °C}.$$

Worked example. A beaker of mass 42 g and specific heat capacity 840 J kg^{-1} K^{-1} contains 120 g of a liquid. A heater supplies energy at 810 W and the temperature of the beaker and liquid rises from 19 °C to 47 °C in 15 s. Find the specific heat capacity of the liquid. The experiment is then repeated with water in place of the liquid; state and explain how the temperature rise differs.

Energy supplied $Q = Pt = 810 \times 15 = 1.22 \times 10^4$ J, $\Delta T = 28$ K. It heats both the beaker and the liquid: $Q = m_{\text{b}} c_{\text{b}} \Delta T + m_{\text{l}} c_{\text{l}} \Delta T$, so $1.215 \times 10^4 = 0.042 \times 840 \times 28 + 0.120 \times c_{\text{l}} \times 28$, i.e. $1.215 \times 10^4 = 988 + 3.36 c_{\text{l}}$, giving $c_{\text{l}} = 3.3 \times 10^3$ J kg^{-1} K^{-1}. Forgetting the beaker is the usual lost mark. With water ($c = 4200$, higher) the same energy in the same time produces a **smaller** temperature rise, since $\Delta T = Q/(mc)$ and mc is larger. If energy is lost to the surroundings the true Q absorbed is less than Pt, so a value of c found this way is an **overestimate**.

Worked example. Two metal blocks X and Y are placed in contact and insulated from the surroundings. X (mass 0.50 kg, initially 80 °C, $c = 390$ J kg^{-1} K^{-1}) and Y (mass 0.50 kg, initially 20 °C, $c = 900$ J kg^{-1} K^{-1}) reach a common final temperature. Explain, in terms of energy, why the final temperature is nearer to Y's starting temperature, and find it.

Thermal energy flows from X (hotter) to Y until they are in thermal equilibrium; the energy lost by X equals the energy gained by Y. Because Y has the larger specific heat capacity, a given amount of energy changes its temperature less than it changes X's, so Y warms by fewer degrees than X cools and the final temperature lies nearer to 20 °C. Numerically: $0.50 \times 390 \times (80 - T) = 0.50 \times 900 \times (T - 20)$, so $390(80 - T) = 900(T - 20)$, $31200 + 18000 = 1290T$, $T = 38$ °C.

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Worked example. An aluminium block has volume 3.612×10^{-3} m^3 and density 2.70×10^3 kg m^{-3}. Find the energy needed to raise its temperature by 40 K ($c = 900$ J kg^{-1} K^{-1}).

Mass $= \rho V = 2.70 \times 10^3 \times 3.612 \times 10^{-3} = 9.75$ kg, so $Q = mc\Delta T = 9.75 \times 900 \times 40 = 3.5 \times 10^5$ J. A mass hidden behind a density and a volume is a common first step.

Specific latent heat

When a substance changes state (solid ↔ liquid, or liquid ↔ gas) at constant temperature, energy must be supplied (or removed) with **no** temperature change. This energy is the **latent heat** 潜热.

The **specific latent heat** 比潜热 L is the energy to change the state of **unit mass** at constant temperature:

$$L = \frac{Q}{m} \qquad \Longleftrightarrow \qquad Q = mL.$$

Unit: J kg^{-1}.

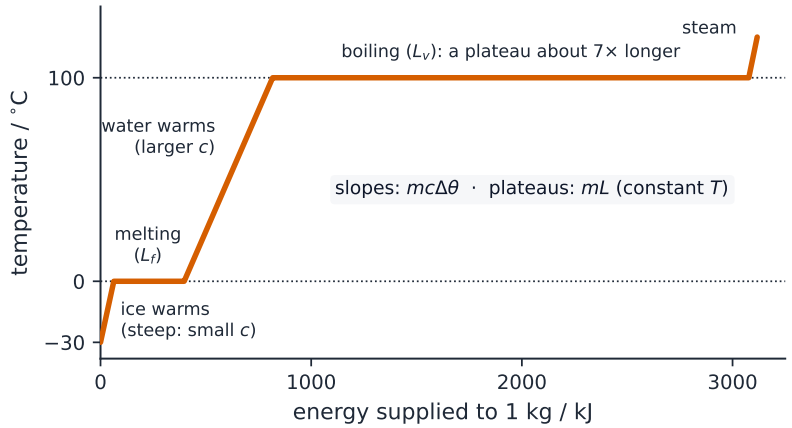
The two-mark definition: *specific latent heat is the energy required per unit mass to change the state of a substance without a change of temperature.* "Without a change in temperature" (or "at constant temperature") is the second mark and the one most often missed. Add "from solid to liquid" for fusion or "from liquid to gas" for vaporisation when a particular one is asked for.

Two kinds:

fusion 熔化 means melting; **vaporisation** 汽化 means changing a liquid into a gas.

- **specific latent heat of fusion** 比熔化潜热 L_{f} – for melting or freezing (solid ↔ liquid).
- **specific latent heat of vaporisation** 比汽化潜热 L_{v} – for boiling or condensing (liquid ↔ gas).

For water at atmospheric pressure: $L_{\text{f}} \approx 3.34 \times 10^5$ J kg^{-1} (at 0 °C); $L_{\text{v}} \approx 2.26 \times 10^6$ J kg^{-1} (at 100 °C). So L_{v} is about 7 times L_{f}.



Heating curve: the sloped parts warm the substance ($mc\Delta T$); the flat plateaus are the phase changes (mL)

> **Worked example.** A 2.0 kW heater boils water already at 100 °C. How long does it take to turn 0.10 kg of this water into steam? ($L_\text{v} = 2.26 \times 10^6$ J kg^{-1}, no heat lost.)

The energy needed is $Q = mL_\text{v} = 0.10 \times 2.26 \times 10^6 = 2.26 \times 10^5$ J. From $Q = Pt$,

$$t = \frac{Q}{P} = \frac{2.26 \times 10^5}{2000} \approx 110 \text{ s}.$$

> **Worked example.** Water in a kettle stays at 100 °C while it boils, even though the element keeps heating it. Explain this with reference to molecular energies (3 marks).

Temperature is a measure of the mean **kinetic energy** of the molecules. During boiling the energy supplied is used to separate the molecules: doing work against the attractive forces between them increases their **potential energy**, and the rest does work pushing back the atmosphere as the vapour expands. None of it increases their kinetic energy. With the mean kinetic energy unchanged, the temperature stays constant until all the water has become steam.

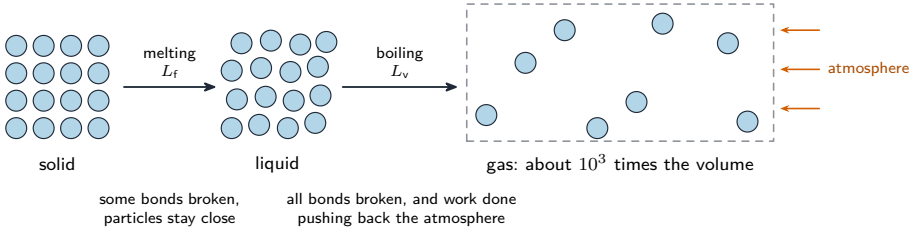
■ Why $L_\text{v} > L_\text{f}$

Two reasons, both from the particle picture of matter:

1. **Bonds**: in melting, only some of the **intermolecular** 分子间 bonds break; the particles stay close as a liquid. In boiling, **all** the bonds must break so the particles can separate. Breaking all of them needs more energy.

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2. **Work against the atmosphere**: when a liquid turns to gas it expands hugely (vapour has about 10^3 times the liquid's **volume** 体积), so it does work pushing back the surrounding **atmospheric pressure** 大气压强. That work comes from the energy supplied.



Melting breaks some bonds; boiling breaks all of them and pushes back the atmosphere

Write both reasons and name the energies: the marks are for *bonds broken (potential energy increased) – some in melting, all in boiling*, and *work done against the atmosphere in the large expansion*. "Boiling needs more energy" restates the question.

> **Worked example.** A dish holds 7.2×10^{-5} m^3 of a liquid of density 710 kg m^{-3} and specific latent heat of vaporisation 3.6×10^5 J kg^{-1}. It evaporates completely. Find the energy absorbed, and suggest, with a reason, whether the substance's specific latent heat of fusion is likely to be smaller or larger than this.

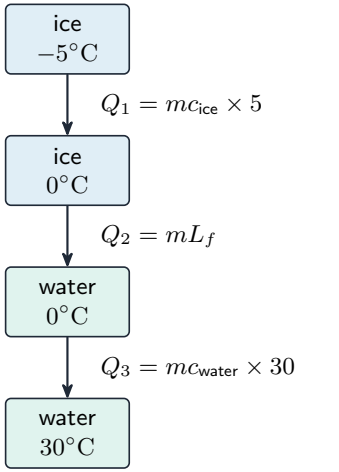
Mass $= \rho V = 710 \times 7.2 \times 10^{-5} = 5.1 \times 10^{-2}$ kg, so the energy absorbed is $Q = mL_\text{v} = 5.1 \times 10^{-2} \times 3.6 \times 10^5 = 1.8 \times 10^4$ J. Its L_f is likely to be **smaller**: melting breaks only some of the intermolecular bonds and the volume barely changes, whereas vaporising breaks them all and does work against the atmosphere – for water L_f is about a seventh of L_v.

■ Multi-step problems

If a problem mixes temperature change and a **phase change** 相变 (e.g. ice at −5 °C warming to water at 30 °C):

1. heat the solid from −5 °C to 0 °C: $Q_1 = mc_\text{ice} \times 5$.
2. melt at 0 °C: $Q_2 = mL_\text{f}$.
3. heat the water from 0 °C to 30 °C: $Q_3 = mc_\text{water} \times 30$.

Total: $Q_1 + Q_2 + Q_3$. A phase change is at constant temperature, so use mL there, not $mc\Delta T$.



Split a mixed problem into stages – warm, change state, warm – then add the energies

When a question gives heater power P and asks for the time, use $Q = Pt$ (assuming no heat loss). Insulating (lagging) the container and using a small mass are common ways to improve the experiment.

> **Worked example.** An ice cube of mass 37.0 g at 0.0 °C is dropped into 208 g of water at 26.4 °C in an insulated beaker. Find the final temperature when all the ice has melted. ($c_{\text{water}} = 4.20$ kJ kg^{-1} K^{-1}, $L_{\text{f}} = 334$ kJ kg^{-1}.)

The ice gains energy twice – to melt, then to warm from 0 to T – and the water loses energy cooling from 26.4 to T. Working in grams and kilojoules: $37.0 \times 0.334 + 37.0 \times 4.20 \times 10^{-3}\,T = 208 \times 4.20 \times 10^{-3}\,(26.4 - T)$, i.e. $12.4 + 0.155T = 23.1 - 0.874T$, so $1.03T = 10.7$ and $T = 10$ °C. Three energy terms, one equation; the commonest error is to forget that the melted ice must also be warmed.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
thermal equilibrium	the state of two objects between which there is no net transfer of thermal energy; they are at the same temperature

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Term	Definition
thermal energy	energy transferred from one object to another because of a temperature difference
thermodynamic temperature scale	a temperature scale that does not depend on the property of any particular substance, fixed by absolute zero and the triple point of water
absolute zero	the lowest possible temperature, 0 K (−273.15 °C), at which a system has its minimum internal energy
kelvin and Celsius	$T/\text{K} = \theta/°\text{C} + 273.15$; a temperature difference is the same number in both
specific heat capacity	the energy required per unit mass of a substance to raise its temperature by one kelvin
specific latent heat	the energy required per unit mass to change the state of a substance without a change of temperature
specific latent heat of fusion	the specific latent heat for the change from solid to liquid
specific latent heat of vaporisation	the specific latent heat for the change from liquid to gas

Exam tips

- **Thermal equilibrium** means no *net* flow of thermal energy (equal temperature); the thermodynamic scale does not depend on any particular substance and is fixed by absolute zero and the triple point.
- Convert temperatures with $T/\text{K} = \theta/°\text{C} + 273.15$ and use **kelvin** in energy and gas equations; a temperature *difference* is the same in K and °C.
- Use $Q = mc\Delta T$ for heating and $Q = mL$ for a change of state (no temperature change) – never mix the two, and in a mixing problem write one equation: energy lost = energy gained, with every term.
- A container heated with the liquid takes its share of the energy: include $m_{\text{beaker}} c_{\text{beaker}} \Delta T$.
- For a "why" about latent heat, name the energies: the potential energy of the molecules rises (bonds broken) and work is done against the atmosphere; their kinetic energy, and hence the temperature, does not change.

■ Common mistakes

- Defining specific heat capacity without "per unit mass" or without "per kelvin". Both are needed; the unit J kg^{-1} K^{-1} is the reminder.
- Leaving "at constant temperature" out of the latent-heat definition. That phrase is what distinguishes latent heat from heating.
- Using $mc\Delta T$ across a change of state, or mL for a temperature rise.
- Forgetting the beaker, the calorimeter or the melted ice's own warming in an energy equation.
- Saying the thermodynamic scale "uses kelvin" as its defining feature. The feature is that it depends on no particular substance.
- Adding 273.15 to a temperature *difference*. Differences are the same in both scales.

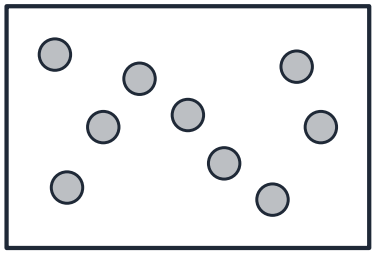
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Ideal gases

A-Level Physics

The mole

Amount of substance 物质的量 is an SI base quantity. Its unit is the **mole** 摩尔 (mol) – one of the seven SI base units, with the kilogram, metre, second, ampere and **kelvin** 开尔文 from Topic 1.



1 mole

$= 6.02 \times 10^{23}$ particles (N_A)

One mole contains the Avogadro number of particles

One mole of any substance has a number of particles equal to the **Avogadro constant** 阿伏伽德罗常量:

$$N_\mathrm{A} = 6.02 \times 10^{23}\ \mathrm{mol}^{-1}.$$

A "particle" means whatever you are counting – **atoms** 原子 for a **monatomic** 单原子 element like helium, **molecules** 分子 for O_2 or H_2O. Always say what you are counting.

For n moles, the number of particles is $N = nN_\mathrm{A}$.

The **molar mass** 摩尔质量 M_m is the mass of one mole (kg mol^{-1} or g mol^{-1}). Mass of n moles is $M = nM_\mathrm{m}$. Mass of one particle is $m_0 = M_\mathrm{m}/N_\mathrm{A}$.

The examiner's wording. *The Avogadro constant is the number of atoms (or molecules) in one mole of a substance* – one mark; adding "in 0.012 kg of carbon-12" is accepted but not needed. Asked for "the relationship between N_A, R and k", write $k = R/N_\mathrm{A}$ (or $R = N_\mathrm{A}k$).

> **Worked example.** Oxygen has a molar mass of 32 g mol^{-1}. Find the mass of one oxygen molecule, and the number of molecules in 8.0 g of oxygen.

$m_0 = \dfrac{M_\mathrm{m}}{N_\mathrm{A}} = \dfrac{0.032}{6.02\times10^{23}} = 5.3\times10^{-26}$ kg. 8.0 g is $n = 8.0/32 = 0.25$ mol, so $N = nN_\mathrm{A} = 1.5\times10^{23}$ molecules. Keep molar masses in kg mol^{-1} when the answer is in kilograms – the factor of 1000 is the usual slip.

Equation of state of an ideal gas



Compressed gas cylinders store a fixed mass of gas at high pressure.

An **ideal gas** 理想气体 obeys $pV \propto T$ exactly, where T is the **thermodynamic temperature** 热力学温度.

The **equation of state** 状态方程 can be written two equal ways:

$$pV = nRT \qquad \text{or} \qquad pV = NkT.$$

Here:

- p – **pressure** 压强 (Pa).
- V – **volume** 体积 (m^3).
- T – thermodynamic temperature in kelvin (never °C).
- n – number of moles; N – number of molecules.
- R – **molar gas constant** 摩尔气体常量, $R = 8.31$ J mol^{-1} K^{-1}.
- k – **Boltzmann constant** 玻尔兹曼常量, $k = 1.38\times10^{-23}$ J K^{-1}.

Since $N = nN_\mathrm{A}$, we get $k = R/N_\mathrm{A}$: k is the gas constant **per molecule**, as R is per mole.

The two-mark definition. *An ideal gas is one that obeys $pV = nRT$ (or $pV \propto T$, with T the thermodynamic temperature) at all values of pressure, volume and temperature.* Both marks need the equation (or the proportionality with T named as thermodynamic) **and** "at all values" or "for all p, V and T". Asked to "state the meaning of each symbol in $pV = NkT$": p is the pressure, V the volume, N the **number of molecules**, k the Boltzmann constant and T the thermodynamic temperature – a real gas is nearly ideal at low pressure and high temperature, where its molecules are far apart.

■ Using the equation of state

List the variables you have, find the unknown, and choose the form that matches your "amount" (moles → nRT; molecules → NkT). Always use SI units: Pa, m^3, K.

> **Worked example.** A cylinder of volume 0.020 m^3 holds gas at 27 °C and a pressure of 2.0×10^5 Pa. How many moles of gas are there? ($R = 8.31$ J mol^{-1} K^{-1}.)

Convert to kelvin: $T = 27 + 273 = 300$ K. Then from $pV = nRT$,

$$n = \frac{pV}{RT} = \frac{(2.0\times10^5)(0.020)}{8.31\times300} \approx 1.6 \text{ mol}.$$

If a fixed amount of gas changes from state 1 to state 2:

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}.$$

> **Worked example.** A fixed mass of gas at 300 K occupies 0.50 m^3. It is heated to 450 K at constant pressure. Find the new volume.

At constant pressure V/T is constant, so

$$V_2 = V_1\frac{T_2}{T_1} = 0.50\times\frac{450}{300} = 0.75 \text{ m}^3.$$

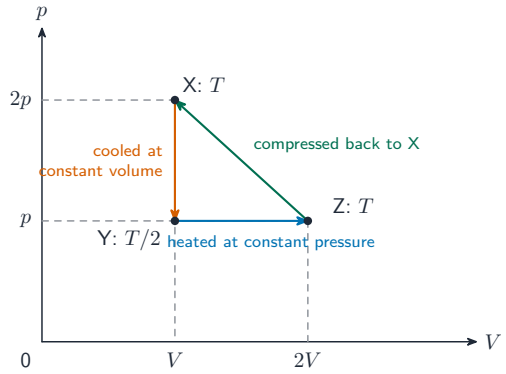
> **Worked example.** A sealed vessel of volume 0.0500 m^3 contains 0.0424 kg of an ideal gas at 227 °C and 1.37×10^5 Pa. Find the amount of gas, the mass of one molecule, and the mean-square speed of its molecules.

$T = 227 + 273 = 500$ K. $n = \dfrac{pV}{RT} = \dfrac{1.37\times10^5\times0.0500}{8.31\times500} = 1.65$ mol. The molar mass is $0.0424/1.65 = 0.0257$ kg mol^{-1}, so one molecule has mass $0.0257/(6.02\times10^{23}) = 4.3\times10^{-26}$ kg. From $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$, $\langle c^2\rangle = \dfrac{3kT}{m} = \dfrac{3\times1.38\times10^{-23}\times500}{4.27\times10^{-26}} = 4.8\times10^5$ m^2 s^{-2} (an r.m.s. speed of about 700 m s^{-1}).

> **Worked example.** Cylinder X (volume 0.0260 m^3, 0.740 mol) and cylinder Y (volume 0.0180 m^3, 0.320 mol) contain ideal gas and are in thermal equilibrium with each other at 290 K. Find the pressure in each, and explain what happens to the number of molecules in each cylinder if a tap joining them is opened.

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$p_X = \dfrac{nRT}{V} = \dfrac{0.740 \times 8.31 \times 290}{0.0260} = 6.86 \times 10^4$ Pa and $p_Y = \dfrac{0.320 \times 8.31 \times 290}{0.0180} = 4.28 \times 10^4$ Pa. Gas flows from the higher pressure (X) to the lower (Y) until the pressures are equal; the temperature is unchanged, so the final pressure is $p = \dfrac{(n_X + n_Y)RT}{V_X + V_Y} = 5.8 \times 10^4$ Pa, and X ends with $n_X = pV_X/RT = 0.63$ mol – it loses about 0.11 mol to Y.



A cycle on a p–V diagram: a vertical line is constant volume, a horizontal line constant pressure, and pV/T is the same at every state

Worked example. A fixed amount of ideal gas at temperature T is in state X, with pressure $2p$ and volume V. It is cooled at constant volume to state Y, where its pressure is p; then heated at constant pressure to state Z, where its volume is $2V$; then returned to X. Find the temperatures at Y and Z, and describe how the internal energy changes round the cycle.

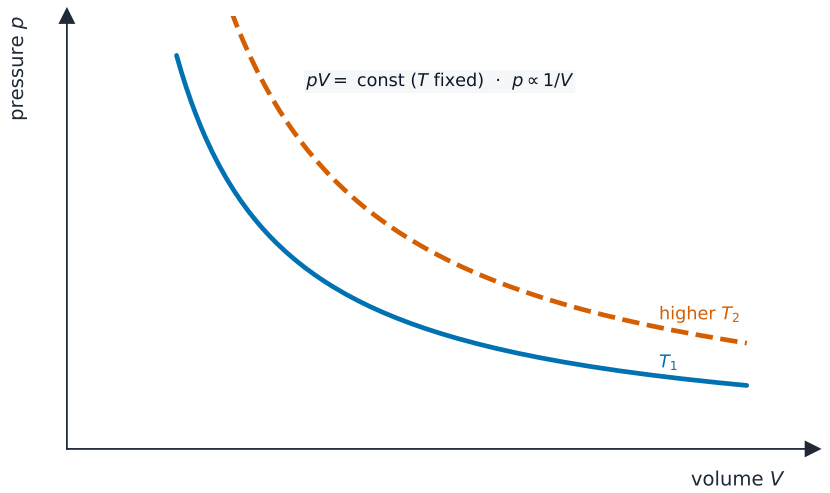
X to Y is at constant volume, so p/T is constant: halving the pressure halves the temperature, $T_Y = T/2$. Y to Z is at constant pressure, so V/T is constant: doubling the volume doubles the temperature, $T_Z = T$. The internal energy of an ideal gas depends only on temperature, so it falls from X to Y (by $\frac{3}{2}nR \cdot T/2$), rises by the same amount from Y to Z, and is the same at X as at Z, so round the whole cycle the net change is zero. Read each leg off the diagram before writing an equation: vertical means constant V, horizontal means constant p.

Special cases:

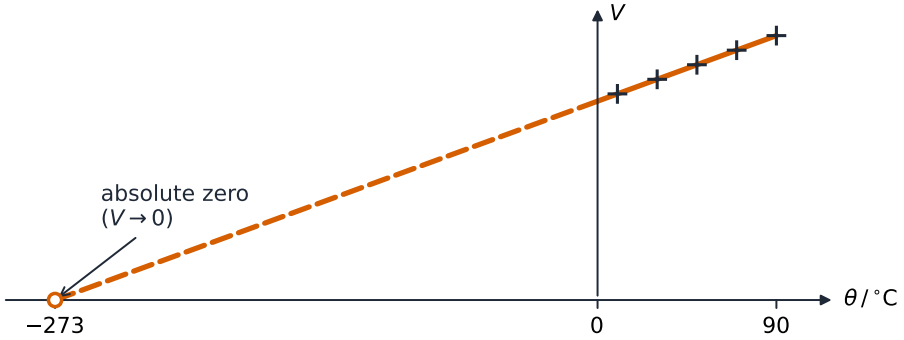
- **constant temperature** (**Boyle's law** 玻意耳定律): $p_1V_1 = p_2V_2$.
- **constant pressure** (**Charles's law** 查理定律): $V/T =$ constant.
- **constant volume** (**pressure law** 气体压强定律): $p/T =$ constant.

A common mistake is using °C instead of K – $pV \propto T$ only holds with T in kelvin.

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Boyle's law: at constant temperature pV is constant, so a p–V graph is a hyperbola



$V \propto T$ · Charles: extrapolates to absolute zero

At constant pressure the volume of a gas rises linearly with temperature, reaching zero at absolute zero (Charles's law)

Kinetic theory of gases



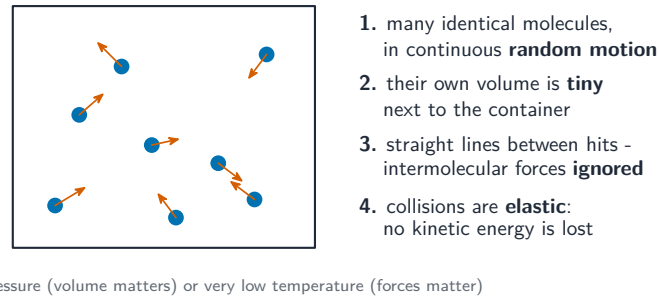
A scuba diver breathes compressed gas; its pressure, volume and temperature are all linked.

The **kinetic theory** 分子动理论 explains a gas's large-scale behaviour from the **random motion** 无规则运动 of its molecules.

Assumptions

For an ideal gas:

1. a **large number** of identical molecules in continuous random motion.
2. the molecules' own volume is too small to matter compared with the container.
3. the time of each collision is too short to matter compared with the time between collisions.
4. **intermolecular** 分子间 forces are ignored except during collisions (molecules go in straight lines between them).
5. collisions (with the walls and with each other) are **elastic** – an **elastic collision** 弹性碰撞 loses no kinetic energy, so the gas does not cool down by itself.
6. Newton's laws apply.



The key assumptions of the kinetic theory of an ideal gas

These assumptions become poor at very high pressure (molecular volume matters) or very low temperature (intermolecular forces matter).

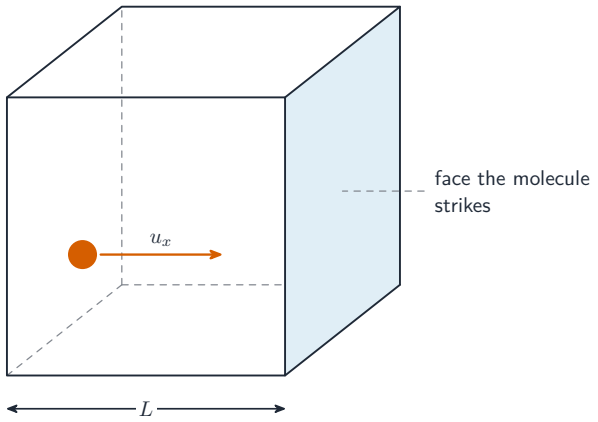
As the exam asks it. "State two (or three) basic assumptions of the kinetic theory": choose from *the molecules are in continuous random motion*; *the volume of the molecules is negligible compared with the volume of the gas*; *there are no forces between the molecules except during collisions*; *the collisions are perfectly elastic*; *the time of a collision is negligible compared with the time between collisions.* "State what is meant by an elastic collision": *one in which the total kinetic energy is conserved* (as well as momentum). "Use the assumptions to suggest why the gas at the surface of a star, at very high pressure, is not ideal": the molecules are so close together that their own volume is not negligible compared with the gas volume, and intermolecular forces act between them – the two assumptions that fail when a gas is compressed.

> **Worked example.** A balloon contains 0.40 mol of hydrogen ($m = 3.34 \times 10^{-27}$ kg per molecule) in 9.8×10^{-3} m^3. Estimate the average separation of the molecules, the gravitational force between neighbouring molecules, and comment on the kinetic-theory assumption this tests.

$N = 0.40 \times 6.02 \times 10^{23} = 2.4 \times 10^{23}$ molecules, so each occupies $V/N = 4.1 \times 10^{-26}$ m^3 and the average separation is $d = (V/N)^{1/3} = 3.4 \times 10^{-9}$ m. The gravitational force between two molecules is $F = \dfrac{Gm^2}{d^2} = \dfrac{6.67 \times 10^{-11} \times (3.34 \times 10^{-27})^2}{(3.4 \times 10^{-9})^2} = 6 \times 10^{-47}$ N – some 10^{20} times smaller than a molecule's own weight ($mg = 3.3 \times 10^{-26}$ N). Forces between molecules really are negligible except during collisions, as the theory assumes.

Pressure of a gas – outline of the derivation

Take a cubic box of side L with N molecules, each of mass m. Look at one molecule moving along the x-axis with velocity u_1.



The pressure derivation considers one molecule's velocity component u_x normal to a face of a cube of gas

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- **one collision with the right wall**: velocity reverses to $-u_1$, change in **momentum** 动量 $\Delta p_x = -2mu_1$. By Newton's third law the wall gets an **impulse** 冲量 of $+2mu_1$.
- **time between hits on that wall**: travel $2L$ there and back, so $\Delta t = 2L/u_1$.
- **average force from this molecule**: $F_1 = \Delta p/\Delta t = mu_1^2/L$.
- **add over all molecules**: $F = (Nm/L)\langle u_x^2\rangle$, where $\langle u_x^2\rangle$ is the **mean square** 均方 of the x-velocity.
- **pressure**: $p = F/L^2 = Nm\langle u_x^2\rangle/V$.

In 3-D, by symmetry $\langle u_x^2\rangle = \frac{1}{3}\langle c^2\rangle$, where $\langle c^2\rangle$ is the **mean-square speed** 均方速率. So

$$pV = \tfrac{1}{3}Nm\langle c^2\rangle.$$

"Explain how molecular movement causes the pressure exerted by a gas" (3 marks). The molecules move randomly and collide with the walls of the container; at each collision a molecule's momentum changes (it rebounds), so by Newton's second law the wall exerts a force on it, and by the third law it exerts an equal force on the wall; the very many collisions each second produce a steady total force on the wall, and the pressure is that force per unit area. The syllabus phrase "a simple model considering one-dimensional collisions" means the derivation above: it is sufficient to follow one molecule bouncing between two opposite faces and then average.

The density form. Since Nm is the total mass of the gas and Nm/V is its density ρ, the same result reads $p = \frac{1}{3}\rho\langle c^2\rangle$ – the version to use when a question gives a density instead of N and m.

> **Worked example.** An ideal gas at a pressure of 1.6×10^5 Pa has a density of 1.9 kg m^{-3}. Show that the r.m.s. speed of its molecules is about 500 m s^{-1}.

$\langle c^2\rangle = \dfrac{3p}{\rho} = \dfrac{3 \times 1.6 \times 10^5}{1.9} = 2.53\times10^5$ m^2 s^{-2}, so $c_{\text{r.m.s.}} = \sqrt{2.53 \times 10^5} = 503$ m s^{-1} $\approx$ 500 m s^{-1}. In a "show that", keep an extra figure (503) before comparing with the value given.

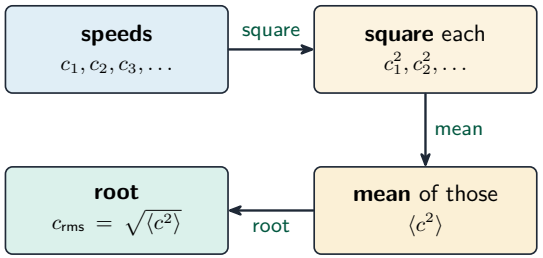
■ Root-mean-square speed

The square root of $\langle c^2\rangle$ is the **root-mean-square** 均方根 (r.m.s.) speed:

$$c_{\text{r.m.s.}} = \sqrt{\langle c^2\rangle}.$$

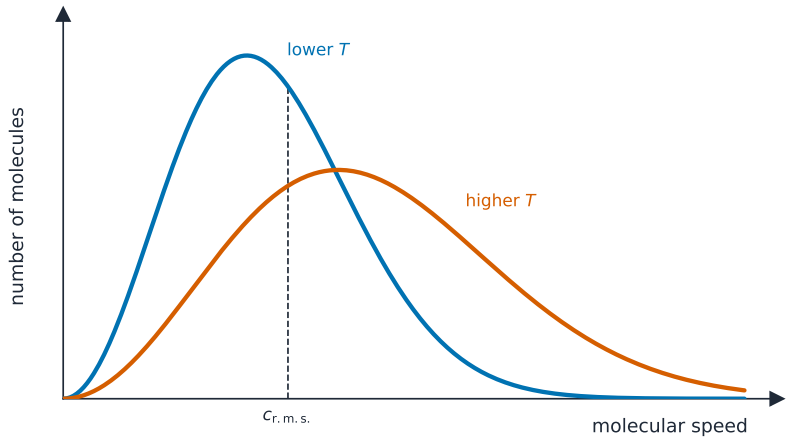
It is a useful single measure of how fast the molecules move, slightly larger than the mean speed (squaring weights fast molecules more).

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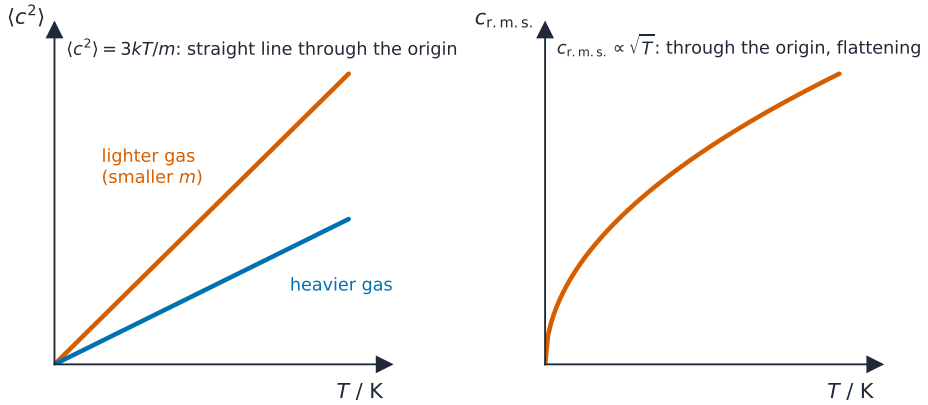
read the name **backwards**: **r**oot of the **m**ean of the **s**quares

Root-mean-square speed: square each speed, take the mean, then the square root



higher T: peak → higher speeds, flatter curve

The spread of molecular speeds: a higher temperature broadens the curve and shifts it to faster speeds



The two graphs the exam asks you to sketch: $\langle c^2\rangle \propto T$ is a straight line through the origin, so $c_{r.m.s.}$ rises as $\sqrt{T}$

Average translational kinetic energy

Compare the two expressions for pV:

$$pV = NkT \quad \text{and} \quad pV = \tfrac{1}{3}Nm\langle c^2\rangle.$$

Set them equal, cancel N, and multiply by $\frac{3}{2}$:

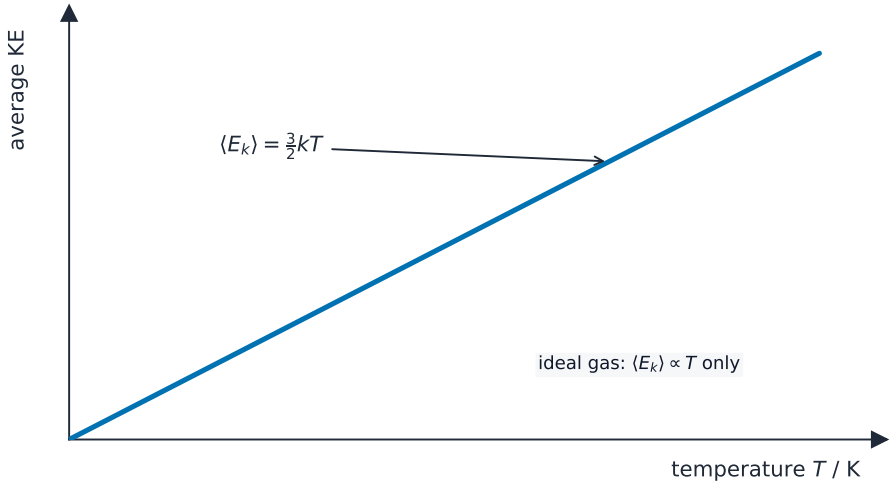
$$\tfrac{3}{2}kT = \tfrac{1}{2}m\langle c^2\rangle.$$

translational kinetic energy 平动动能 is the energy of motion from place to place. The right side is the **average translational kinetic energy** 平均平动动能 $\langle E_k\rangle$ of one molecule. So

$$\langle E_k\rangle = \tfrac{1}{2}m\langle c^2\rangle = \tfrac{3}{2}kT.$$

This is a key result: **the average translational kinetic energy of an ideal-gas molecule depends only on the thermodynamic temperature**, not on the type of gas or its pressure.

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Average molecular KE is proportional to thermodynamic temperature: $\langle E_k\rangle = \frac{3}{2}kT$

Worked example. Find the root-mean-square speed of oxygen molecules at 300 K. (Mass of one O_2 molecule $= 5.3 \times 10^{-26}$ kg, $k = 1.38 \times 10^{-23}$ J K^{-1}.)

From $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$, the mean-square speed is $\langle c^2\rangle = 3kT/m$:

$$c_{\text{r.m.s.}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38\times10^{-23})(300)}{5.3\times10^{-26}}} \approx 480 \text{ m s}^{-1}.$$

Worked example. The surface of the Moon reaches about 400 K in sunlight. Calculate the r.m.s. speed of hydrogen molecules ($m = 3.34 \times 10^{-27}$ kg) at this temperature, and use the Moon's escape speed of 2.4 km s^{-1} to suggest why the Moon has no hydrogen atmosphere.

$c_{\text{r.m.s.}} = \sqrt{\dfrac{3kT}{m}} = \sqrt{\dfrac{3\times1.38\times10^{-23}\times400}{3.34\times10^{-27}}} = 2.2\times10^{3}$ m s^{-1}. This r.m.s. speed is close to the escape speed, and the speeds are spread widely about it, so a large fraction of the molecules move faster than 2.4 km s^{-1} at any moment and escape; over time the hydrogen is lost.

Worked example. The gas at the surface of a star is mainly hydrogen atoms ($m = 1.67 \times 10^{-27}$ kg) with an r.m.s. speed of 9300 m s^{-1}. Find the temperature of the surface.

$$T = \frac{m\langle c^2\rangle}{3k} = \frac{1.67\times10^{-27}\times9300^2}{3\times1.38\times10^{-23}} = 3.5\times10^{3} \text{ K}.$$

> **Worked example.** For a fixed sample of gas, a graph of pV against kT is a straight line through the origin of gradient 7.2×10^{22}. When $pV = 270$ J the r.m.s. speed of the molecules is 1900 m s^{-1}. Find the number of molecules and the mass of one molecule in u.

From $pV = NkT$ the gradient is $N = 7.2 \times 10^{22}$. From $pV = \frac{1}{3}Nm\langle c^2\rangle$: $m = \dfrac{3pV}{N\langle c^2\rangle} = \dfrac{3 \times 270}{7.2 \times 10^{22} \times 1900^2} = 3.1 \times 10^{-27}$ kg $= \dfrac{3.1 \times 10^{-27}}{1.66 \times 10^{-27}} = 1.9$ u – hydrogen molecules, within the rounding.

Comparing gases and samples. At the same temperature every gas has the same average kinetic energy per molecule, so $\langle c^2\rangle \propto 1/m$: hydrogen ($M_\mathrm{m} = 2$) and oxygen ($M_\mathrm{m} = 32$) have $\dfrac{c_\mathrm{H}}{c_\mathrm{O}} = \sqrt{\dfrac{32}{2}} = 4$. Two samples at the same T – X with N molecules of mass m in volume V, Y with $2N$ molecules of mass $2m$ in volume $2V$ – have the *same pressure* ($p = NkT/V$ and both N and V double), the same average kinetic energy per molecule, but Y's molecules have half the mean-square speed ($3kT/2m$) and Y has twice the internal energy (twice as many molecules).

■ Consequences

- **doubling the absolute temperature doubles the average KE** of each molecule, so $\langle c^2\rangle$ doubles and $c_\mathrm{r.m.s.}$ grows by $\sqrt{2}$.
- for **two gases at the same temperature**, the lighter gas has a larger $\langle c^2\rangle$. Hydrogen molecules move faster on average than oxygen molecules in the same room.
- **total translational KE** of N molecules: $\frac{3}{2}NkT = \frac{3}{2}nRT$.

■ Internal energy of an ideal gas

For an ideal gas the molecules are point particles with no intermolecular potential energy and (in this simple model) no rotation or vibration. So the **internal energy** 内能 is just the total **kinetic energy** 动能:

$$U = \tfrac{3}{2}NkT = \tfrac{3}{2}nRT.$$

So the internal energy of an ideal gas is proportional to the thermodynamic temperature – doubling T doubles U.

As the exam asks it. "Use one of the basic assumptions to explain what can be deduced about the potential energy of the molecules": there are *no forces between the molecules* (except in collisions), so there is *no potential energy* associated with their separation – the random-motion energy is entirely kinetic. "Explain why the internal energy of an ideal gas is directly proportional to thermodynamic temperature" (2 marks): the internal energy is the sum of the kinetic and potential energies of the molecules; the potential energy is zero (no intermolecular forces), and the average kinetic energy of a molecule is $\frac{3}{2}kT$, proportional to T; so the total, $U = \frac{3}{2}NkT$, is proportional to T. "Derive $\frac{3}{2}kT$" (2 marks): equate $pV = NkT$ with $pV = \frac{1}{3}Nm\langle c^2\rangle$, cancel N, and rearrange to $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$ – show the cancelling and the factor of $\frac{3}{2}$.

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> **Worked example.** A sample of 0.26 m^3 of an ideal gas is at 2.0×10^5 Pa and 290 K. Find the number of molecules, the average translational kinetic energy of a molecule, and the internal energy of the gas.

$N = \dfrac{pV}{kT} = \dfrac{2.0 \times 10^5 \times 0.26}{1.38 \times 10^{-23} \times 290} = 1.3 \times 10^{25}$. $\langle E_\mathrm{k}\rangle = \frac{3}{2}kT = 1.5 \times 1.38 \times 10^{-23} \times 290 = 6.0 \times 10^{-21}$ J. $U = N\langle E_\mathrm{k}\rangle = 1.3 \times 10^{25} \times 6.0 \times 10^{-21} = 7.8 \times 10^4$ J – or directly $U = \frac{3}{2}pV = 7.8 \times 10^4$ J, a neat check.

■ Changing pressure at fixed temperature

Doubling p at fixed T (by squeezing the gas to half its volume) does **not** change $\langle E_\mathrm{k}\rangle$ – that depends only on T. There are more wall collisions per second, but each molecule has the same average kinetic energy.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
mole	the amount of substance containing a number of particles equal to the Avogadro constant
Avogadro constant	the number of atoms or molecules in one mole of a substance, 6.02×10^{23} mol^{-1}
ideal gas	a gas that obeys $pV = nRT$ (equivalently $pV \propto T$, T thermodynamic) at all values of p, V and T
Boltzmann constant	the gas constant per molecule, $k = R/N_\mathrm{A}$
elastic collision	a collision in which the total kinetic energy is conserved
mean-square speed	the mean of the squares of the speeds of the molecules, $\langle c^2\rangle$
root-mean-square speed	the square root of the mean-square speed, $c_\mathrm{r.m.s.} = \sqrt{\langle c^2\rangle}$
average translational kinetic energy of a molecule	$\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$, depending only on thermodynamic temperature

Term	Definition
internal energy of an ideal gas	the total kinetic energy of its molecules, $\frac{3}{2}NkT$, since there is no potential energy

Exam tips

- Use $pV = nRT$ (n in mol) or $pV = NkT$ (N molecules), with temperature in **kelvin** and volume in m^3 ($1\ \mathrm{cm}^3 = 10^{-6}\ \mathrm{m}^3$).
- Learn the **kinetic-theory assumptions** word for word (random motion, negligible molecular volume, no intermolecular forces except in collisions, elastic collisions, negligible collision time), and know which two fail at high pressure.
- Mean translational KE $= \frac{3}{2}kT$ – it depends **only on temperature**; at the same T a lighter molecule is faster, $\langle c^2\rangle \propto 1/m$.
- For a "show that" about pressure, give the chain: collisions with the wall, change of momentum, force (Newton's second and third laws), force per unit area.
- $p = \frac{1}{3}\rho\langle c^2\rangle$ when a density is given; $U = \frac{3}{2}NkT = \frac{3}{2}pV$ when the internal energy is asked.

Common mistakes

- Using °C in $pV = nRT$ or in $\langle E_\mathrm{k}\rangle = \frac{3}{2}kT$. Convert first; a temperature *ratio* only works in kelvin.
- Defining an ideal gas by "obeys $pV = nRT$" alone. Add "at all values of p, V and T".
- Confusing n (moles) with N (molecules), or R with k. $N = nN_\mathrm{A}$ and $k = R/N_\mathrm{A}$.
- Giving "molecules move randomly" as the reason for pressure. The mark is for the change of momentum at the wall and the resulting force per unit area.
- Sketching $c_\mathrm{r.m.s.}$ against T as a straight line. $\langle c^2\rangle$ is linear in T; $c_\mathrm{r.m.s.}$ rises as $\sqrt{T}$.
- Saying a molecule's kinetic energy changes when the pressure is doubled at constant temperature. Only the number of collisions per second changes.

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Thermodynamics

A-Level Physics

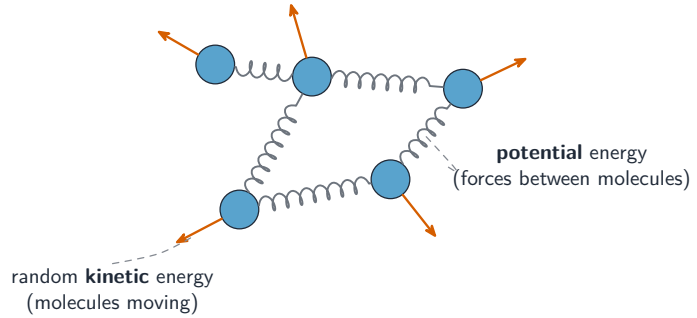
Internal energy

The **internal energy** 内能 U of a system is the sum of:

- the random **kinetic energies** 动能 of its **molecules** 分子 – they fly through space (**translational** 平动 motion), and unless they are single atoms they also spin (**rotational** 转动) and shake (**vibrational** 振动), and
- the **potential energies** from the forces between the molecules.

For a real solid, liquid or gas, both parts matter. In the **ideal-gas** 理想气体 model the **intermolecular** 分子间 forces are ignored, so the molecular potential energy is zero and the internal energy is purely kinetic.

The two-mark definition. *The internal energy of a system is the sum of the random distribution of the kinetic and potential energies of its molecules (or atoms).* Both marks need "sum of kinetic and potential energies" **and** "random" (or "of the molecules") – leaving out "random" describes the energy of a moving object, not its internal energy. For an ideal gas, "with reference to kinetic and potential energy": the internal energy is the total *kinetic* energy of the molecules only, because there are no intermolecular forces and therefore no potential energy.



Internal energy is the molecules' random kinetic energy (the arrows) plus the potential energy of the forces between them (the springs)

Two key points:

1. U **depends only on the state** of the system (its **temperature** 温度, **pressure** 压强, **volume** 体积, **amount of substance** 物质的量) – not on the path taken to get there.

2. U **is a sum over the molecules**, not the kinetic energy of the whole object moving. A moving train of gas has bulk kinetic energy, but that is separate from U – U is the energy of the *random* molecular motion.

Temperature and internal energy

Raising an object's temperature raises the random kinetic energy of its molecules, and so raises its internal energy.

For an ideal gas every molecule has average translational kinetic energy $\frac{3}{2}kT$ (Topic 15). With zero intermolecular potential energy, the total internal energy is

$$U = \tfrac{3}{2}NkT = \tfrac{3}{2}nRT.$$

So the internal energy of an ideal gas is directly proportional to the **thermodynamic temperature** 热力学温度. Doubling T doubles U. This is **only** exact for an ideal gas.

During a **phase change** 相变 (melting or boiling) of a real substance, U rises because the molecular potential energy rises (bonds breaking), even though the temperature stays constant.

Describing a change in internal energy – always name both energies. A three-mark "describe and explain, with reference to molecular kinetic and potential energies" answer says what happens to *each*:

- **a gas heated at constant volume:** the kinetic energy of the molecules increases, because temperature is a measure of their mean kinetic energy; the potential energy is unchanged (no work is done, the molecules' separation does not change); so the internal energy increases.
- **a wire stretched within its elastic limit at constant temperature:** the kinetic energy of the atoms is unchanged (same temperature); the potential energy increases, because the atoms are pulled further apart against the interatomic forces; so the internal energy increases.
- **ice melting at 0 °C:** the kinetic energy is unchanged (constant temperature); the potential energy increases as the bonds between molecules are broken and the separation grows; so the internal energy increases by the latent heat supplied.

> **Worked example.** A fixed mass of ideal gas is heated at constant pressure. Sketch the variation of its internal energy U with its volume V.

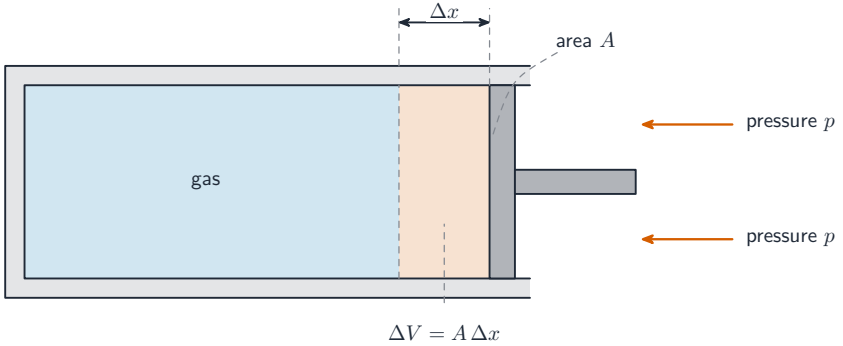
At constant pressure $V \propto T$ (Charles's law) and for an ideal gas $U \propto T$, so $U \propto V$: a **straight line through the origin**. The origin is on the line because at absolute zero both V (extrapolated) and U are zero.

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Work done on or by a gas



A steam turbine does work as expanding steam pushes its blades around.



A gas pushing a piston of area A out by Δx does work $W = p\,\Delta V$ on the surroundings (swept volume $\Delta V = A\,\Delta x$)

When a gas changes volume against an outside pressure, mechanical work is done. At constant pressure p with a small volume change ΔV, the size of the work is

$$W = p\Delta V.$$

> **Worked example.** A gas at a constant pressure of 1.0×10^5 Pa expands from 2.0×10^{-3} m^3 to 5.0×10^{-3} m^3. Find the work done by the gas.

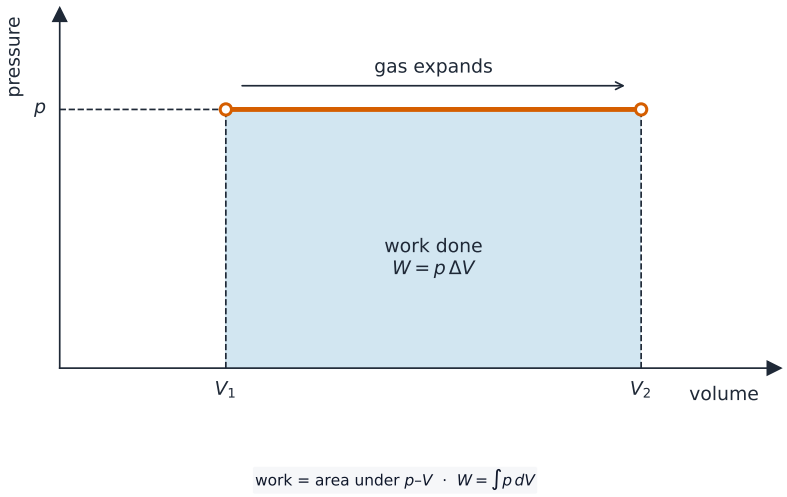
$$W = p\,\Delta V = (1.0 \times 10^5)(5.0 \times 10^{-3} - 2.0 \times 10^{-3}) = 300 \text{ J}.$$

Worked example. An ideal gas of mass 0.35 kg is heated at a constant pressure of 2.0×10^5 Pa; its internal energy rises by 7600 J and its volume increases by 1.2×10^{-2} m^3. Find the work done by the gas and the thermal energy supplied. The gas is then heated at constant *volume* until its internal energy rises by the same 7600 J; explain why less thermal energy is needed.

Work done *by* the gas $= p\Delta V = 2.0 \times 10^5 \times 1.2 \times 10^{-2} = 2400$ J, so the work done *on* it is $W = -2400$ J. First law: $q = \Delta U - W = 7600 - (-2400) = 1.0 \times 10^4$ J. At constant volume no work is done ($\Delta V = 0$, $W = 0$), so the whole of the thermal energy goes into internal energy: $q = \Delta U = 7600$ J. The extra 2400 J at constant pressure was the work the gas did pushing back the surroundings as it expanded.

Worked example. An aluminium block of volume 3.612×10^{-3} m^3 is heated from 0 °C to 40 °C; its volume increases by 1.0×10^{-5} m^3 against atmospheric pressure (1.0×10^5 Pa). Compare the work it does on the atmosphere with the thermal energy it receives ($m = 9.75$ kg, $c = 900$ J kg^{-1} K^{-1}).

Work done by the block $= p\Delta V = 1.0 \times 10^5 \times 1.0 \times 10^{-5} = 1.0$ J; thermal energy $q = mc\Delta T = 9.75 \times 900 \times 40 = 3.5 \times 10^5$ J. The work is about 3×10^{-6} of the heating, so for a solid $\Delta U \approx q$: solids and liquids barely expand, and $p\Delta V$ only matters for a gas.

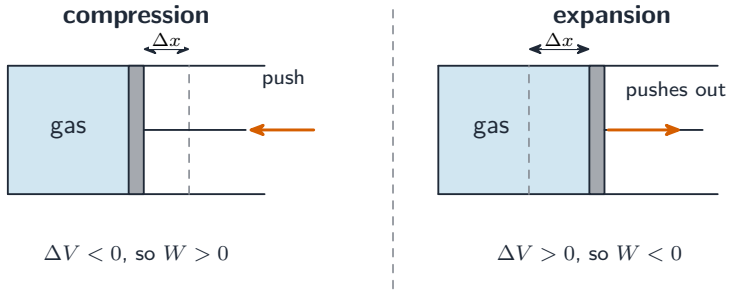


On a pressure–volume graph, the work done at constant pressure is the area under the line: $W = p\,\Delta V$

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Sign convention in this syllabus

This syllabus writes the first law as $\Delta U = q + W$, where W is the work done **on** the gas and q is the energy put in **by heating**.



Work done on the gas: positive when it is compressed, negative when it expands

- when the gas is **compressed**, ΔV is negative and the work done **on** the gas is positive – the gas gains energy.
- when the gas **expands**, ΔV is positive and the work done **on** the gas is negative – the gas loses energy (it does work on the surroundings).

Watch which form a question wants:

- "work done **on** the gas" – positive when compressing.
- "work done **by** the gas" – the opposite sign, positive when expanding.

At constant volume ($\Delta V = 0$), no work is done.

First law of thermodynamics



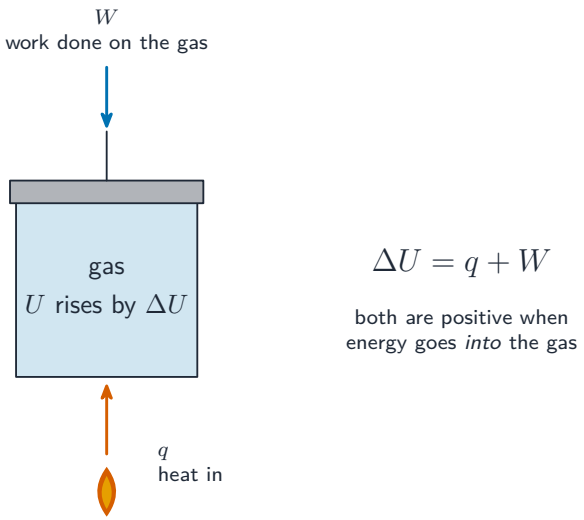
A power station is a heat engine: it converts heat into useful work.

The **first law of thermodynamics** 热力学第一定律 says that **energy is conserved when heat and work pass between a system and its surroundings**:

$$\Delta U = q + W,$$

where ΔU is the rise in internal energy, q is the energy added **by heating** (positive in, negative out), and W is the work done **on** the gas (positive when compressed). This is **conservation of energy** 能量守恒 for a gas.

As the exam asks it. "State the first law of thermodynamics, identifying any symbols" (2 marks): *the increase in internal energy of a system, ΔU, is equal to the sum of the thermal energy transferred to the system by heating, q, and the work done on the system, W*: $\Delta U = q + W$. Every symbol must be defined with its **direction** – "to the system" and "on the system" are what make the signs mean something. "State two ways in which the first law says the internal energy of a system may be changed": by *heating* (thermal energy transferred into or out of the system) and by *doing work* on or by the system.



$$\Delta U = q + W$$

both are positive when energy goes *into* the gas

Both heating (q) and work done on the gas (W) put energy in, raising the internal energy by ΔU

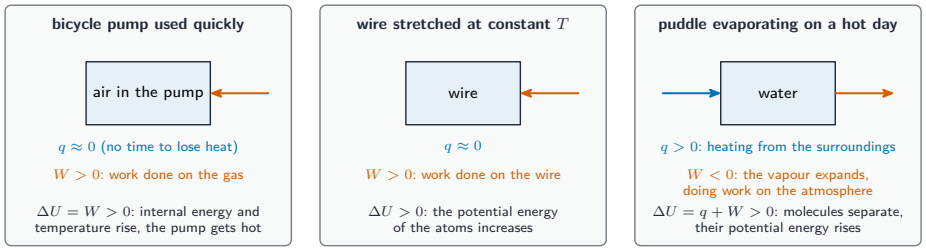
Worked example. A gas absorbs 500 J of heat while it expands and does 200 J of work on its surroundings. Find the change in its internal energy.

The gas does work, so the work done *on* it is $W = -200$ J:

$$\Delta U = q + W = 500 + (-200) = 300 \text{ J}.$$

Explaining with the first law. A three-mark "use the first law to explain" answer has three steps: say what q is (and its sign), say what W is (and its sign), then combine them for ΔU – and, if asked, say what the change in internal energy means for the molecules.

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The three classic first-law explanations: give q with its sign, W with its sign, then ΔU

- **Why a bicycle pump gets hot when used quickly:** the air is compressed, so work is done *on* it ($W > 0$); the compression is fast, so there is no time for thermal energy to leave ($q \approx 0$); therefore $\Delta U = W > 0$ – the internal energy, and so the temperature, of the air rises, and the pump warms up.
- **A spring stretched at constant temperature within its elastic limit:** work is done on the spring by the stretching force ($W > 0$); there is no thermal energy transfer ($q = 0$); so the internal energy increases – stored as the elastic potential energy of the atoms, which are pulled further apart.
- **Water evaporating from a puddle on a hot day:** thermal energy is transferred to the water from the surroundings ($q > 0$); the vapour formed occupies a far larger volume than the liquid, so the system does work on the atmosphere ($W < 0$); the internal energy still increases (q is larger than the work done), and the increase is potential energy – the molecules are separated against the attractive forces between them.

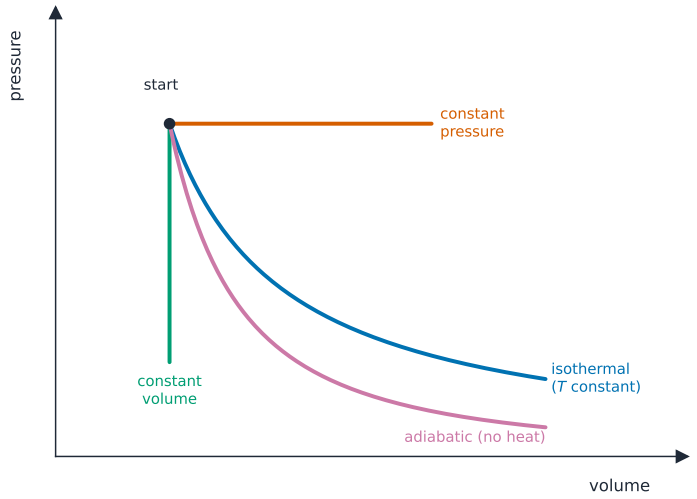
Reading the equation

ΔU is fixed by the change of state (for an ideal gas, by the change in temperature). The same ΔU can come from different mixes of q and W:

- all heat, no work: $\Delta U = q$ (constant-volume heating).
- all work, no heat: $\Delta U = W$ (insulated compression or expansion).

Standard processes

For an ideal gas, $\Delta U = \frac{3}{2} nR\Delta T$ – it depends only on ΔT.



The four standard processes, all starting from the same state

Process	What stays constant	ΔU	W (on gas)	q
Isothermal	T	0	W	$-W$
Constant volume	V	$\frac{3}{2}nR\Delta T$	0	ΔU
Constant pressure	p	$\frac{3}{2}nR\Delta T$	$-p\Delta V$	$\Delta U - W$
Adiabatic	(no heat)	W	W	0

Read each row with the first law $\Delta U = q + W$:

- **Isothermal** 等温 (constant T): $\Delta T = 0$, so $\Delta U = 0$. Then $q = -W$ – any heat that goes in comes straight back out as work.
- **Adiabatic** 绝热 (no heat flow): $q = 0$, so $\Delta U = W$. The gas warms up only because work is done on it.
- **Constant volume** (sealed rigid container): no work is done ($\Delta V = 0$), so all the heat goes into internal energy: $q = \Delta U$.
- **Constant pressure** (gas pushing a **piston** 活塞): the gas does work as it expands, so the heat you supply does two jobs – it raises the internal energy *and* does the expansion work.

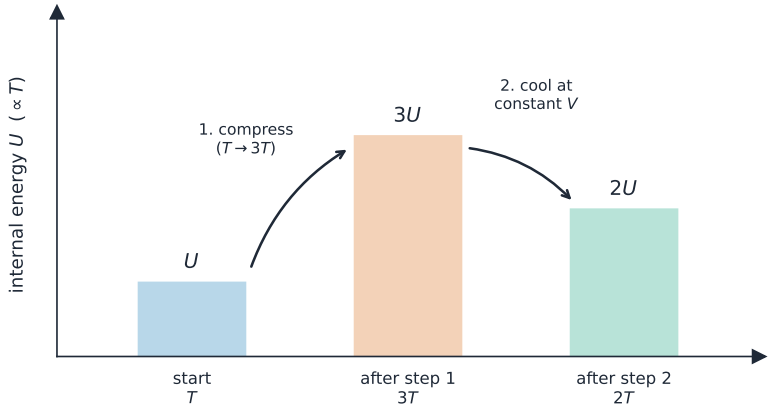
So for the **same rise in internal energy**, heating at constant pressure needs *more* thermal energy than heating at constant volume, by exactly the work $p\Delta V$ the gas does while expanding – the constant-volume gas keeps every joule; the constant-pressure gas hands some back to the surroundings.

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Worked example: two-step process

A sample of ideal gas at temperature T with internal energy U goes through:

1. **compression** to temperature $3T$; work W is done on the gas.
2. **cooling at constant volume** to temperature $2T$.



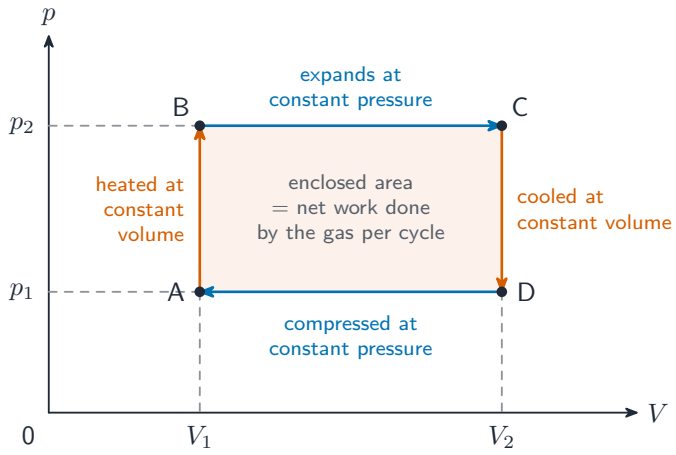
Internal energy tracks temperature: $U \to 3U$ on compressing to $3T$, then $3U \to 2U$ on cooling to $2T$

Step 1 ($T \to 3T$): $U = \frac{3}{2}nRT$, so $U \to 3U$, giving $\Delta U_1 = 2U$. $W_1 = +W$. So $q_1 = \Delta U_1 - W_1 = 2U - W$.

Step 2 ($3T \to 2T$, constant volume): $\Delta U_2 = -U$. $W_2 = 0$. So $q_2 = -U$ (heat flows out).

Check: total $\Delta U = 2U - U = U$, taking the gas from T to $2T$ ($U \to 2U$) – consistent.

Worked example: a cycle on a p–V diagram



A cycle: for each leg decide W from the volume change and ΔU from the temperature change, and q follows from the first law

A fixed mass of ideal gas is taken round the cycle ABCDA: A to B at constant volume V_1 (pressure $p_1 \to p_2$), B to C at constant pressure p_2 (volume $V_1 \to V_2$), C to D at constant volume, D to A at constant pressure p_1. Complete a table of the signs of q, W and ΔU for each leg, and explain the internal-energy change from B to C.

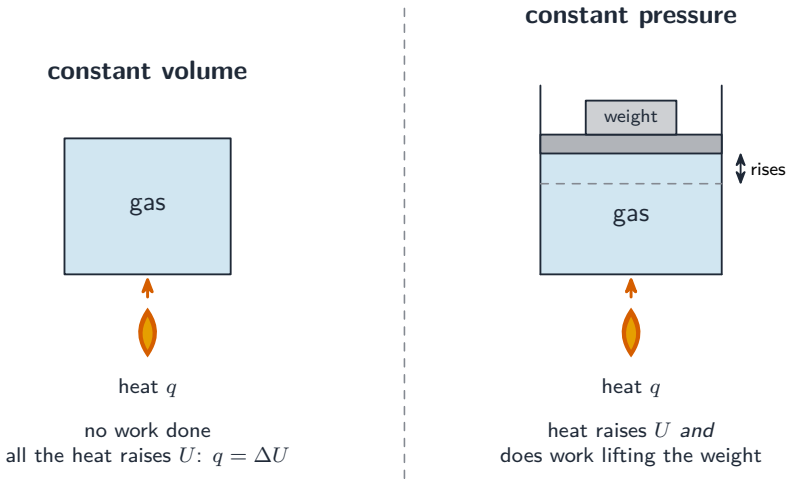
Leg	Process	W (on gas)	ΔU	q
A $\to$ B	constant volume, pressure rises	0 (no volume change)	+ (temperature rises, pV larger)	+ ($q = \Delta U$)
B $\to$ C	constant pressure, expands	− (gas does work $p_2(V_2 - V_1)$)	+ ($T \propto V$ at constant p)	+ (and larger than ΔU)
C $\to$ D	constant volume, pressure falls	0	−	− (thermal energy leaves)
D $\to$ A	constant pressure, compressed	+ (work done on the gas $p_1(V_2 - V_1)$)	−	− (larger in size than ΔU)

From B to C the gas expands at constant pressure, so its temperature rises (V/T constant) and its internal energy increases; it does work on the surroundings (W negative), so the thermal energy supplied must cover both: $q = \Delta U - W$, larger than the rise in internal energy. Round the whole cycle $\Delta U = 0$ (the gas returns to its starting state), so the net thermal energy in equals the net work done by the gas – the area enclosed by the rectangle, $(p_2 - p_1)(V_2 - V_1)$.

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Heat capacity at constant volume

For constant-volume heating of an ideal gas, $q = \Delta U = \frac{3}{2}nR\Delta T$. So the molar **heat capacity** 热容 at constant volume is $\frac{3}{2}R$ for a **monatomic** 单原子 ideal gas. (You are not required to use the symbol C_V, but the result $q = \frac{3}{2}nR\Delta T$ for constant-volume heating is.)



At constant volume all the heat raises U ($q = \Delta U$); at constant pressure the gas also does work

Heating without a temperature change

If heat is supplied during a phase change at constant pressure (e.g. boiling water), the temperature stays constant but the internal energy still rises (the **latent heat** 潜热 separates the molecules), and the gas does expansion work. The first law still holds: $\Delta U = q + W$.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
internal energy	the sum of the random distribution of the kinetic and potential energies of the molecules of a system
internal energy of an ideal gas	the total random kinetic energy of its molecules, since with no intermolecular forces there is no potential energy
first law of thermodynamics	the increase in internal energy of a system equals the thermal energy transferred to the system by heating plus the work done on the system: $\Delta U = q + W$
work done on a gas at constant pressure	$W = p\Delta V$ in size; positive when the gas is compressed, negative when it expands (the gas then does work on its surroundings)
thermal energy transfer (heating)	energy transferred to or from a system because of a temperature difference; q is positive when it enters
isothermal change	a change at constant temperature, so for an ideal gas $\Delta U = 0$ and $q = -W$
adiabatic change	a change with no thermal energy transfer, $q = 0$, so $\Delta U = W$

Exam tips

- **First law:** $\Delta U = q + W$ – define every symbol with its direction: q is energy transferred *to* the system by heating, W is work done *on* the system.
- For an ideal gas, **internal energy depends only on temperature** ($\Delta U \propto \Delta T$) and is kinetic energy only; for a solid or liquid the potential energy changes too.
- Work done by a gas at constant pressure = $p\Delta V$; read the sign from expansion (by) or compression (on), and remember that at constant volume $W = 0$.
- A "describe and explain" answer names both kinetic and potential energy and says what happens to each; an "explain using the first law" answer gives q, W and then ΔU, each with its sign.
- Round a complete cycle $\Delta U = 0$, so net heating equals net work done by the gas – the area enclosed on the p–V diagram.

Common mistakes

- Defining internal energy as "the total energy of the molecules" or forgetting "random" or "potential". The mark scheme wants the sum of random kinetic and potential energies.
- Using $W = p\Delta V$ with the wrong sign. When the gas expands the work done *on* it is negative.

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- Saying the internal energy of an ideal gas rises when it is compressed isothermally. At constant temperature $\Delta U = 0$: the work done on it leaves again as heat.
- Treating a phase change as $\Delta U = 0$ because the temperature is constant. The potential energy rises; ΔU is the latent heat minus any expansion work.
- Forgetting that a gas heated at constant pressure does work, so it needs more heating than at constant volume for the same ΔU.
- Applying $\Delta U = \frac{3}{2}nR\Delta T$ to a real gas, a liquid or a solid. It is the ideal-gas result only.

Oscillations

A-Level Physics

Simple harmonic motion: definition



A pendulum clock keeps time using simple harmonic motion.

A particle moves with **simple harmonic motion** 简谐运动 (SHM) when its **acceleration** 加速度 is:

- **proportional to its displacement** from a fixed **equilibrium** 平衡 point, and
- **directed back towards that point** – opposite in sign to the displacement.

The defining equation is

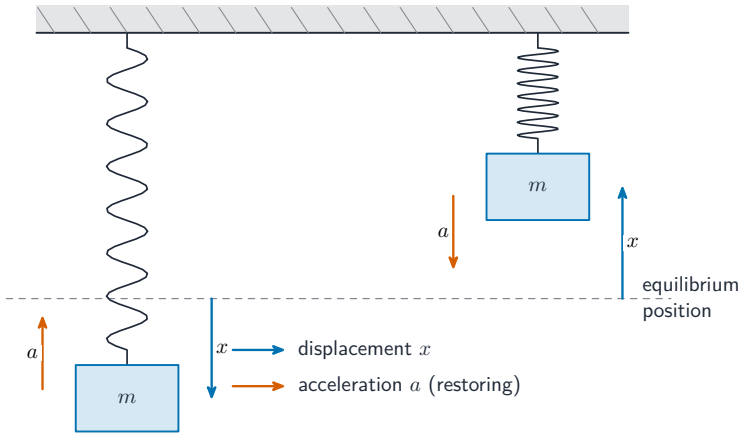
$$a = -\omega^2 x,$$

where x is the **displacement** 位移 from equilibrium and ω is a positive constant, the **angular frequency** 角频率. The minus sign means "directed back towards equilibrium".

The two-mark definition. *Simple harmonic motion is motion in which the acceleration is proportional to the displacement from a fixed point and is always directed towards that point* (in the opposite direction to the displacement). Both halves are needed. "The significance of the minus sign": the acceleration is in the **opposite direction** to the displacement – always towards the equilibrium position. "State what is meant by the frequency of the oscillations": *the number of complete oscillations per unit time.*

Many systems do this near a stable equilibrium: a mass on a **spring** 弹簧, a **pendulum** 单摆 (small swing), a floating block pushed down, the charge on a **capacitor** 电容器 in an LC circuit, atoms in a solid.

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Acceleration always points back towards equilibrium, opposite to the displacement

Key terms

- **displacement** x – distance from equilibrium at a moment (a vector along the line of motion).
- **amplitude** 振幅 x_0 – the largest displacement from equilibrium. Always positive.
- **period** 周期 T – the time for one full oscillation.
- **frequency** 频率 f – the number of oscillations per second; $f = 1/T$. Unit: Hz.
- **angular frequency** ω – $\omega = 2\pi/T = 2\pi f$. Unit: rad s^{-1}.
- **phase difference** 相位差 – the fraction of a cycle (in radians) by which one oscillation leads or lags another. A quarter-cycle apart is a phase difference of $\pi/2$.

So $T = 2\pi/\omega$ and $f = \omega/(2\pi)$ – given any one of ω, f, T you can find the others.

> **Worked example.** A mass on a spring oscillates with SHM of amplitude 0.050 m and frequency 2.5 Hz. Find its maximum acceleration.

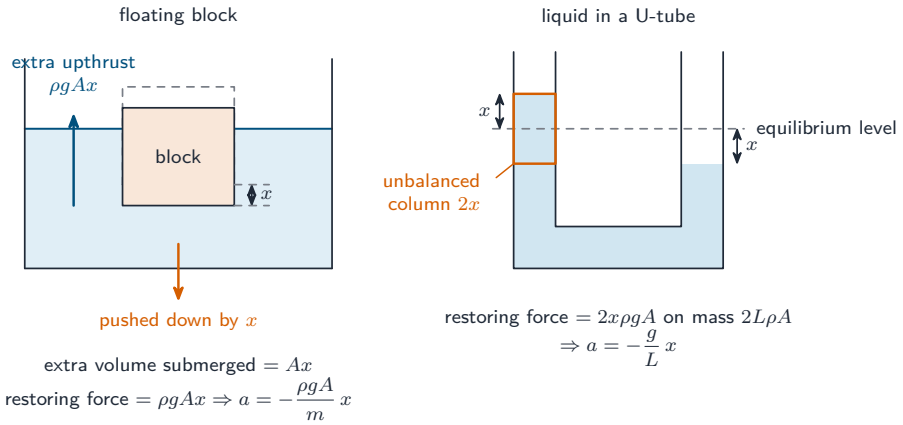
The angular frequency is $\omega = 2\pi f = 2\pi \times 2.5 = 15.7$ rad s^{-1}. The acceleration is largest at the extremes, where $|a| = \omega^2 x_0$:

$$a_{\max} = \omega^2 x_0 = 15.7^2 \times 0.050 \approx 12 \text{ m s}^{-2}.$$

Showing that a motion is simple harmonic

The exam's favourite four-mark derivation, and the recipe is always the same: (1) displace the object by x from equilibrium; (2) find the **resultant restoring force**, which comes out as $F = -(\text{constant}) \times x$; (3) apply Newton's second law, $a = F/m = -(\text{constant}/m)\,x$; (4) this has the form $a = -\omega^2 x$, so the motion is simple harmonic with $\omega^2 = \text{constant}/m$, and $T = 2\pi/\omega$. Say in words *why* the force points back towards

equilibrium and *why* it is proportional to x – those are the marks, not the algebra.



Two "show that it is SHM" set-ups: find the extra force when the system is displaced by x, and show it is proportional to x and directed back

> **Worked example.** A mass m hangs from a spring of spring constant k. Show that, when displaced and released, it moves with simple harmonic motion, and find the period.

At equilibrium the spring tension balances the weight. Displace the mass a distance x downwards: the tension increases by kx (Hooke's law), so there is a resultant force kx **upwards**, towards equilibrium. Newton's second law: $a = -\dfrac{k}{m}x$. This is $a = -\omega^2 x$ with $\omega^2 = k/m$, so the motion is simple harmonic and $T = 2\pi/\omega = 2\pi\sqrt{m/k}$. (A trolley held between *two* identical springs has both pulling it back, $F = -2kx$, so $\omega^2 = 2k/m$.)

> **Worked example.** A cuboidal block of cross-sectional area A and mass m floats in a liquid of density ρ with its base a depth h below the surface. It is pushed down a small distance x and released. Show that it oscillates with simple harmonic motion and find the period in terms of h and g.

When the block is pushed down by x an extra volume Ax is submerged, so the **upthrust** 浮力 increases by the weight of that extra liquid, ρgAx; this extra force acts **upwards**, towards the equilibrium position. Hence $a = -\dfrac{\rho gA}{m}x$: proportional to x and opposite in direction, so the motion is simple harmonic with $\omega^2 = \rho gA/m$. At equilibrium the upthrust on the submerged volume Ah equals the weight, $\rho gAh = mg$, so $m = \rho Ah$ and $\omega^2 = g/h$: $T = 2\pi\sqrt{h/g}$ – the same form as a pendulum of length h.

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> **Worked example.** A U-tube of cross-sectional area A contains liquid of density ρ; the column in each arm has length L. The liquid is displaced so that one surface is x above its equilibrium level and the other x below. Show that the liquid oscillates with simple harmonic motion.

The two surfaces now differ in height by $2x$, so a column of liquid of height $2x$ is unbalanced: the restoring force on the liquid is the weight of that column, $2x\rho gA$, directed so as to level the surfaces. The mass being moved is the whole liquid, $2L\rho A$. So $a = -\dfrac{2x\rho gA}{2L\rho A} = -\dfrac{g}{L}x$: simple harmonic, with $\omega^2 = g/L$ and $T = 2\pi\sqrt{L/g}$. Note that ρ and A cancel – the period depends on neither the liquid nor the tube.

Displacement, velocity, acceleration in SHM

If the particle starts at $x = 0$ moving in the positive direction at $t = 0$, then

$$x = x_0 \sin(\omega t).$$

Differentiating once gives the **velocity** 速度:

$$v = x_0\omega\cos(\omega t) = v_0\cos(\omega t),$$

where $v_0 = x_0\omega$ is the **maximum speed** (as the particle passes through equilibrium). Differentiating again gives the acceleration:

$$a = -x_0\omega^2\sin(\omega t) = -\omega^2 x,$$

which is the SHM defining equation again. (If the particle instead starts at the extreme position $x = x_0$ at $t = 0$, use $x = x_0\cos(\omega t)$. Choose the one that fits the start conditions.)

Velocity in terms of displacement

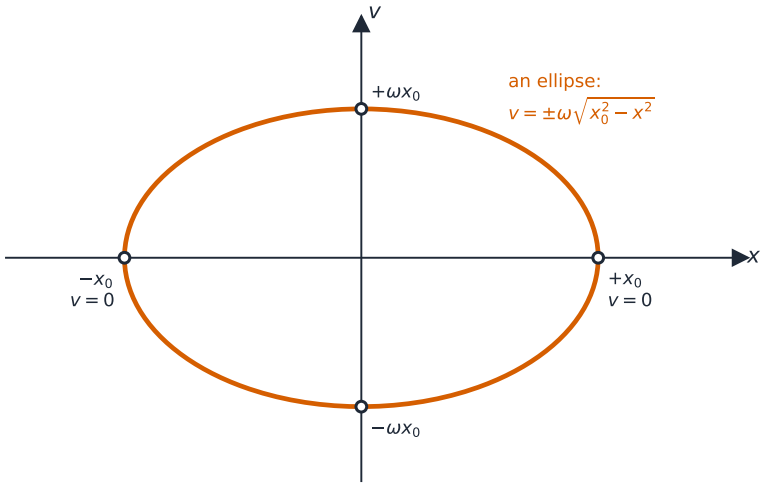
A useful relation that does not use time:

$$v = \pm\omega\sqrt{x_0^2 - x^2}.$$

- at equilibrium ($x = 0$): $v = \pm\omega x_0$ (maximum speed). Both signs, because the particle passes through equilibrium twice each cycle.
- at the extremes ($x = \pm x_0$): $v = 0$ (at rest for an instant).

> **Worked example.** The same oscillation has amplitude $x_0 = 0.050$ m and angular frequency $\omega = 15.7$ rad s^{-1}. Find the speed when the displacement is $x = 0.030$ m.

$$v = \omega\sqrt{x_0^2 - x^2} = 15.7\sqrt{0.050^2 - 0.030^2} = 15.7 \times 0.040 \approx 0.63 \text{ m s}^{-1}.$$



Velocity against displacement is an ellipse: $v = 0$ at the extremes and $v = \pm\omega x_0$ through equilibrium – read x_0 and v_0 from where it cuts the axes, then $\omega = v_0/x_0$

Reading the v–x graph. Its width gives the amplitude ($x = \pm x_0$ where $v = 0$) and its height the maximum speed ($v = \pm\omega x_0$ at $x = 0$), so $\omega = v_0/x_0$ and the period follows. Asked to sketch a against x on the same axes, draw the straight line through the origin with negative gradient $-\omega^2$, reaching $\mp\omega^2 x_0$ at $x = \pm x_0$.

> **Worked example.** The velocity of an oscillating block varies with time as $v = 0.56\sin(5.1t)$, with v in m s^{-1} and t in seconds. Find the amplitude, the period and the maximum acceleration.

Compare with $v = v_0\sin\omega t$: $v_0 = 0.56$ m s^{-1} and $\omega = 5.1$ rad s^{-1}. Since $v_0 = \omega x_0$, the amplitude is $x_0 = 0.56/5.1 = 0.11$ m; $T = 2\pi/\omega = 1.2$ s; $a_{\max} = \omega^2 x_0 = \omega v_0 = 5.1 \times 0.56 = 2.9$ m s^{-2}. (Because this v is a *sine*, the block was at an extreme at $t = 0$: its displacement is a cosine.)

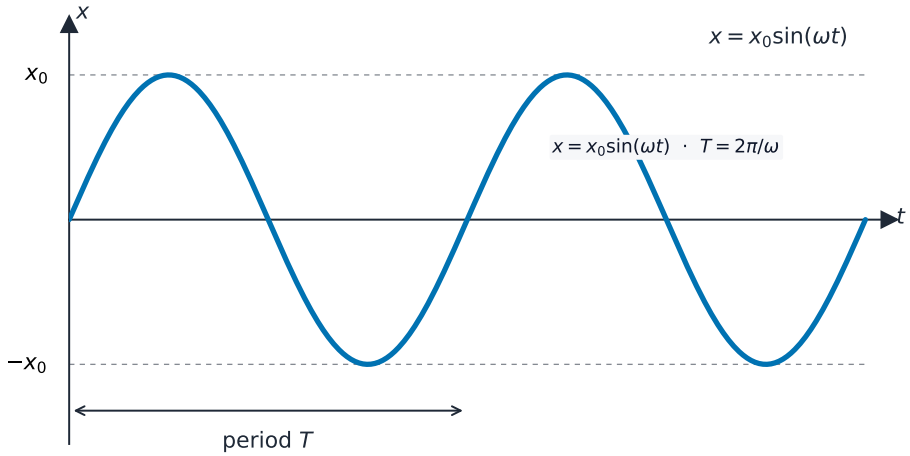
SHM as the shadow of circular motion. A ball moving round a circle of radius R at angular speed ω, lit from the side, casts a shadow that moves back and forth with simple harmonic motion of amplitude R and the same ω: the shadow's displacement is $x = R\cos\omega t$ (if it starts at an extreme), its maximum speed is $R\omega$ and its maximum acceleration $R\omega^2$. For a circle of diameter 0.46 m and $\omega = 1.9$ rad s^{-1} the shadow has amplitude 0.23 m, $v_0 = 0.44$ m s^{-1} and $a_{\max} = 0.83$ m s^{-2} – greatest in size at the two ends of its path and always directed back towards the centre.

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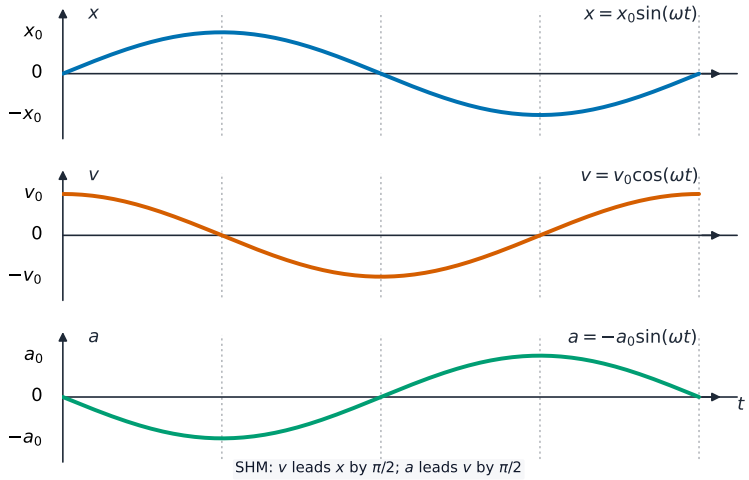
Graphs against time

For $x = x_0\sin\omega t$:

- x vs t – a sine curve, amplitude x_0, period $T = 2\pi/\omega$.
- v vs t – a cosine curve, **leading** x **by** $\pi/2$, amplitude ωx_0.
- a vs t – a negative sine curve, **out of phase with** x **by** π (180°), amplitude $\omega^2 x_0$.



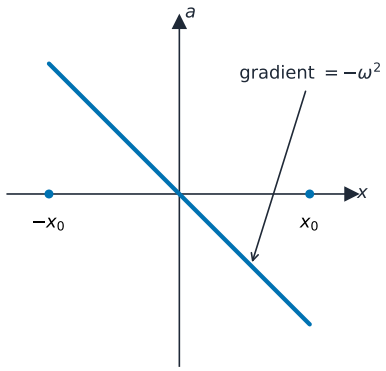
Displacement varies sinusoidally with time in simple harmonic motion



Velocity leads displacement by a quarter cycle; acceleration is exactly out of phase with displacement

Graph of a against x

A straight line through the origin with **negative gradient** $-\omega^2$. So you can read ω from the graph: gradient $= -\omega^2$, so $\omega = \sqrt{|\text{gradient}|}$, then $T = 2\pi/\omega$. This is a common exam pattern.



SHM: $a = -\omega^2 x$ · gradient $= -\omega^2$

The acceleration–displacement graph is a straight line through the origin with gradient $-\omega^2$

> **Worked example.** A graph of the height h of an object of mass 36 kg undergoing vertical simple harmonic motion shows h varying between 2 m and 10 m, with one complete oscillation every 4.0 s. Find the amplitude, the angular frequency, the maximum speed, the maximum acceleration and the maximum kinetic energy.

Amplitude $x_0 = (10 - 2)/2 = 4.0$ m (half the peak-to-peak range; the equilibrium height is 6 m). $\omega = 2\pi/T = 2\pi/4.0 = 1.57$ rad s^{-1}. $v_{\max} = \omega x_0 = 6.3$ m s^{-1}; $a_{\max} = \omega^2 x_0 = 9.9$ m s^{-2}; $E_{\mathrm{K,max}} = \frac{1}{2} m v_{\max}^2 = \frac{1}{2} \times 36 \times 6.28^2 = 7.1 \times 10^2$ J. Reading the amplitude as the full height range instead of half of it is the commonest slip here.

> **Worked example.** A small object rests on a horizontal platform that oscillates vertically with simple harmonic motion of amplitude 6.0 mm. Find the highest frequency at which the object stays in contact with the platform throughout, and state where in the motion contact is first lost.

The object leaves the platform when the platform's downward acceleration exceeds g – at the **top** of the oscillation, where the acceleration is greatest and directed downwards: the platform then falls away faster than the object can. The limit is $\omega^2 x_0 = g$: $\omega = \sqrt{9.81/0.0060} = 40$ rad s^{-1}, so $f = \omega/2\pi = 6.4$ Hz. Below this the normal contact force at the top is small but positive; above it, zero.

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Energy in simple harmonic motion

A simple harmonic oscillator keeps swapping energy between two forms:

- **kinetic energy** 动能 $E_{\mathrm{K}} = \frac{1}{2} m v^2$.
- **potential energy** E_{P} (elastic for a spring, gravitational for a pendulum).

With no **damping** 阻尼, the **total energy** is constant (this is **conservation of energy** 能量守恒).

Maximum and minimum

- at **equilibrium** ($x = 0$): v is largest, so E_{K} is largest and E_{P} is smallest (zero, by choice).
- at the **extremes** ($x = \pm x_0$): $v = 0$, so $E_{\mathrm{K}} = 0$ and E_{P} is largest.

Total energy

Using $v_{\max} = \omega x_0$:

$$E_{\text{total}} = \tfrac{1}{2} m v_{\max}^2 = \tfrac{1}{2} m \omega^2 x_0^2.$$

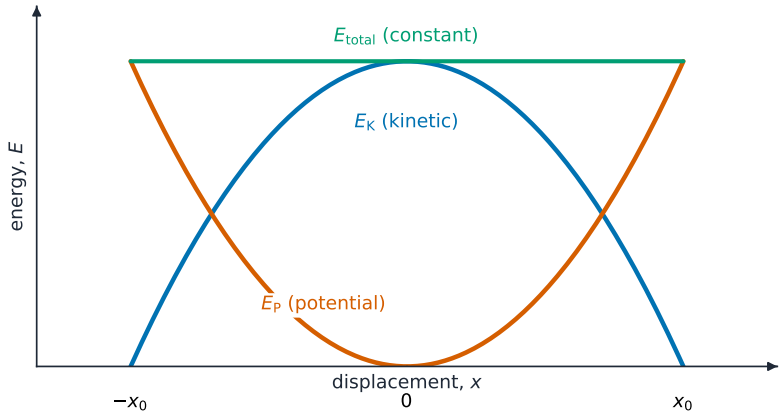
Two key facts: the total energy is proportional to the **square of the amplitude** (doubling x_0 gives four times the energy), and to ω^2.

Energy against displacement

Using $v^2 = \omega^2 (x_0^2 - x^2)$:

$$E_{\mathrm{K}} = \tfrac{1}{2} m \omega^2 (x_0^2 - x^2), \qquad E_{\mathrm{P}} = \tfrac{1}{2} m \omega^2 x^2.$$

So E_{K} is a downward parabola (peak at $x = 0$, zero at $x = \pm x_0$) and E_{P} is an upward parabola (zero at $x = 0$, largest at $x = \pm x_0$). Their sum is constant.



SHM: $E_k + E_p =$ const · max E_k at centre

Kinetic and potential energy swap over a cycle while the total energy stays constant

> **Worked example.** A 0.20 kg mass oscillates with amplitude $x_0 = 0.050$ m and angular frequency $\omega = 15.7$ rad s^{-1}. Find the total energy of the oscillation.

The total energy equals the maximum kinetic energy, as the mass passes through $x = 0$ at $v_{\max} = \omega x_0$:

$$E = \tfrac{1}{2} m\omega^2 x_0^2 = \tfrac{1}{2}(0.20)(15.7)^2(0.050)^2 \approx 0.062 \text{ J}.$$

This stays constant, swapping between kinetic and potential form twice each cycle.

Describing the interchange without calculation. At an extreme of the motion the oscillator is momentarily at rest: its kinetic energy is zero and its potential energy is a maximum. As it moves towards equilibrium the restoring force does work on it, so potential energy is converted to kinetic energy; at the equilibrium position the kinetic energy is a maximum and the potential energy a minimum. Beyond it the kinetic energy is converted back to potential energy until the other extreme. The total stays constant, and the exchange happens **twice per oscillation**, so the energy graphs against time have twice the frequency of the displacement graph.

> **Worked example.** A pendulum bob of mass 0.81 kg swings with small oscillations of amplitude 32 mm and period 1.9 s. Find the total energy of the oscillation, and state what happens to it if the amplitude falls to 16 mm.

$\omega = 2\pi/1.9 = 3.31$ rad s^{-1}; $E = \tfrac{1}{2} m\omega^2 x_0^2 = \tfrac{1}{2} \times 0.81 \times 3.31^2 \times 0.032^2 = 4.5 \times 10^{-3}$ J. Halving the amplitude quarters the energy ($E \propto x_0^2$), to 1.1×10^{-3} J; the difference has been dissipated as thermal energy by air resistance.

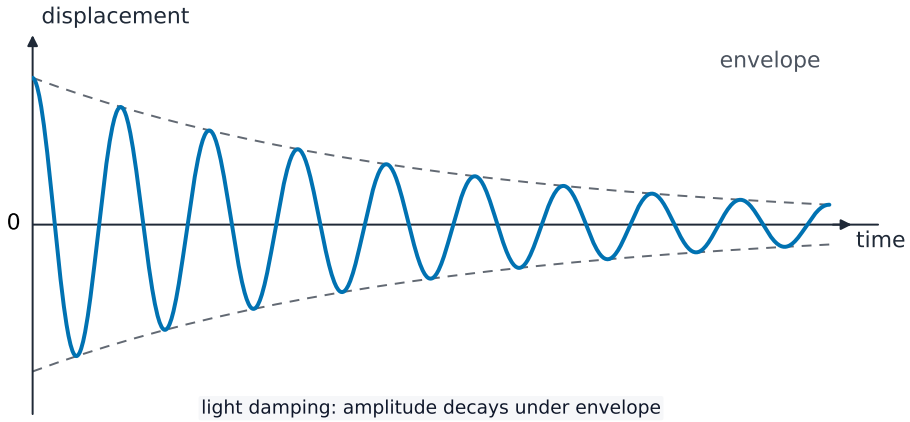
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Damped oscillations

A resistive force (**friction** 摩擦力, **drag** 阻力, **air resistance** 空气阻力) causes **damping** – the amplitude shrinks over time as energy is lost as heat. Three named cases:

Light damping

The amplitude **shrinks slowly** over many cycles (a **light damping** 轻阻尼 case). The system still oscillates near its natural frequency, but each cycle is smaller than the last. A car's suspension is light-to-medium damped, so bumps die away but the ride stays smooth.



light damping: amplitude decays under envelope

In light damping the amplitude dies away slowly over many cycles

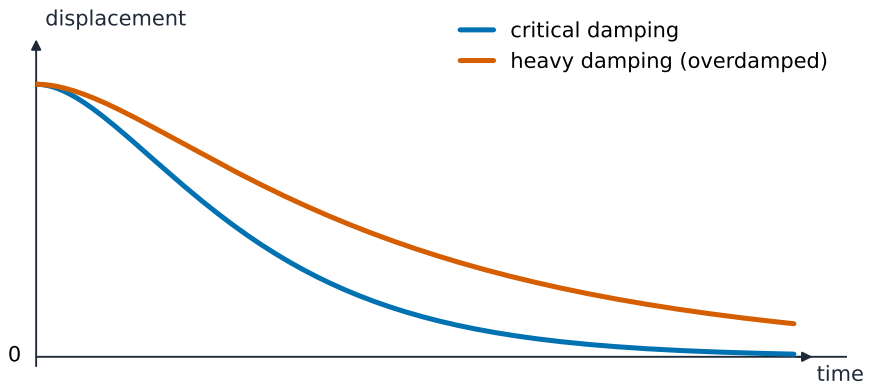
Critical damping

The least damping that brings the system back to equilibrium **without overshooting and without oscillating** – a **critical damping** 临界阻尼 case. It returns in the shortest time. A **galvanometer** 检流计 or analogue **voltmeter** 电压表 is critically damped so the needle settles quickly.

Heavy damping

So much resistance that the system returns slowly, with no oscillation, but more slowly than the critical case – a **heavy damping** 过阻尼 case. A door with a strong closer is heavily damped.

On a displacement–time graph: light damping is a wave whose size dies away smoothly; critical damping returns quickly with no overshoot; heavy damping returns slowly.



critical: fastest no-oscillation return · heavy: overdamped

Critical damping returns to equilibrium fastest without overshoot; overdamping returns more slowly

As the exam asks it. *Damping* is the decrease in the amplitude of an oscillation caused by a resistive force that dissipates the oscillator's energy. In **light damping** the system oscillates with an amplitude that decreases gradually (exponentially) with time; the period is almost unchanged. In **critical damping** the system returns to equilibrium in the shortest possible time without oscillating. In **heavy damping** it returns to equilibrium slowly, without oscillating. Sketching them: the light-damping curve is a sine wave inside a shrinking envelope; the critical and heavy curves both fall to zero without crossing the axis, the heavy one more slowly.

> **Worked example.** A bar magnet hangs from a spring with one pole inside a coil that is connected through a switch to a resistor. The magnet is set oscillating. Explain why the oscillations die away more quickly when the switch is closed.

The moving magnet changes the magnetic flux linking the coil, so an e.m.f. is induced (Faraday's law); with the switch closed a current flows through the resistor and dissipates energy as heat. By Lenz's law the induced current opposes the motion of the magnet – an extra resistive force. That energy comes from the oscillation, so the amplitude decreases faster: the circuit adds damping. With the switch open there is an e.m.f. but no current, so no extra energy loss.

Forced oscillations and resonance

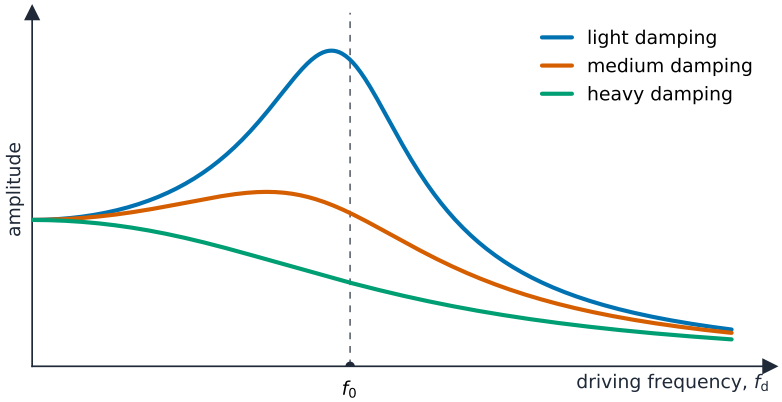
A **forced oscillation** 受迫振动 is driven by an outside periodic force at a frequency f_d chosen by the experimenter. The system then oscillates at this **driving frequency** 驱动频率 f_d, not at its own natural frequency. A plot of amplitude against f_d is a **resonance curve** 共振曲线 with a peak.



A plucked guitar string vibrates at its resonant frequencies.

Resonance

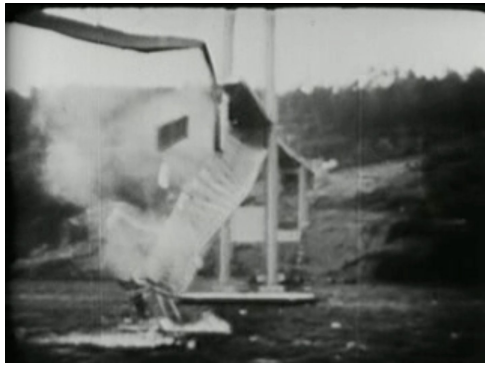
Resonance 共振 happens when the driving frequency equals the system's **natural frequency** 固有频率 f_0. At resonance the amplitude is **largest** and the energy transfer from the driver is most efficient.



resonance: max amplitude near f_0 · lighter damping ⇒ sharper peak

The amplitude of a forced oscillation peaks at resonance, when the driving frequency equals the natural frequency

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Resonance can destroy. In 1940 the wind pushed the Tacoma Narrows Bridge close to its natural frequency; with little damping the twisting grew and grew until the deck ripped apart. Engineers now design bridges and buildings so their natural frequencies avoid such driving forces

Examples:

- a swing pushed at the right rate builds up a large amplitude.
- a wine glass broken by a sound at its natural ringing frequency.
- a building shaken by an earthquake whose frequency matches a natural frequency – engineers design buildings so their natural frequencies avoid the main earthquake range.

The peak's shape depends on damping: **lighter damping → a sharper, higher peak**; **heavier damping → a broader, lower peak**, shifted slightly to lower frequency.

The two-mark definition. *Resonance is the condition in which a system is forced to oscillate at its natural frequency, and the amplitude of the oscillation is a maximum* (the transfer of energy from the driver is greatest). Both parts – driving frequency equal to natural frequency, and maximum amplitude – are needed.

> **Worked example.** A ball on a stretched string has a natural frequency of 4.0 Hz. The string is driven by a vibration generator whose frequency is increased slowly from 0 to 10 Hz. Sketch the variation of the amplitude of the ball with the driving frequency, and state the effect of adding damping.

The curve starts at a small, non-zero amplitude at low frequency, rises to a sharp peak at 4.0 Hz, and falls away towards zero at high frequency; the peak is at the natural frequency, and only there is the amplitude a maximum. More damping makes the peak **lower and broader** and shifts it slightly to a lower frequency; the amplitude far from resonance hardly changes.

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Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
simple harmonic motion	motion in which the acceleration is proportional to the displacement from a fixed point and is always directed towards that point
amplitude	the maximum displacement from the equilibrium position
period	the time for one complete oscillation
frequency	the number of complete oscillations per unit time
angular frequency	$\omega = 2\pi f = 2\pi/T$; the rate at which the phase of the oscillation changes
phase difference	the fraction of a cycle, expressed as an angle, by which one oscillation leads or lags another
damping	the reduction in amplitude of an oscillation caused by a resistive force that dissipates its energy
critical damping	the damping that returns a displaced system to equilibrium in the shortest time without oscillation
natural frequency	the frequency at which a system oscillates when displaced and left to oscillate freely
resonance	the forced oscillation of a system at its natural frequency, at which the amplitude is a maximum

Exam tips

- The **SHM condition is** $a = -\omega^2 x$ (acceleration proportional to displacement, directed back to equilibrium); to *show* a motion is SHM, find the restoring force, show it is proportional to x and directed back, and read off ω^2.
- Learn $x = x_0 \sin \omega t$ (or cos), $v_{max} = \omega x_0$, $a_{max} = \omega^2 x_0$ and $v = \pm\omega\sqrt{x_0^2 - x^2}$; KE and PE interchange with the **total energy constant**, $E = \frac{1}{2}m\omega^2 x_0^2 \propto x_0^2$.
- Velocity is **zero at the extremes** and maximum at the centre; acceleration is greatest at the extremes and zero at the centre.
- The amplitude is **half** the peak-to-peak range of a displacement graph; ω comes from the period, or from the gradient of a–x, or from v_0/x_0.

- **Resonance** occurs when the driving frequency equals the natural frequency; damping lowers and broadens the peak.

Common mistakes

- Defining SHM as "oscillation about an equilibrium position". The mark scheme wants acceleration proportional to displacement *and* directed towards the fixed point.
- Saying the minus sign means the acceleration is negative. It means the acceleration is opposite in direction to the displacement.
- Sketching v against x as a straight line. It is an ellipse; a against x is the straight line.
- Taking the amplitude as the whole range of the displacement. It is half the range.
- Defining resonance as "a large amplitude". It is forced oscillation *at the natural frequency*, giving the *maximum* amplitude.
- Calling critical damping "no damping", or heavy damping "the fastest return". Critical damping is the fastest return without oscillation; heavy damping is slower.

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Electric fields

A-Level Physics



Lightning is a giant spark driven by a huge electric field.

An **electric field** 电场 is a region where a charge feels a **force** 力 from other charges. The **electric field strength** 电场强度 E at a point is the **force per unit positive charge** on a small positive **test charge** 检验电荷 placed there:

$$E = \frac{F}{q}.$$

Unit: N C^{-1} (the same as V m^{-1}, as we will see). E is a **vector** 矢量, pointing the way the force acts on a **positive** charge. The force on a charge q is

$$F = qE,$$

opposite to the field if q is negative.

The two-mark definition. *Electric field strength at a point is the force per unit positive charge acting on a small (test) charge placed at that point.* Three words carry the marks: **force per unit charge** (not "force on a charge"), **positive** (which fixes the direction), and **small** or **test** (so the charge does not disturb the field it is measuring). The multiple-choice version offers "force per unit charge acting on a small mass" and "force per unit mass" as distractors: a field is defined by a **charge**, never by a mass.

> **Worked example.** A proton accelerates at 2.00 m s^{-2} in an electric field, with no other force acting. Find the field strength. ($m_p = 1.67 \times 10^{-27}$ kg, $e = 1.60 \times 10^{-19}$ C.)

$qE = ma$, so

$$E = \frac{ma}{q} = \frac{(1.67 \times 10^{-27})(2.00)}{1.60 \times 10^{-19}} = 2.09 \times 10^{-8}\ \text{V m}^{-1} = 20.9\ \text{nV m}^{-1}.$$

The same rearrangement the other way round gives the acceleration of an electron in a field of 1500 V m^{-1}: $a = eE/m_e = (1.60 \times 10^{-19})(1500)/(9.11 \times 10^{-31}) = 2.6 \times 10^{14}$ m s^{-2}, enormous because the electron's mass is so small. Gravity (9.81 m s^{-2}) is negligible beside it, which is why exam questions about electrons between plates ignore weight.

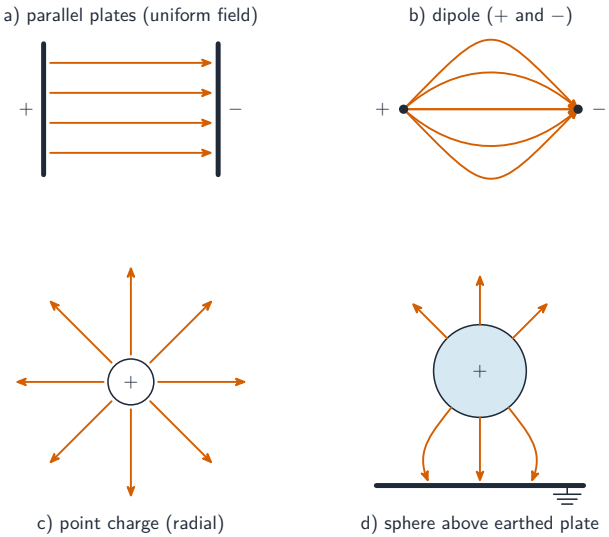
Field lines

- **field lines** 场线 point the way the force acts on a positive test charge.
- lines start on **positive** charges and end on **negative** charges (or go to infinity).
- lines never cross; closer lines mean a stronger field.
- every line carries an **arrow**; lines meet the surface of a conductor at right angles.

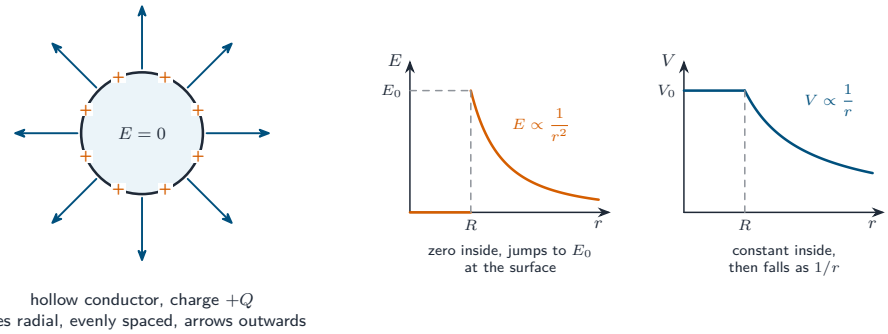
What a field line represents. Asked "state what is represented by an electric field line", give both halves: its **direction** shows the direction of the force on a **positive** charge placed there, and the **spacing** of the lines shows the strength of the field (closer lines, stronger field). A sketch is marked on exactly these points: arrows drawn, lines not crossing, even spacing where the field is uniform, and radial lines for a point charge or a sphere.

Examples: a positive **point charge** 点电荷 has radial lines pointing out; a negative one has lines pointing in; two opposite charges (a **dipole** 偶极子) have lines curving from + to −; two parallel charged plates give a **uniform field** 匀强场 of equally spaced parallel lines.

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Field-line patterns for parallel plates, a dipole, a point charge, and a charged sphere above an earthed plate



A charged hollow conductor: the field outside is that of a point charge at the centre; inside, the field is zero and the potential is constant

A charged conducting sphere

The charge on an **isolated** 孤立的 **conductor** 导体 sits on its **outer surface**, spread evenly when the sphere is on its own. Outside the sphere the field lines are **radial**, evenly spaced and pointing outwards (for positive charge), exactly the pattern of a point charge at the centre. **Inside** a hollow (or solid) conductor the field is **zero**: the fields of all the surface charges cancel everywhere inside, so no field line enters. Two

graphs follow, and both are exam favourites. The field strength E is zero out to the radius R, **jumps** to its surface value E_0 and then falls as $1/r^2$; the potential V is **constant** inside (a zero field means a zero potential gradient) and then falls as $1/r$.

Reading the graphs. Given an E–x graph for a charged sphere, the **radius** is the distance at which E jumps from zero, and the **charge** comes from any point on the curve: $Q = 4\pi\varepsilon_0 r^2 E$. Given "$E_0$ at the surface", the value at $2R$ is $E_0/4$ and at $3R$ is $E_0/9$: sketch the curve through those points, starting at E_0 on the surface line, never from the origin.

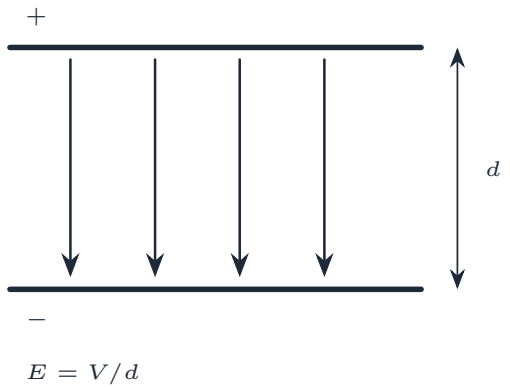


A Van de Graaff generator stores a large static charge on its metal dome, making a strong electric field around it

Uniform electric fields

Between two parallel plates a distance d apart with **potential difference** 电势差 V between them, the field is uniform (apart from edge effects) with size

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Between parallel plates the field is uniform, $E = V/d$

$$E = \frac{V}{d}.$$

It points from the higher-potential plate to the lower one. The unit V m^{-1} comes straight from this and equals N C^{-1}.

> **Worked example.** Two parallel plates 5.0 mm apart have a p.d. of 200 V between them. Find the field strength, and the force on an electron in the gap. ($e = 1.6 \times 10^{-19}$ C.)

$$E = \frac{V}{d} = \frac{200}{5.0 \times 10^{-3}} = 4.0 \times 10^4 \text{ V m}^{-1}, \qquad F = qE = (1.6 \times 10^{-19})(4.0 \times 10^4) = 6.4 \times 10^{-15} \text{ N}.$$

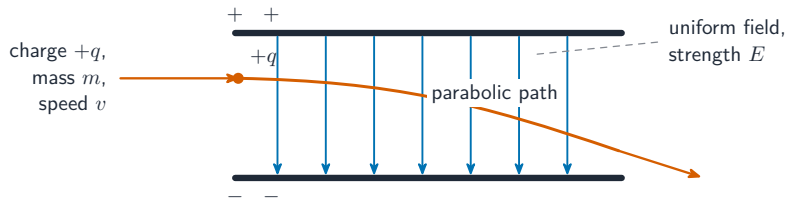
Which changes the field? Two things only: $E = V/d$ rises if the p.d. is **increased** or the plates are moved **closer**. A resistor in series with the supply changes nothing: no current flows once the plates are charged, so there is no p.d. across the resistor and the plates sit at the full supply voltage. Between plates at +800 V and +1300 V the field points from the **higher** potential to the lower; two positive potentials change nothing about the direction rule.

A charged particle in a uniform field

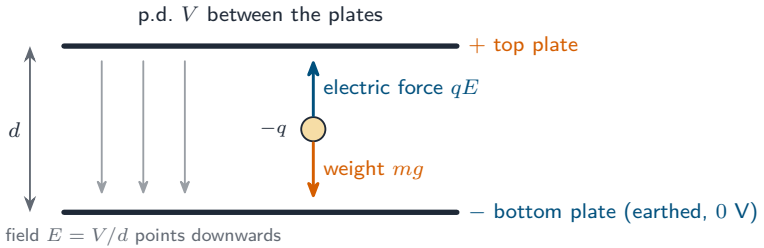
A charge q in a uniform field feels a **constant force** $F = qE$, so a **constant acceleration** $a = qE/m$ – just like a mass in a uniform gravitational field.

- released **at rest**, it speeds up along the field (positive charge) or against it (negative charge), gaining **kinetic energy** 动能.

- entering **at right angles** to the field, it follows a **parabolic** 抛物线 path – like a **projectile** 抛体 in gravity. This is how a **cathode-ray tube** 阴极射线管 used to steer its beam.



A charge entering a uniform field at right angles follows a parabolic path, like a projectile



$$\text{stationary} \Rightarrow qE = mg \Rightarrow q = \frac{mgd}{V}$$

A charged oil drop held stationary between horizontal plates: the electric force balances the weight, so $q = mgd/V$

Two forces, not one. A charged oil drop "held stationary" between horizontal plates is the exam's favourite **equilibrium** 平衡: **both** an electric force and its weight act on it (the multiple-choice distractor is "electric force only"), and they are equal and opposite. For a **negative** drop the electric force is opposite to the field, so the top plate must be **positive** to hold it up.

> **Worked example.** An oil drop of mass 2.6×10^{-15} kg carries a charge of -4.8×10^{-19} C and is held stationary in a vacuum between horizontal plates 2.0 cm apart. Find the p.d. between the plates, and say which plate is positive.

$$qE = mg \;\Rightarrow\; \frac{qV}{d} = mg \;\Rightarrow\; V = \frac{mgd}{q} = \frac{(2.6\times10^{-15})(9.81)(0.020)}{4.8\times10^{-19}} = 1.1\times10^{3}\ \text{V}.$$

The drop is negative, so the force on it is against the field; for the force to be upwards

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the field must point **down**, so the **top** plate is positive. Notice that the charge is $3e$, three excess electrons, which is how Millikan showed that charge comes in multiples of e.

> **Worked example.** Two parallel plates in a vacuum are 0.041 m apart with a p.d. of 250 V between them. An electron is released from rest at the negative plate. Find (a) the field strength, (b) the force on the electron and its acceleration, (c) the time it takes to reach the positive plate, (d) its kinetic energy on arrival. ($m_e = 9.11 \times 10^{-31}$ kg.)

(a) $E = V/d = 250/0.041 = 6.1\times10^{3}$ V m^{-1}. (b) $F = eE = (1.60\times10^{-19})(6.1\times10^{3}) = 9.8\times10^{-16}$ N; $a = F/m_e = 1.07\times10^{15}$ m s^{-2}, towards the positive plate. (c) From rest with constant acceleration, $d = \frac{1}{2}at^2$, so $t = \sqrt{2d/a} = \sqrt{2\times0.041/(1.07\times10^{15})} = 8.8\times10^{-9}$ s. (d) Energy method, no kinematics needed: $E_k = qV = (1.60\times10^{-19})(250) = 4.0\times10^{-17}$ J, so $v = \sqrt{2E_k/m_e} = 9.4\times10^{6}$ m s^{-1}. (Check: $v = at = (1.07\times10^{15})(8.8\times10^{-9}) = 9.4\times10^{6}$ m s^{-1}.)

> **Worked example (deflection).** An electron travelling horizontally at 2.0×10^{7} m s^{-1} enters the field between two horizontal plates 18 mm apart with a p.d. of 400 V. The plates are 30 mm long. Find the **deflection** 偏转 of the electron by the time it leaves the field.

The horizontal motion is unaffected: time in the field $t = 0.030/(2.0\times10^{7}) = 1.5\times10^{-9}$ s. Vertically, $a = eE/m_e = eV/(m_e d) = (1.60\times10^{-19})(400)/[(9.11\times10^{-31})(0.018)] = 3.9\times10^{15}$ m s^{-2}, so the deflection is $y = \frac{1}{2}at^2 = \frac{1}{2}(3.9\times10^{15})(1.5\times10^{-9})^2 = 4.4\times10^{-3}$ m, less than the half-gap of 9 mm, so the electron does leave the field. Its vertical velocity on exit is $at = 5.9\times10^{6}$ m s^{-1}, so it leaves at $\tan\theta = 5.9/20$, about 16°; the path is a parabola, and **after** the plates (no field) it travels in a straight line in the direction it had on exit.

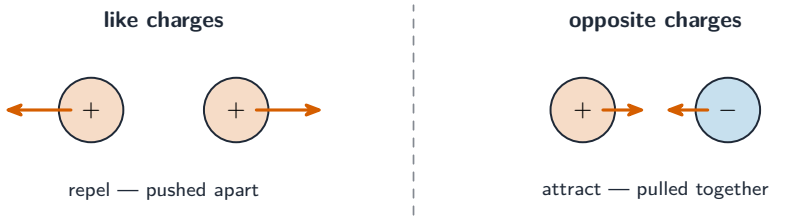
Coulomb's law

For two point charges Q_1 and Q_2 a distance r apart in free space, each feels a force of size

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}.$$

This is **Coulomb's law** 库仑定律. Here $\varepsilon_0 = 8.85\times10^{-12}$ F m^{-1} is the **permittivity of free space** 真空电容率, and $1/(4\pi\varepsilon_0) \approx 9.0\times10^{9}$ N m^2 C^{-2}. The force is along the line joining the charges: **repulsive** for like charges, **attractive** for opposite charges.

Stating the law in words. "State Coulomb's law" wants a sentence, not a formula: *the (electric) force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of their separation.* Both proportionalities are needed, and the charges must be **point** charges (or spheres treated as point charges at their centres) in a vacuum.



Like charges repel; opposite charges attract – the force is along the line joining them

Worked example. Find the electrostatic force between point charges of +2.0 nC and +3.0 nC placed 4.0 cm apart. ($1/(4\pi\varepsilon_0) = 9.0 \times 10^9$ N m^2 C^{-2}.)

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2} = \frac{(9.0\times10^9)(2.0\times10^{-9})(3.0\times10^{-9})}{(0.040)^2} \approx 3.4\times10^{-5}\ \text{N (repulsive)}.$$

Worked example. In a hydrogen atom the proton and the electron may be treated as point charges 120 pm apart. Find the electric force between them, and compare it with their gravitational attraction. ($m_\text{p} = 1.67\times10^{-27}$ kg, $m_\text{e} = 9.11\times10^{-31}$ kg, $G = 6.67\times10^{-11}$ N m^2 kg^{-2}.)

$$F_\text{E} = \frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{(8.99\times10^9)(1.60\times10^{-19})^2}{(1.20\times10^{-10})^2} = 1.6\times10^{-8}\ \text{N (attractive)}.$$

Gravity: $F_\text{G} = Gm_\text{p}m_\text{e}/r^2 = (6.67\times10^{-11})(1.67\times10^{-27})(9.11\times10^{-31})/(1.20\times10^{-10})^2 = 7.0\times10^{-48}$ N, about 10^{39} times smaller. Inside atoms, gravity is irrelevant.

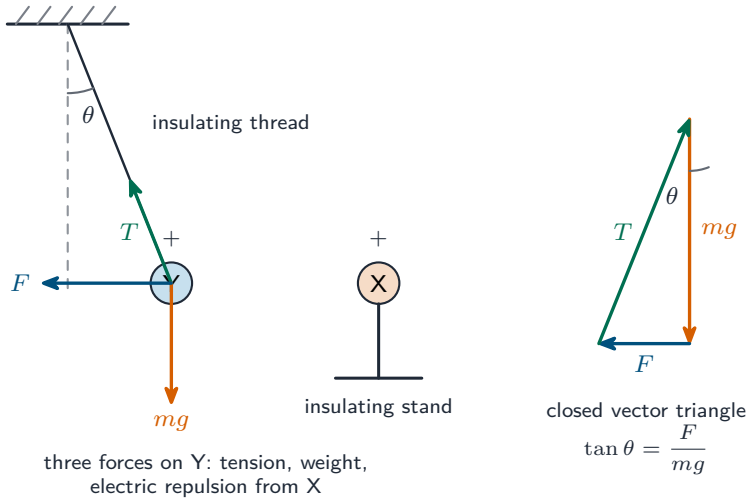
Worked example. Two identical oil droplets in a vacuum have their centres 3.8×10^{-6} m apart. Each carries the same charge, and they repel with a force of 1.0×10^{-15} N. Find the charge on each droplet, and the number of excess electrons it carries.

$$F = \frac{Q^2}{4\pi\varepsilon_0 r^2} \Rightarrow Q = \sqrt{4\pi\varepsilon_0 F r^2} = \sqrt{\frac{(1.0\times10^{-15})(3.8\times10^{-6})^2}{8.99\times10^9}} = 1.3\times10^{-18}\ \text{C},$$

that is $1.3\times10^{-18}/1.60\times10^{-19} \approx 8$ electrons. Identical objects with the same sign of charge always **repel**; do not write "attract" for the direction.

Worked example. A charged sphere X is fixed on an insulating stand. A second sphere Y, of mass 2.0 g, hangs beside it on an insulating thread and settles in equilibrium with the thread at 12° to the vertical, its centre 5.0 cm from the centre of X on the same horizontal level. Find the electric force on Y and, if the spheres carry equal charges, the charge on each.

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A charged sphere hanging near a charged sphere: three forces balance, the tension along the thread, the weight and the horizontal electric repulsion, so tan θ = F/mg

Three forces act on Y: its weight mg (down), the **tension** 张力 T (along the thread) and the electric force F (horizontal, away from X because like charges repel). Resolving, $T\cos\theta = mg$ and $T\sin\theta = F$, so

$$F = mg\tan\theta = (2.0\times10^{-3})(9.81)\tan12^\circ = 4.2\times10^{-3}\ \text{N}.$$

Then $F = Q^2/(4\pi\varepsilon_0 r^2)$ gives $Q = \sqrt{4\pi\varepsilon_0 F r^2} = \sqrt{(4.2\times10^{-3})(0.050)^2/(8.99\times10^9)} = 3.4\times10^{-8}$ C. A closed vector triangle of the three forces is an equally good method; the exam accepts either, but it must show all **three** forces.

Worked example (helium). A helium atom may be modelled as a nucleus of charge $+2e$ with two electrons in diametrically opposite circular orbits of radius 170 pm. Find the **resultant force** 合力 on one electron, and hence its orbital speed.

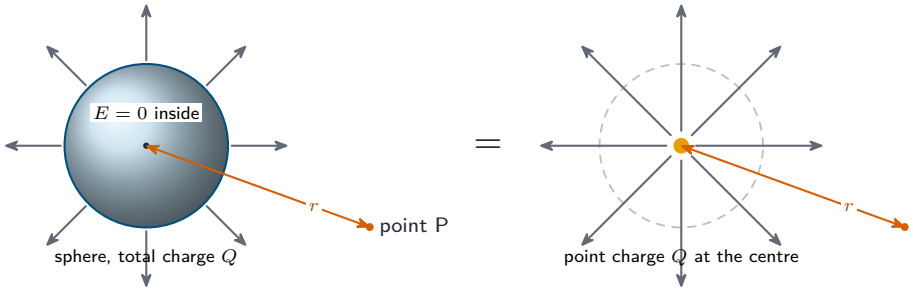
Two forces act on the electron: attraction to the nucleus, at distance r, and repulsion from the other electron, at distance $2r$ on the far side:

$$F_\text{res} = \frac{2e^2}{4\pi\varepsilon_0 r^2} - \frac{e^2}{4\pi\varepsilon_0 (2r)^2} = \frac{e^2}{4\pi\varepsilon_0 r^2}\left(2 - \tfrac{1}{4}\right) = 1.75\times\frac{(8.99\times10^9)(1.60\times10^{-19})^2}{(1.70\times10^{-10})^2} = 1.4\times10^{-8}\ \text{N},$$

towards the nucleus. This resultant is the centripetal force, $F = m_\text{e}v^2/r$, so $v = \sqrt{Fr/m_\text{e}} = \sqrt{(1.4\times10^{-8})(1.70\times10^{-10})/(9.11\times10^{-31})} = 1.6\times10^6$ m s^{-1}. The trap is to forget the second electron, or to put it at distance r rather than $2r$.

Spheres treated as point charges

A spherical conductor with total charge Q gives, at any point **outside**, the same field as a point charge Q at its centre (measure r from the centre). **Inside** a hollow charged conductor the field is zero, so the conductor is an **equipotential** 等势面.



at P, outside: $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$ in both, with r from the centre

Outside the sphere, use a point charge at its centre: measure r from the centre, not the surface

Electric field due to a point charge

Coulomb's law gives the force between **two** charges. Divide it by the test charge ($E = F/q$) and you are left with the field of the source charge Q **alone**. The field at distance r from a point charge Q is

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}.$$

It points out from a positive Q, in towards a negative Q, and falls as $1/r^2$ – just like **gravitational field** 重力场 strength, except gravity is always attractive. For several charges, add the fields as a **vector sum** 矢量和.

> **Worked example.** The Earth may be treated as a uniform conducting sphere of radius 6.37×10^6 m carrying a total charge of -4.80×10^5 C spread over its surface. Find the electric field strength at the surface, and compare it with the gravitational field strength there. ($M_\mathrm{E} = 5.98 \times 10^{24}$ kg.)

$$E = \frac{Q}{4\pi\varepsilon_0 R^2} = \frac{(8.99 \times 10^9)(4.80 \times 10^5)}{(6.37 \times 10^6)^2} = 106 \text{ V m}^{-1},$$

directed **towards** the centre (the charge is negative). Gravity: $g = GM/R^2 = 9.83$ N kg^{-1}, so $E/g \approx 11$; in symbols $E/g = Q/(4\pi\varepsilon_0 GM)$, which shows that the ratio depends only on the charge-to-mass ratio of the sphere. A charged sphere of mass m and charge q would need $qE = mg$ to float in this field: $q/m = g/E = 0.093$ C kg^{-1}.

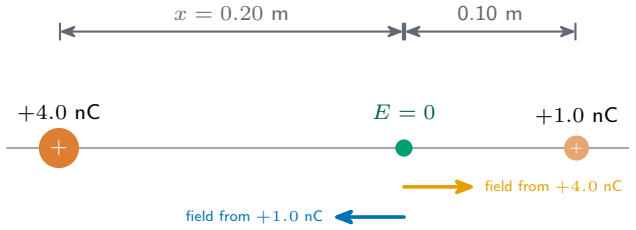
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> **Worked example.** Point charges of +4.0 nC and +1.0 nC are 30 cm apart. Where on the line joining them is the resultant field zero?

Between the charges the two fields point in opposite directions; call the distance from the 4.0 nC charge x. Zero resultant needs equal magnitudes:

$$\frac{4.0}{x^2} = \frac{1.0}{(0.30 - x)^2} \Rightarrow \frac{0.30 - x}{x} = \frac{1}{2} \Rightarrow x = 0.20 \text{ m},$$

twice as far from the larger charge. For **opposite** charges the fields between them point the same way and never cancel: the zero lies **outside**, beyond the smaller charge.



$4.0/x^2 = 1.0/(0.30 - x)^2$: twice as far from the larger charge

Between like charges the fields oppose; they cancel where the inverse squares balance.

Electric potential

Electric potential 电势 V at a point is the **work done per unit positive charge** in bringing a small positive test charge from **infinity** 无穷远 to that point:

$$V = \frac{W}{q}.$$

Unit: V. The potential is **zero at infinity**. For a positive source charge $V > 0$ everywhere outside; for a negative source charge $V < 0$.

The two-mark definition. *Electric potential at a point is the work done per unit positive charge in bringing a small test charge from infinity to the point.* The marks are for **work done per unit (positive) charge** and **from infinity to the point**. The potential of a positive charge is positive because work must be **done on** a positive test charge to push it in against the repulsion; near a negative charge the field does the work, so the potential is negative. "Explain why the potential near an isolated proton is positive" is answered in exactly those words.

Potential due to a point charge

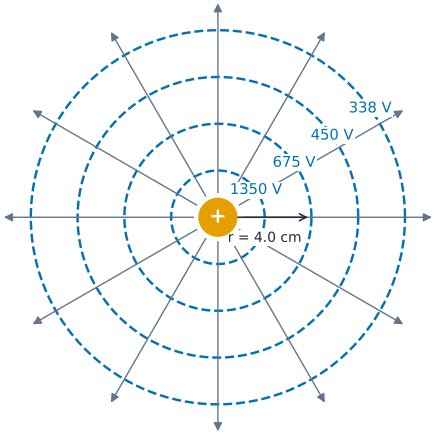
$$V = \frac{Q}{4\pi\varepsilon_0 r}.$$

Note the $1/r$ here (compared with $1/r^2$ for the field). V is a **scalar** 标量; for several charges, add the potentials (with sign).

> **Worked example.** Find the electric potential 4.0 cm from a point charge of +3.0 nC. $(1/(4\pi\varepsilon_0) = 9.0 \times 10^9$ N m^2 C^{-2}.)

$$V = \frac{Q}{4\pi\varepsilon_0 r} = \frac{(9.0\times10^9)(3.0\times10^{-9})}{0.040} \approx 6.8\times10^2 \text{ V}.$$

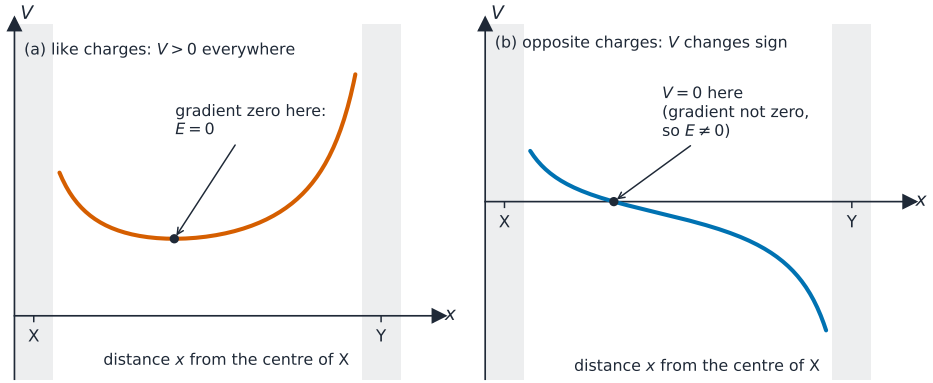
Because potential is a scalar, the potential from several charges is simply their sum (each with its own sign) – no directions to resolve.



+3.0 nC: blue circles are equipotentials, grey arrows field lines

Equipotentials around the +3.0 nC charge: V falls as $1/r$, and every equipotential crosses the field lines at right angles

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Potential along the line between two charged spheres: like charges give a minimum (field zero there); opposite charges give a zero crossing

Two charged spheres on a line

The exam's favourite structured question puts two charged spheres X and Y some distance apart and draws the potential V along the line joining their centres (or asks you to reason from it). Everything follows from two facts: potential is a **scalar** that adds with sign, and the field is the **negative gradient** of the graph.

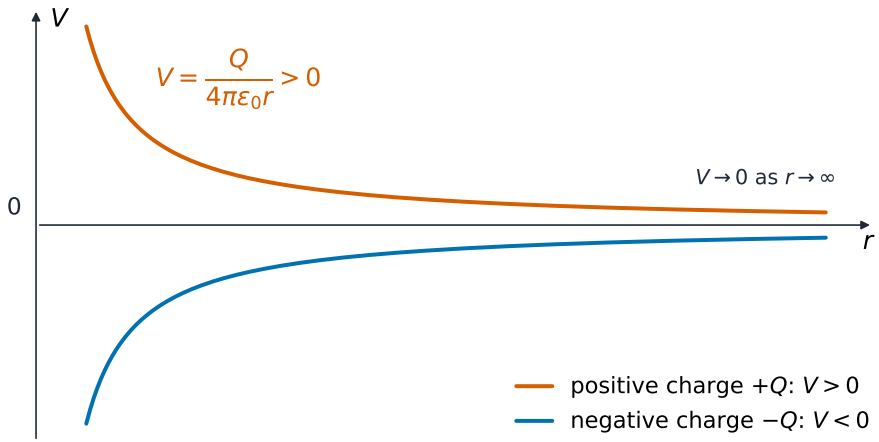
- **Sign of the charges.** If V is positive along the whole line, both charges are positive; if V changes sign, the charges are opposite (positive near the positive sphere).
- **Where the field is zero.** Where the curve has a **minimum** (gradient zero). For like charges this lies between them, closer to the smaller charge: $Q_X/x^2 = Q_Y/(d-x)^2$. For opposite charges there is **no** such point between them: the curve crosses zero there, but its gradient is not zero, so the field is not.
- **Ratio of the charges.** Where $V = 0$ between opposite charges, $Q_X/x = Q_Y/(d-x)$: the distances give the ratio directly. For like charges use the minimum instead (the square root of the field condition).
- **Force on a charge placed at P.** Read the **gradient** at P by drawing a **tangent** 切线; then E is minus the gradient, and $F = qE$.

> **Worked example.** Two isolated charged metal spheres X and Y have their centres 1.2 m apart in a vacuum. The potential along the line between them is positive everywhere and has a minimum at $x = 0.50$ m from the centre of X. State three conclusions, and then find the force on a proton held at $x = 0.60$ m, where the gradient of the graph is +180 V m^{-1}.

Conclusions: (1) both spheres are **positively** charged, because the potential is positive everywhere; (2) the electric field is **zero** at $x = 0.50$ m, where the gradient is zero; (3) the charge on Y is larger, because the zero-field point lies closer to X; in fact

$Q_Y/Q_X = (0.70/0.50)^2 = 2.0$. At $x = 0.60$ m: $E = -180$ V m^{-1} (pointing towards X, since V rises towards Y), so $F = eE = (1.60 \times 10^{-19})(180) = 2.9 \times 10^{-17}$ N towards X. Released, the proton accelerates towards X, down the slope; the gradient, and so the acceleration, **decreases** to zero at the minimum ($x = 0.50$ m), where the proton moves fastest. Beyond it the field points the other way and slows the proton, so it oscillates about the zero-field point. An external force is needed to hold it still at $x = 0.60$ m.

Describing the motion. "A positively charged particle is placed at P and released. Describe and explain its motion." The examiner wants: the direction (towards **lower** potential, down the slope of the V graph); that the force, and so the acceleration, is **not constant** (it follows the gradient, which changes along the line); and that the particle **speeds up** while it moves down the slope, since the force is then in the direction of motion. If it reaches a minimum of the curve (the zero-field point between two positive charges) it passes through at its greatest speed and is then slowed by the reversed field. Placed exactly at the zero-field point, it stays at rest.



The potential near a point charge varies as $1/r$ – positive for a positive charge, negative for a negative one

Link between field and potential

The **potential gradient** 电势梯度 is the change in potential per unit distance along a chosen direction.

The field equals the **negative potential gradient** 负电势梯度:

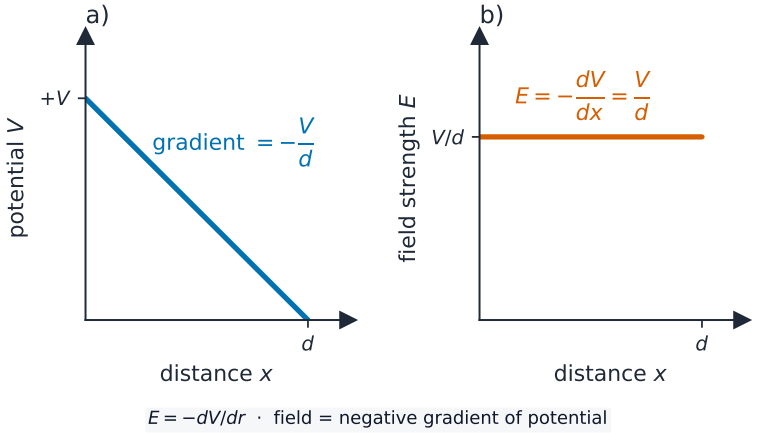
$$E = -\frac{dV}{dx}.$$

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Between parallel plates V changes evenly with position, giving $E = V/d$ as before. The minus sign means the field points towards **lower** potential. For a point charge, $-\frac{dV}{dr} = \frac{Q}{4\pi\varepsilon_0 r^2} = E$.

The one-line relationship. "State the relationship between electric field and electric potential": *the field strength is equal to the negative of the potential gradient*, $E = -\Delta V/\Delta x$. On a V–x graph E is minus the **gradient** of a tangent; on a V–r graph for a point charge the gradient is steepest close to the charge, where the field is strongest, and the sign of E is fixed by "the field points towards **lower** potential". The uniform-field version $E = V/d$ is the same statement with a constant gradient.

Equipotentials. An **equipotential** surface joins points of equal potential; no work is done moving a charge along it, so the field is always at **right angles** to it. Around a point charge the equipotentials are concentric spheres, growing further apart as V falls; between parallel plates they are planes parallel to the plates. The surface of a conductor is an equipotential, which is why field lines meet it at right angles.



In a uniform field the potential falls steadily with distance, so the field strength V/d is constant

Electric potential energy

A charge q at a point of potential V has **electric potential energy** 电势能 $E_P = qV$. For two point charges Q and q a distance r apart:

$$E_P = \frac{Qq}{4\pi\varepsilon_0 r}.$$

- **like** charges: $E_P > 0$ – stored energy that would be released if they flew apart.
- **opposite** charges: $E_P < 0$ – a **bound** 束缚 system; energy must be supplied to separate them.

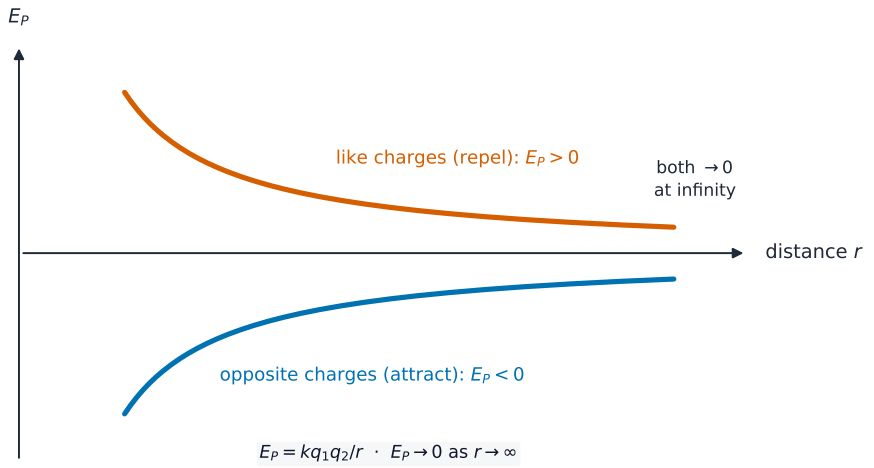
In both cases $E_P \to 0$ as $r \to \infty$.

> **Worked example.** The proton and the electron in a hydrogen atom are 5.3×10^{-11} m apart. Find the electric potential energy of the pair.

$$E_P = \frac{Qq}{4\pi\varepsilon_0 r} = \frac{(8.99\times10^9)(+1.60\times10^{-19})(-1.60\times10^{-19})}{5.3\times10^{-11}} = -4.3\times10^{-18}\ \text{J} = -27\ \text{eV}.$$

The **negative** sign is part of the answer: the charges are opposite, so energy (4.3×10^{-18} J) must be **supplied** to pull them apart to infinity. (The electron also has kinetic energy of half that size, so the energy needed to **ionise** 电离 the atom is 13.6 eV; the "worked-example pattern" below shows why.)

Charge through a p.d. A charge q moved through a potential difference V changes its potential energy by qV, which is why an electron accelerated from rest through 250 V arrives with 250 eV of kinetic energy whatever the shape of the field. For an MCQ about "moving P a distance x along the field lines", the work done is qEx, and its potential energy **falls** if it moves the way the force pushes it.



Electric PE of two charges: positive for like charges, a negative well for opposite charges

Worked-example pattern

An **electron** 电子 orbits a **nucleus** 原子核 of charge $+Ze$ at distance r. The Coulomb attraction provides the **centripetal force** 向心力:

$$\frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{m_e v^2}{r}, \qquad v = \sqrt{\frac{Ze^2}{4\pi\varepsilon_0 m_e r}}.$$

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The total energy is kinetic plus potential:

$$E_{\text{total}} = \tfrac{1}{2}m_e v^2 - \frac{Ze^2}{4\pi\varepsilon_0 r} = -\frac{Ze^2}{8\pi\varepsilon_0 r},$$

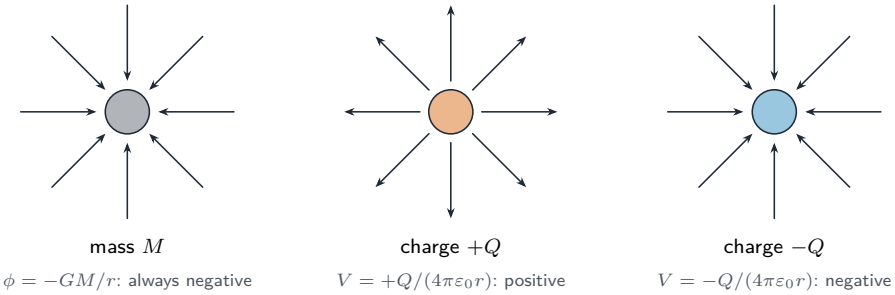
which is negative (a bound state).

Gravitational versus electric

The two field theories look alike:

Quantity	Gravitational	Electric
Source	mass M (always positive)	charge Q (can be $\pm$)
Field strength	$g = GM/r^2$	$E = Q/(4\pi\varepsilon_0 r^2)$
Force on test object	$F = mg$	$F = qE$
Potential	$\phi = -GM/r$	$V = Q/(4\pi\varepsilon_0 r)$
PE of two	$-GMm/r$	$Qq/(4\pi\varepsilon_0 r)$
Nature	always attractive	attractive or repulsive

The minus sign in the **gravitational potential** 引力势 reflects that gravity is always attractive; the electric potential takes the sign of the source charge.



Gravitational field lines only point inwards; electric field lines follow the sign of the charge

Similarity and difference (a standard two-marker). Both potentials are proportional to $1/r$, both are zero at infinity, and both are scalars; but gravitational potential is **always negative** (the force is always attractive) whereas electric potential takes the **sign of the charge** and can be positive. For the fields: both obey an inverse-square law and both are drawn as radial lines around a point source, but gravitational field lines only ever point **inwards** (attraction), while electric field lines point outwards from a positive charge and inwards to a negative one.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
electric field	a region in which a charge experiences an (electric) force
electric field strength	the force per unit positive charge acting on a small (test) charge placed at the point
electric field line	a line whose direction shows the direction of the force on a positive charge; the spacing of the lines shows the field strength
Coulomb's law	the force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of their separation
electric potential	the work done per unit positive charge in bringing a small test charge from infinity to the point
field and potential	the electric field strength is equal to the negative of the potential gradient: $E = -\Delta V/\Delta x$
electric potential energy (of two charges)	the work done in bringing the charges from infinity to their separation; $Qq/(4\pi\varepsilon_0 r)$
equipotential	a surface (or line) on which every point has the same potential; the field is at right angles to it
uniform field	a field with the same strength and direction at every point, shown by parallel, evenly spaced field lines

Exam tips

- Four formulae, two with r^2 and two with r: $F = Q_1Q_2/4\pi\varepsilon_0 r^2$ and $E = Q/4\pi\varepsilon_0 r^2$ (inverse square), $V = Q/4\pi\varepsilon_0 r$ and $E_\mathrm{P} = Qq/4\pi\varepsilon_0 r$. Decide which one before you write.
- Distinguish a **uniform field** ($E = V/d$, between plates) from a **radial field** ($E = Q/4\pi\varepsilon_0 r^2$, outside a point charge or a sphere); r for a sphere is measured from the **centre**, not the surface.
- The field points from high to low potential and E is minus the potential gradient: draw a tangent on a V–x graph to find it.
- Field is a vector (add components, subtract opposing ones); potential is a scalar (add with sign). Between like charges there is a zero-field point; between opposite charges a zero-potential point.

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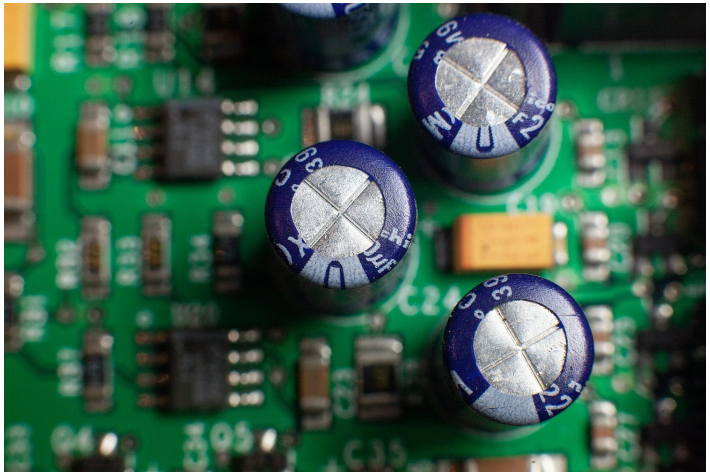
- A charged particle in a uniform field: $F = qE$, $a = qE/m$, $E_\mathrm{k} = qV$; along the field it is the vertical half of projectile motion, and motion at right angles to the field is unaffected.
- The data sheet gives both $\varepsilon_0 = 8.85\times10^{-12}$ F m^{-1} and $1/(4\pi\varepsilon_0) = 8.99\times10^{9}$ m F^{-1}; either works, do not mix them up.

Common mistakes

- Defining field strength as "the force on a charge" or "force per unit mass". It is the force per unit **positive charge** on a small positive charge.
- Forgetting to square r in Coulomb's law and in the field, or squaring it in the potential ($V \propto 1/r$).
- Using the diameter, or the distance from the **surface** of a sphere, instead of the centre-to-centre distance.
- Sketching E for a sphere from the origin. It is zero inside and starts at E_0 on the surface; V is constant inside, not zero.
- Field lines without arrows, crossing, or unevenly spaced in a uniform field; lines not at right angles to a conductor.
- Dropping the negative sign on the potential energy of opposite charges, or taking the direction of E from the sign of V instead of from its gradient.
- Forgetting the second electron in the helium atom, or placing it at distance r rather than $2r$.
- Using $E = V/d$ for a point charge, or $E = Q/4\pi\varepsilon_0 r^2$ between plates.
- Giving an electron weight, or using the proton's mass for an electron; in electrons-between-plates questions the weight is negligible.
- Leaving out the weight in an "oil drop held stationary" question, or getting the polarity of the top plate backwards.

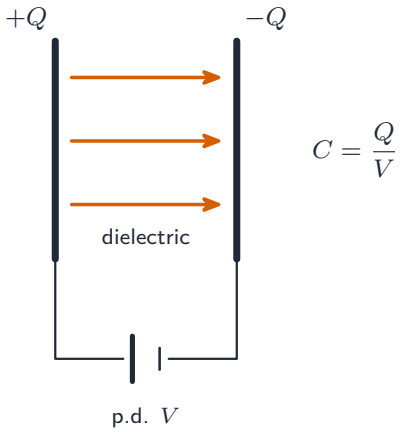
Capacitance

A-Level Physics



Capacitors store electric charge in a circuit.

A **capacitor** 电容器 stores charge. The simplest one is two parallel **conductor** 导体 plates with an **insulator** 绝缘体 (a **dielectric** 电介质, or just **vacuum** 真空 / air) between them. Connected to a battery, charge $+Q$ builds up on one plate and $-Q$ on the other, with a **potential difference** 电势差 V across the gap.



$$C = \frac{Q}{V}$$

A parallel-plate capacitor: equal and opposite charges on two plates with a p.d. V across the gap

How the plates become charged. Connect the plates to a battery and electrons flow, through the battery, from the plate joined to the positive **terminal** 接线端 to the plate joined to the negative terminal. One plate is left with charge $+Q$ and the other gains $-Q$: the charges are always **equal and opposite**, so the capacitor as a whole stays **neutral** 电中性. The flow stops when the p.d. between the plates equals the **e.m.f.** 电动势 of the battery. Nothing crosses the gap; the insulator is what keeps the charge separated.

The **capacitance** 电容 C of any capacitor (or any **isolated** 孤立的 conductor) is

$$C = \frac{Q}{V}.$$

This applies to:

- an **isolated sphere** holding charge Q at potential V (zero at infinity). For radius r, $V = Q/(4\pi\varepsilon_0 r)$, so $C = 4\pi\varepsilon_0 r$.
- a **parallel-plate capacitor** 平行板电容器: charges $\pm Q$ on the plates, p.d. V between them.

Unit: **farad** 法拉 (F) = C V^{-1}. A farad is huge, so real capacitors run from pF to mF.

Capacitance is **constant** for a given capacitor (set by its size and dielectric). Doubling the charge doubles the voltage, so $C = Q/V$ stays the same.

The two-mark definitions. For a **parallel-plate capacitor**: *the charge on one plate per unit potential difference between the plates.* For an **isolated sphere** (or any isolated conductor): *the charge on the conductor per unit potential.* The examiner looks for "charge per unit p.d." (or "per unit potential") and, for the plates, that the charge is the charge on **one** plate and the p.d. is that **between the plates**. "The charge stored" is accepted; "the total charge on both plates" is not, because that total is zero.

What sets the capacitance. For parallel plates the capacitance is proportional to the plate **area** and **inversely proportional** 成反比 to the **separation** 间距 x ($C \propto 1/x$: halve the gap and the capacitance doubles), and it rises when a dielectric replaces the air. A question that states "C is inversely proportional to x" and then moves the plates apart at constant charge is asking for $V = Q/C \propto x$: the p.d. rises in proportion. At constant p.d. (still connected to the supply) it is the charge that falls instead.

Worked example. A 100 μF capacitor is charged to 12 V. Find the charge stored.

$$Q = CV = (100 \times 10^{-6})(12) = 1.2 \times 10^{-3} \text{ C } (= 1.2 \text{ mC}).$$

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Worked example. An isolated metal sphere of radius 15 cm is charged to a potential of 9.0×10^3 V. Find its capacitance and its charge. It is then **discharged** 放电 to **earth** 接地 through a 120 MΩ resistor: find the time constant of the discharge.

$$C = 4\pi\varepsilon_0 r = \frac{0.15}{8.99 \times 10^9} = 1.7 \times 10^{-11}\ \text{F} = 17\ \text{pF}, \qquad Q = CV = (1.67 \times 10^{-11})(9.0 \times 10^3) = 1.5 \times 10^{-7}\ \text{C}.$$

The time constant is $RC = (1.2 \times 10^8)(1.67 \times 10^{-11}) = 2.0 \times 10^{-3}$ s: even through an enormous resistance the sphere loses its charge in milliseconds, because its capacitance is so small. Notice that $4\pi\varepsilon_0 = 1/(8.99 \times 10^9)$, so $C = r/(8.99 \times 10^9)$ with r in metres.



Real capacitors range from large electrolytic cans (high capacitance) down to tiny film and ceramic types – from pF up to mF

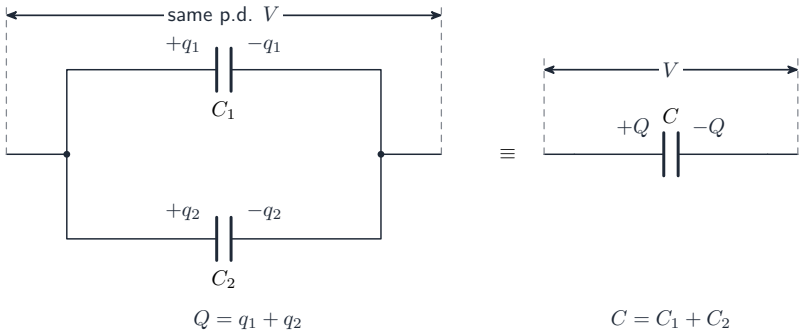
Combining capacitors

Capacitors in parallel 并联 share the same p.d. V. The total charge is the sum:

$$Q_{\text{total}} = C_1V + C_2V + \ldots, \qquad \text{so} \qquad C_{\text{parallel}} = C_1 + C_2 + \ldots$$

A parallel combination has **larger** capacitance than any one capacitor.

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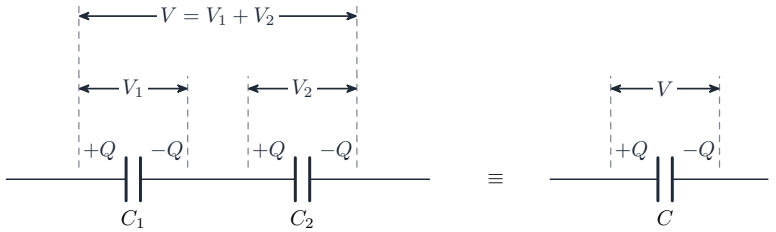
$Q = q_1 + q_2$ $\qquad$ $C = C_1 + C_2$

Capacitors in parallel share the same p.d.; the charges add

Capacitors in series 串联 carry the **same charge** Q. The total p.d. is the sum:

$$V_{\text{total}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \ldots, \qquad \text{so} \qquad \frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$

A series combination has **smaller** capacitance than any one capacitor.



Capacitors in series carry the same charge; the p.d.s add

Note: these rules are the **opposite** of those for resistors (resistors sum in series; capacitors sum in parallel), because $C = Q/V$ has V on the bottom while $R = V/I$ has I on the bottom.

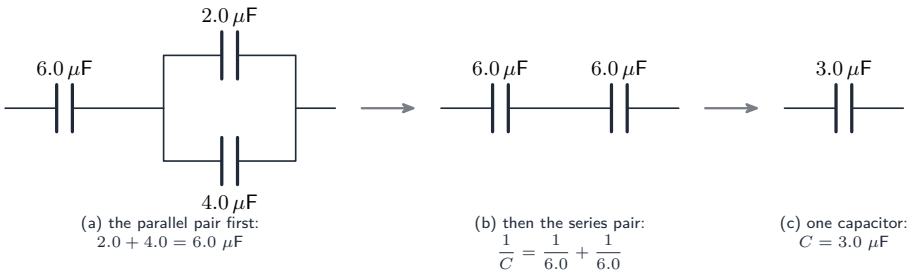
Deriving the two rules

"Derive an expression for the combined capacitance" is a **show that** question, and the mark scheme wants the physics stated, not just the algebra.

- **Series.** The capacitors carry the **same charge** Q: the plate joined to the positive terminal loses electrons to the far plate of the next capacitor, so every plate in the chain holds $\pm Q$. The p.d.s **add**: $V = V_1 + V_2$. With $V_1 = Q/C_1$ and $V_2 = Q/C_2$, and the combination defined by $V = Q/C$, dividing through by Q gives $1/C = 1/C_1 + 1/C_2$.

- **Parallel.** The capacitors have the **same p.d.** V, because each is connected directly across the supply. The charges **add**: $Q = Q_1 + Q_2 = C_1V + C_2V$, and $Q = CV$ for the combination, so $C = C_1 + C_2$.

Two capacitors in series always give **less** than the smaller one; two equal capacitors C give $C/2$ in series and $2C$ in parallel.



A network is reduced one step at a time: combine the parallel pair first, then the series pair

> **Worked example.** A 6.0 μF capacitor is in series with a parallel pair of 2.0 μF and 4.0 μF. Find the total capacitance, and the charge on each capacitor when 12 V is applied across the network.

Parallel pair: $2.0 + 4.0 = 6.0$ μF. In series with 6.0 μF: $1/C = 1/6.0 + 1/6.0$, so $C = 3.0$ μF. Total charge $Q = CV = (3.0 \times 10^{-6})(12) = 3.6 \times 10^{-5}$ C: this is the charge on the 6.0 μF capacitor (series, same charge), and it is shared by the parallel pair in the ratio of their capacitances (same p.d., 6.0 V across each): the 2.0 μF holds 1.2×10^{-5} C and the 4.0 μF holds 2.4×10^{-5} C. Check: they add to 3.6×10^{-5} C.

> **Worked example.** Capacitors X and Y, each of capacitance C, are connected in series across a supply of voltage V; capacitors P and Q, each C, are connected in parallel across an identical supply. Compare the charge drawn from each supply and the energy stored in each arrangement.

Series: $C_\text{s} = C/2$, charge $Q_\text{s} = CV/2$, energy $\frac{1}{2}C_\text{s}V^2 = CV^2/4$. Parallel: $C_\text{p} = 2C$, charge $2CV$, energy CV^2. The parallel arrangement draws **four times** the charge and stores **four times** the energy, because at the same supply voltage the energy is $\frac{1}{2}CV^2$ and the capacitance is four times larger. Asked which arrangement stores more energy from a given supply, the answer is always parallel.

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Energy stored in a capacitor

Charging a capacitor from 0 to Q needs work, because each extra bit of charge is pushed against the p.d. already there. When the charge is q, the p.d. is $V(q) = q/C$, so adding a small charge dq needs work $V\,dq$. The total work is

$$W = \int_0^Q \frac{q}{C}\,dq = \frac{Q^2}{2C}.$$

Using $V = Q/C$, this is the **energy** 能量 stored:

$$W = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}.$$

> **Worked example.** Find the energy stored in a 100 μF capacitor charged to 12 V.

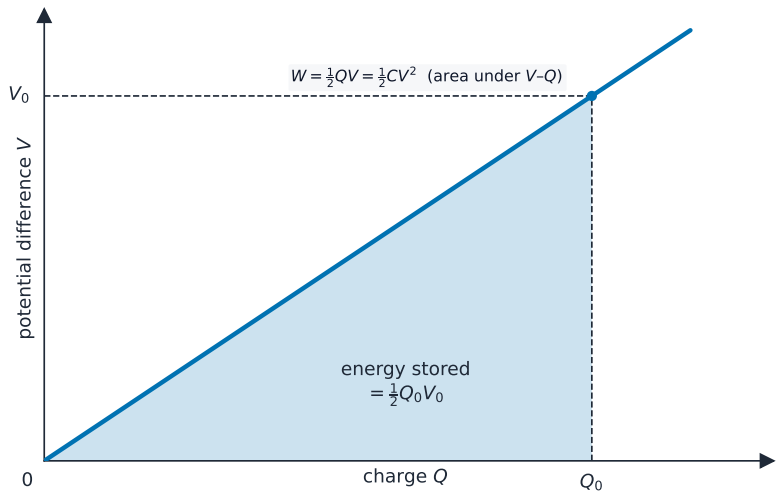
$$W = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(100 \times 10^{-6})(12)^2 = 7.2 \times 10^{-3}\text{ J } (= 7.2\text{ mJ}).$$

> **Worked example.** The graph of charge Q against p.d. V for a capacitor is a straight line through the origin passing through (10 V, 1.2×10^{-3} C). Find the capacitance and the energy stored at 10 V, and then the extra energy stored when the p.d. is raised to 12 V.

The **gradient** 斜率 of Q against V is $C = 1.2 \times 10^{-3}/10 = 1.2 \times 10^{-4}$ F $= 120$ μF. Energy is the area under the line: $\frac{1}{2}QV = \frac{1}{2}(1.2 \times 10^{-3})(10) = 6.0 \times 10^{-3}$ J. At 12 V: $\frac{1}{2}CV^2 = \frac{1}{2}(1.2 \times 10^{-4})(12)^2 = 8.6 \times 10^{-3}$ J, so the extra energy is 2.6×10^{-3} J. Do not find the extra energy from the extra charge as $\frac{1}{2}(\Delta Q)V$: energy is not proportional to charge, so subtract the two areas.

■ Reading the Q–V graph

A plot of V against Q is a straight line through the origin with gradient $1/C$. The energy stored is the **area** under the line up to a given charge Q, which is the triangle $\frac{1}{2}QV$. The factor $\frac{1}{2}$ is there because the average p.d. during charging is $V/2$ (it grows from zero to V), not V.

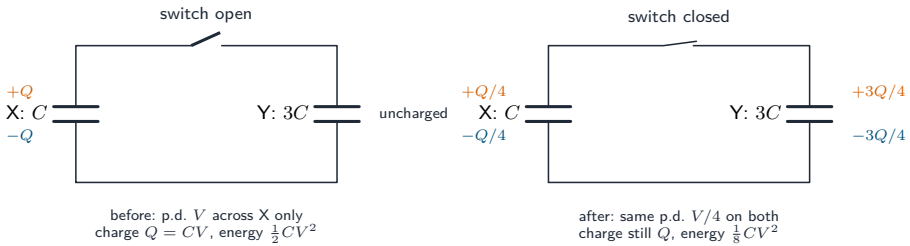


The energy stored is the area under the potential–charge line (the triangle $\frac{1}{2}QV$)

Why charging is "half efficient"

Connect a capacitor C to an ideal battery of e.m.f. V through a wire. The capacitor stores $\frac{1}{2}CV^2$, but the battery supplies charge $Q = CV$ at e.m.f. V, giving out $QV = CV^2$. The other half is lost as heat in the wire – whatever the wire's resistance.

Sharing charge between capacitors



Connecting a charged capacitor to an uncharged one: charge is conserved, the p.d. equalises, and energy is lost

A fully charged capacitor X (capacitance C, p.d. V, charge $Q = CV$) is disconnected from its supply and connected across an uncharged capacitor Y of capacitance $3C$. Charge flows until the two p.d.s are equal. Two principles settle everything:

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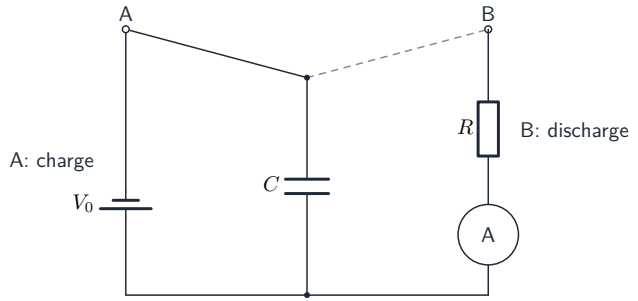
1. **Charge is conserved.** The total charge is still Q, now spread over a parallel pair of total capacitance $4C$, so the common p.d. is $V' = Q/(4C) = V/4$. X keeps $Q/4$ and Y takes $3Q/4$ (charge in the ratio of the capacitances, since the p.d. is the same).
2. **Energy is not conserved.** Before: $\frac{1}{2}CV^2$. After: $\frac{1}{2}(4C)(V/4)^2 = \frac{1}{8}CV^2$. Three-quarters of the energy has gone: it is **dissipated** 耗散 as heat in the connecting wires (and as a **spark** 电火花 or **electromagnetic radiation** 电磁辐射) while the charge moves, however small the resistance.

> **Worked example.** A 220 μF capacitor charged to 9.0 V is connected across an uncharged 440 μF capacitor. Find the final p.d. and the energy lost.

$Q = CV = (220 \times 10^{-6})(9.0) = 1.98 \times 10^{-3}$ C; total capacitance 660 μF, so $V' = 1.98 \times 10^{-3}/(660 \times 10^{-6}) = 3.0$ V. Energy before: $\frac{1}{2}(220 \times 10^{-6})(9.0)^2 = 8.9 \times 10^{-3}$ J; after: $\frac{1}{2}(660 \times 10^{-6})(3.0)^2 = 3.0 \times 10^{-3}$ J; lost: 5.9×10^{-3} J, two-thirds of the original. A capacitor connected to another of capacitance kC always loses the fraction $k/(k+1)$.

Capacitor discharging through a resistor

A capacitor C charged to V_0 is connected through a switch to a **resistor** 电阻器 of **resistance** 电阻 R. When the switch closes at $t = 0$, the capacitor discharges.



A capacitor charges through switch A, then discharges through the resistor via switch B

Setting up the equation

By **Kirchhoff's second law** 基尔霍夫第二定律 around the loop, $V_C = V_R$. Using $V_C = Q/C$, $V_R = IR$ and $I = -dQ/dt$:

$$\frac{Q}{C} = -R\frac{dQ}{dt}.$$

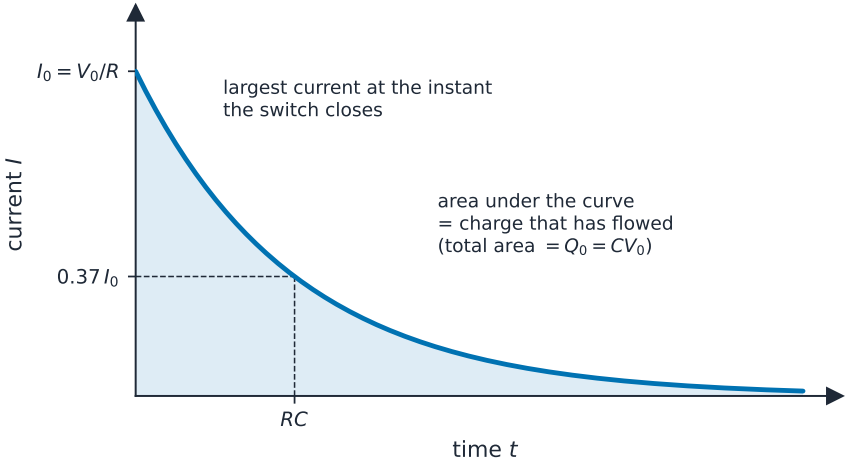
This is solved by an **exponential decay** 指数衰减 with time constant RC.

Discharge equations

Charge Q, p.d. V and current I all decay exponentially with the same time constant:

$$Q = Q_0 e^{-t/(RC)}, \qquad V = V_0 e^{-t/(RC)}, \qquad I = I_0 e^{-t/(RC)},$$

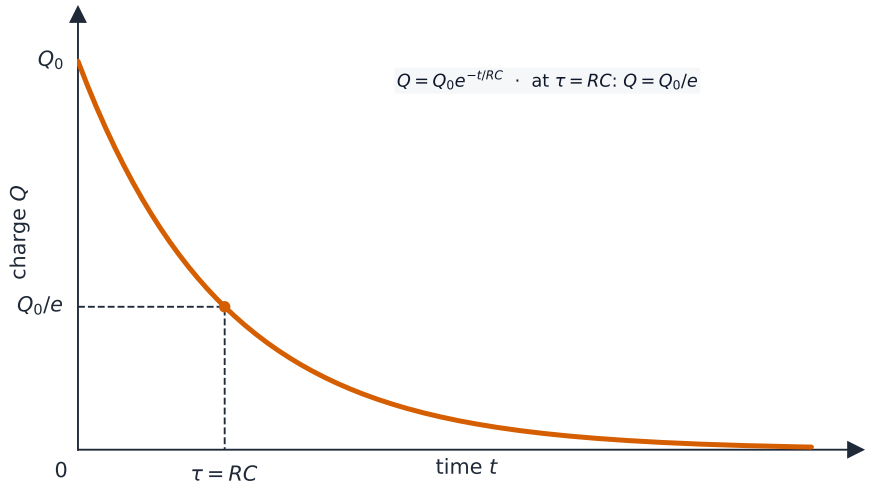
with $I_0 = V_0/R$.



The discharge current starts at V_0/R and decays with the same time constant; the area under the curve is the charge that has flowed

Reading the three graphs. All three curves have the same shape and the same time constant, so one measurement of τ from any of them gives RC. The **current** is largest the instant the switch closes, $I_0 = V_0/R$, because the full p.d. V_0 is then across the resistor; it falls as the p.d. falls. The **area** under the I–t graph is the charge that has flowed, so the total area is $Q_0 = CV_0$, and the charge that flows in the first time constant is $Q_0(1 - e^{-1}) = 0.63\,Q_0$. The **gradient** of the Q–t graph is $-I$: steepest at the start, flattening as the current dies away. (During **charging** through a resistor the current has the same decaying shape, while V and Q rise as $V_0(1 - e^{-t/RC})$, the mirror image of the discharge curve.)

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Charge decays exponentially during discharge, falling to Q_0/e after one time constant RC

Time constant

$$\tau = RC$$

is the **time constant** 时间常数 (in seconds: $\Omega \cdot \mathrm{F} = \mathrm{s}$). It is the time for a decaying quantity to fall to $1/e \approx 0.37$ (about 37%) of its starting value. After 2τ it is at about 13.5%; after 5τ, below 1%.

> **Worked example.** A 100 μF capacitor charged to 12 V is discharged through a 47 kΩ resistor. Find the time constant and the voltage after one time constant.

$$\tau = RC = (47 \times 10^3)(100 \times 10^{-6}) = 4.7 \text{ s}.$$

After one time constant the voltage falls to $1/e$ of its start: $V = 12 \times 0.37 \approx 4.4$ V.

> **Worked example.** A 470 μF capacitor is charged to 24 V and then discharged through a 5.6 kΩ resistor. Find (a) the initial charge and energy, (b) the time constant and the initial current, (c) the p.d. after 4.0 s, (d) the time for the p.d. to fall to 6.0 V.

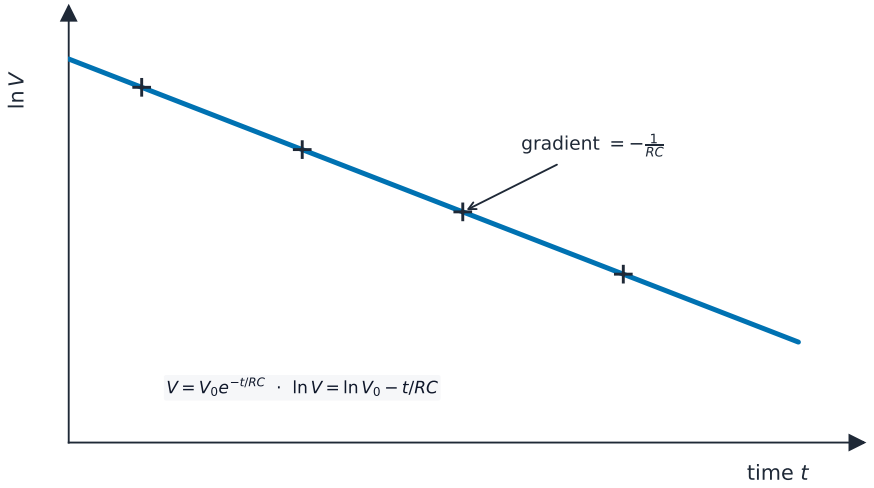
(a) $Q_0 = CV_0 = (470 \times 10^{-6})(24) = 1.1 \times 10^{-2}$ C; $W = \frac{1}{2}CV_0^2 = \frac{1}{2}(470 \times 10^{-6})(24)^2 = 0.135$ J. (b) $\tau = RC = (5.6{\times}10^3)(470{\times}10^{-6}) = 2.6$ s; $I_0 = V_0/R = 24/5600 = 4.3$ mA. (c) $V = V_0 e^{-t/RC} = 24\,e^{-4.0/2.63} = 24{\times}0.219 = 5.3$ V. (d) Rearrange with **logarithms** 对数: $t = -RC\ln(V/V_0) = -2.63\ln(6.0/24) = 2.63 \times 1.386 = 3.6$ s.

Keep the time constant unrounded (2.632 s) inside the calculation and round only the answers. The rearrangement in (d) is the one most often asked and most often botched: $\ln(V/V_0)$ is negative, so t comes out positive.

Worked example. From a discharge graph, the p.d. across a 2200 μF capacitor falls from 6.0 V to 2.2 V in 4.5 s. Find the resistance of the resistor.

$2.2/6.0 = 0.367 \approx 1/e$, so 4.5 s is one time constant: $R = \tau/C = 4.5/(2200 \times 10^{-6}) = 2.0 \times 10^3\ \Omega$. If the fall is not to $1/e$, use $RC = -t/\ln(V/V_0)$; the answer is the same either way.

To find τ from a curve: read the time to fall to $1/e$ of the start. Or take logs: $\ln(V/V_0) = -t/(RC)$, so a plot of $\ln V$ against t is a straight line with gradient $-1/(RC)$.



A graph of $\ln V$ against time is a straight line of gradient $-1/(RC)$

Using the logarithmic graph. In an experiment, plot $\ln V$ (or $\ln I$) on the y-axis against t. The points should lie on a **straight line**: its gradient is $-1/(RC)$, so $RC = -1/\text{gradient}$, and its **intercept** 截距 on the $\ln V$ axis is $\ln V_0$. A straight line is the test that the decay really is exponential; a curve means it is not. Take the gradient from two points far apart on the best-fit line, not from two data points, and quote RC in seconds.

Reading graphs during discharge

- Q against V_C: since $Q = CV$ always, this is a straight line through the origin with gradient C. Discharge moves the point from (V_0, Q_0) down to $(0, 0)$.
- I against V_C: since $I = V_C/R$, this is a straight line through the origin with gradient $1/R$, so you can find R.

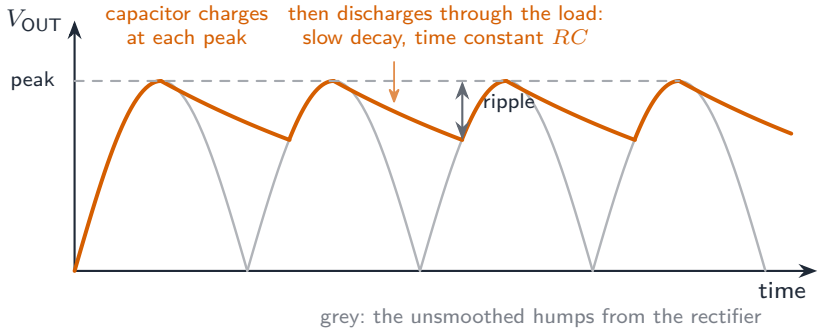
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Common exam questions

Given a discharge curve $V(t)$ or $Q(t)$:

- read the start value V_0 or Q_0 at $t = 0$.
- read the time to fall to $V_0/e \rightarrow$ time constant $\tau = RC$.
- given R, find $C = \tau/R$ (or the other way round).
- predict a later value with the exponential formula.

A capacitor as a smoothing component



A capacitor across the load of a rectifier charges at each peak and discharges through the load in between, turning the humps into a nearly steady output with a small ripple

A **rectifier** 整流器 (Topic 21) turns an **alternating voltage** 交变电压 into a series of positive humps. A capacitor connected across the **load** 负载 **smooths** 平滑 them. Near each **peak** 峰值 the **diodes** 二极管 conduct and the capacitor charges to the peak voltage; as the supply voltage falls away the diodes stop conducting and the capacitor **discharges through the load resistor**, holding the output up until the next hump rises above it and recharges the capacitor. The output therefore falls only a little between peaks; the rise and fall that remains is the **ripple** 纹波.

How much it falls depends on the time constant of the discharge, RC, compared with the time between peaks. A **larger capacitance**, or a **larger load resistance** (a smaller load current), gives a larger RC, a slower decay and a smaller ripple; a heavily loaded supply (small R) has a larger ripple. Asked to sketch the smoothed output, draw it touching each peak, decaying slightly between them, and never falling to zero.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
capacitance (parallel-plate capacitor)	the charge on one plate per unit potential difference between the plates
capacitance (isolated conductor)	the charge on the conductor per unit potential
farad	one coulomb per volt
capacitors in series	the same charge on each; the p.d.s add; $1/C = 1/C_1 + 1/C_2$
capacitors in parallel	the same p.d. across each; the charges add; $C = C_1 + C_2$
energy stored	the work done in charging the capacitor, equal to the area under the potential–charge graph; $W = \frac{1}{2}QV = \frac{1}{2}CV^2$
time constant	the product RC; the time for the charge (or p.d., or current) to fall to $1/e$ of its initial value

Exam tips

- $C = Q/V$ defines capacitance; for an isolated sphere $C = 4\pi\varepsilon_0 r$. Energy stored $W = \frac{1}{2}QV = \frac{1}{2}CV^2 = Q^2/2C$: pick the form that uses what you know, and remember that energy is **not** proportional to charge.
- Combine capacitors the **opposite way to resistors** (parallel add, series reciprocal), and reduce a network one step at a time.
- Charge sharing: **charge is conserved, energy is not**. Find the common p.d. from the total charge and the total capacitance first, then the energies.
- Discharge: $x = x_0 e^{-t/RC}$ for Q, V and I alike; $t = -RC\ln(x/x_0)$; $\tau = RC$ is in seconds ($\Omega \cdot \mathrm{F} = \mathrm{s}$) and is the time to fall to 37%, not to zero.
- Graphs: the initial current is V_0/R; the area under I–t is charge; the gradient of Q–t is $-I$; $\ln V$ against t is a straight line of gradient $-1/(RC)$ and intercept $\ln V_0$.
- Convert μF and kΩ before multiplying: k$\Omega \times \mu$F gives milliseconds, not seconds.

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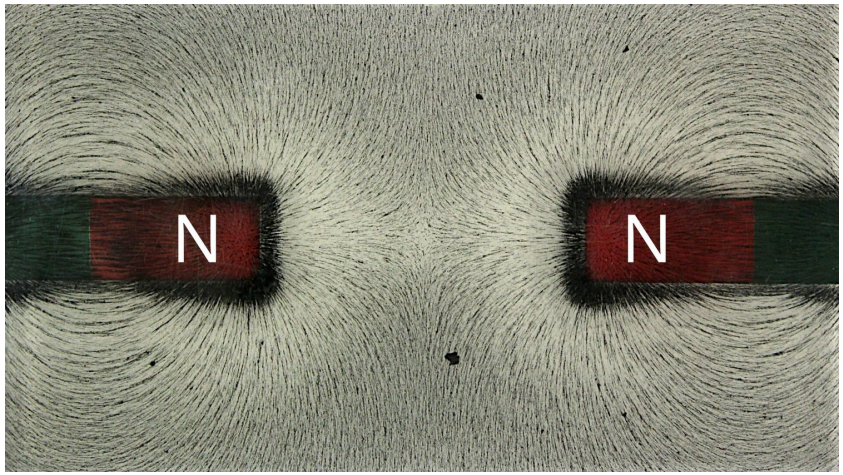
Common mistakes

- Defining capacitance as "the charge stored" or "the charge on both plates". It is the charge on **one** plate per unit p.d. between the plates.
- Adding capacitors in series as if they were resistors, or forgetting to invert the reciprocal sum at the end.
- After charge sharing, using the **supply** voltage in $\frac{1}{2}CV^2$ instead of the new common p.d., or claiming the energy is conserved.
- Drawing the discharge current starting from zero. It starts at its largest value, V_0/R.
- Treating τ as the time to discharge completely; after 5τ about 1% remains.
- Dropping the minus sign in $t = -RC\ln(V/V_0)$ and reporting a negative time.
- Finding the extra energy on raising the p.d. from the extra charge alone: subtract two areas (or two $\frac{1}{2}CV^2$ values).
- Sketching a smoothed output that falls to zero between peaks, or saying a larger load resistance gives a larger ripple.

Magnetic fields

A-Level Physics

The magnetic field



Iron filings trace the field lines around bar magnets.

A **magnetic field** 磁场 is a region where a moving charge (or a **current** 电流) feels a **force** 力. It is made by:

- **moving charges** (usually a current in a wire), or
- **permanent magnets** 永磁体 (where it comes from tiny atomic currents).

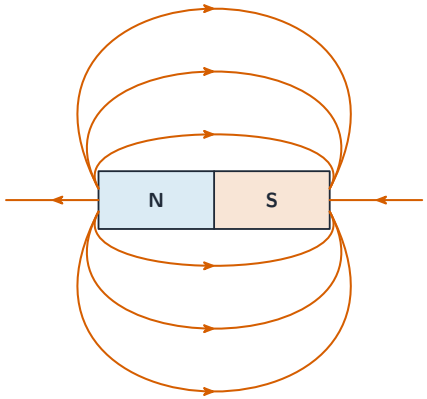
"State what is meant by a magnetic field." *A region in which a force acts on a moving charge, a current-carrying conductor or a magnetic material.* The mark scheme wants the word **force** and one of those three objects; "a region around a magnet" on its own scores nothing.

Field lines

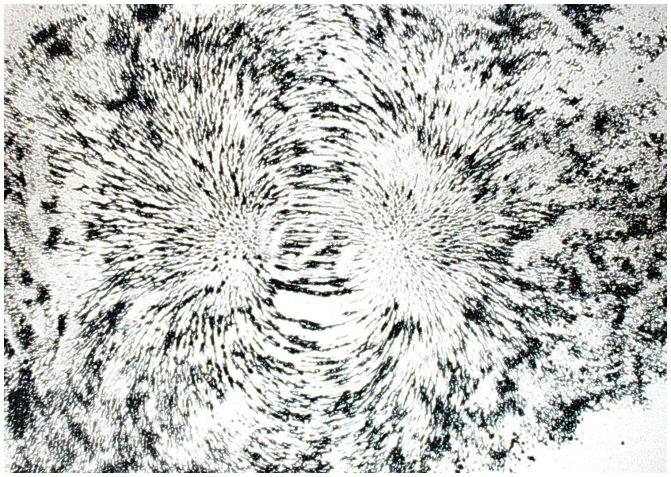
- **field lines** 场线 point from **N** to **S** outside a magnet, and S to N inside (so they form closed loops).
- lines never cross; closer lines mean a stronger field.

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field lines run from **N** to **S** outside the magnet



The field of a bar magnet: lines run from N to S, strongest near the poles

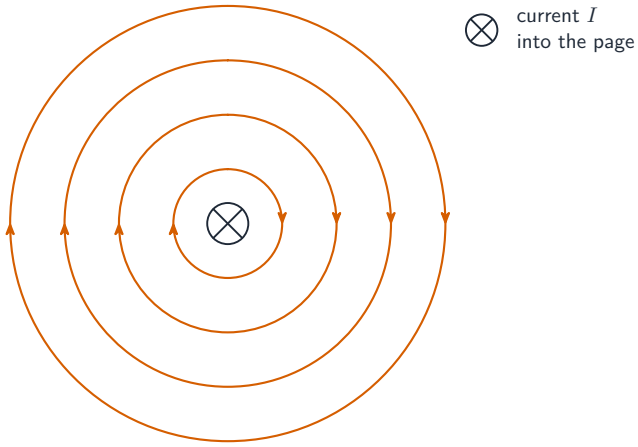


Iron filings around a bar magnet line up along the field, showing its real shape

Patterns to know:

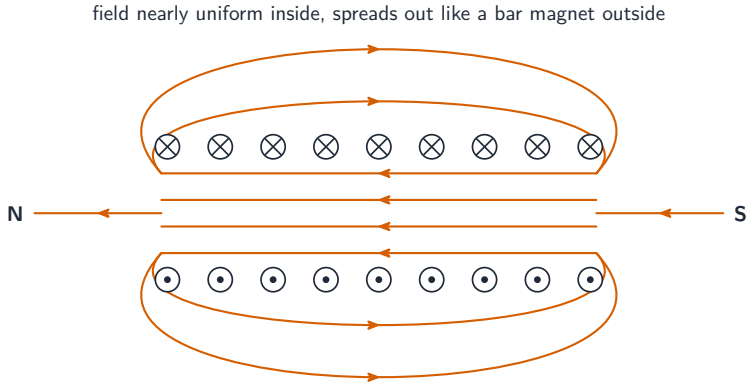
- **bar magnet** 条形磁铁 – curved lines from N to S outside, strongest near the poles.
- **long straight wire** – circles around the wire; the direction comes from the **right-hand grip rule** 右手定则 (thumb along the current, fingers curl the way the field points).

- **flat circular coil** – the field through the centre is at right angles to the coil; the coil acts like a small bar magnet.
- **long solenoid** 螺线管 – the field inside is nearly **uniform** along the axis, like a stretched bar magnet; outside it falls off fast.



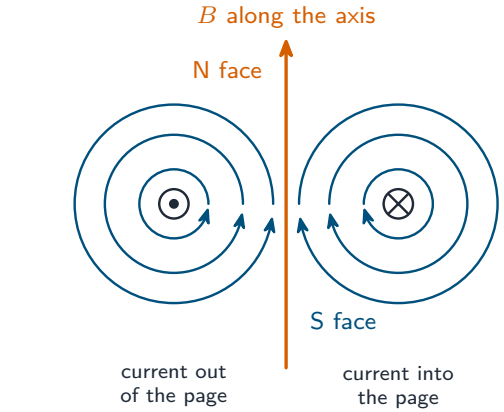
circular field lines (right-hand grip rule)

Field around a long straight wire



Field of a solenoid

Drawing the wire's field. "Draw four field lines" around a straight wire is marked on three things: the lines are **concentric circles** centred on the wire; their spacing **increases** with distance (the field weakens with distance from the wire); and every line carries an **arrow** in the direction given by the right-hand grip rule (current **into** the page: **clockwise**; current out of the page: anticlockwise). Draw them as circles, not free-hand ovals.



coil seen edge-on: the loops around both sides pass through the centre the same way, at right angles to the coil

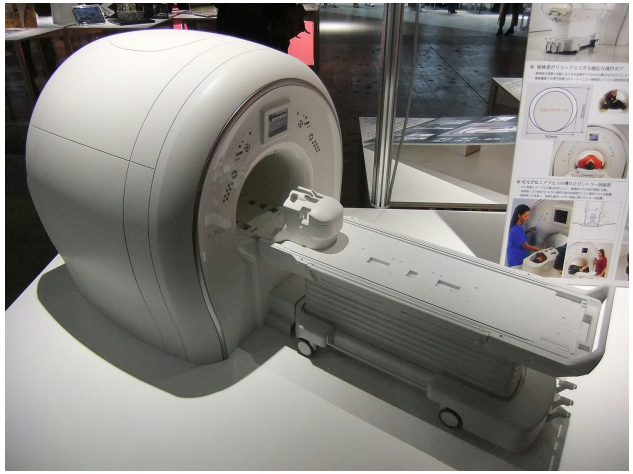
A flat circular coil seen edge-on: the field loops around each side of the coil and passes through the centre at right angles to the coil's plane

For a **flat circular coil** the loops around the two sides of the coil reinforce through the centre, so the field there is at right angles to the plane of the coil and strongest at the centre; seen from one face the coil is a north pole (the field leaves it), from the other a south pole. A long **solenoid** is many such coils in a row: the field inside is uniform and parallel to the axis, and the lines emerge from one end (its N end) and return outside to the other.

An **iron core** 铁芯 inside a solenoid greatly **increases** the field, because the iron's atomic magnets line up and add to it. This is why **electromagnets** 电磁铁 and **transformers** 变压器 have iron cores.

The syllabus calls this a **ferrous** 含铁的 core: iron, steel or another magnetic material. Asked why a core increases the field, say that the core becomes **magnetised** 被磁化 by the solenoid's field and that its own field adds to the field of the current.

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An MRI scanner is built around a very strong, very uniform magnetic field – a solenoid the size of a room

Force on a current-carrying conductor

A current I in a wire of length L in a magnetic field of flux density B feels a force

$$F = BIL\sin\theta,$$

where θ is the angle between the wire and the field. The force is largest when the wire is at right angles to the field ($F = BIL$) and zero when the wire is along the field.

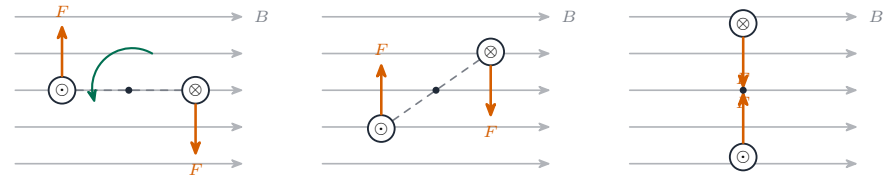
> **Worked example.** A wire of length 0.20 m carries a current of 3.0 A at right angles to a magnetic field of flux density 0.50 T. Find the force on it.

$$F = BIL = 0.50 \times 3.0 \times 0.20 = 0.30 \text{ N}.$$

Two conditions for a force. A copper wire in a magnetic field feels a force only if (1) it carries a **current**, and (2) the current is **not parallel** to the field (some part of the wire is at right angles to B). A wire with no current, or one lying along the field lines, feels nothing: $\sin\theta = 0$.

> **Worked example (the current balance).** A horizontal wire of length 5.0 cm lies at right angles to the field between the poles of a magnet standing on a top-pan balance. With no current the balance reads 102.30 g; with a current of 2.5 A it reads 102.86 g. Find the flux density.

By Newton's third law the force on the wire is matched by an equal and opposite force on the magnet, so the balance reading changes by the magnetic force: $F = \Delta m\, g = (0.56 \times 10^{-3})(9.81) = 5.5 \times 10^{-3}$ N. Then $B = F/(IL) = 5.5 \times 10^{-3}/(2.5 \times 0.050) = 4.4 \times 10^{-2}$ T (44 mT). The reading **rose**, so the force on the magnet is downwards and the force on the wire is upwards; reversing the current would make the reading fall by the same amount.



(a) plane parallel to the field: the couple (green) turns the coil most

(b) tilted: same forces, smaller separation

(c) plane at right angles: forces in line, no turning

A rectangular coil in a uniform field, seen end-on: the forces on the two sides that cross the field form a couple whose turning effect is largest when the coil's plane is parallel to the field and zero when it is perpendicular

> **Worked example (forces on a coil).** A rectangular coil PQRS with sides PS and QR of length 8.0 cm carries a current of 1.2 A in a horizontal field of 0.30 T, with the plane of the coil parallel to the field. Describe the forces on the coil.

Sides PS and QR are at right angles to the field, so each feels $F = BIL = 0.30 \times 1.2 \times 0.080 = 2.9 \times 10^{-2}$ N; the currents in them run in opposite directions, so by Fleming's rule the forces are **equal and opposite**, one up and one down. Sides PQ and SR are **parallel** to the field and feel no force. The two forces form a **couple** 力偶 that turns the coil about its axis; as the coil turns the forces stay the same size but their perpendicular separation shrinks, so the turning effect falls to zero when the plane of the coil is at right angles to the field. This is the principle of the electric motor and of the moving-coil meter.

Magnetic flux density

This equation also **defines** the **magnetic flux density** 磁通密度 B:

$$B = \frac{F}{IL} \quad \text{(wire at right angles to the field).}$$

So B is the force per unit current per unit length on a wire at right angles to the field. Unit: **tesla** 特斯拉, $\text{T} = \text{N A}^{-1}\text{ m}^{-1}$.

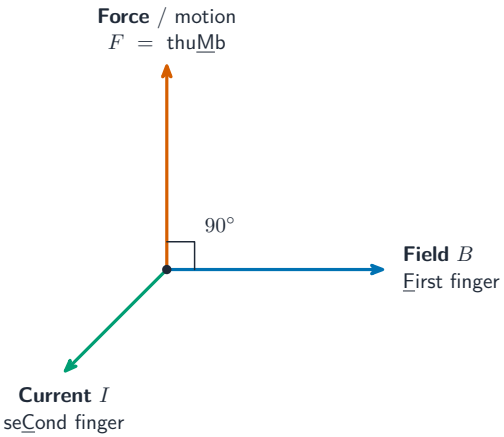
The two-mark definitions. *Magnetic flux density is the force per unit current per unit length acting on a (straight) conductor placed at right angles to the field.* Both "per unit current" and "per unit length" are needed, and so is "at right angles" (or "perpendicular"): without it the definition is false, because the force also depends on the angle. *The tesla is the flux density that produces a force of one newton on*

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one metre of conductor carrying a current of one ampere at right angles to the field: $1\ \mathrm{T} = 1\ \mathrm{N\ A^{-1}\ m^{-1}}$.

Direction – Fleming's left-hand rule

Use the **left hand** (**Fleming's left-hand rule** 弗莱明左手定则): first finger = **F**ield, second finger = **C**urrent, thumb = force (thrust). Hold the three at right angles.



use the **left** hand; hold the three at right angles

Fleming's left-hand rule: thumb = force, first finger = field, second finger = current

Force on a moving charge

A charge Q moving at **velocity** 速度 v through a field feels

$$F = BQv\sin\theta,$$

with θ the angle between v and B. Same left-hand rule (the second finger is the motion of a **positive** charge – reverse it for a negative charge). The force is largest when v is at right angles to B, and zero when v is along B.

Two situations with no force. A charged particle in a magnetic field feels no force when it is **stationary** (no v) or when it moves **parallel or antiparallel** to the field ($\sin\theta = 0$). Contrast the electric field, which pushes a stationary charge just as hard: that difference is the whole point of the velocity selector below.

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Circular motion in a uniform field

A charge moving at right angles to a **uniform field** 匀强场 feels a force at right angles to both v and B. This force does **no work** (always at right angles to the motion), so the **kinetic energy** 动能 and speed stay constant – the particle moves in a **circle**. Set the magnetic force equal to the **centripetal force** 向心力:

$$BQv = \frac{mv^2}{r}, \qquad r = \frac{mv}{BQ}.$$

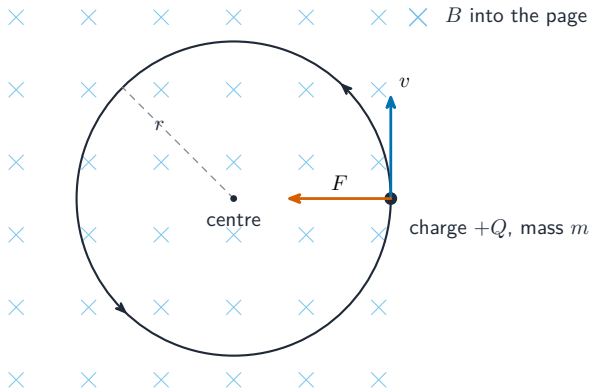
Worked example. A proton (mass 1.7×10^{-27} kg, charge 1.6×10^{-19} C) moves at 2.0×10^{6} m s^{-1} at right angles to a 0.50 T field. Find the radius of its circular path.

$$r = \frac{mv}{BQ} = \frac{(1.7\times10^{-27})(2.0\times10^{6})}{(0.50)(1.6\times10^{-19})} \approx 0.043\ \mathrm{m}.$$

So the radius depends on the **momentum** 动量 mv. The period is

$$T = \frac{2\pi m}{BQ},$$

which does **not** depend on the speed – a faster particle goes in a bigger circle but takes the same time per turn. If v also has a part along B, that part is unchanged, and the path is a **helix** 螺旋.



$$BQv = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{BQ}$$

Circular path of a charged particle in a magnetic field

"Explain why the path is circular." The magnetic force is always at **right angles** to the velocity, so it changes the direction of motion but not the speed, and its mag-

nitude BQv is **constant** because B, Q and v are constant. A force of constant size that stays perpendicular to the velocity is exactly a **centripetal force**, so the path is a circle. Give all three ideas: perpendicular, constant magnitude, speed unchanged.

> **Worked example.** Electrons moving at 1.7×10^7 m s^{-1} enter a uniform field of flux density 4.8 mT at right angles to it. Find the radius of their path. The electrons are then replaced by **positrons** 正电子 (same mass, charge $+e$) moving at 3.4×10^7 m s^{-1} along the same initial line: describe how the path changes.

$$r = \frac{mv}{BQ} = \frac{(9.11 \times 10^{-31})(1.7 \times 10^7)}{(4.8 \times 10^{-3})(1.60 \times 10^{-19})} = 2.0 \times 10^{-2} \text{ m}.$$

The positrons are deflected the **opposite** way (positive charge, so Fleming's rule with the second finger along the motion gives a force in the opposite direction), and since $r \propto v$ their radius is **twice** as large, 4.0 cm. Their period is the same ($T = 2\pi m/BQ$ has no v in it): they go round a bigger circle at twice the speed.

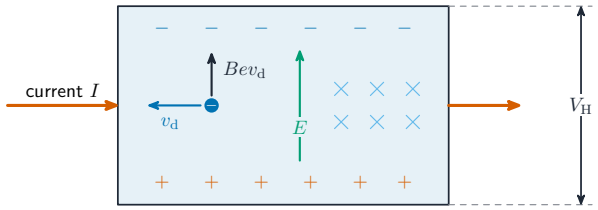
Hall effect

A slab of conductor carrying current I, in a field B at right angles to the current, develops a voltage across its faces – the **Hall voltage** 霍尔电压 V_H (the **Hall effect** 霍尔效应).

The moving charges feel a magnetic force BQv_d (v_d is the **drift velocity** 漂移速度), so they build up on one face, making an **electric field** 电场 E that opposes more build-up. At steady state $eE = Bev_\text{d}$, so $E = Bv_\text{d}$. With $V_\text{H} = Ew$ and $I = nev_\text{d}wt$:

$$V_\text{H} = \frac{BI}{ntq},$$

where q is the carrier charge. A **Hall probe** 霍尔探头 uses this to measure B: pass a known current through a thin **semiconductor** 半导体 slab and read V_H (largest when the slab is at right angles to B).



× B into the page; electrons drift against I

$$V_\text{H} = \frac{B\,I}{n\,t\,q}$$

The Hall effect

Deriving $V_\text{H} = BI/(ntq)$ (the exam asks for this in full). Take a slice of width d, across which the Hall voltage appears, and thickness t along B, so the current passes through a cross-section $A = dt$. (1) At steady state the electric force on a carrier balances the magnetic force: $qE = Bqv$, so $E = Bv$. (2) The Hall field is uniform across the width, so $V_\text{H} = Ed = Bvd$. (3) The current is $I = nAvq = n\,dt\,vq$, so $v = I/(n\,dt\,q)$. Substituting: $V_\text{H} = Bd \cdot I/(n\,dt\,q) = BI/(ntq)$. Two things to notice: the voltage appears across the faces **perpendicular to the magnetic force**, not across the ends where the current enters; and it is **larger for a semiconductor**, whose number density n is millions of times smaller than a metal's, which is why probes use a semiconductor slice.

Using a Hall probe. The reading is proportional to B but also depends on the **orientation**: it is a maximum when the plane of the slice is at **right angles** to the field and zero when the field lies in the plane of the slice. So (1) rotate the probe until the reading is largest, and (2) **calibrate** 校准 it in a known field, since $V_\text{H} \propto B$ at fixed current.

> **Worked example.** A Hall probe gives a maximum reading of 24 mV in a uniform field of 32 mT. In a second field, with the probe again turned for a maximum, the reading is 15 mV. Find the second flux density.

$V_\text{H} \propto B$ (same probe, same current): $B = 32 \times 15/24 = 20$ mT. If the probe had been left at some other angle the reading would be smaller and B would be underestimated, which is why "rotate for a maximum" is part of the method.

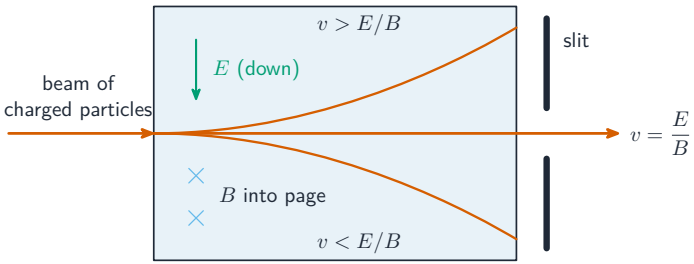
Velocity selector

A **velocity selector** 速度选择器 uses crossed electric and magnetic fields to let through only one speed. With the electric force qE and magnetic force qvB set to oppose each other, the net force is zero only when

$$qE = qvB \quad \Rightarrow \quad v = \frac{E}{B}.$$

Particles at speed E/B go straight through; faster or slower ones are deflected.

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goes straight when $qE = qvB$, i.e. $v = E/B$

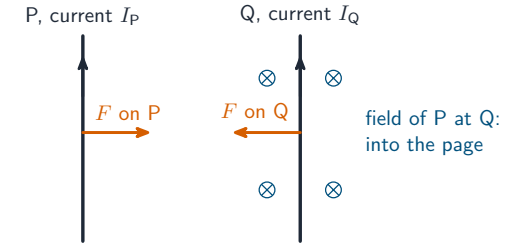
Velocity selector

Deriving and explaining the selector. The magnetic force on a positive ion moving through the crossed fields is BQv, at right angles to its motion; the electric field is applied so that the electric force QE is in the **opposite** direction (the plates must be arranged so that the field points against the magnetic force). For the ion to pass straight through, $QE = BQv$, so $v = E/B$, independent of the charge and the mass. A faster ion has a larger magnetic force and bends towards the magnetic-force side; a slower one is pushed the other way by the unchanged electric force. Asked "explain how an electric field can make the particle reach Z", say which direction the electric force must have, that it must equal the magnetic force in size, and give $E = Bv$. This is **velocity selection** 速度选择.

Force between parallel currents

Two long parallel wires each sit in the other's magnetic field. Using Fleming's left-hand rule: **parallel currents (same direction) attract; antiparallel currents (opposite directions) repel**. This force was the basis of the old SI definition of the ampere; since 2019 the ampere has been defined from the fixed value of the elementary charge e.

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currents in the same direction: the wires attract
currents in opposite directions: they repel
(equal and opposite forces, Newton's third law)

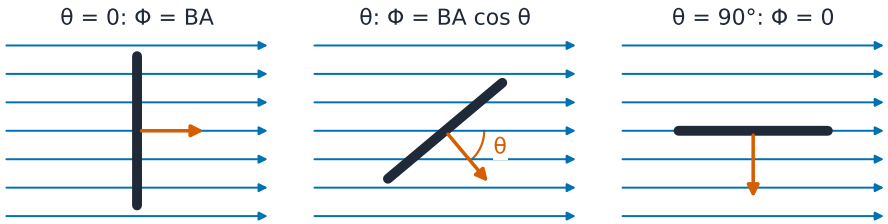
Parallel currents attract: each wire sits in the other's field, and Fleming's left-hand rule gives a force towards the other wire

Explaining the force (a standard three-marker). (1) The current in P produces a magnetic field around it (circles centred on P). (2) At Q this field is at right angles to the current in Q, so Q, a current-carrying conductor in a magnetic field, feels a force BIL. (3) Fleming's left-hand rule gives its direction: towards P for currents in the same direction, away for opposite currents. By Newton's third law the force on P is equal and opposite, so both wires attract (or both repel) with the same force. Doubling the current in **one** wire doubles the force on **both**: the force on Q rises because I_Q has doubled, the force on P because the field it sits in has doubled.

Electromagnetic induction

Magnetic flux

The **magnetic flux** 磁通量 Φ through a flat area A at right angles to B is



edge-on view: black = coil, red = normal to the coil, blue = field B

Only the part of the field along the normal passes through the coil

$$\Phi = BA.$$

If the area's normal is at angle θ to B, use $\Phi = BA\cos\theta$. Unit: **weber** 韦伯, Wb = T m^2. For a **coil** 线圈 of N turns, the **flux linkage** 磁链 is $N\Phi = NBA$.

The two-mark definitions. *Magnetic flux is the product of the magnetic flux density and the area (of the circuit) perpendicular to the field. Flux linkage is the product of the flux and the number of turns of the coil,* $N\Phi$, in weber-turns. Say "perpendicular" (or "normal to the field"): $\Phi = BA$ is only true for the component of the area at right angles to B.

> **Worked example.** A small coil of 64 turns and cross-sectional area 0.71 cm^2 is placed inside a long solenoid, on its axis, where the flux density is 2.5 mT. Find the flux linkage of the coil.

The flux through the coil uses the **coil's own** area, not the solenoid's: $\Phi = BA = (2.5\times10^{-3})(0.71\times10^{-4}) = 1.78\times10^{-7}$ Wb, so the flux linkage is $N\Phi = 64\times1.78\times10^{-7} = 1.1\times10^{-5}$ Wb. When the solenoid's current changes, the e.m.f. induced in the small coil is the rate of change of this linkage.

Faraday's and Lenz's laws

When the flux linkage through a circuit **changes**, an **electromotive force** 电动势 (e.m.f.) is induced – this is **electromagnetic induction** 电磁感应.

Faraday's law 法拉第定律: the induced e.m.f. equals the rate of change of flux linkage:

$$|\varepsilon| = N\frac{d\Phi}{dt}.$$

> **Worked example.** A coil of 200 turns and area 0.010 m^2 sits with its plane at right angles to a 0.50 T field. The field falls steadily to zero in 0.20 s. Find the average induced e.m.f.

The flux linkage changes from $N\Phi = NBA = 200\times0.50\times0.010 = 1.0$ Wb to zero, so

$$|\varepsilon| = \frac{\Delta(N\Phi)}{\Delta t} = \frac{1.0}{0.20} = 5.0\text{ V}.$$

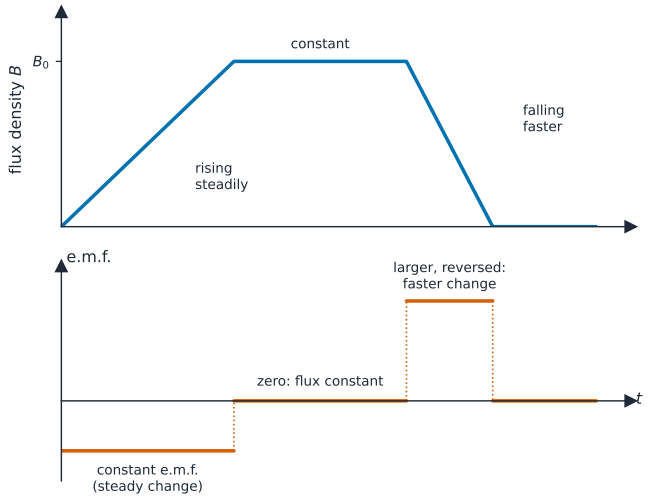
Lenz's law 楞次定律: the induced e.m.f. acts to **oppose the change** that makes it. This is **conservation of energy** 能量守恒 – if it reinforced the change, energy would come from nothing. Combined:

$$\varepsilon = -\frac{d(N\Phi)}{dt}.$$

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What changes the flux?

- **changing** B (moving a magnet near a coil),
- **changing area** A (a rod sliding along rails),
- **changing orientation** (a coil turning in a field – the a.c. generator, next topic).



The induced e.m.f. is minus the gradient of the flux-linkage graph: constant while the flux changes steadily, zero while it is constant, and larger and reversed when the flux falls faster

Reading an e.m.f. off a flux graph. The induced e.m.f. is the **gradient** of the flux-linkage graph (with a minus sign for Lenz). A steadily rising flux gives a **constant** e.m.f.; a constant flux gives **zero** e.m.f. however large the flux is; a faster fall gives a larger e.m.f. of the opposite sign. "Sketch the variation with time of the e.m.f." is answered by differentiating the flux graph by eye, section by section.

> **Worked example.** The 64-turn coil above sits in a solenoid whose flux density rises uniformly from 0 to 2.5 mT in 40 ms and is then held steady. Find the e.m.f. induced in the coil during the rise, and afterwards.

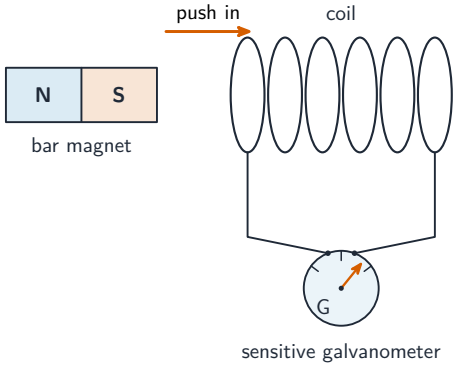
During the rise the flux linkage grows steadily to 1.1×10^{-5} Wb: $\varepsilon = \Delta(N\Phi)/\Delta t = 1.1\times10^{-5}/0.040 = 2.8\times10^{-4}$ V (0.28 mV), constant while the rise lasts. Afterwards the flux linkage is constant, so the e.m.f. is **zero**, even though the coil is still in a strong field: it is the **change** that induces.

> **Worked example (flux cutting).** An aircraft with a **wingspan** 翼展 of 60 m flies horizontally at 250 m s^{-1} where the vertical component of the Earth's field is 45 μT. Find the e.m.f. between the wingtips.

In time Δt the wing sweeps out an area $Lv\Delta t$, cutting flux $BLv\Delta t$, so $\varepsilon = BLv = (45 \times 10^{-6})(60)(250) = 0.68$ V. Only the component of B **perpendicular** to the swept area counts (here the vertical one). There is a p.d. between the wingtips but no current, because there is no complete circuit; Fleming's rules give which tip is positive.

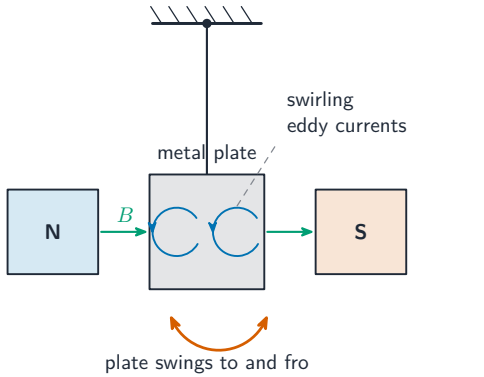
Demonstrations

- moving a bar magnet into a coil deflects a **galvanometer** 检流计; the deflection reverses when the magnet is pulled out (Lenz's law), and is larger for faster motion (Faraday's law).



Demonstrating electromagnetic induction

- a copper disc swinging into a field is quickly slowed – **eddy currents** 涡流 are induced that oppose the motion.



Eddy-current damping

Electromagnetic braking, explained with Lenz's law. A vehicle's aluminium disc rotates between the poles of an electromagnet. The disc **cuts** the field lines, so an e.m.f. is induced in it; the disc is a conductor, so **eddy currents** flow; by Lenz's law these currents produce forces that **oppose the motion** of the disc, slowing the vehicle. The kinetic energy is converted to **thermal energy** in the disc. The braking is stronger at high speed (a larger rate of flux cutting, so larger currents) and fades as the vehicle slows; it cannot hold a stationary vehicle, because no motion means no induced current.

A magnet oscillating in a coil. A bar magnet on a spring oscillates in and out of a coil connected to a resistor. The changing flux linkage induces an e.m.f.; with the circuit complete a current flows and, by Lenz's law, the coil's field opposes the magnet's motion, so the oscillation is **damped**: energy leaves the oscillation as heat in the resistor and the amplitude decays. Open the switch and the e.m.f. is still induced but no current flows, so the coil no longer damps the motion.

What makes the induced e.m.f. larger

From $\varepsilon = N\,d\Phi/dt$ with $\Phi = BA$: more turns N, a stronger B, a larger area A, or a faster change – each gives a larger induced e.m.f.

Finding the direction with Lenz's law. Push the N pole of a magnet towards a coil and the induced current makes the near face of the coil a **north** pole, to repel the approaching magnet; pull it away and the near face becomes a **south** pole, to attract it. In each case the person moving the magnet does work against that force, and that work is the source of the electrical energy: Lenz's law is conservation of energy in disguise. Two sentences to learn: *the induced e.m.f. acts in such a direction as to produce effects that oppose the change producing it* (Lenz), and *the induced e.m.f. is proportional to the rate of change of flux linkage* (Faraday).

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
magnetic field	a region in which a force acts on a moving charge, a current-carrying conductor or a magnetic material
magnetic flux density	the force per unit current per unit length on a straight conductor placed at right angles to the field
tesla	the flux density producing a force of 1 N on 1 m of conductor carrying 1 A at right angles to the field
magnetic flux	the product of the flux density and the area perpendicular to the field, $\Phi = BA$

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Term	Definition
flux linkage	the product of the flux through a coil and its number of turns, $N\Phi$
weber	one tesla metre squared
Faraday's law	the induced e.m.f. is proportional to the rate of change of flux linkage
Lenz's law	the induced e.m.f. acts in such a direction as to produce effects that oppose the change producing it
Hall voltage	the p.d. that develops across a current-carrying slice in a magnetic field, $V_{\mathrm{H}} = BI/(ntq)$

Exam tips

- Force on a current $F = BIL\sin\theta$; on a moving charge $F = BQv\sin\theta$; direction from **Fleming's left-hand rule** with the second finger along the **conventional** current (reverse it for an electron).
- A charge moving at right angles to B moves in a circle: $BQv = mv^2/r$, so $r = mv/(BQ)$ and $T = 2\pi m/(BQ)$, independent of speed. A component of v along B is unchanged: a helix.
- Crossed fields: $v = E/B$ passes undeflected. Hall probe: $V_{\mathrm{H}} = BI/(ntq)$, largest with the slice at right angles to B; semiconductor because n is small.
- Induction: e.m.f. = rate of change of **flux linkage** $N\Phi$ (Faraday); its direction opposes the change (Lenz). On a flux–time graph the e.m.f. is the gradient: constant change gives a constant e.m.f., no change gives zero.
- Flux cutting: $\varepsilon = BLv$ for a conductor of length L moving at v at right angles to B (area swept per second times B).
- Definitions carry the words "per unit current per unit length", "at right angles", "area perpendicular to the field" and "rate of change of flux linkage": each is a mark.

Common mistakes

- Leaving "at right angles to the field" out of the definition of flux density, or defining the tesla without the 1 N, 1 m, 1 A.
- Using the right hand for the force on a current (the left-hand rule), or pointing the second finger along the electron's motion instead of the conventional current.
- Saying a stationary charge feels a magnetic force, or that a charge moving along the field lines does.
- Explaining circular motion without saying the force is perpendicular to the velocity and constant in size; or claiming the magnetic force changes the speed.

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- Using the solenoid's area instead of the small coil's when finding its flux linkage.
- Saying "a large flux induces an e.m.f.": only a **changing** flux linkage does. A coil at rest in a steady field has no e.m.f.
- Quoting Lenz's law as "the current opposes the magnet" without "the change producing it", or Faraday's law without "rate of change" or without "flux linkage".
- Drawing straight-wire field lines as evenly spaced circles or without arrows; drawing a solenoid's field lines crossing inside it.
- Forgetting that doubling the current in one of two parallel wires doubles the force on both (Newton's third law).

Alternating currents

A-Level Physics

Alternating current basics

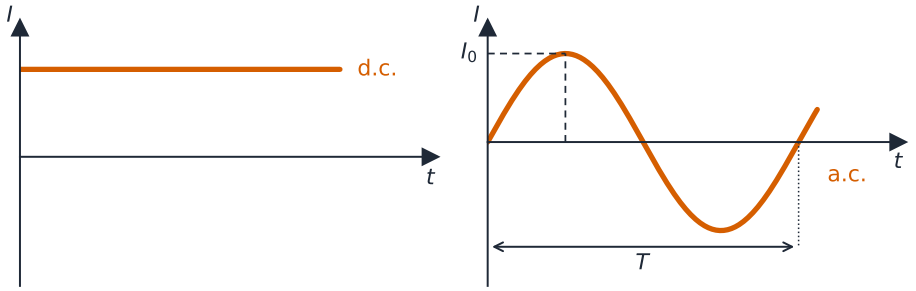


A substation's transformers step alternating voltage up or down.

An **alternating current** 交流电 (a.c.) keeps reversing direction. Mains supply is sinusoidal a.c.: I or V follows a sine wave in time:

$$I = I_0 \sin(\omega t), \qquad V = V_0 \sin(\omega t).$$

(For a purely resistive load the **voltage** 电压 and **current** 电流 are in phase, which is the case in this syllabus.)



d.c. constant polarity · a.c. reverses each cycle

A steady direct current compared with a sinusoidal alternating current of peak I_0 and period T

Key terms

- **period** 周期 T – the time for one full cycle. Unit: s.
- **frequency** 频率 f – cycles per second; $f = 1/T$. Mains is often 50 Hz or 60 Hz.
- **angular frequency** 角频率 $\omega = 2\pi f = 2\pi/T$.
- **peak value** 峰值 I_0 or V_0 – the largest value in a cycle (also called the amplitude).
- **peak-to-peak value** 峰峰值 $2I_0$ – from $+I_0$ to $-I_0$. Useful when reading an oscilloscope.

The two-mark definitions. *The frequency of an alternating current is the number of complete cycles per unit time* (say "per second" or "per unit time", and "complete cycles" or "oscillations"). *The period is the time for one complete cycle*, and *the peak value is the maximum value of the current or voltage during a cycle.* Note that the **mean** value of a sinusoidal current over a cycle is **zero**, which is exactly why the r.m.s. value is needed to describe it.

Reading the equation. In $V = V_0 \sin(\omega t)$ the number in front is the peak value and the number multiplying t is $\omega = 2\pi f$; the angle ωt is in **radians** 弧度, so set the calculator to radians before evaluating. A supply written with cos instead of sin is the same wave starting at its peak rather than at zero.

> **Worked example.** The output of a supply is $V = 320 \sin(100\pi t)$ (volts, seconds). Find the peak value, the frequency, the period and the r.m.s. value, and the first time after $t = 0$ at which $V = 160$ V.

Peak $V_0 = 320$ V. $\omega = 100\pi$ rad s^{-1}, so $f = \omega/2\pi = 50$ Hz and $T = 1/f = 0.020$ s. $V_{\text{r.m.s.}} = 320/\sqrt{2} = 226$ V. For $V = 160$ V: $\sin(100\pi t) = 0.5$, so $100\pi t = \pi/6$ and $t = 1/600 = 1.7 \times 10^{-3}$ s. (For $V = 18\cos(40\pi t)$ the same reading gives $V_0 = 18$ V, $f = 20$ Hz, $T = 50$ ms, and a sketch that starts at +18 V.)

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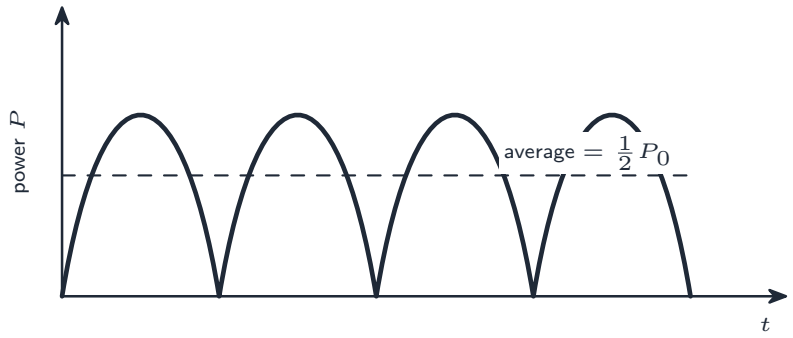
■ Reading a CRO trace

Same as for any wave (Topic 7), using a **cathode-ray oscilloscope** 示波器:

- horizontal divisions × **time-base** 时基 → period T, so $f = 1/T$.
- vertical divisions × y-gain → peak voltage V_0 (measure centre to peak, or peak-to-peak then halve).

Power delivered to a resistor

For a resistive load R, the instant **power** 功率 is $P(t) = I(t)^2 R$. With $I = I_0 \sin(\omega t)$:



The power in a resistor pulses; its average is half the peak

$$P(t) = I_0^2 R \sin^2(\omega t).$$

This is always positive, with peak $I_0^2 R$ and minimum zero, oscillating at twice the frequency of I. The mean of $\sin^2(\omega t)$ over a cycle is $\frac{1}{2}$, so the **average power** is

$$\langle P \rangle = \tfrac{1}{2} I_0^2 R = \tfrac{1}{2} P_{\text{peak}}.$$

Average a.c. power in a resistor is **half the peak power**.

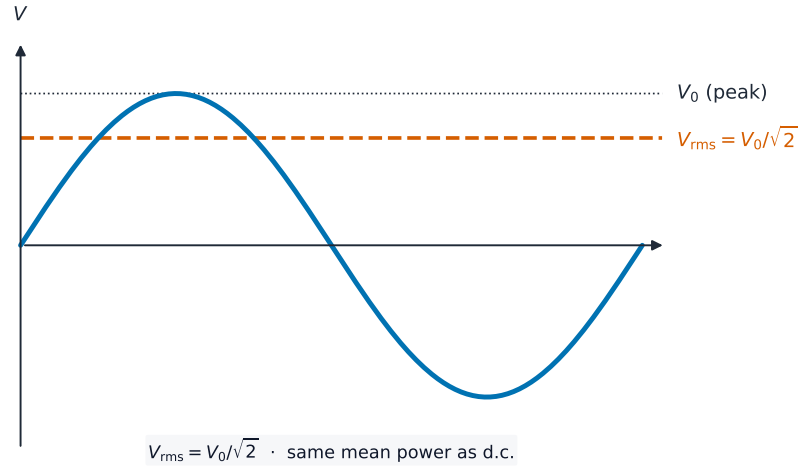
"Show by calculation that the mean power is half the peak power." A supply of peak value 12 V drives a 680 Ω resistor. Peak power: $P_0 = V_0^2/R = 12^2/680 = 0.212$ W, the instantaneous power at the moment the voltage is at its peak. Mean power: use the r.m.s. value, $V_{\text{r.m.s.}} = 12/\sqrt{2} = 8.49$ V, so $\langle P \rangle = V_{\text{r.m.s.}}^2/R = 8.49^2/680 = 0.106$ W, exactly half. The two calculations must be shown separately; writing "$\frac{1}{2}$ of 0.212" scores nothing, because that is the thing being shown. The $\frac{1}{2}$ is $(1/\sqrt{2})^2$: squaring the r.m.s. factor.

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Root-mean-square (r.m.s.) values

The r.m.s. current $I_{\text{r.m.s.}}$ is the **steady direct current** that would give the same average power in the same **resistance** 电阻 R. From $\langle P \rangle = I_{\text{r.m.s.}}^2 R = \frac{1}{2} I_0^2 R$:

$$I_{\text{r.m.s.}} = \frac{I_0}{\sqrt{2}}, \qquad V_{\text{r.m.s.}} = \frac{V_0}{\sqrt{2}}.$$



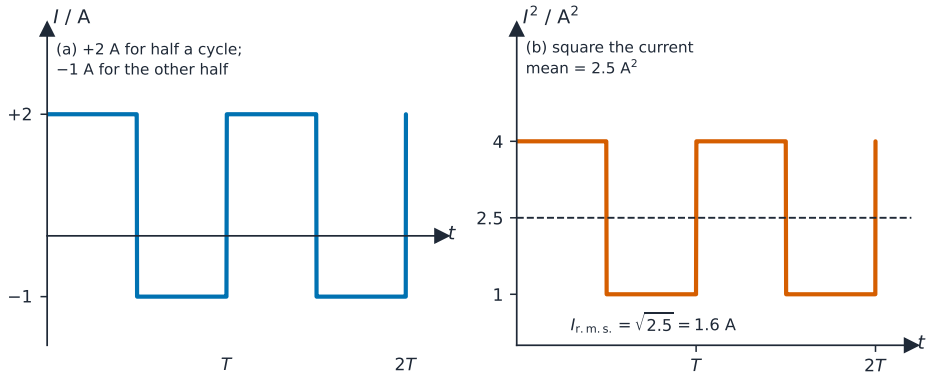
The r.m.s. value is the steady d.c. level ($V_0/\sqrt{2}$) that delivers the same average power as the a.c.

Worked example. An a.c. supply has a peak voltage of 12 V. Find its r.m.s. voltage.

$$V_{\text{r.m.s.}} = \frac{V_0}{\sqrt{2}} = \frac{12}{\sqrt{2}} \approx 8.5 \text{ V}.$$

The $\sqrt{2}$ comes from the name **root-mean-square** 均方根: $I_{\text{r.m.s.}} = \sqrt{\langle I^2 \rangle}$ and $\langle \sin^2 \rangle = \frac{1}{2}$. (Only the sinusoidal case is needed.)

The two-mark definition. *The r.m.s. value of an alternating current is the value of the direct (steady) current that would dissipate the same (mean) power in the same resistor.* "By reference to the heating effect" means exactly this sentence: same resistor, same power (or same heating), direct current. A definition that only says "$I_0/\sqrt{2}$" scores nothing, because that formula is true only for a sine wave.



Root-mean-square, taken literally: square the current, average the squares over a cycle, take the square root. For a non-sinusoidal current this is the only way; $I_0/\sqrt{2}$ applies to a sine wave alone

Worked example (non-sinusoidal). A current through a resistor is +2.0 A for the first half of each cycle and −1.0 A for the second half, a **square wave** 方波. Find its r.m.s. value and the mean power in a 10 Ω resistor.

Square the current: 4.0 A^2 for half the time and 1.0 A^2 for the other half, so the mean square is $(4.0 + 1.0)/2 = 2.5$ A^2 and $I_{\text{r.m.s.}} = \sqrt{2.5} = 1.6$ A. Mean power $= I_{\text{r.m.s.}}^2 R = 2.5 \times 10 = 25$ W. The sign of the current does not matter to the heating (it is squared away), and $I_0/\sqrt{2}$ would have given the wrong answer, 1.4 A: that shortcut is for sine waves only.

■ Why r.m.s. matters

Quoted a.c. values are r.m.s. values. "230 V mains" means $V_{\text{r.m.s.}} = 230$ V, with peak $V_0 = 230\sqrt{2} \approx 325$ V. Components must be rated for the peak, not the r.m.s. Average power then takes the d.c. form:

$$\langle P \rangle = I_{\text{r.m.s.}}^2 R = V_{\text{r.m.s.}}^2 / R = V_{\text{r.m.s.}} I_{\text{r.m.s.}}.$$

Worked example. A heater of resistance 50 Ω is connected to the 230 V r.m.s. mains. Find the r.m.s. current and the average power dissipated.

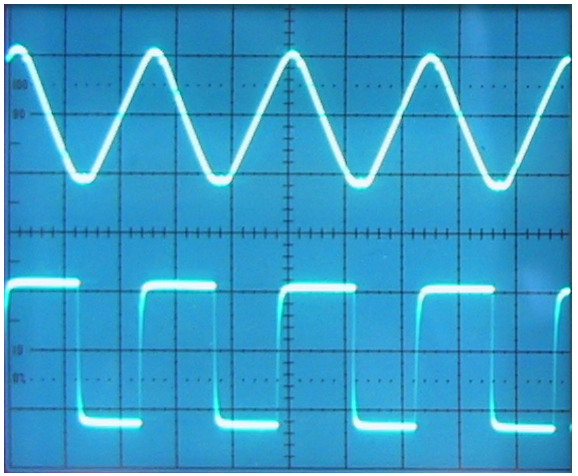
$$I_{\text{r.m.s.}} = \frac{V_{\text{r.m.s.}}}{R} = \frac{230}{50} = 4.6 \text{ A}, \qquad \langle P \rangle = V_{\text{r.m.s.}} I_{\text{r.m.s.}} = 230 \times 4.6 \approx 1.1 \times 10^3 \text{ W}.$$

Worked example. A sinusoidal supply of r.m.s. value 4.2 V and frequency 50 kHz is connected across a 150 Ω resistor. Find the peak current, the mean power, and write down an equation for the current.

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$V_0 = 4.2\sqrt{2} = 5.9$ V, so $I_0 = V_0/R = 5.9/150 = 0.040$ A (or $I_{\text{r.m.s.}} = 4.2/150 = 0.028$ A and then $\times\sqrt{2}$). Mean power $= V_{\text{r.m.s.}}^2/R = 4.2^2/150 = 0.12$ W. With $\omega = 2\pi f = 2\pi \times 5.0 \times 10^4 = 3.1 \times 10^5$ rad s^{-1}: $I = 0.040 \sin(3.1 \times 10^5 t)$ (amps, seconds). Keep the two families apart: peak values go into the equation and into component ratings; r.m.s. values go into power.

Rectification



An oscilloscope shows how a voltage varies with time.

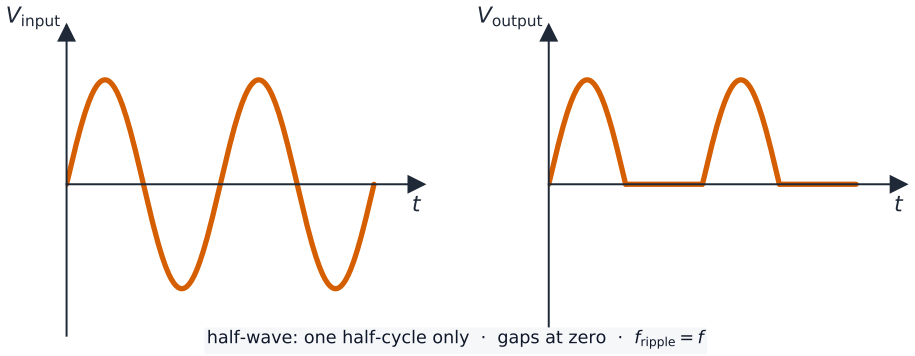
Rectification 整流 turns an alternating voltage into a one-direction (d.c.-like) voltage, using **diodes** 二极管 (which conduct in only one direction).

"State what is meant by rectification." *The conversion of an alternating current (or voltage) into a direct current (or voltage)*: one that flows in one direction only, however much it varies. A **diode** conducts only when it is **forward-biased** 正向偏置, that is, when its anode (the flat end of the symbol's triangle) is more positive than its cathode (the bar); otherwise it is reverse-biased and behaves like an open switch. Treat it as ideal: zero resistance one way, infinite the other.

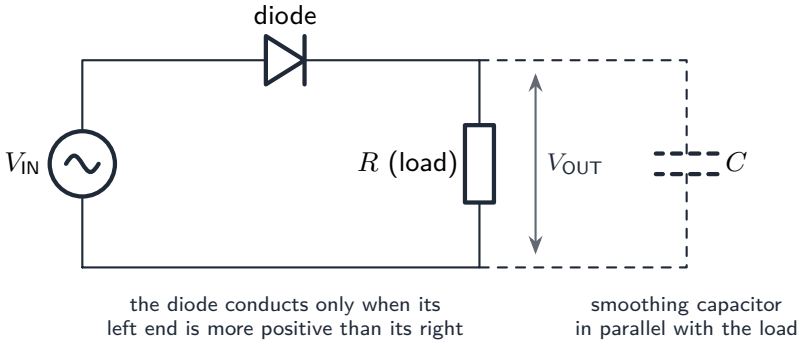
■ Half-wave rectification

A **single diode** in series with the load passes only the positive half of each cycle; in the negative half the diode is **reverse-biased** 反向偏置 and no current flows. This is **half-wave rectification** 半波整流.

Output: positive half-waves with flat zero gaps. The mean output is $V_0/\pi \approx 0.32 V_0$. Drawback: half the input is wasted and the output is very uneven.



In half-wave rectification a single diode passes only the positive half-cycles

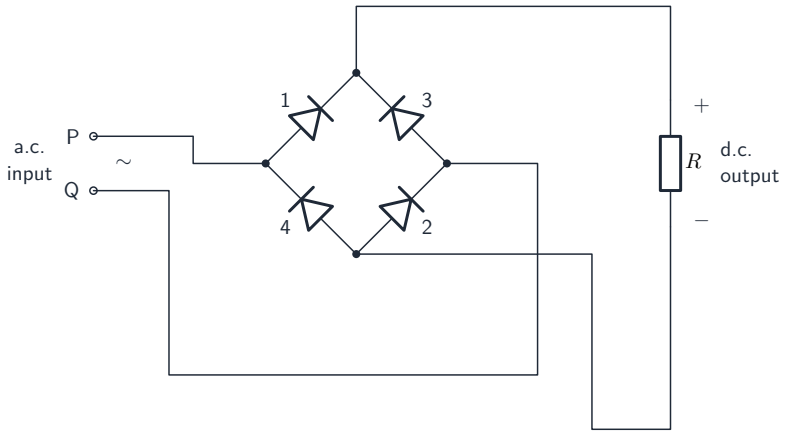


The half-wave rectifier the exam asks you to draw: source, diode, load in series, output across the load; a smoothing capacitor, if wanted, goes in parallel with the load, never in series

Completing the circuit. "Complete the diagram to produce half-wave rectification" needs one diode in **series** between the supply and the load, with the triangle pointing the way the output current must flow, and V_{OUT} taken across the **load**. If a capacitor is to smooth the output it goes **across the load** (in parallel); a capacitor in series would block the d.c. altogether.

■ Full-wave rectification (bridge rectifier)

A **bridge rectifier** 桥式整流器 uses **four diodes** arranged so the current through the load always flows the same way, whichever a.c. terminal is positive – **full-wave rectification** 全波整流. On each half-cycle a different pair of diodes conducts, but the load always sees the same direction.



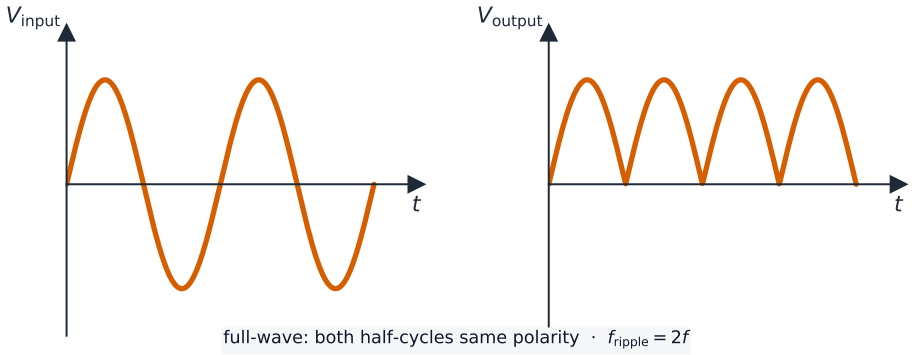
A four-diode bridge sends the load current the same way whichever a.c. terminal is positive

Explaining the bridge (the standard four-marker). When terminal P is positive, current leaves P, passes through the diode pointing away from P to the **top** of the load, flows **down** through the load and returns to Q through the diode pointing towards Q; the other two diodes are reverse-biased and carry nothing. When Q is positive the other pair conducts, but they are arranged so that the current still enters the load at the top: the load current is in the **same direction** in both half-cycles. Name the diodes in each half-cycle and say which end of the load is positive.

Completing a bridge. Given a bridge with diodes missing, remember the rule for every diode: conventional current flows through it in the direction of the triangle. Both diodes joined to the **positive** output terminal must point **towards** it; both joined to the **negative** output terminal must point **away** from it. A bridge with a diode the wrong way round either short-circuits the supply on one half-cycle or passes nothing.

Output: a continuous run of positive half-waves (no gaps), at **twice** the input frequency. The mean output is $2V_0/\pi \approx 0.64V_0$ – double the half-wave value. It uses all the input and is smoother and easier to filter.

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In full-wave rectification every half-cycle is used, giving a continuous run of positive humps

■ Drawing the diagrams

- half-wave: a.c. source – single diode – load R, in series.
- full-wave bridge: four diodes as the arms of a "diamond"; the a.c. input goes to one pair of opposite corners, the load R across the other pair. The diode directions make the load terminals keep the same polarity for either input polarity.

"State the difference between half-wave and full-wave rectification." In half-wave rectification only **one** half of each input cycle appears at the output and the other half is blocked (the output is zero for half the time); in full-wave rectification **both** halves appear, one of them inverted, so there are no gaps and the output has twice the input frequency. A sketch should show, for half-wave, positive humps separated by flat zero sections of equal length; for full-wave, positive humps joined at the zero line.

Smoothing with a capacitor

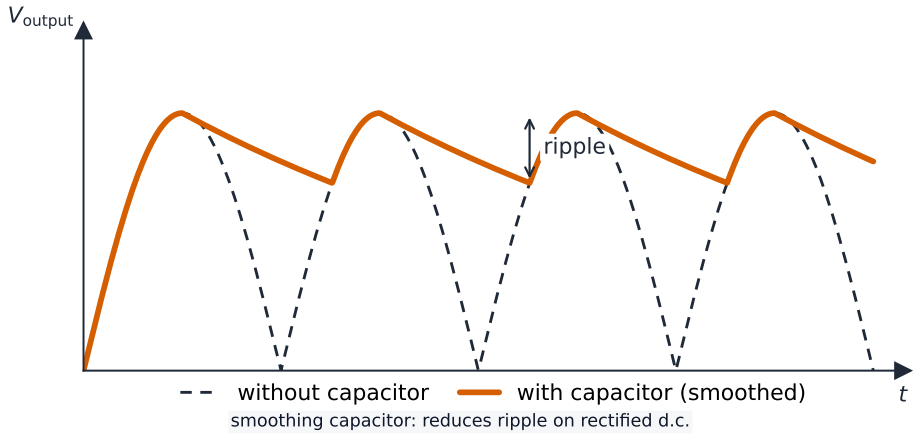
A rectifier's output is still bumpy. To smooth it, put a **capacitor** 电容器 C **in parallel with the load** R.

■ How it works

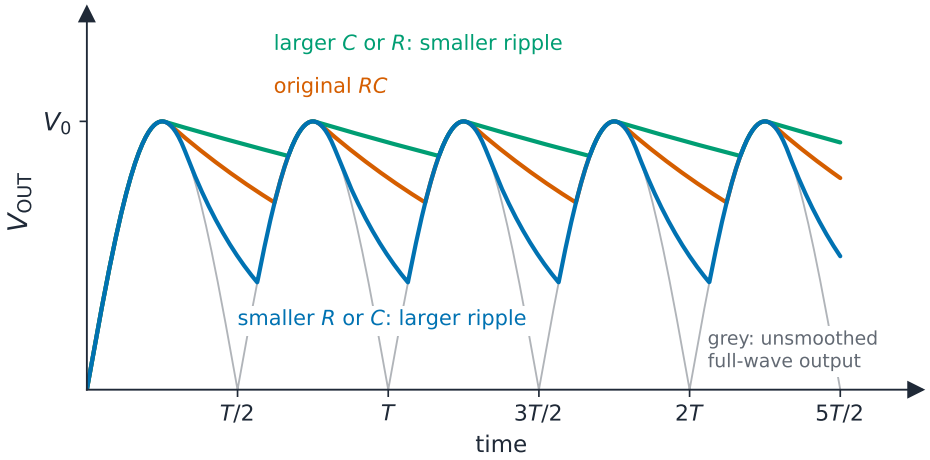
- on the **rising part** of each pulse, the capacitor charges up to near the peak.
- on the **falling part** (and any gap), the diodes are reverse-biased, so the capacitor **discharges through the load**, keeping current flowing. The voltage falls with **time constant** 时间常数 RC (Topic 19).
- at the next peak, the capacitor charges again, and the cycle repeats.

The output now sits near the peak with small dips. The size of the dips is the **ripple** 纹波 (this whole step is called **smoothing** 平滑).

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A capacitor across the load smooths the rectified output, leaving only a small ripple



The same rectified supply smoothed with different time constants: a larger RC (more capacitance, or a larger load resistance) decays less between peaks, so the ripple is smaller

Worked example. A half-wave rectifier fed from a 50 Hz supply has a 470 μF capacitor across its 1.2 kΩ **load** 负载. Estimate the fractional fall in the output between peaks, and say how it changes with a bridge rectifier.

The peaks are $T = 1/50 = 20$ ms apart (half-wave: one peak per cycle). The time constant is $RC = (1.2 \times 10^3)(470 \times 10^{-6}) = 0.56$ s. Between peaks the capacitor discharges to $V_0 e^{-t/RC} = V_0 e^{-0.020/0.56} = 0.965\,V_0$, a fall of about 3.5%. With full-

wave rectification the peaks are 10 ms apart, so the fall is only 1.8%; doubling the capacitance would halve it again. Because $t \ll RC$ the fall is approximately $V_0\, t/(RC)$: the ripple is proportional to the time between peaks and inversely proportional to both R and C. (A network of capacitors across the output combines by the rules of Topic 19 before it is used here.)

What reduces the ripple

- **larger** C → more stored charge → smaller dip between peaks → smaller ripple.
- **larger** R → smaller load current → slower discharge → smaller ripple.
- **higher rectified frequency** (full-wave is twice the input) → less time to discharge between peaks → smaller ripple.

In short, a large RC compared with the time between peaks gives a smoother output.

Sketching the smoothed output. Draw the unsmoothed humps faintly first. The smoothed curve **touches each peak**, then falls along a gentle curve (steepest just after the peak) until the next hump rises to meet it, where it turns sharply upwards and follows the hump to the peak. It never falls to zero, and it never rises above the peak. For half-wave rectification the decay has to last through the missing half-cycle as well, so the ripple is about twice that of full-wave for the same RC. If the question changes C or R, redraw on the same axes: a larger RC hugs the peak line more closely; a smaller RC sags further.

Purpose in summary

The smoothing capacitor reduces the ripple, giving a steadier d.c. voltage suitable for sensitive electronics.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
alternating current	a current that reverses its direction periodically (varies sinusoidally with time about zero)
period	the time for one complete cycle
frequency	the number of complete cycles per unit time
peak value	the maximum value of the current or voltage in a cycle
r.m.s. value	the value of the direct (steady) current that would dissipate the same power in the same resistor
rectification	the conversion of an alternating current into a direct (one-direction) current

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Term	Definition
half-wave rectification	only one half of each cycle is passed to the output; the other half is blocked
full-wave rectification	both halves of each cycle appear at the output in the same direction
smoothing	using a capacitor across the load to reduce the variation (ripple) of a rectified output

Exam tips

- $V = V_0 \sin \omega t$ with $\omega = 2\pi f$ in radians per second: read V_0 and ω straight off the equation; radian mode for any time calculation.
- $I_{\text{r.m.s.}} = I_0/\sqrt{2}$ and $V_{\text{r.m.s.}} = V_0/\sqrt{2}$ for a **sine wave only**; otherwise square, average, root. Quoted mains values are r.m.s.; components are rated for the peak.
- Mean power in a resistor is $I^2_{\text{r.m.s.}} R = V^2_{\text{r.m.s.}}/R = \frac{1}{2} I_0^2 R$, half the peak power; the mean **current** is zero, the mean **power** is not.
- One diode in series gives half-wave rectification; a four-diode bridge gives full-wave, with the load current always in the same direction and the output at twice the input frequency.
- Smoothing: capacitor **across the load**; ripple falls with a larger C, a larger R (smaller load current) and a higher rectified frequency, because the decay between peaks is $V_0 e^{-t/RC}$.
- Sketches are marked on shape: humps that touch the peak line, a decay that never reaches zero, flat zero gaps for half-wave, none for full-wave.

Common mistakes

- Using the peak value in a power calculation, or the r.m.s. value as the amplitude in $V = V_0 \sin \omega t$.
- Writing $\omega = f$ or $\omega = 2\pi/f$; it is $2\pi f = 2\pi/T$.
- Defining the r.m.s. value as "the peak divided by $\sqrt{2}$", or leaving out "same power" and "same resistor".
- Applying $I_0/\sqrt{2}$ to a square or triangular wave.
- Drawing a diode backwards, or a bridge in which two diodes joined to the same output terminal point opposite ways.
- Putting the smoothing capacitor in series with the load.
- A smoothed output sketched falling to zero between peaks or above the peak.
- A larger load resistance gives a **smaller** ripple (slower discharge), not a larger one.
- Unsmoothed full-wave rectification is not steady d.c.: it is a train of humps.

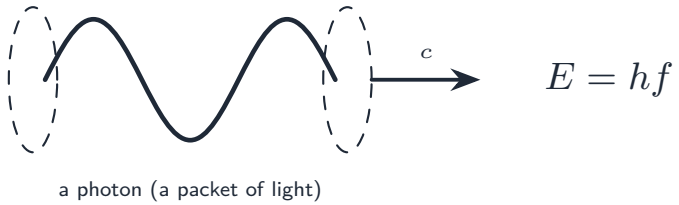
Quantum physics

A-Level Physics

Photons: the particle nature of light

Electromagnetic radiation behaves like **particles** as well as like a wave. The particles of EM radiation are **photons** 光子 – small packets ("**quanta**" 量子) of EM energy that travel at the speed of light.

"State what is meant by a photon" (two marks). *A photon is a quantum (a discrete packet) of energy of electromagnetic radiation.* Both halves score: **quantum** or **packet** (or "discrete amount"), and **of electromagnetic radiation** (or "of light"). "A particle of light" alone is not enough. That radiation comes in such packets is what the syllabus calls its **particulate nature** 粒子性: energy is delivered in lumps of hf, never in smaller pieces.



A photon is a packet of energy, $E = hf$

Energy of a photon

A photon of **frequency** 频率 f has energy

$$E = hf,$$

where $h = 6.63 \times 10^{-34}$ J s is the **Planck constant** 普朗克常量. Using $c = f\lambda$:

$$E = \frac{hc}{\lambda}.$$

Worked example. Find the energy of a photon of green light of wavelength 500 nm. ($h = 6.63 \times 10^{-34}$ J s, $c = 3.0 \times 10^{8}$ m s^{-1}.)

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34})(3.0 \times 10^{8})}{500 \times 10^{-9}} \approx 4.0 \times 10^{-19} \text{ J } (\approx 2.5 \text{ eV}).$$

Higher-frequency (shorter-**wavelength** 波长) photons carry more energy: one γ-ray photon carries far more than one radio photon.

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The electronvolt

The **electronvolt** 电子伏特 (eV) is a handy energy unit on the atomic scale:

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}.$$

It is the **kinetic energy** 动能 an **electron** 电子 gains moving through a **potential difference** 电势差 of 1 V. For example, a visible photon ($\lambda \approx 500$ nm) has energy ≈ 2.5 eV. To go eV $\to$ J multiply by 1.60×10^{-19}; J $\to$ eV divide.

Using the electronvolt. Photon energies, work functions and energy levels are all a few eV, so the exam quotes them that way and expects you to move between units without fuss: a 2.0 eV work function is $2.0 \times 1.60 \times 10^{-19} = 3.2 \times 10^{-19}$ J; a photon of 4.0×10^{-19} J is 2.5 eV. A useful shortcut for wavelengths: $hc = 1.99 \times 10^{-25}$ J m = 1240 eV nm, so a 2.0 eV photon has $\lambda = 1240/2.0 = 620$ nm (red) and a 400 nm photon carries 3.1 eV.

Momentum of a photon

A photon also carries **momentum** 动量:

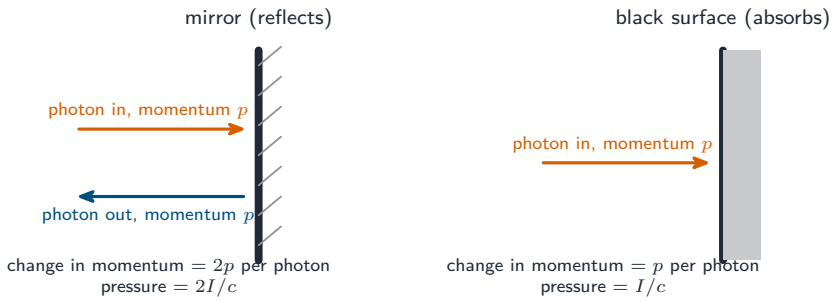
$$p = \frac{E}{c} = \frac{h}{\lambda}.$$

It has zero rest mass but a non-zero momentum E/c. Radiation pressure (photons pushing on a surface) follows from this.

"Show that $p = h/\lambda$." Start from the two photon relations: $E = hf$ and $p = E/c$. Then $p = hf/c$, and since $c = f\lambda$, $f/c = 1/\lambda$, so $p = h/\lambda$. Give both starting equations and the wave equation; the mark is for the chain, not the result.

Worked example. A photon in free space has momentum 9.5×10^{-28} N s. Show that it is a photon of red light.

$\lambda = h/p = 6.63 \times 10^{-34}/9.5 \times 10^{-28} = 7.0 \times 10^{-7}$ m = 700 nm, which lies at the red end of the visible spectrum (400–700 nm). The energy is $pc = 2.9 \times 10^{-19}$ J = 1.8 eV.



force = (photons per second) × (change in momentum per photon); photons per second $= IA/(hf)$

Radiation pressure: force equals the number of photons arriving per second times the change in momentum of each; a mirror doubles the change, an absorber does not

Radiation pressure 辐射压. A beam of intensity I (power per unit area) falling on area A delivers $IA/(hf)$ photons per second, each with momentum h/λ. Force is the rate of change of momentum. On a **mirror** each photon bounces back, so its momentum changes by $2p$ and the force is $F = 2IA/c$ (pressure $2I/c$); on a **black** surface each photon is absorbed, the change is p, and the pressure is I/c. Two things follow, and both are examined: the pressure depends on the **intensity**, not on the colour, because blue light of the same intensity has fewer photons per second but each carries proportionally more momentum; and even sunlight ($I \approx 1$ kW m^{-2}) exerts only a few micropascals.

> **Worked example.** Red light of intensity 160 W m^{-2} falls normally on a plane mirror; each photon has momentum 9.5×10^{-28} N s. Find the number of photons hitting 1.0 m^2 of the mirror per second, and the pressure on it.

Photon energy $E = pc = (9.5 \times 10^{-28})(3.00 \times 10^8) = 2.85 \times 10^{-19}$ J. Photons per second on 1.0 m^2: $160/2.85 \times 10^{-19} = 5.6 \times 10^{20}$ s^{-1}. Each is reflected, so the force is $F = 5.6 \times 10^{20} \times 2 \times 9.5 \times 10^{-28} = 1.1 \times 10^{-6}$ N on 1.0 m^2: a pressure of 1.1×10^{-6} Pa (check: $2I/c = 320/3.00 \times 10^8 = 1.1 \times 10^{-6}$ Pa). Replace the beam with blue light of the same intensity and the pressure is **unchanged**.

> **Worked example.** A laser emits 2.0 mW of light of wavelength 650 nm. Find the number of photons it emits per second, and the force on a surface that absorbs the beam completely.

$E = hc/\lambda = (6.63 \times 10^{-34})(3.00 \times 10^8)/(650 \times 10^{-9}) = 3.06 \times 10^{-19}$ J, so the rate is $P/E = 2.0 \times 10^{-3}/3.06 \times 10^{-19} = 6.5 \times 10^{15}$ s^{-1}. The force is the momentum delivered per second: $F = P/c = 2.0 \times 10^{-3}/3.00 \times 10^8 = 6.7 \times 10^{-12}$ N (or $6.5 \times 10^{15} \times h/\lambda$, the same thing).

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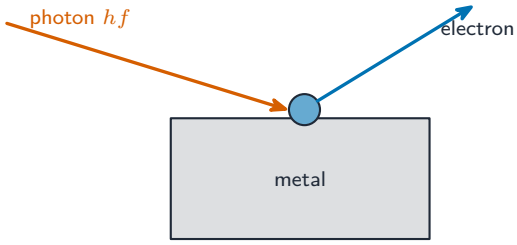
Photoelectric effect



Solar cells use the photoelectric effect to turn light into electricity.

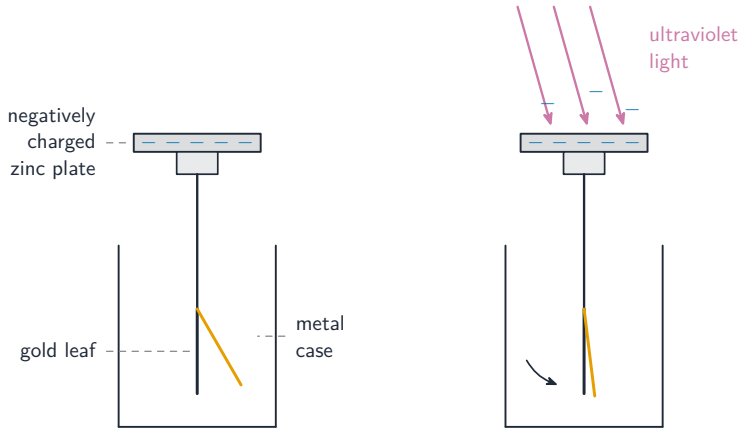
When EM radiation of high enough frequency hits a metal, **electrons are emitted**. These are **photoelectrons** 光电子, and the effect is the **photoelectric effect** 光电效应.

"State what is meant by the photoelectric effect." *The emission of electrons from (the surface of) a metal when electromagnetic radiation of high enough frequency is incident on it.* Two marks: **emission of electrons** and **from a metal surface illuminated by electromagnetic radiation** (or "when light is shone on it"). It is the syllabus's evidence for the particulate nature of radiation, because a wave could not explain what follows.



$hf = \Phi + \mathrm{KE}_{\max}$
(escape energy Φ + leftover KE)

One photon gives its energy hf to one electron: part frees it (the work function Φ), the rest is the electron's KE



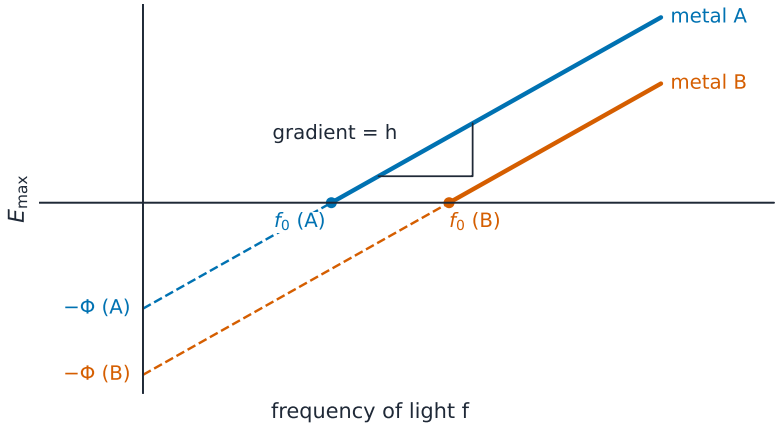
(a) charged plate: leaf stays up

(b) UV frees electrons: plate discharges, leaf falls

A charged zinc plate loses its charge – the gold leaf falls – when ultraviolet light shines on it

Threshold frequency and work function

Each metal has a lowest photon frequency, the **threshold frequency** 极限频率 f_0, below which **no** electrons come out, however bright the light. The **work function** 逸出功 Φ is the **least energy** needed to free an electron from the surface:



Every metal gives a line of gradient h; the intercepts give f_0 and $-\Phi$

$$\Phi = hf_0.$$

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Different metals have different work functions (about 2–5 eV).

The two-mark definitions. *The work function energy of a metal is the minimum energy needed to remove an electron from the surface of the metal.* Both "minimum" and "from the surface" carry marks: an electron deeper in the metal needs more, which is why the equation gives a **maximum** kinetic energy. *The threshold frequency is the minimum frequency of radiation for which photoelectrons are emitted*, and the **threshold wavelength** 极限波长 $\lambda_0 = c/f_0 = hc/\Phi$ is the corresponding **maximum** wavelength: longer wavelengths do nothing.

> **Worked example.** Light of wavelength 400 nm falls on four metals whose work functions are: caesium 2.1 eV, sodium 2.3 eV, zinc 4.3 eV, platinum 5.6 eV. Which emit photoelectrons, and with what maximum kinetic energy?

The photon energy is $hc/\lambda = 1240/400 = 3.1$ eV. Emission needs $hf \geq \Phi$, so **caesium** ($3.1 - 2.1 = 1.0$ eV) and **sodium** (0.8 eV) emit; zinc and platinum do not, however intense the light. To make zinc emit, the wavelength must fall below $\lambda_0 = 1240/4.3 = 290$ nm, in the ultraviolet, which is why the electroscope demonstration needs a UV lamp on zinc.

> **Worked example.** A polished magnesium sheet in a vacuum emits electrons only when the ultraviolet frequency is at least 8.8×10^{14} Hz. It is illuminated at 1.2×10^{15} Hz. Find the work function and the maximum speed of the photoelectrons.

$\Phi = hf_0 = (6.63 \times 10^{-34})(8.8 \times 10^{14}) = 5.8 \times 10^{-19}$ J (3.6 eV). Then $\frac{1}{2}mv_{max}^2 = h(f - f_0) = (6.63 \times 10^{-34})(1.2 \times 10^{15} - 8.8 \times 10^{14}) = 2.1 \times 10^{-19}$ J, so $v_{max} = \sqrt{2 \times 2.1 \times 10^{-19}/9.11 \times 10^{-31}} = 6.8 \times 10^5$ m s^{-1}. Subtract the frequencies **before** multiplying by h; rounding hf and Φ separately loses a significant figure in the small difference.

Einstein's photoelectric equation

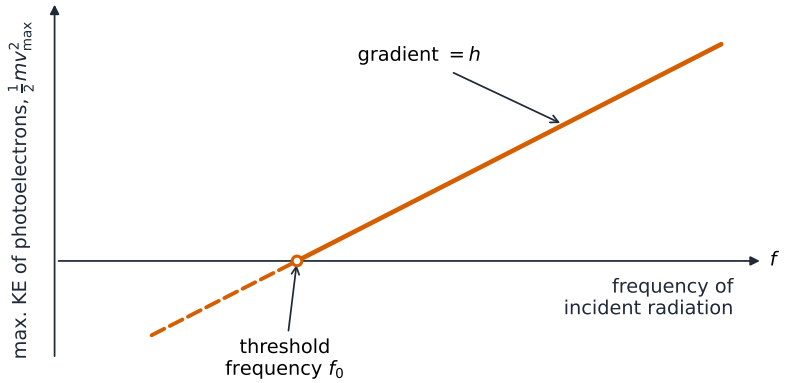
One photon gives all its energy to one electron. If the photon energy hf is more than the work function, the electron escapes with kinetic energy up to a maximum:

$$hf = \Phi + \tfrac{1}{2}mv_{max}^2, \qquad \text{so} \qquad \tfrac{1}{2}mv_{max}^2 = h(f - f_0).$$

> **Worked example.** A metal has a work function of 2.0 eV. Light made of photons of energy 3.5 eV shines on it. Find the maximum kinetic energy of the photoelectrons.

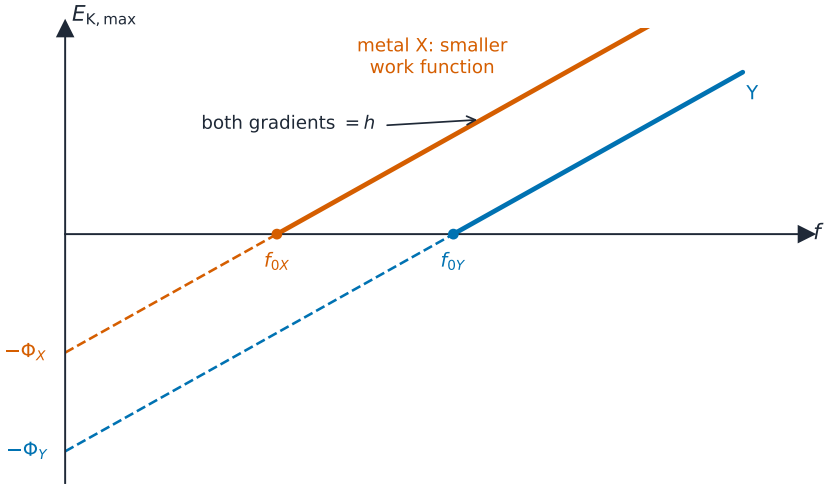
$$\tfrac{1}{2}mv_{max}^2 = hf - \Phi = 3.5 - 2.0 = 1.5 \text{ eV } (= 2.4 \times 10^{-19} \text{ J}).$$

So the maximum KE of photoelectrons depends **linearly on frequency**, not on brightness.



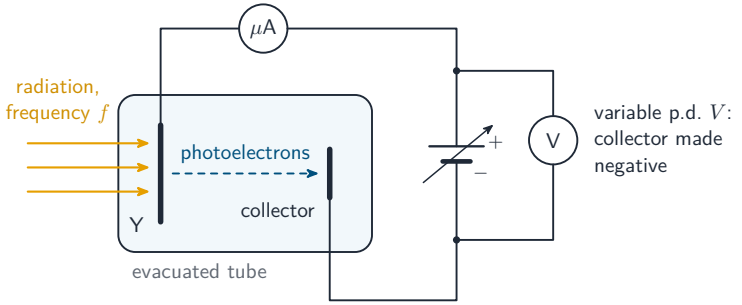
photoelectric: $KE_{max} = hf - \phi$ · threshold $f_0 = \phi/h$

The maximum kinetic energy of photoelectrons rises linearly with frequency, reaching zero at the threshold frequency f_0



Two metals on one graph: parallel lines (the gradient is the Planck constant for both), each cutting the frequency axis at its own threshold and the energy axis at minus its own work function

Reading the graph. Write the equation as $E_{\mathrm{K,max}} = hf - \Phi$: a straight line of gradient h, intercept $-\Phi$ on the energy axis and $f_0 = \Phi/h$ on the frequency axis. So a graph for two metals shows two **parallel** lines (same gradient h for every metal), the metal with the larger work function cutting the frequency axis further to the right; the intensity of the light moves neither line. This is how the Planck constant is measured, and "sketch the line for metal Y" is marked on exactly those two features.



increase the reverse p.d. until the current is zero:
this is the stopping potential V_s, and $eV_s = E_{\mathrm{K,max}}$

Investigating the effect: the current measures the rate of emission, and the reverse p.d. that just stops the fastest electrons measures their maximum kinetic energy

Measuring the maximum kinetic energy. In a photocell the photoelectrons cross a vacuum to a collector and the **current** is a count of electrons per second. Make the collector **negative** and the electrons must climb a potential hill; raise the reverse p.d. until the current just reaches zero, the **stopping potential** 遏止电势 V_s, and then $eV_s = E_{\mathrm{K,max}}$. Two results, both examined: at fixed frequency, doubling the intensity doubles the current at low reverse p.d. but leaves V_s unchanged; raising the frequency raises V_s but, at fixed intensity, does not raise the current.

Why the wave model fails

A wave model predicts that brightness should set the electrons' kinetic energy, and that emission should happen at any frequency given enough time. But experiments show:

- **no emission below the threshold frequency**, however bright.
- **immediate emission** at or above the threshold, even when dim.
- **maximum KE depends on frequency**, not brightness.
- the **number of photoelectrons** (the current) depends on brightness.

The photon model explains this: light arrives as photons each of energy hf. One photon–electron interaction either has enough energy to free the electron ($hf \geq \Phi$) or it does not.

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Why max KE is fixed but current grows with brightness

A brighter beam of the **same frequency** has **more photons per second**, but each still carries hf. So the maximum KE of any electron is $hf - \Phi$ (set by f only), while the rate of emission (the current) grows with the number of photons, i.e. with brightness. Doubling the brightness doubles the current but does not change the maximum KE.

Writing the explanation (a standard three-marker). (1) Each photon interacts with, and gives all its energy to, **one** electron. (2) The photon energy hf depends only on the **frequency**, so the maximum energy an electron can leave with, $hf - \Phi$, is fixed by the frequency. (3) Increasing the **intensity** at the same frequency increases the **number of photons per second**, so more electrons are emitted per second (a larger current), but each still receives the same energy. A wave, by contrast, would spread its energy over the surface, so a brighter wave should have given faster electrons and a dim one should have needed a delay to accumulate energy; neither happens.

Wave–particle duality

The photoelectric effect is strong evidence for the **particle** nature of light. But **interference** 干涉 (Young's double slit, the **diffraction grating** 衍射光栅) and **diffraction** 衍射 show its **wave** nature. So light has **both** wave and particle sides – this is **wave–particle duality** 波粒二象性.

"Describe what is meant by wave–particle duality" (two marks). *Electromagnetic radiation (and matter) can exhibit both wave properties, such as interference and diffraction, and particle properties, such as the photoelectric effect (or, for matter, discrete collisions).* Asked for **one piece of evidence** for each nature of radiation, give: **particulate**, the photoelectric effect (the threshold frequency and the immediate emission); **wave**, diffraction or interference (Young's slits, a diffraction grating). For **matter**, the wave evidence is electron diffraction.

De Broglie hypothesis

If a wave can act like particles, perhaps particles can act like waves. De Broglie proposed that any moving particle has a **de Broglie wavelength** 德布罗意波长:

$$\lambda = \frac{h}{p},$$

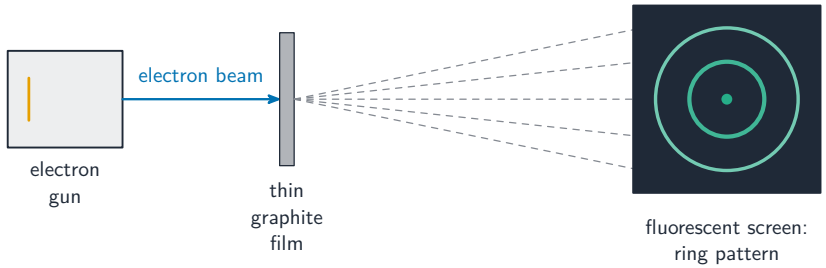
where $p = mv$. Example: an electron at $v = 4.9 \times 10^7$ m s^{-1} has $p = 4.46 \times 10^{-23}$ kg m s^{-1}, so $\lambda = 1.49 \times 10^{-11}$ m ≈ 0.015 nm – close to atomic spacings.

"State what is meant by the de Broglie wavelength." *The wavelength associated with a moving particle*, or *the wavelength of the wave associated with a particle of momentum p*, given by $\lambda = h/p$ where h is the Planck constant. In "state the formula and the meaning of any other symbol", name h as the Planck constant and p as the momentum of the particle. The wavelength is small because h is small: a 0.10 kg ball at 10 m s^{-1} has $\lambda = 6.6 \times 10^{-34}$ m, far below any slit or lattice spacing, which is why everyday objects show no diffraction.

Electron diffraction

When electrons are fired at a **crystal lattice** 晶格 (e.g. thin graphite), they make a **diffraction pattern** of bright rings on a screen – exactly what waves of wavelength $\lambda = h/p$ would do. This is direct evidence for the **wave nature of particles** (**electron diffraction** 电子衍射): only waves diffract, yet electrons do.

A faster electron has more momentum, so a **shorter** de Broglie wavelength, which diffracts less – the rings move **closer together**. Slowing the electrons spreads the rings apart. To calculate: $p = \sqrt{2mE_K}$, and for an electron accelerated through p.d. V, $E_K = eV$, so $\lambda = h/\sqrt{2m_e eV}$.



Electrons fired at graphite form a ring diffraction pattern – only waves diffract, so electrons behave as waves

> **Worked example.** An electron is accelerated from rest through a p.d. of 2500 V. Find its de Broglie wavelength. ($m_e = 9.11 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C, $h = 6.63 \times 10^{-34}$ J s.)

Its kinetic energy is $E_K = eV$, so $\lambda = \dfrac{h}{\sqrt{2m_e eV}}$:

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2(9.11 \times 10^{-31})(1.6 \times 10^{-19})(2500)}} \approx 2.5 \times 10^{-11} \text{ m}.$$

This is close to the spacing between atoms in a crystal, which is why the electrons diffract off the graphite.

Deriving λ for an accelerated electron. An electron of mass m and charge q accelerated from rest through a p.d. V gains kinetic energy $qV = \frac{1}{2}mv^2$, so $v = \sqrt{2qV/m}$ and $p = mv = \sqrt{2mqV}$; hence $\lambda = h/\sqrt{2mqV}$. Two consequences the exam asks for: **increasing** V increases the momentum and **shortens** the wavelength, so the diffraction rings shrink towards the centre; halving the wavelength needs four times the p.d.

Describing electron diffraction (four marks). (1) Electrons from a heated fila-

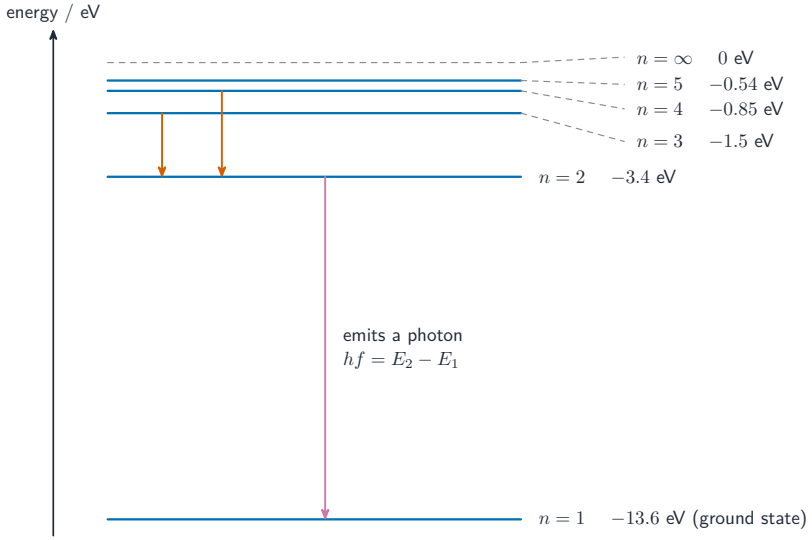
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ment are accelerated through a high p.d. into a beam. (2) The beam meets a thin **polycrystalline** 多晶的 graphite film; the regular spacing of the carbon atoms, about 10^{-10} m, acts as a diffraction grating. (3) On a fluorescent screen the electrons produce a bright central spot surrounded by **concentric rings**. (4) Rings are a diffraction pattern, and diffraction is a wave property, so the electrons are behaving as waves; the ring radii match a wavelength h/p, which confirms de Broglie's relation. Sketch the pattern as rings, not spots or a fringe pattern. A faster beam gives rings of **smaller** radius.

Energy levels in atoms

In an isolated atom, electrons can only sit at certain **discrete** 分立 **energy levels** 能级 – never in between. The lowest is the **ground state** 基态; the others are **excited states** 激发态.

By convention, energies are written **negative**, with $E = 0$ for an electron just free of the atom. For hydrogen the ground state is $E_1 = -13.6$ eV; higher states approach zero.



The electron energy levels of hydrogen are discrete and negative, with the ground state at −13.6 eV

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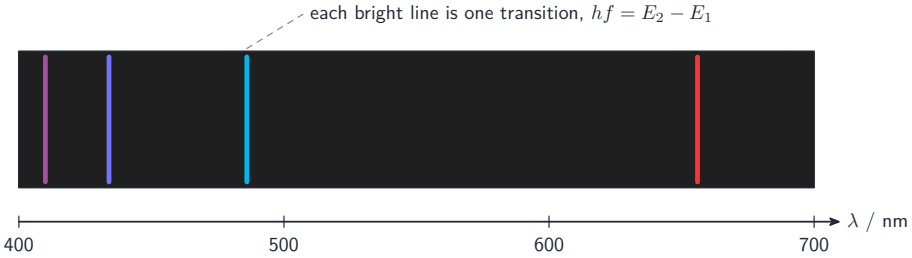
Emission spectrum

When an electron drops from a higher level E_2 to a lower level E_1, it emits one photon of energy

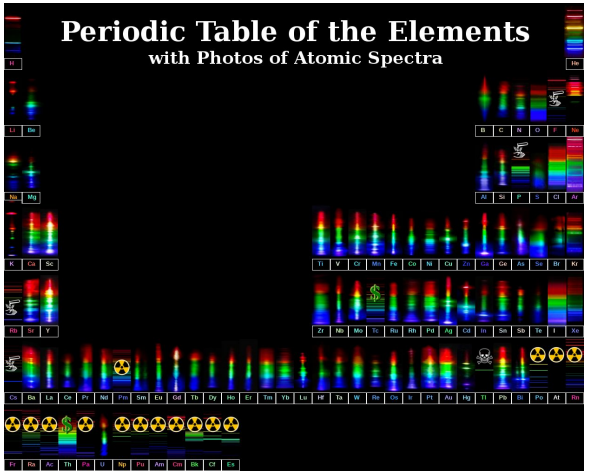
$$hf = E_2 - E_1.$$

(Both energies are negative; their difference is positive.) Because the levels are discrete, only certain photon energies – and so certain wavelengths – come out. The **emission spectrum** 发射光谱 is a set of **sharp bright lines** on a dark background, one line per **transition** 跃迁. The pattern is a "fingerprint" of the element.

Explaining a line spectrum (the standard four-marker). (1) The electrons in an isolated atom can only occupy **discrete** energy levels. (2) An electron in an excited state falls to a lower level and the energy it loses is emitted as **one photon**. (3) The photon energy equals the **difference** between the two levels, $hf = E_2 - E_1$, so only certain frequencies (wavelengths) are emitted. (4) Each possible transition gives one **line**; the same set of levels gives the same lines every time, which is why a spectrum identifies the element. Asked to match lines to transitions: the **largest** energy gap gives the line of **highest frequency** and **shortest wavelength**.



The emission spectrum of hydrogen is a set of sharp bright lines on a dark background

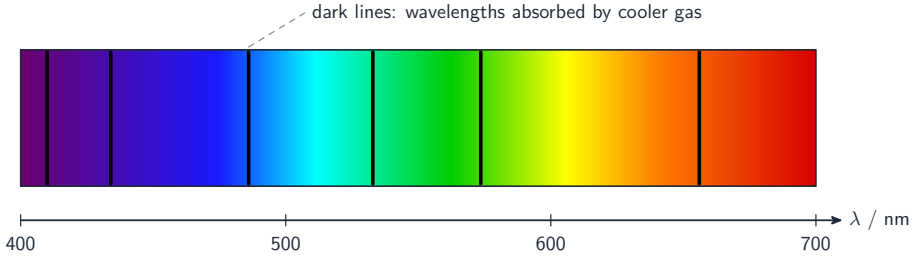


Real emission spectra of the elements: each one is a unique set of bright lines – a fingerprint of that element

Absorption spectrum

When white light passes through a cool gas, photons whose energy exactly matches an upward transition are absorbed. The light then shows **dark lines on a bright background** – the **absorption spectrum** 吸收光谱. The dark lines sit at the **same wavelengths** as the emission lines of the same gas.

Explaining the dark lines. Photons whose energy equals the difference between two levels are **absorbed**, raising an electron to the higher level; the excited electron soon falls back and re-emits a photon of the same energy, but **in a random direction**, so almost none of that light continues along the original path. The rest of the white light, whose photons match no gap, passes through unchanged. Hence dark lines at exactly the wavelengths the same gas would emit: the Sun's spectrum shows the absorption lines of the cooler gases in its outer layers.



Dark absorption lines in the Sun's spectrum mark the wavelengths absorbed by cooler gas

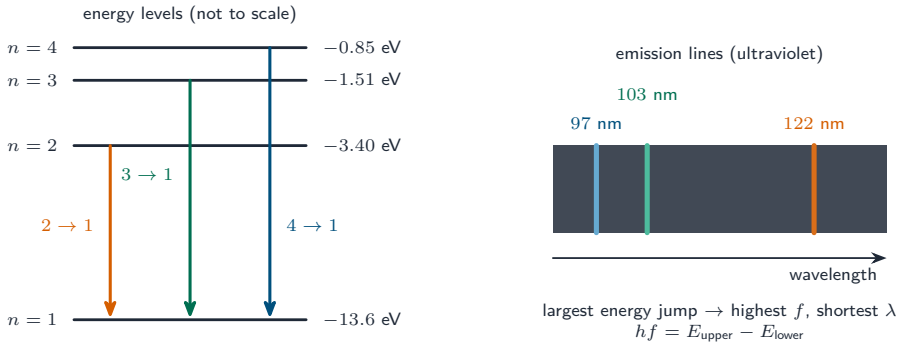
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Calculations

For a transition between two known levels:

$$hf = E_2 - E_1, \qquad \lambda = \frac{hc}{E_2 - E_1}.$$

Work in consistent units – convert eV to joules ($\times\ 1.60 \times 10^{-19}$) before finding λ in metres, or use $hc \approx 1240$ eV nm for a quick estimate.



From levels to lines: each transition down to the ground state produces one line, and the biggest jump lands furthest towards the short-wavelength end

> **Worked example.** The lowest four energy levels of hydrogen are -13.6, -3.40, -1.51 and -0.85 eV. Find the wavelengths of the three lines produced by transitions to the ground state, and the number of lines these four levels can produce altogether.

Transition $2 \to 1$: $\Delta E = 13.6 - 3.40 = 10.2$ eV, so $\lambda = 1240/10.2 = 122$ nm. $3 \to 1$: 12.09 eV, $\lambda = 103$ nm. $4 \to 1$: 12.75 eV, $\lambda = 97.3$ nm: all ultraviolet, the largest jump giving the shortest wavelength. Four levels allow $4 \to 3$, $4 \to 2$, $4 \to 1$, $3 \to 2$, $3 \to 1$ and $2 \to 1$: **six** lines. The visible red line of hydrogen is $3 \to 2$: 1.89 eV, 656 nm. (Working in joules: $10.2\ \text{eV} = 1.63 \times 10^{-18}$ J, $\lambda = hc/E = 1.99 \times 10^{-25}/1.63 \times 10^{-18} = 1.22 \times 10^{-7}$ m.)

> **Worked example.** A laser emits red light of wavelength 650 nm when electrons drop from one level to another. Find the energy gap between the two levels.

$\Delta E = hc/\lambda = 1240/650 = 1.91\ \text{eV} = 3.1 \times 10^{-19}$ J. The gap, not either level, fixes the colour: two atoms with different levels but the same gap emit the same line.

> **Worked example (annihilation).** An electron and a positron, each moving slowly, meet and **annihilate** 湮灭, producing two identical photons. Find the wavelength of each photon. ($m_e = 9.11 \times 10^{-31}$ kg.)

The rest energy of each particle is $E = mc^2 = (9.11\times10^{-31})(3.00\times10^{8})^2 = 8.2\times10^{-14}$ J (0.51 MeV), and momentum conservation shares the energy between two photons moving in opposite directions, so each carries 8.2×10^{-14} J: $\lambda = hc/E = 1.99 \times 10^{-25}/8.2 \times 10^{-14} = 2.4 \times 10^{-12}$ m, a gamma ray. (Any kinetic energy the pair had is added to the photon energies.)

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
photon	a quantum (discrete packet) of energy of electromagnetic radiation
electronvolt	the energy gained by an electron accelerated through a potential difference of one volt; 1.60×10^{-19} J
photoelectric effect	the emission of electrons from a metal surface when electromagnetic radiation of high enough frequency is incident on it
work function energy	the minimum energy needed to remove an electron from the surface of the metal
threshold frequency	the minimum frequency of radiation that causes photoelectric emission from a metal
threshold wavelength	the maximum wavelength of radiation that causes photoelectric emission, $\lambda_0 = hc/\Phi$
wave–particle duality	radiation and matter show both wave properties (diffraction, interference) and particle properties (photoelectric effect, discrete collisions)
de Broglie wavelength	the wavelength associated with a moving particle, $\lambda = h/p$
energy level	one of the discrete energies an electron in an isolated atom may have
emission line spectrum	a set of bright lines of definite wavelengths, each from a transition between two energy levels
absorption line spectrum	dark lines on a continuous spectrum at the wavelengths absorbed by transitions to higher levels

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Exam tips

- Photon: $E = hf = hc/\lambda$, $p = E/c = h/\lambda$. Use $hc = 1240$ eV nm to move between wavelength and energy in eV, then convert to joules only if the answer demands it.
- Photoelectric equation $hf = \Phi + \frac{1}{2}mv^2_{\max}$: maximum energy, one photon to one electron; the graph of $E_{\mathrm{K,max}}$ against f has gradient h and intercepts f_0 and $-\Phi$.
- Intensity changes the **number** of photons per second (the current); frequency changes the **energy** of each (the maximum kinetic energy). Keep those two sentences apart and every explanation writes itself.
- Evidence: photoelectric effect for particles, diffraction and interference for waves; electron diffraction for the wave nature of matter, with $\lambda = h/p = h/\sqrt{2mqV}$.
- Line spectra: discrete levels, one photon per transition, $hf = E_2 - E_1$; the biggest gap gives the shortest wavelength; absorption lines sit where emission lines would, because the absorbed light is re-emitted in all directions.
- Radiation pressure: force = photons per second × momentum change per photon; $2p$ for a mirror, p for an absorber; pressure depends on intensity, not colour.

Common mistakes

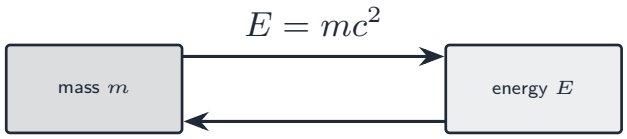
- Defining a photon as "a particle of light" without "quantum/packet of energy", or the work function without "minimum" and "from the surface".
- Saying brighter light gives faster photoelectrons, or that below the threshold frequency emission happens eventually.
- Mixing eV and joules in one equation; forgetting to subtract the work function; using $\frac{1}{2}mv^2$ with the maximum kinetic energy in eV.
- Writing the de Broglie wavelength for an accelerated electron as $h/(mv)$ with v guessed, instead of $h/\sqrt{2mqV}$.
- Describing electron diffraction as bright fringes or spots; the pattern is concentric rings, and a higher p.d. makes them smaller.
- Getting the direction of a transition wrong: emission is a fall to a lower level, absorption a rise; the photon energy is the difference, never the energy of one level.
- Claiming the largest energy gap gives the longest wavelength.
- Forgetting that a reflected photon changes momentum by $2p$, or that the pressure of blue light of the same intensity is the same as red.

Nuclear physics

A-Level Physics

Mass-energy equivalence

Einstein's special relativity gives the famous link (**mass-energy equivalence** 质能等价):



Mass and energy are equivalent and can change into each other

$$E = mc^2,$$

where $c = 3.00 \times 10^8$ m s^{-1}. A mass m matches an **energy** 能量 E – the two can change into each other. For a mass change Δm:

> **Worked example.** The star Sirius loses mass through nuclear fusion at 1.09×10^{11} kg s^{-1}. Find the power it radiates.

Every kilogram that disappears leaves as energy: $P = c^2 \times$ (mass lost per second) $= (3.00 \times 10^8)^2(1.09 \times 10^{11}) = 9.8 \times 10^{27}$ W. This is the star's **luminosity** 光度 (Topic 25); the Sun's is 3.8×10^{26} W, so it loses about 4 million tonnes a second. The same idea in reverse: a power station producing 1 GW for a year converts $E/c^2 = 3.2 \times 10^{16}/9.0 \times 10^{16} = 0.35$ kg of mass, which is why the fuel weighs almost the same afterwards.

$$\Delta E = c^2 \Delta m.$$

In nuclear physics the masses are tiny but c^2 is huge, so a small mass change means a large energy. A mass change of 1 u (1.661×10^{-27} kg) matches $\Delta E \approx 1.49 \times 10^{-10}$ J. This gives a conversion you will use again and again:

$$1 \text{ u} \longleftrightarrow 931 \text{ MeV}.$$

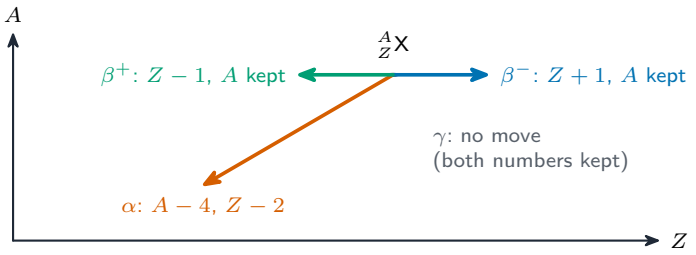
Nuclear reactions

A **nuclear reaction** 核反应 is written like

$$^{14}_{7}\text{N} + ^{4}_{2}\text{He} \rightarrow ^{17}_{8}\text{O} + ^{1}_{1}\text{H},$$

with **nucleon number** 核子数 conserved (top numbers: $14 + 4 = 17 + 1$) and charge conserved (bottom numbers: $7 + 2 = 8 + 1$) – this is **conservation of charge** 电荷守恒. Use these to fill in an unknown: identify the species, then balance the top and bottom numbers.

The decay equations. Alpha decay removes $^{4}_{2}$He, so A falls by 4 and Z by 2: $^{211}_{84}\text{Po} \rightarrow ^{207}_{82}\text{Pb} + ^{4}_{2}\text{He}$. Beta-minus decay turns a neutron into a proton, emitting an electron and an antineutrino: $^{15}_{6}\text{C} \rightarrow ^{15}_{7}\text{N} + ^{0}_{-1}\text{e} + \bar{\nu}$ (A unchanged, Z up by 1). Beta-plus decay turns a proton into a neutron, emitting a **positron** 正电子 and a **neutrino** 中微子: $^{18}_{9}\text{F} \rightarrow ^{18}_{8}\text{O} + ^{0}_{+1}\text{e} + \nu$ (Z down by 1). Gamma emission changes neither number. In a chain of decays just keep the books: a nucleus W that emits β^-, then α, then β^- ends with $A - 4$ and $Z + 1 - 2 + 1 = Z$, an **isotope of W**. Check both lines of every equation you complete, and remember the neutrino: the mark scheme includes it.



Each decay is a fixed move on the chart: keep the books on A and Z.

Mass defect and binding energy

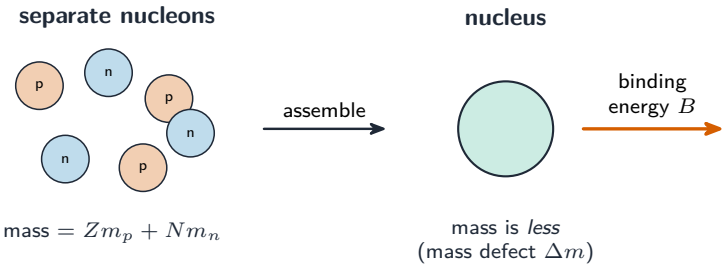
The mass of a **nucleus** 原子核 is **less than** the total mass of its separate **protons** 质子 and **neutrons** 中子. The difference is the **mass defect** 质量亏损 Δm:

$$\Delta m = (Zm_\text{p} + Nm_\text{n}) - m_\text{nucleus}.$$

By $E = mc^2$, this "missing" mass was released as energy when the nucleus formed. To pull the nucleus fully apart you must put that energy back – the **binding energy** 结合能 B:

$$B = \Delta m \cdot c^2.$$

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The assembled nucleus has less mass than its separate nucleons; the missing mass is released as binding energy

> **Worked example.** A helium-4 nucleus has a mass defect of $\Delta m = 0.0304$ u. Find its binding energy. (1 u corresponds to 931 MeV.)

$$B = \Delta m \cdot c^2 = 0.0304 \times 931 \approx 28 \text{ MeV}.$$

A more tightly bound nucleus has a larger mass defect and larger binding energy. The **binding energy per nucleon** 比结合能 is B/A (usually in MeV per nucleon) – a measure of how tightly each nucleon is held, useful for comparing nuclides.

The two-mark definitions. *The mass defect of a nucleus is the difference between the total mass of its separate nucleons and the mass of the nucleus. The binding energy is the minimum energy required to separate the nucleus into its individual nucleons* (equivalently, the energy released when the nucleus is formed from separate nucleons). Say "separate nucleons" or "individual protons and neutrons"; "the energy holding the nucleus together" scores nothing. Binding energy is **released**, not stored: the nucleus has *less* energy than its parts.

> **Worked example.** The masses are: proton 1.007 276 u, neutron 1.008 665 u, polonium-212 nucleus 211.945 4 u. Find the mass defect and the binding energy per nucleon of $^{212}_{84}$Po.

$Z = 84$ protons and $N = 212 - 84 = 128$ neutrons: total $84 \times 1.007\,276 + 128 \times 1.008\,665 = 84.611\,2 + 129.109\,1 = 213.720\,3$ u. Mass defect $\Delta m = 213.720\,3 - 211.945\,4 = 1.774\,9$ u. Binding energy $= 1.774\,9 \times 931.5 = 1653$ MeV, so per nucleon $1653/212 = 7.80$ MeV, on the falling part of the curve. Keep every decimal place until the subtraction: the defect is a small difference of two large numbers.

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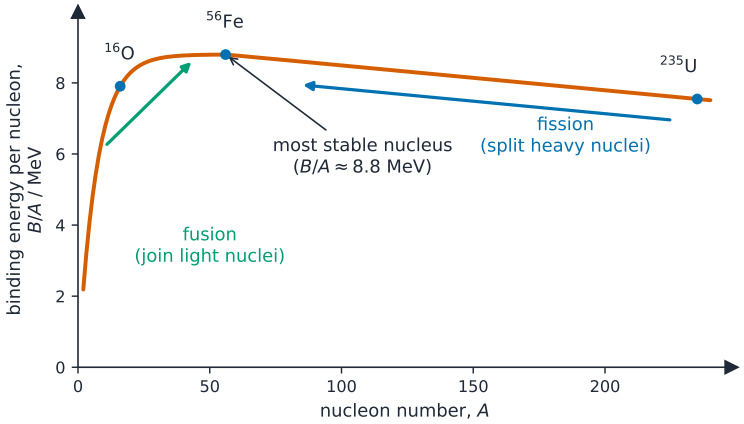
Binding energy per nucleon vs nucleon number

A graph of B/A against A has a typical shape:

- for light nuclei ($A < 20$), B/A rises quickly (with a spike at the very stable $^4_2$He).
- around $A \sim 56$ (iron), B/A reaches its **maximum** of about 8.8 MeV. **Iron-56 is taken as the most stable nucleus** (nickel-62 is very slightly higher, but the syllabus uses iron).
- for heavy nuclei ($A > 100$), B/A falls slowly, to about 7.5 MeV for uranium.

So the curve is dome-shaped, rising to iron then falling.

Sketching the curve. The exam gives blank axes (A from 1 to 250, B/A up to about 9 MeV) and marks: a steep rise from near zero at $A = 1$, a **maximum** near $A = 56$ at about 8.8 MeV, then a **slow, gentle fall** to about 7.5 MeV at $A = 238$. Do not start the curve at the origin exactly (hydrogen-1 has no binding energy but is a single point), do not make the fall as steep as the rise, and do not let the curve reach zero on the right. Asked to mark a nucleus that undergoes **alpha decay**, put X on the far right, $A > 200$; a nucleus that undergoes **fusion** goes at the far left, $A < 10$; both are at low B/A, moving up the curve when they react.



binding energy per nucleon peaks near ^{56}Fe

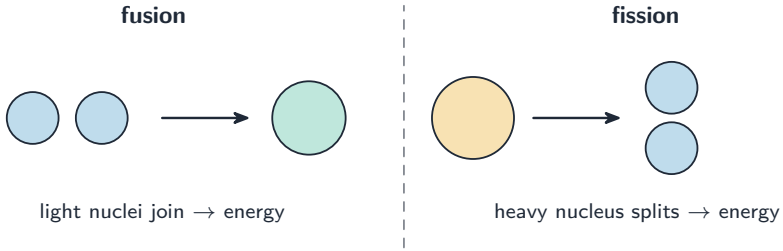
Binding energy per nucleon peaks near iron ($A \approx 56$); lighter and heavier nuclei are less tightly bound

Nuclear fusion and fission



A nuclear power station releases energy by nuclear fission.

Energy is **released** when nuclei move **towards** the iron peak – by joining light nuclei or splitting heavy ones.



Fusion joins light nuclei; fission splits a heavy nucleus – both release energy by moving towards the iron peak

Nuclear fusion

Nuclear fusion 核聚变 joins two light nuclei into one heavier nucleus:

$$^{2}_{1}\mathrm{H} + ^{3}_{1}\mathrm{H} \rightarrow ^{4}_{2}\mathrm{He} + ^{1}_{0}\mathrm{n} + \text{energy}.$$

The product has greater binding energy per nucleon than the reactants, so energy is released. Fusion powers stars. It needs very high temperatures (millions of kelvin) so the nuclei have enough **kinetic energy** 动能 to beat their **electrostatic** 静电 repulsion and get close enough for the **strong nuclear force** 强核力 to take over.

Nuclear fission

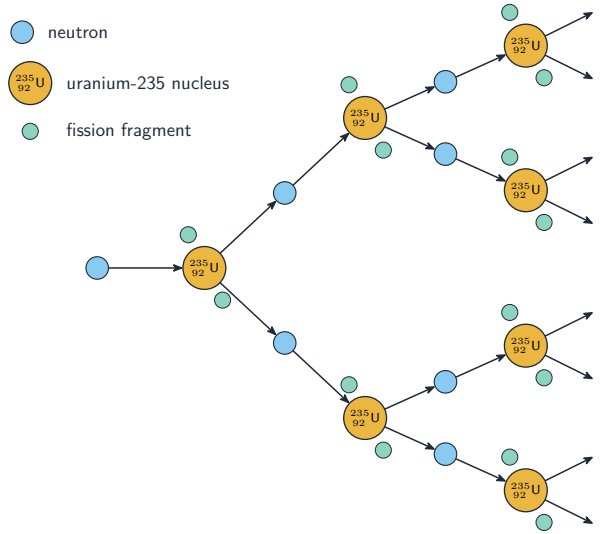
Nuclear fission 核裂变 splits a heavy nucleus into two lighter ones:

$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \rightarrow ^{141}_{56}\mathrm{Ba} + ^{92}_{36}\mathrm{Kr} + 3\,^{1}_{0}\mathrm{n} + \text{energy}.$$

The products have higher binding energy per nucleon than ^{235}U, so energy is released. The extra neutrons can cause more fissions – a **chain reaction** 链式反应 in a large enough mass of fuel (the **critical mass** 临界质量). This is the basis of nuclear power and weapons.

"Describe the differences between fission and fusion." Fission: a **heavy** nucleus (large A) splits into two **lighter** nuclei of roughly similar mass, usually after absorbing a neutron, releasing further neutrons. Fusion: two **light** nuclei (small A) join to form one heavier nucleus; it needs very high temperature and pressure to overcome the electrostatic repulsion between the nuclei. Both release energy, but per kilogram of fuel fusion releases more.

"Explain, with reference to the curve, why energy is released." In both processes the products lie **higher** on the binding-energy-per-nucleon curve than the reactants: each nucleon ends up more tightly bound, so the total binding energy **increases**. The extra binding energy is released (as the kinetic energy of the products and as photons), and the total mass of the products is **less** than that of the reactants by $\Delta E/c^2$. A nucleus near the peak (iron) can release energy by neither process, which is why the stars' fusion stops at iron.



In an uncontrolled chain reaction each fission of uranium-235 releases neutrons that cause more fissions

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Calculating the energy released

1. find the total mass of the reactants.
2. find the total mass of the products.
3. mass change $\Delta m = m_{\text{reactants}} - m_{\text{products}}$ (positive when energy is released).
4. energy released $\Delta E = c^2 \Delta m$.

In kg this gives joules; in atomic mass units use ΔE (MeV) $= \Delta m$ (u) $\times$ 931.

> **Worked example.** In a nuclear reaction the total mass decreases by 0.020 u. Find the energy released.

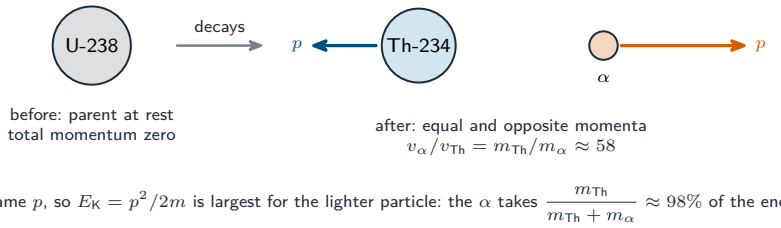
$$\Delta E = \Delta m \text{ (u)} \times 931 = 0.020 \times 931 \approx 19 \text{ MeV}.$$

> **Worked example (fusion).** The mass defect of deuterium $^{2}_{1}\text{H}$ is 0.002 388 u and that of helium-4 is 0.030 377 u. Find the energy released when two deuterium nuclei fuse to form one helium-4 nucleus.

Energy released = (binding energy of the products) − (binding energy of the reactants) $= [0.030\,377 - 2 \times 0.002\,388] \times 931.5 = 0.025\,601 \times 931.5 = 23.8$ MeV $(3.82 \times 10^{-12}$ J). Mass defects can be used directly like this because the number of nucleons is the same on both sides; the difference in the defects **is** the mass converted. Per kilogram of deuterium this is 5.7×10^{14} J, about ten million times as much as a kilogram of a chemical fuel such as petrol ($\approx 5 \times 10^{7}$ J).

> **Worked example (fission).** $^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \to {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 3\,{}^{1}_{0}\text{n}$. Masses: U-235 235.043 9 u, n 1.008 665 u, Ba-141 140.914 4 u, Kr-92 91.926 2 u. Find the energy released.

Reactants: $235.043\,9 + 1.008\,665 = 236.052\,6$ u. Products: $140.914\,4 + 91.926\,2 + 3 \times 1.008\,665 = 235.866\,6$ u. $\Delta m = 0.186\,0$ u, so $\Delta E = 0.186\,0 \times 931.5 = 173$ MeV $= 2.8 \times 10^{-11}$ J per fission. Count the three neutrons on the right and the one on the left: forgetting one changes the answer by a whole nucleon mass.



Alpha decay from rest: equal and opposite momenta, so the light alpha particle carries about 98% of the energy released and the heavy nucleus barely recoils

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> **Worked example (alpha decay and momentum).** A stationary $^{238}_{92}\text{U}$ nucleus decays to $^{234}_{90}\text{Th}$ by emitting an α-particle; the total kinetic energy released is 4.27 MeV. Find the kinetic energy of the α-particle.

Momentum is conserved and the parent was at rest, so the α and the thorium nucleus have **equal and opposite** momenta p. With $E_K = p^2/2m$, the energies are in the inverse ratio of the masses: $E_\alpha / E_{\text{Th}} = m_{\text{Th}} / m_\alpha = 234/4$. So $E_\alpha = 4.27 \times 234/238 = 4.20$ MeV and the thorium **recoil** 反冲 takes only 0.07 MeV. The α-particles from a given decay are all emitted with this one energy, which is the sign that the energy is shared between just two bodies.

> **Worked example (a radioactive power source).** A space probe is powered by 0.874 kg of plutonium-238, half-life 87.7 years, each decay releasing 5.59 MeV. Find the power available at launch.

Number of nuclei: $N = 0.874/(238 \times 1.661 \times 10^{-27}) = 2.21 \times 10^{24}$. Decay constant: $\lambda = 0.693/(87.7 \times 3.156 \times 10^{7}) = 2.50 \times 10^{-10}$ s^{-1}. Activity: $A = \lambda N = 5.53 \times 10^{14}$ Bq. Power $= A \times E = 5.53 \times 10^{14} \times 5.59 \times 1.60 \times 10^{-13} = 490$ W, falling to half after 87.7 years. A nuclide with a **shorter** half-life would give more power per kilogram but would not last the mission; polonium-210 (138 days) would be far more powerful at first and useless within two years.

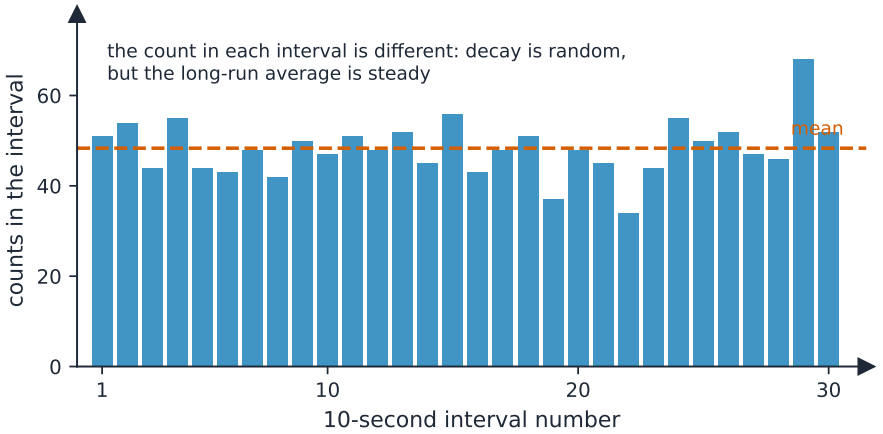
Radioactive decay

Random and spontaneous

Radioactive decay is:

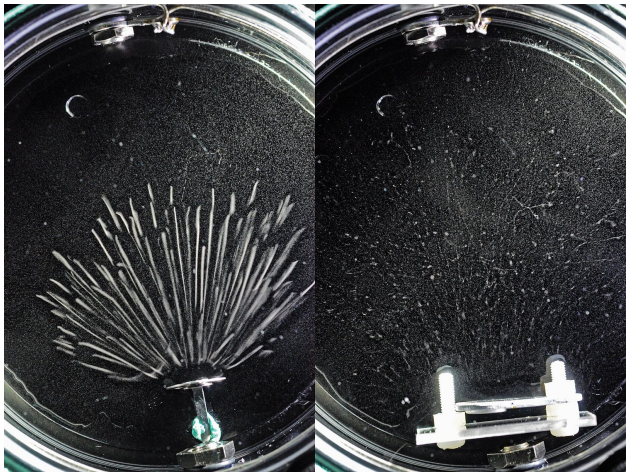
- **spontaneous** 自发 – it happens with no outside trigger, and the rate is not changed by temperature, pressure or chemical state; and
- **random** 随机 – you cannot predict when a given nucleus will decay, only the probability that it decays in a time.

Evidence for randomness: the count rate **fluctuates**. A **Geiger counter** 盖革计数器 next to a source clicks at uneven intervals – never a steady stream – although the long-run mean rate is well-defined.



Counts in equal time intervals from the same source are never the same twice: the fluctuation is the evidence that decay is random, while the steady mean shows the probability is constant

The two-mark definitions. *Radioactive decay is the spontaneous and random emission of a particle (α or β) or a photon (γ) from an unstable nucleus. Spontaneous* means the decay is **not affected by external factors**: temperature, pressure, chemical state, or the presence of other nuclei. *Random* means it is **impossible to predict which** nucleus will decay next, or **when** a given nucleus will decay; only a probability can be given. Evidence for randomness: the **count rate fluctuates** from one interval to the next, even for a source whose activity is not changing over the experiment.



Each beta particle from the source leaves a thin track in a cloud chamber – direct evidence of separate, random decays

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Activity and decay constant

For N undecayed nuclei of a **radionuclide** 放射性核素, the rate of decay is

$$A = \lambda N.$$

- A is the **activity** 活度 – decays per unit time. Unit: **becquerel** 贝克勒尔 (Bq) $= \mathrm{s}^{-1}$.
- λ is the **decay constant** 衰变常数 – the probability per unit time that a nucleus decays. Unit: s^{-1}.

λ is fixed for a nuclide; a larger sample (larger N) has proportionally larger activity.

The one-mark definitions. *Activity is the number of decays (of nuclei) per unit time*, or the rate of decay. *The decay constant is the probability per unit time that a (given) nucleus will decay.* Not "the rate of decay": that is the activity. Note the units are the same, s^{-1}, but A counts events and λ is a probability per second.

> **Worked example.** Fluorine-18 has a half-life of 110 minutes. Show that its decay constant is 1.05×10^{-4} s^{-1}, and find the activity of 2.1×10^{-12} kg of fluorine-18.

$\lambda = 0.693/(110 \times 60) = 1.05 \times 10^{-4}$ s^{-1}. The number of nuclei is the mass divided by the mass of one nucleus: $N = 2.1 \times 10^{-12}/(18 \times 1.661 \times 10^{-27}) = 7.0 \times 10^{13}$. So $A = \lambda N = 1.05 \times 10^{-4} \times 7.0 \times 10^{13} = 7.4 \times 10^{9}$ Bq. A tiny mass gives a huge activity because the half-life is short; the same mass of uranium-238 (half-life 4.5×10^{9} years) would give about 10^{-5} Bq.

Exponential decay

Since λ is the fractional decay rate, $\dfrac{dN}{dt} = -\lambda N$, whose solution is an **exponential decay** 指数衰减:

$$N = N_0 e^{-\lambda t}.$$

Because $A = \lambda N$, the activity (and any **count rate** 计数率 proportional to it) decays the same way:

$$A = A_0 e^{-\lambda t}.$$

Why exponential? For each nucleus, λ is a fixed probability per unit time, independent of the others and of the nucleus's age. So the same **fraction** decays in each time interval, which gives exponential decay.

Writing the explanation (three marks). (1) The decay constant is the **probability per unit time** of decay and is the **same for every nucleus** of the nuclide, whatever its age. (2) So the rate of decay, the activity, is **proportional to the number of undecayed nuclei** present: $A = \lambda N$. (3) A rate of change proportional to the quantity itself gives an **exponential** change; equivalently, the **same fraction** of

the remaining nuclei decays in every equal time interval, so the number never reaches zero but halves in every half-life.

Half-life

The **half-life** 半衰期 $t_{1/2}$ is the time for the number of undecayed nuclei (or the activity, or the count rate) to fall to **half**. From $N = N_0 e^{-\lambda t}$ with $N = N_0/2$:

$$\ln 2 = \lambda t_{1/2}, \qquad \lambda = \frac{\ln 2}{t_{1/2}} \approx \frac{0.693}{t_{1/2}}.$$

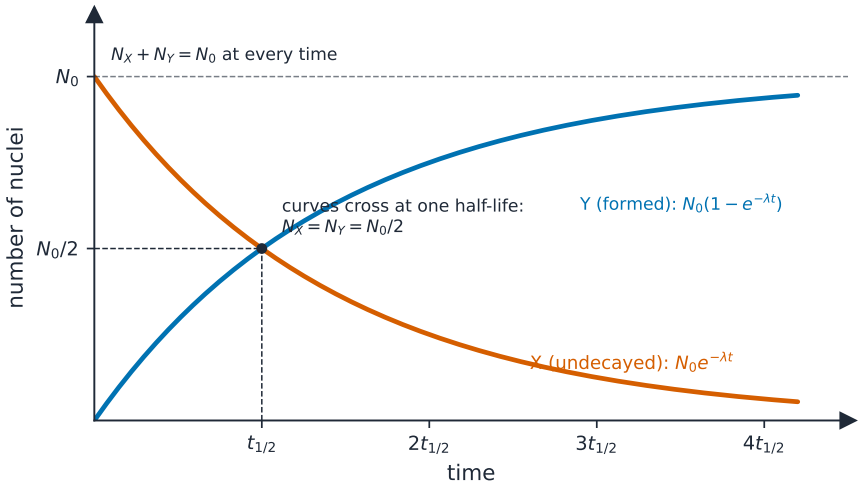
> **Worked example.** A radioactive isotope has a half-life of 6.0 hours. Find its decay constant.

Convert the half-life to seconds: 6.0 h = 21 600 s. Then

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{21\,600} \approx 3.2 \times 10^{-5}\ \text{s}^{-1}.$$

A larger decay constant means a shorter half-life. After n half-lives the surviving fraction is $(1/2)^n$; after 5 half-lives only about 3% remains.

The definition of half-life. *The time taken for the number of undecayed nuclei (or the activity) of a sample to fall to half its initial value.* "Half the atoms decay" is accepted; "half the sample disappears" is not, since the decayed nuclei are still there as the daughter product.



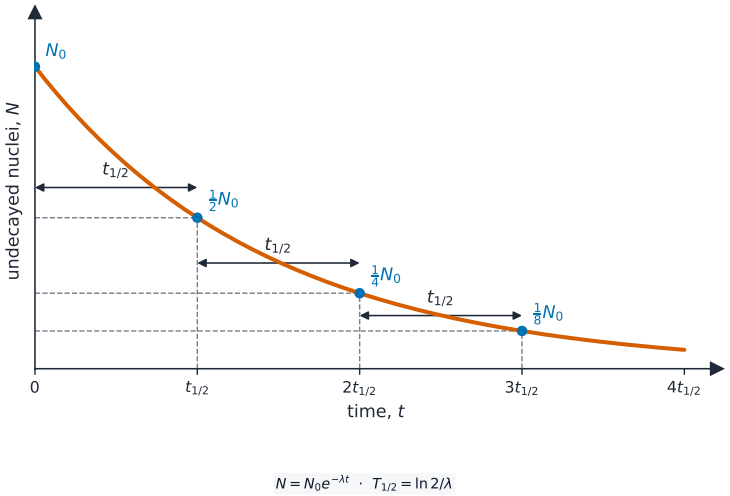
A parent decaying into a stable daughter: what X loses, Y gains, so the curves cross after one half-life and the half-life can be read off the crossing

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Reading a two-isotope graph. When X decays to a **stable** Y, $N_Y = N_0 - N_X = N_0(1 - e^{-\lambda t})$. The half-life is the time at which the curves **cross** ($N_X = N_Y = N_0/2$), or the time for N_X to halve. If the graph gives the initial number N_0 and the initial mass m, the nucleon number follows from $m = N_0 A u$: for $m = 7.3 \times 10^{-4}$ kg and $N_0 = 2.0 \times 10^{21}$, $A = m/(N_0 u) = 7.3 \times 10^{-4}/(2.0 \times 10^{21} \times 1.661 \times 10^{-27}) = 220$.

> **Worked example.** A sample of a single radioactive isotope has an activity of 180 Bq at $t = 0$ and 45 Bq at $t = 8.4$ minutes. Find the half-life and the decay constant, and the activity after a further 8.4 minutes.

$45/180 = 1/4 = (1/2)^2$: two half-lives in 8.4 minutes, so $t_{1/2} = 4.2$ min $= 252$ s and $\lambda = 0.693/252 = 2.8 \times 10^{-3}$ s^{-1}. After another two half-lives the activity is $45/4 = 11$ Bq. When the ratio is not a neat power of two, use $t = \ln(A_0/A)/\lambda$: for the activity to fall from 180 to 50 Bq takes $\ln(3.6)/2.75 \times 10^{-3} = 466$ s. Always convert the half-life to **seconds** before finding λ if the activity is in becquerels.



The number of undecayed nuclei falls by half in each half-life

Finding λ from data

Given A_0 and A at time t:

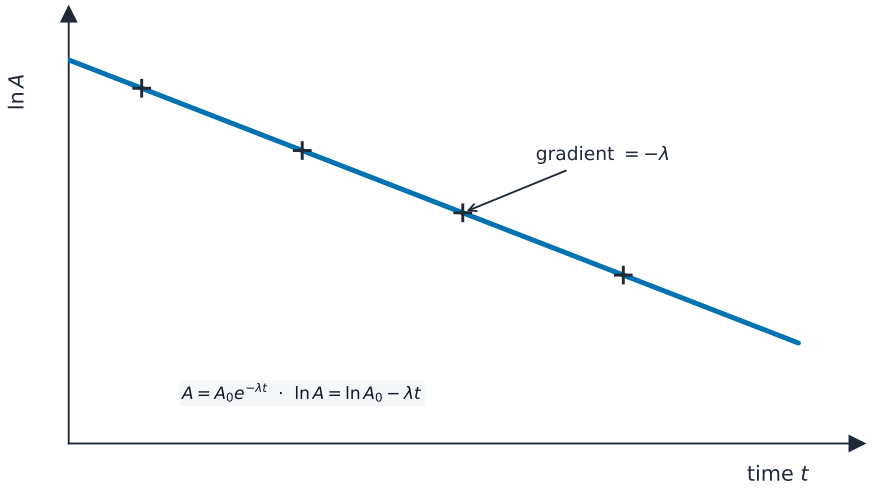
$$\lambda = \frac{1}{t} \ln \frac{A_0}{A}, \qquad t_{1/2} = \frac{\ln 2}{\lambda}.$$

Taking logs of $A = A_0 e^{-\lambda t}$ gives $\ln A = \ln A_0 - \lambda t$, so a plot of $\ln A$ against t is a straight line with gradient $-\lambda$. Use this with several data points.

Count rate is not activity. A detector records only the radiation that reaches it and is absorbed in it: a fraction set by the solid angle it covers, by absorption in the air

and in the source itself, and by its efficiency. So the measured count rate is **smaller** than the activity, but **proportional** to it, and the half-life obtained from a count-rate graph is correct. Subtract the **background radiation** 本底辐射 (measured with the source removed) from every reading before taking ratios or logarithms; an unsubtracted background makes the curve flatten and the half-life appear too long.

Tracers in medicine. Fluorine-18 and oxygen-15 are β^+ emitters used as **tracers** 示踪剂 in PET scanning (Topic 24): the positron annihilates with an electron, giving two gamma photons that leave in opposite directions and reveal where the tracer is. A **short** half-life (110 minutes, 2 minutes) is chosen so that the activity falls quickly after the scan and the dose to the patient stays small, at the price of having to make the isotope close to the hospital and use it at once.



A graph of $\ln A$ *against time is a straight line of gradient* $-\lambda$

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
mass defect	the difference between the total mass of the separate nucleons and the mass of the nucleus
binding energy	the minimum energy needed to separate a nucleus into its individual nucleons (the energy released when it forms from them)

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Term	Definition
nuclear fusion	two light nuclei combine to form a single heavier nucleus
nuclear fission	a heavy nucleus splits into two lighter nuclei of similar mass (usually after absorbing a neutron)
radioactive decay	the spontaneous and random emission of α, β or γ radiation from an unstable nucleus
spontaneous	not affected by external factors such as temperature, pressure or chemical state
random	it cannot be predicted which nucleus will decay, or when a given nucleus will decay
activity	the number of nuclei decaying per unit time
decay constant	the probability per unit time that a nucleus decays
half-life	the time for the number of undecayed nuclei (or the activity) to fall to half its initial value

Exam tips

- $E = mc^2$ with 1 u = 931.5 MeV: work in u and MeV for reactions, then convert to joules only if asked (1 MeV = 1.60×10^{-13} J).
- Energy released = (total mass before − total mass after) $\times c^2$, or (binding energy after − binding energy before). Mass defects can be subtracted directly when the nucleon count is unchanged.
- The curve: steep rise, peak 8.8 MeV near $A = 56$, gentle fall. Products higher on the curve means energy released; fusion on the left, fission (and α-decay) on the right.
- Balance every equation twice: nucleon numbers along the top, proton numbers along the bottom; β decays carry a neutrino or antineutrino.
- $A = \lambda N$, $\lambda = 0.693/t_{1/2}$ with $t_{1/2}$ in **seconds**; N from mass: $N = m/(Au)$. Ratios of 1/2, 1/4, 1/8 mean whole half-lives; otherwise $t = \ln(x_0/x)/\lambda$.
- Explanations are marked on the words: constant probability, rate proportional to number, same fraction per interval; fluctuating count rate for randomness; unaffected by external conditions for spontaneous.

Common mistakes

- Defining binding energy as "the energy holding the nucleus together" or "the energy stored in the nucleus"; it is the energy to **separate** the nucleons.
- Subtracting masses the wrong way round and reporting a negative energy release, or forgetting the neutron(s) on one side of a fission equation.
- Drawing the binding-energy curve falling as steeply as it rises, or reaching zero at large A.
- Mixing a half-life in minutes or years with an activity in becquerels; use seconds.
- Confusing the decay constant (a probability per second) with the activity (decays per second).
- Saying "half the sample disappears" for a half-life, or that after two half-lives nothing is left.
- Giving "random" as "it happens at any time" without "cannot predict which nucleus or when", or "spontaneous" without "unaffected by external factors".
- Treating the count rate as the activity, or forgetting to subtract background.
- The α-particle and the recoil nucleus share **momentum** equally, not energy.

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Medical physics

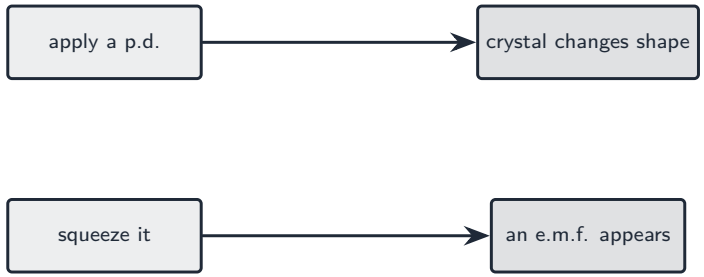
A-Level Physics

Ultrasound

Piezo-electric effect

A **piezo-electric** 压电 crystal changes shape a little when a p.d. is put across it, and the reverse: it makes an **electromotive force** 电动势 (e.m.f.) across itself when its shape is changed. Quartz and PZT are common examples. Two linked effects:

- **apply a p.d.** → the crystal changes shape (used to make vibrations).
- **change the shape** (a wave squeezes it) → an e.m.f. appears (used to detect vibrations).



Applying a p.d. deforms the crystal; squeezing it makes a p.d.

Piezo-electric transducer

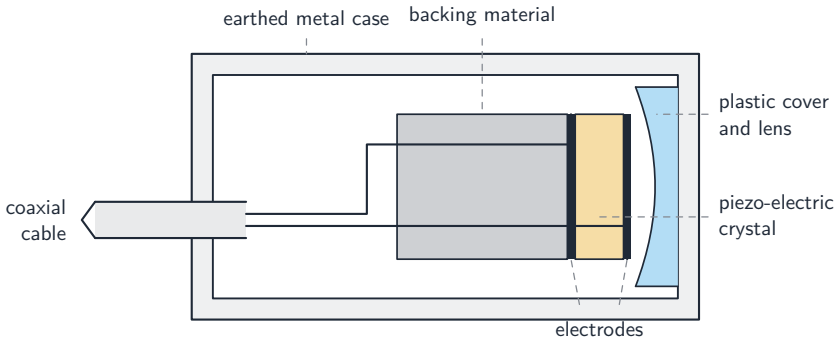
A **transducer** 换能器 uses this effect to both **make** and **detect ultrasound** 超声波.

- an alternating p.d. (a few MHz) makes the crystal vibrate at the same frequency, sending out **longitudinal** 纵波 waves above 20 kHz (1–15 MHz for medical imaging).
- the same crystal then detects: returning ultrasound makes it vibrate and produce an e.m.f.

Generating ultrasound (three marks). (1) An **alternating** p.d. is applied across the crystal. (2) The crystal changes shape, expanding and contracting **at the frequency of the p.d.**, so it vibrates. (3) The p.d. is chosen at the crystal's **resonant** frequency, so the vibration is large and the crystal's faces push on the tissue, sending out a longitudinal wave of that frequency. **Detecting ultrasound (two marks).** The returning wave's pressure variations **change the shape** of the crystal; a changing shape **generates an e.m.f.** across it, alternating at the frequency of the wave,

which is amplified and recorded. The same crystal is used for both, switched between transmitting and receiving.

So one transducer is both emitter and detector, switching between sending and listening.



A piezo-electric transducer both sends and detects ultrasound using a vibrating crystal

Pulse-echo imaging

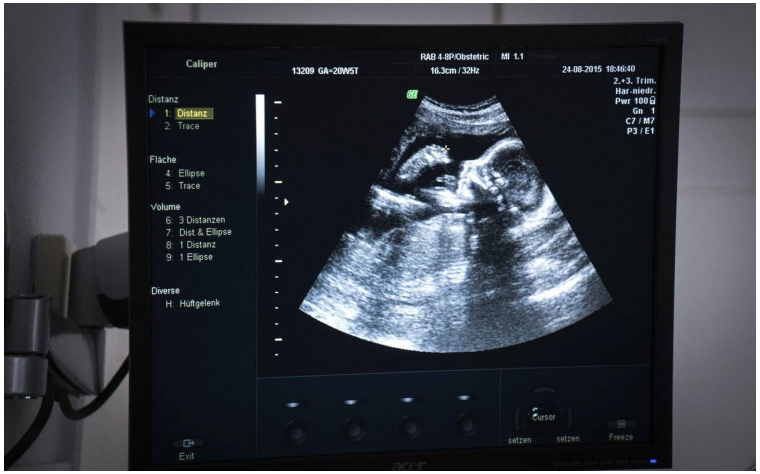
To see inside the body (**pulse-echo** 脉冲回波 imaging):

1. the transducer sends a **short pulse** into the body.
2. at each tissue boundary, part of the pulse is **reflected** and part goes on.
3. the transducer detects each reflected pulse.
4. the **time delay** gives the depth: $d = ct/2$ (there and back). The echo's amplitude gives the strength of the reflection.
5. sweeping across the body builds a 2-D image.

> **Worked example.** An ultrasound pulse returns to the transducer 60 μs after it was sent. The speed of sound in the tissue is 1500 m s^{-1}. Find the depth of the reflecting boundary.

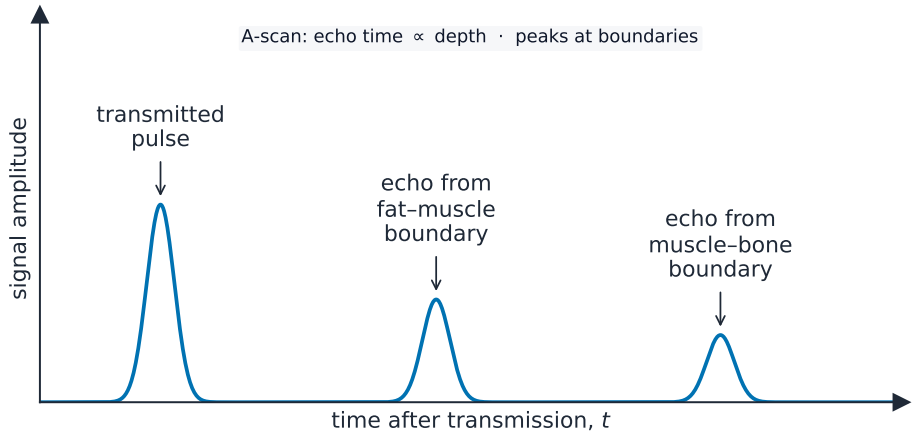
The pulse travels there and back, so $d = \dfrac{ct}{2}$:

$$d = \frac{1500 \times 60 \times 10^{-6}}{2} = 0.045 \text{ m } (= 4.5 \text{ cm}).$$



A real ultrasound image – the fan shape comes from the transducer sweeping across the body. Every bright speck is an echo from a boundary between tissues, and its depth was worked out from the echo's time delay, exactly as in the worked example above

A **coupling gel** 耦合剂 is put between the transducer and the skin to push out the air; without it almost all the ultrasound would reflect at the skin–air boundary and never enter the body.



An A-scan shows the transmitted pulse and the echoes from each tissue boundary

"Outline the use of ultrasound to obtain diagnostic information" (four marks). (1) A **pulse** of ultrasound is sent into the body by the transducer, which is coupled to the skin with gel. (2) At each **boundary** between tissues part of the pulse is **reflected**; the reflected pulse returns to the transducer and is detected. (3) The

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time delay between emission and echo, with the speed of sound, gives the **depth** of the boundary ($d = ct/2$). (4) The **intensity** of the echo, set by the change in acoustic impedance at the boundary, shows what kind of boundary it is. Sweeping the transducer, or using an array, builds up an image.

Why pulses, and why megahertz. The transducer sends a short pulse and then listens: the echo from every boundary must arrive **before** the next pulse leaves, otherwise echoes from different pulses could not be told apart, and the crystal cannot transmit and receive at the same moment. The frequency is a compromise: a **higher** frequency means a shorter wavelength and so finer detail (**resolution** 分辨率), but it is attenuated more strongly, so a deep organ needs a lower frequency than a shallow one. Typical medical scanning uses 1–15 MHz, wavelengths of a fraction of a millimetre in tissue.

■ Specific acoustic impedance

The **specific acoustic impedance** 声阻抗 of a medium is

$$Z = \rho c,$$

where ρ is the **density** 密度 and c the speed of sound. Unit: kg m^{-2} s^{-1}. Bone has large Z; air has small Z; soft tissue is in between.

> **Worked example.** Find the specific acoustic impedance of soft tissue. (Density 1060 kg m^{-3}, speed of sound 1540 m s^{-1}.)

$$Z = \rho c = 1060 \times 1540 \approx 1.6 \times 10^6 \text{ kg m}^{-2} \text{ s}^{-1}.$$

The definition (two marks). *The specific acoustic impedance of a medium is the product of its density and the speed of sound in it.* Both quantities must be named; "how hard sound finds it to pass" scores nothing. Because Z combines density and speed, it is Z, not density alone, that decides how much sound reflects: water and soft tissue have similar densities and similar speeds of sound, so their Z values are close and little sound reflects between them, while bone, about twice as dense and with nearly three times the speed, has a Z several times larger.

■ Reflection at a boundary

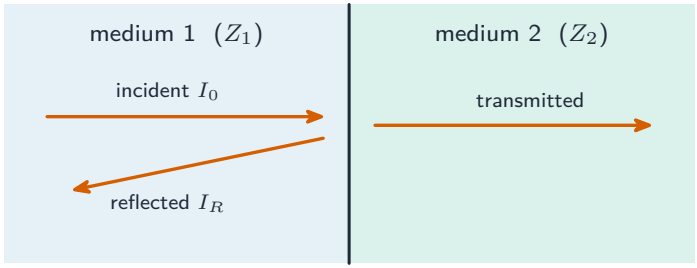
At a boundary between media of impedance Z_1 and Z_2, the **intensity reflection coefficient** 强度反射系数 (fraction reflected) is

$$\frac{I_\text{R}}{I_0} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2.$$

- very different impedances: almost all is reflected (skin/air – hence the gel).
- very similar impedances: almost nothing is reflected, so the boundary cannot be seen.

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- best for imaging: different enough to give an echo, but not so different that nothing passes on.



$\frac{I_R}{I_0} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2$ — a bigger impedance difference gives a bigger echo

At a boundary, part of the pulse reflects and part transmits – a bigger impedance difference gives a bigger echo

> **Worked example (why gel).** Air: density 1.29 kg m^{-3}, speed of sound 343 m s^{-1}. Gel: $Z = 1.50 \times 10^6$ kg m^{-2} s^{-1}. Soft tissue: $Z = 1.63 \times 10^6$ kg m^{-2} s^{-1}. Find the intensity reflection coefficient at an air–tissue boundary and at a gel–tissue boundary.

$Z_\text{air} = 1.29 \times 343 = 442$ kg m^{-2} s^{-1}. Air–tissue: $\left(\frac{442 - 1.63 \times 10^6}{442 + 1.63 \times 10^6}\right)^2 = 0.9989$: 99.9% of the intensity reflects and only 0.1% enters the body. Gel–tissue: $\left(\frac{1.50 - 1.63}{1.50 + 1.63}\right)^2 = 0.0017$: only 0.2% reflects. So the gel, whose impedance is close to that of skin, replaces the air layer that would otherwise reflect nearly everything, and almost all of the pulse enters the body (and the echoes can get back out again by the same route).

> **Worked example.** Water: density 1000 kg m^{-3}, speed 1480 m s^{-1}. Glass: density 2500 kg m^{-3}, speed 5600 m s^{-1}. What fraction of the intensity is reflected at a water–glass boundary?

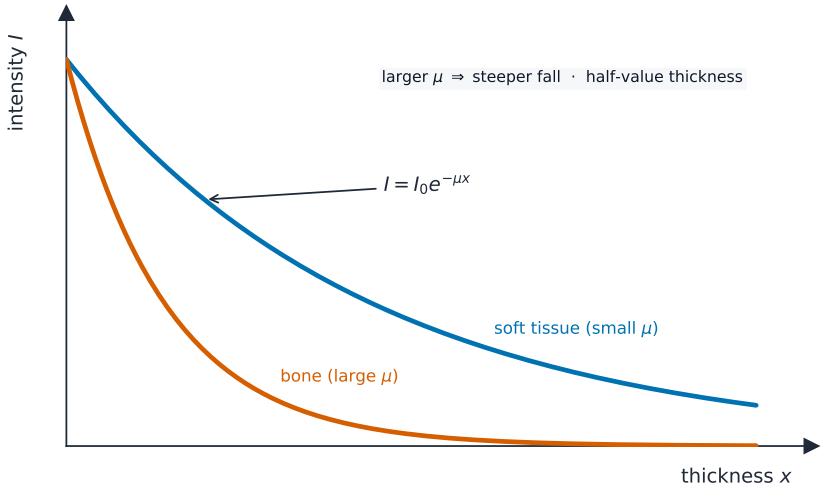
$Z_\text{water} = 1.48 \times 10^6$, $Z_\text{glass} = 1.40 \times 10^7$ kg m^{-2} s^{-1}. Coefficient $= \left(\frac{1.48 - 14.0}{1.48 + 14.0}\right)^2 = 0.65$: two-thirds reflected, one-third transmitted. Working in units of 10^6 keeps the arithmetic clean, since the ratio is what matters. A steel implant in tissue ($Z_\text{steel} = 4.7 \times 10^7$) gives 0.87: nearly all the pulse reflects from its surface, so nothing behind it can be imaged, but the implant itself shows up very brightly.

Attenuation

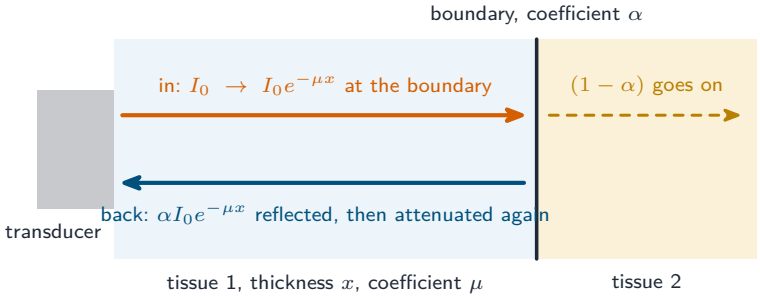
As ultrasound goes through tissue, its intensity falls with distance:

$$I = I_0 e^{-\mu x},$$

where μ is the **attenuation coefficient** 衰减系数 (unit m^{-1}). The same form applies to X-rays.



Intensity falls exponentially with thickness; a larger attenuation coefficient (bone) falls off faster



echo received: $I = \alpha\, I_0\, e^{-2\mu x}$ (two passes through the tissue, one reflection)

One echo, three factors: attenuation in, reflection, attenuation out. The tissue is crossed twice, so the exponent doubles

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> **Worked example (an echo's intensity).** A pulse of intensity I_0 passes through 4.0 cm of tissue with attenuation coefficient 0.50 cm^{-1} to a boundary whose intensity reflection coefficient is 0.010. Find the intensity of the echo returning to the transducer, as a fraction of I_0.

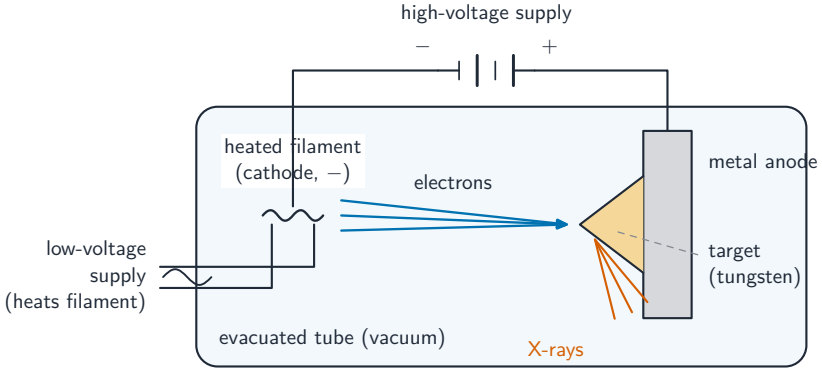
To the boundary: $e^{-\mu x} = e^{-0.50\times4.0} = 0.135$. Reflected: $\times0.010$. Back through the same 4.0 cm: $\times0.135$ again. Echo: $I/I_0 = 0.135 \times 0.010 \times 0.135 = 1.8 \times 10^{-4}$, about 0.02%. Two things the examiner checks: the tissue is crossed **twice**, and the reflection coefficient multiplies the intensity **at the boundary**, not I_0. This is why echoes from deep boundaries are amplified more than shallow ones before display.

X-rays

Production

X-rays come from an **X-ray tube** X 射线管:

1. a heated **cathode** 阴极 emits electrons by **thermionic emission** 热电子发射.
2. a high p.d. (tens to hundreds of kV) accelerates the electrons across a **vacuum** 真空 to a metal **target** 靶 (the **anode** 阳极, often **tungsten** 钨).
3. the electrons hit the target and slow sharply. Most of their **kinetic energy** 动能 becomes heat; a small part is emitted as X-ray **photons** 光子 (**Bremsstrahlung** 轫致辐射, "braking radiation"). Some electrons knock out inner electrons of the metal atoms, and the refilling emits **characteristic** 特征 X-ray lines.



In an X-ray tube electrons from the heated cathode are accelerated onto a metal target anode

"Explain how X-rays are produced" (four marks). (1) Electrons are emitted from a **heated filament** (cathode) by thermionic emission. (2) They are **accelerated** through a **high potential difference** (tens of kV) across an evacuated tube towards a metal target (the anode). (3) They **decelerate rapidly** when they strike the target,

and (4) the kinetic energy lost is emitted as **X-ray photons** (with most of it becoming heat in the target, which is why the anode is cooled or rotated).

Controlling the beam. The **intensity** (energy per unit area per second) is set by the **number of electrons** hitting the target per second, so it is controlled by the **filament current** (a hotter filament emits more electrons); the **hardness** 硬度, meaning the penetrating power, is set by the **photon energies**, so it is controlled by the **accelerating p.d.**: a larger p.d. gives higher-energy, shorter-wavelength, more penetrating X-rays. A metal filter removes the softest X-rays, which would be absorbed in the patient's skin without reaching the detector.

> **Worked example (heating of the target).** In a tube run at 75 kV the electron beam current is 30 mA and 99% of the electrons' energy becomes heat in a tungsten target of mass 15 g (specific heat capacity 134 J kg^{-1} K^{-1}). Find the rate of temperature rise if no heat were lost.

Power delivered $= IV = 0.030 \times 75\,000 = 2250$ W; heating power $= 0.99 \times 2250 = 2230$ W. Then $mc\,\Delta\theta/\Delta t = 2230$, so $\Delta\theta/\Delta t = 2230/(0.015 \times 134) = 1.1 \times 10^3$ K s^{-1}. A target would melt within seconds, which is why exposures are short and anodes rotate to spread the heat.

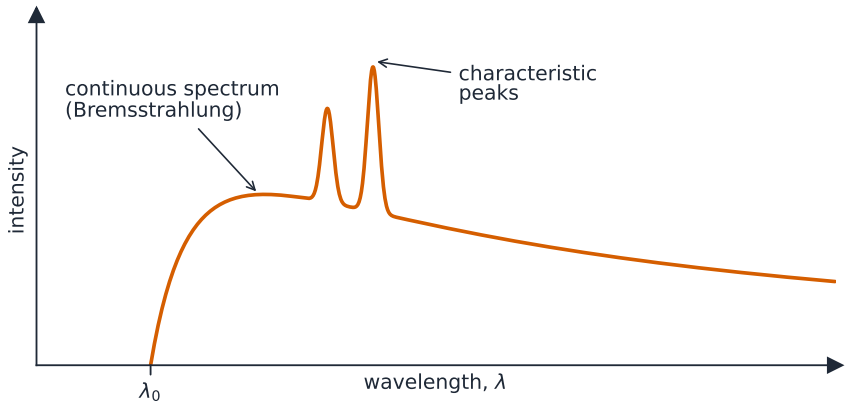
Minimum wavelength

The most energy one X-ray photon can have is the full kinetic energy of one accelerated electron, lost in a single event. For accelerating p.d. V, the KE is eV, so

$$hf_{\max} = eV, \qquad \lambda_{\min} = \frac{hc}{eV}.$$

This is the short-wavelength cut-off. The continuous Bremsstrahlung spectrum tails off above $\lambda_{\min}$, with sharp characteristic peaks set by the target metal.

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X-ray: continuous bremsstrahlung + characteristic peaks

A typical X-ray spectrum – a continuous Bremsstrahlung curve cut off at λ_0, with sharp characteristic peaks

> **Worked example (minimum wavelength).** Electrons are accelerated through 75 kV. Find the maximum photon energy and the minimum wavelength of the X-rays.

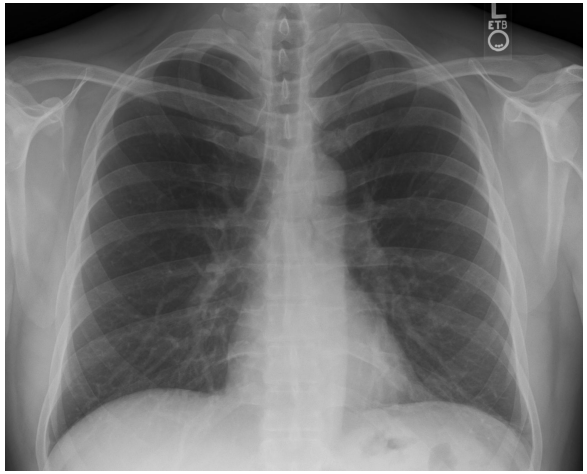
Maximum energy $= eV = 75$ keV $= 75\,000 \times 1.60 \times 10^{-19} = 1.2 \times 10^{-14}$ J (or 0.075 MeV). Then $\lambda_{\min} = hc/(eV) = (6.63 \times 10^{-34})(3.00 \times 10^8)/(1.2 \times 10^{-14}) = 1.7 \times 10^{-11}$ m. The maximum photon **momentum** is $p = E/c = 4.0 \times 10^{-23}$ N s. Doubling the p.d. halves the minimum wavelength.

Explaining the shape of the spectrum. The spectrum is **continuous** because each electron may lose any fraction of its energy in one or several decelerations, so photons of every energy up to the maximum are produced. It has a sharp **cut-off** at $\lambda_{\min}$ because a photon cannot carry more energy than one electron has, eV, and that happens only when an electron loses all its energy in a single event. The **peaks** are characteristic of the target metal: an incoming electron knocks an inner electron out of a target atom, and an outer electron falls into the vacancy, emitting a photon whose energy is the difference between the two levels (Topic 22).

Imaging with X-rays

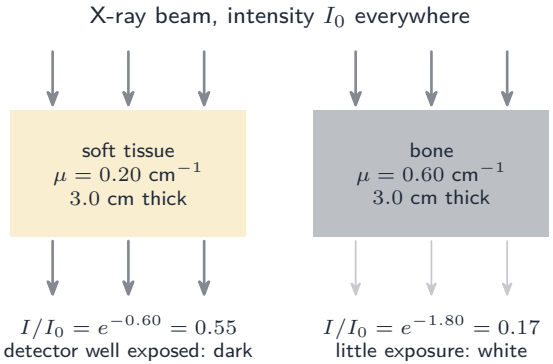
X-rays pass through the patient onto a detector. Tissues that **attenuate** 衰减 more (bone, high Z) cast a stronger shadow and look lighter; tissues that attenuate less (soft tissue, lung) look darker.

The **contrast** 对比度 is the difference in attenuation between tissues. A **contrast medium** 造影剂 (e.g. a barium meal) can be given to make soft tissues stand out.



A real chest X-ray: dense bone absorbs more X-rays and looks white; the air-filled lungs let X-rays through and look dark

Sharpness and contrast. **Sharpness** 清晰度 is how well defined the **edges** of structures are in the image: a small X-ray source (a small spot on the target), a stationary patient and a detector close to the patient all improve it. **Contrast** is the difference in **degree of blackening** (or brightness) between neighbouring regions of the image, produced by a difference in attenuation. The two are independent: an image can be sharp but have poor contrast, or the reverse.



contrast comes from the difference in μ: the two exposures differ by a factor of 3

Contrast is a difference in attenuation: bone attenuates about three times more strongly than soft tissue per centimetre, so equal thicknesses give very different exposures

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"Explain why X-ray images of internal structures have good contrast." Bone and soft tissue have very different **attenuation coefficients**, so equal thicknesses transmit very different intensities and the detector is exposed very differently behind each. Where two soft tissues have similar coefficients (the stomach and its surroundings), a **contrast medium** with a high attenuation coefficient, such as barium, is swallowed or injected to outline one of them. Using a lower p.d. (softer X-rays) also increases contrast, at the cost of a larger dose.

> **Worked example.** X-rays pass through 3.0 cm of soft tissue ($\mu = 0.20$ cm^{-1}) in one region of the body and 3.0 cm of bone ($\mu = 0.60$ cm^{-1}) in another. Compare the transmitted intensities.

Soft tissue: $I/I_0 = e^{-0.20\times3.0} = e^{-0.60} = 0.55$. Bone: $e^{-0.60\times3.0} = e^{-1.80} = 0.17$. The soft-tissue region receives more than three times the exposure of the bone region, so the bone appears white and the tissue dark. The ratio of exposures, $e^{(\mu_{\text{bone}}-\mu_{\text{tissue}})x}$, grows with thickness: contrast improves for thicker structures, but so does the total attenuation.

■ Attenuation law

$$I = I_0 e^{-\mu x}.$$

Higher-energy X-rays penetrate further (smaller μ); bone has a much larger μ than soft tissue. To find the thickness for a given fraction, take logs: $x = \dfrac{1}{\mu}\ln\dfrac{I_0}{I}$. The **half-value thickness** 半值厚度 $x_{1/2} = \ln 2/\mu$ halves the intensity (like half-life in decay).

> **Worked example.** X-rays pass through 3.0 cm of tissue with attenuation coefficient $\mu = 40$ m^{-1}. Find the fraction of the intensity that gets through.

$$\frac{I}{I_0} = e^{-\mu x} = e^{-40\times0.030} = e^{-1.2} \approx 0.30.$$

> **Worked example (percentage absorbed).** X-rays pass through 2.8 cm of a medium with attenuation coefficient 1.4 cm^{-1}. What percentage of the X-ray energy is absorbed?

Transmitted fraction $e^{-1.4\times2.8} = e^{-3.92} = 0.020$, so 98% **is absorbed**. The question asks for what is absorbed, not what gets through: read it twice. The half-value thickness here is $\ln 2/\mu = 0.50$ cm, so 2.8 cm is 5.6 half-value thicknesses and $(1/2)^{5.6} = 0.02$ checks the answer.

> **Worked example (two layers).** A beam of intensity I_0 passes through 2.0 cm of material P ($\mu = 0.35$ cm^{-1}) and then 1.5 cm of material Q ($\mu = 0.90$ cm^{-1}). Find the transmitted intensity.

The exponentials multiply: $I = I_0\,e^{-0.35\times2.0}\,e^{-0.90\times1.5} = I_0\,e^{-(0.70+1.35)} = I_0\,e^{-2.05} =$

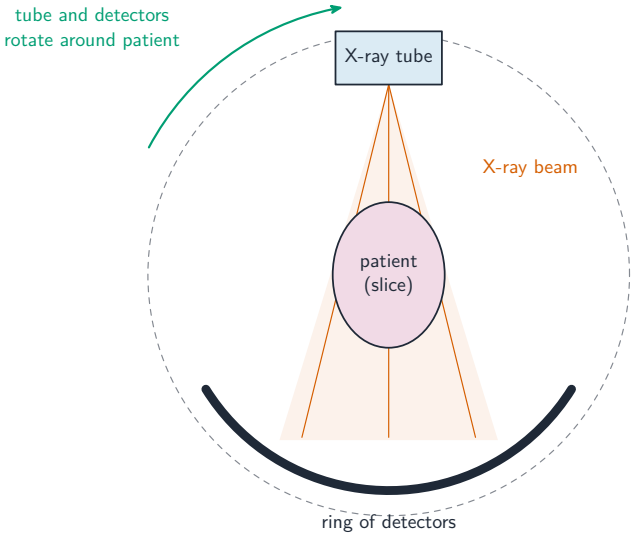
$0.13\,I_0$. Add the exponents $(\mu_1 x_1 + \mu_2 x_2)$; never add the thicknesses or the coefficients. The order of the layers makes no difference to the total transmitted intensity.

Computed tomography (CT)

A **computed tomography** 计算机断层扫描 (CT) scan builds a 3-D image:

1. the tube and detectors rotate around the patient, taking many images of one thin slice from **different angles**.
2. a computer combines these into a 2-D cross-section of the slice.
3. the patient is moved along, and the next slice is imaged.
4. the slices are stacked into a 3-D image.

CT shows far more than a single X-ray, because overlapping soft tissues are separated by the many-angle reconstruction.



In a CT scan the X-ray tube and detectors rotate around the patient to image a slice from many angles

"Explain how CT scanning produces a three-dimensional image" (five marks). (1) The X-ray tube (and a ring of detectors) **rotates around the patient**, so that (2) many X-ray images of **one slice** (section) are taken **from different angles**. (3) A computer **combines** them to produce a two-dimensional image of that slice. (4) The patient is moved and the process is repeated for **successive slices**. (5) The slices are combined to build a **3-D image**, which can be rotated and viewed from any angle. **Purpose:** to image a section of the body without the overlapping of structures that a single X-ray suffers, revealing structures that would be hidden and

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their positions in depth. The price is a much larger dose than a single X-ray.

PET scanning

Tracer

A **tracer** 示踪剂 is a substance with radioactive nuclei put into the body. It is taken up more by the tissue being studied (e.g. a tumour takes up more glucose-tagged tracer due to its high **metabolism** 代谢). Its decay is detected from outside.

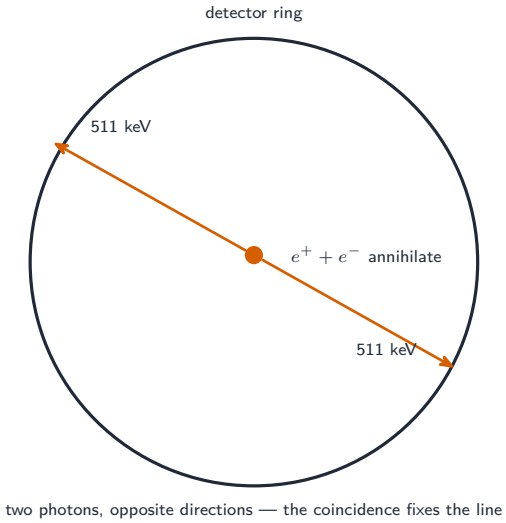
In **positron emission tomography** 正电子发射断层扫描 (PET), the tracer is a β^+ emitter – it gives out a **positron** 正电子. A common one is fluorine-18 on a glucose analogue (FDG).

"Explain what is meant by a tracer and how it is used" (three marks). A tracer is a substance containing **radioactive nuclei** (for PET, a β^+ emitter such as fluorine-18) that is **introduced into the body** (injected or swallowed), usually bound to a molecule such as glucose, and is **absorbed by the tissue being studied**; the radiation it emits leaves the body and is detected, so its distribution shows where that tissue is active. The nuclide is chosen with a **short** half-life, so the activity is high during the scan and the patient's dose afterwards is small, but not so short that it decays before it reaches the tissue: fluorine-18 (110 minutes) and oxygen-15 (2 minutes, made and used on the spot) are typical.

Annihilation

When a particle meets its **antiparticle** 反粒子 they **annihilate** 湮灭: their mass turns into electromagnetic energy. In PET:

- a positron travels a few mm before meeting an **electron** 电子.
- they annihilate. Energy and **momentum** 动量 are conserved.
- since the total momentum is about zero, two photons are produced going in **opposite directions**, each 511 keV ($= m_e c^2$).



PET: annihilation gives two 511 keV photons in opposite directions; a coincidence fixes the line

Energy of the annihilation photons

By energy conservation, the total photon energy equals the pair's rest **energy** 能量:

$$2hf = 2m_e c^2, \qquad hf = m_e c^2.$$

Each photon has $hf = m_e c^2 \approx 8.2 \times 10^{-14}$ J ≈ 0.51 MeV, with $\lambda \approx 2.4 \times 10^{-12}$ m.

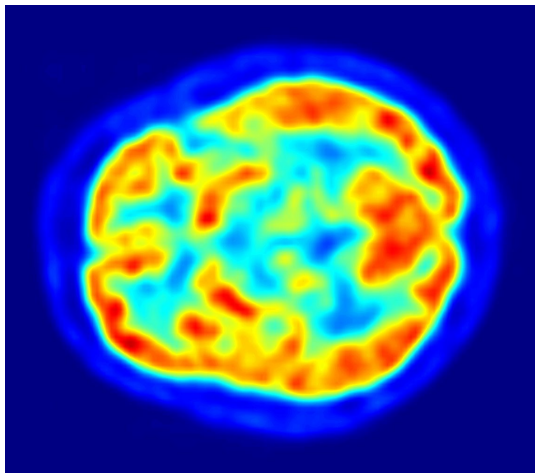
Explaining the annihilation (four marks). (1) The positron emitted by the tracer travels a short distance and **meets an electron** in the tissue. (2) The pair **annihilates**: their mass is converted to energy, (3) emitted as **two gamma-ray photons**, since (4) **momentum** must be conserved: the pair had almost no momentum, so the two photons must travel in **opposite directions** with equal momenta. Each photon carries the rest energy of one particle, 0.51 MeV; a single photon could not conserve momentum.

> **Worked example.** Find the total energy released when a positron and an electron, each moving slowly, annihilate, and the wavelength of each photon.

$E = 2m_e c^2 = 2{\times}9.11{\times}10^{-31}{\times}(3.00{\times}10^8)^2 = 1.64{\times}10^{-13}$ J (1.02 MeV), shared equally: $8.2{\times}10^{-14}$ J each. $\lambda = hc/E = (6.63{\times}10^{-34})(3.00{\times}10^8)/(8.2{\times}10^{-14}) = 2.4{\times}10^{-12}$ m, a gamma ray. If instead the electron and positron each move at 4.9×10^7 m s^{-1} in opposite directions, each has kinetic energy $\frac{1}{2}mv^2 = 1.1 \times 10^{-15}$ J, about 1% of its rest energy, and the photons carry 8.3×10^{-14} J each: the rest energy dominates, which is why the annihilation photons always have very nearly the same energy.

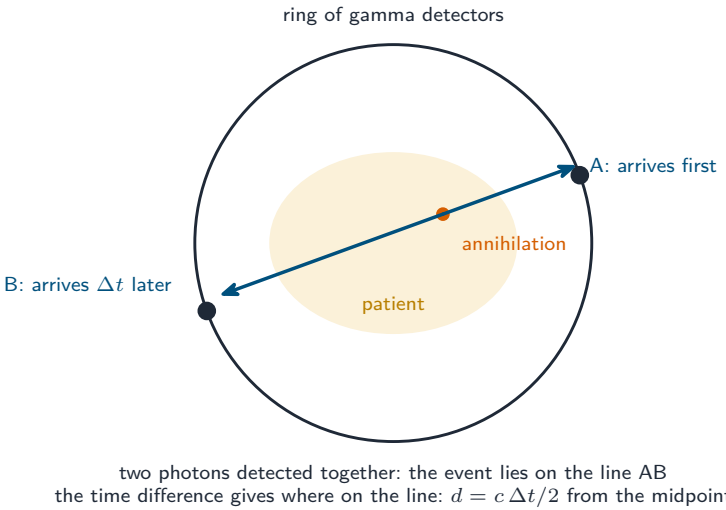
Reconstructing the image

The two photons leave the body in opposite directions and hit **detector rings** around the patient. Recording the **two simultaneous arrivals** (a "coincidence") fixes the line the annihilation happened on. Many coincidences from many angles let the computer build a 3-D map of the tracer – showing tissues with high metabolic activity. Comparing the two **arrival times** can refine the position along that line (time-of-flight PET).



A finished PET image of the brain: warm colours (red, yellow) mark where the tracer collected – the most active tissue

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Locating an annihilation: two photons detected together fix the line, and the difference in their arrival times fixes where on the line

"Explain how the gamma photons are used to form an image" (four marks). (1) The two photons leave the body in opposite directions and are detected by the **ring of detectors**. (2) Two photons arriving at (almost) the **same time** are taken to come from one annihilation, which therefore lies on the **line** joining the two detectors. (3) The small **difference in arrival times** gives the **position** along that line ($d = c\,\Delta t/2$ from the midpoint; a 1 ns difference is 15 cm, so the timing must be very precise). (4) A computer collects many such events and maps the **concentration of tracer** in the tissue, producing an image; a higher concentration means more active tissue, such as a tumour. Photons that arrive singly (their partner absorbed in the body) are rejected.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
piezo-electric effect	a crystal changes shape when a p.d. is applied across it, and generates an e.m.f. when its shape is changed
specific acoustic impedance	the product of the density of a medium and the speed of sound in it, $Z = \rho c$

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Term	Definition
intensity reflection coefficient	the fraction of the incident intensity reflected at a boundary, $(Z_1 - Z_2)^2/(Z_1 + Z_2)^2$
attenuation coefficient	the constant μ in $I = I_0 e^{-\mu x}$; the larger it is, the faster the intensity falls with thickness
hardness (of X-rays)	the penetrating power of the beam, set by the photon energies (the accelerating p.d.)
contrast	the difference in degree of blackening between neighbouring regions of an image
sharpness	how well defined the edges of structures are in an image
CT scanning	X-ray images of one section taken from many angles are combined by computer into a 2-D image of the slice; successive slices give a 3-D image
tracer	a substance containing radioactive nuclei that is introduced into the body and absorbed by the tissue under study
annihilation	a particle and its antiparticle interact and their mass is converted to energy (photons), with mass–energy and momentum conserved

Exam tips

- Ultrasound: pulse in, echo back; depth = $ct/2$ (there **and** back); $Z = \rho c$; reflection coefficient from the two impedances; gel matches impedance to the skin. Attenuation is exponential and an echo crosses the tissue twice.
- X-rays: production is heated filament, high p.d., sudden deceleration at the target; intensity by filament current, hardness by p.d.; $\lambda_{\min} = hc/(eV)$; $I = I_0 e^{-\mu x}$ with exponents added for layers; contrast is a difference in μ.
- CT: many angles, one slice, computer, successive slices, 3-D. Say all five.
- PET: β^+ tracer, annihilation with an electron, two 0.51 MeV photons in opposite directions (momentum), coincidence gives the line, timing gives the position.
- Know the three "explain" answers word for word: how the crystal generates and detects, how X-rays are produced, how the photons locate the tracer.
- Write units with every impedance (kg m^{-2} s^{-1}) and coefficient (cm^{-1} or m^{-1}), and convert cm to m only if μ is in m^{-1}.

■ Common mistakes

- Forgetting the factor of 2 in the depth of a reflecting boundary, or the second pass through the tissue in an echo's intensity.
- Defining acoustic impedance as "the resistance to sound" instead of density times speed of sound.
- Quoting the reflection coefficient for gel–skin as "zero"; it is small, and that is the point.
- Saying the p.d. controls the intensity of the X-ray beam, or that the filament current controls the hardness; it is the other way round.
- Explaining the minimum wavelength without saying that one electron gives all its kinetic energy to a single photon.
- Adding thicknesses of different materials before applying $e^{-\mu x}$; add the products μx instead.
- Describing CT as "an X-ray from several angles" without the computer reconstruction, the slice, or the successive slices.
- Giving one photon for annihilation, or two photons in the same direction; momentum conservation demands two in opposite directions.
- Confusing the tracer's half-life reasoning: short so that the dose is small, but long enough to reach the tissue and be scanned.

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Astronomy and cosmology

A-Level Physics

Luminosity and radiant flux intensity

The **luminosity** 光度 L of a star is the total **power** 功率 of radiation it gives out – the **energy** 能量 radiated per second in all directions. Unit: watt (W).

The one-mark definition. *The luminosity of a star is the total power of radiation emitted by the star* (or the total energy emitted per unit time). It is a property of the star alone; how bright it **looks** from the Earth depends also on how far away it is. Asked for **two reasons why some stars appear brighter than others**, give exactly those two: a **greater luminosity**, and a **smaller distance** (the flux falls as $1/d^2$).

At distance d, this power has spread over a sphere of area $4\pi d^2$. The **radiant flux intensity** 辐射通量密度 F (power per unit area) at distance d is

$$F = \frac{L}{4\pi d^2}.$$

Worked example. The Sun's luminosity is $L = 3.8 \times 10^{26}$ W. Find the radiant flux intensity at the Earth, a distance $d = 1.5 \times 10^{11}$ m away.

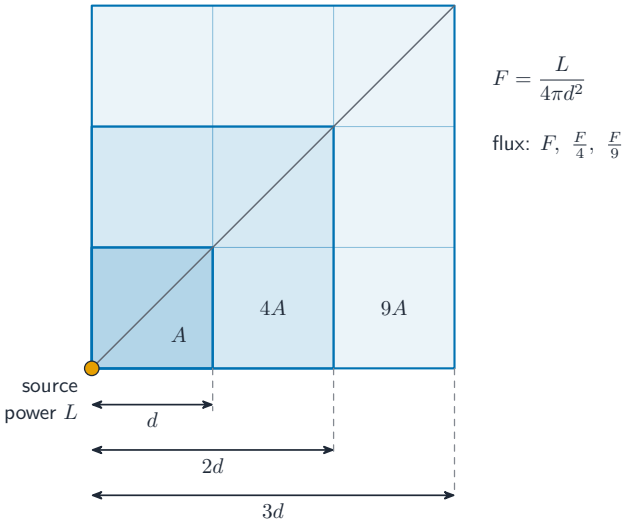
$$F = \frac{L}{4\pi d^2} = \frac{3.8 \times 10^{26}}{4\pi (1.5 \times 10^{11})^2} \approx 1.4 \times 10^{3}\ \text{W m}^{-2}.$$

Unit: W m^{-2}. This is the **inverse-square law** 平方反比定律 for flux: doubling the distance cuts the flux to a quarter. A telescope measures F; if L is known, the distance follows:

$$d = \sqrt{\frac{L}{4\pi F}}.$$

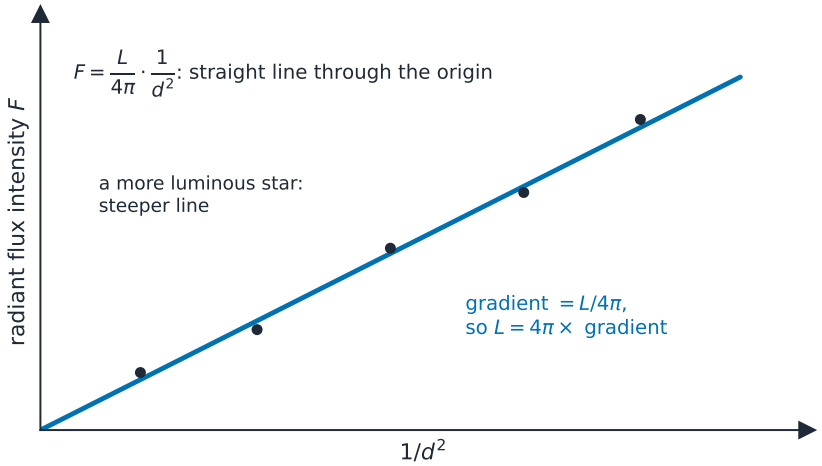


The four units of ESO's Very Large Telescope in Chile, used to measure the flux from distant stars



The same power spreads over a larger area as distance grows, so flux falls as $1/d^2$

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Plot F against $1/d^2$ and the inverse-square law becomes a straight line through the origin; its gradient is $L/4\pi$

Reading the graph. A graph of F against $1/d^2$ for one star is a straight line **through the origin** with gradient $L/4\pi$, so $L = 4\pi \times$ gradient; a more luminous star gives a steeper line. A star whose galaxy is receding still obeys the inverse-square law, but the light we receive is redshifted, so the flux at each wavelength is shifted along the spectrum, and a detector sensitive to one band may see a different fraction of it.

> **Worked example.** The Sun has radius 6.96×10^8 m and surface temperature 5780 K. A space probe carrying a 2.0 m^2 solar panel is 4.5×10^{10} m from the Sun's centre. Find the power falling on the panel when it faces the Sun.

Luminosity: $L = 4\pi\sigma r^2 T^4 = 4\pi(5.67 \times 10^{-8})(6.96 \times 10^8)^2(5780)^4 = 3.85 \times 10^{26}$ W. Flux at the probe: $F = L/(4\pi d^2) = 3.85 \times 10^{26}/[4\pi(4.5 \times 10^{10})^2] = 1.5 \times 10^4$ W m^{-2}, eleven times the flux at the Earth. Power on the panel: $P = FA = 1.5 \times 10^4 \times 2.0 = 3.0 \times 10^4$ W. The flux is power per unit area **perpendicular** to the radiation; a tilted panel receives $FA\cos\theta$.

Standard candles

A **standard candle** 标准烛光 is an object whose **luminosity is known** from its type. Once you find one in a distant galaxy and measure the flux F from it, you get its distance from $d = \sqrt{L/(4\pi F)}$.

Examples:

- **Cepheid variables** 造父变星 (pulsating stars) – the pulsation period is tightly linked to the luminosity, so the period gives L.

supernovae 超新星 are powerful stellar explosions. Only suitable types can be used as standard candles.

- **Type Ia supernovae** Ia 型超新星 – an explosion involving a white dwarf; its peak luminosity can be standardised for distance estimates.

A standard candle gives L without first knowing the distance, so it reaches galaxies far beyond **parallax** 视差.

The definition. *A standard candle is an object (a star or a supernova) of known luminosity.* The luminosity is known because it is fixed by a property that can be measured from any distance: the pulsation period of a Cepheid variable, or the type of a supernova.

distance to a galaxy

identify a standard candle (Cepheid: period → L) → measure the flux F received at the Earth → $F = \dfrac{L}{4\pi d^2}$ so $d = \sqrt{\dfrac{L}{4\pi F}}$

radius of a star

peak wavelength $\lambda_{\max}$ Wien: $T = b/\lambda_{\max}$ → flux F and distance d $L = 4\pi d^2 F$ → Stefan–Boltzmann: $L = 4\pi\sigma r^2 T^4$ so $r = \sqrt{L/(4\pi\sigma T^4)}$

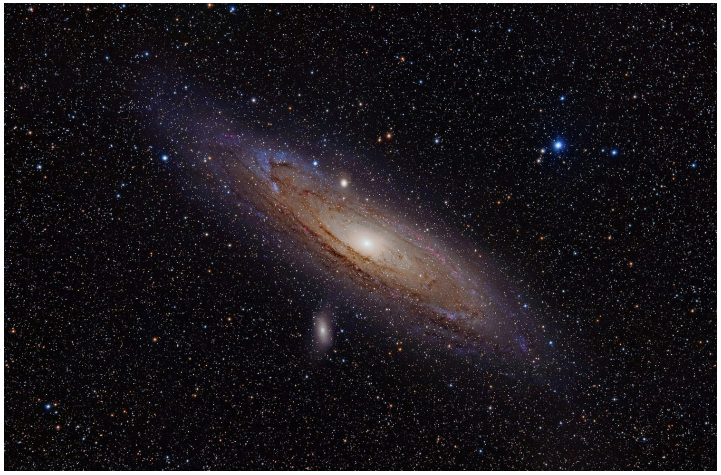
The two recipes of this topic: a distance from a standard candle, and a radius from a spectrum and a flux. Every calculation in the exam is one of them, or a step of one

"Explain how a standard candle is used to determine the distance of a galaxy" (three marks). (1) A standard candle in the galaxy is identified (a Cepheid variable, whose **period** gives its luminosity, or a Type Ia supernova) so that its **luminosity** L is known. (2) The **radiant flux intensity** F of its light arriving at the Earth is measured. (3) The distance follows from the inverse-square law, $d = \sqrt{L/(4\pi F)}$. The order matters: the luminosity comes from the candle's type, **not** from the distance, which is the thing being found.

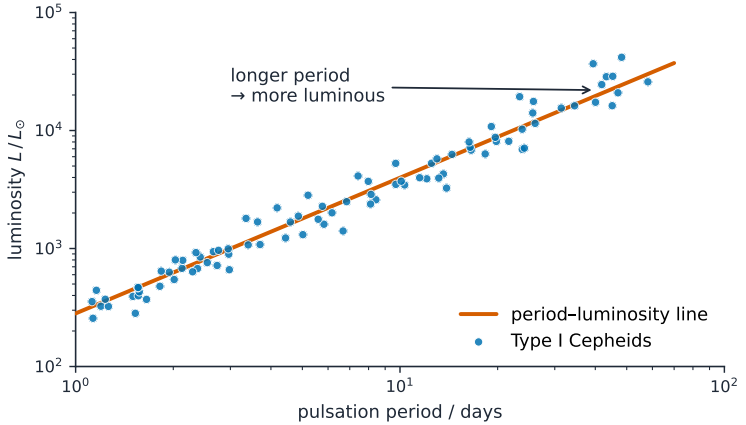
> **Worked example.** A Type Ia supernova, luminosity 1.0×10^{36} W at its peak, is observed with a peak flux of 2.0×10^{-14} W m^{-2}. How far away is its galaxy?

$d = \sqrt{L/(4\pi F)} = \sqrt{1.0 \times 10^{36}/(4\pi \times 2.0 \times 10^{-14})} = 2.0 \times 10^{24}$ m, about 210 million **light years** 光年 (1 ly $= 9.5 \times 10^{15}$ m). A Cepheid, with a luminosity of order 10^{30} W, could not be seen at that distance; supernovae are the candles for the far Universe.

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The Andromeda Galaxy, our nearest large galaxy, about 2.5 million light-years away – Cepheids in it are standard candles



Cepheid: longer period ⇒ brighter · standard candles

For Cepheid variables the pulsation period sets the luminosity, making them standard candles

Stellar surface temperature

Wien's displacement law

A hot body gives out a continuous (**blackbody** 黑体) spectrum with a peak at a **wavelength** 波长 $\lambda_{\max}$ set by its **temperature** 温度. **Wien's displacement law** 维恩位移定律:

$$\lambda_{\max} T = \text{constant}, \qquad b \approx 2.90 \times 10^{-3} \text{ m K}.$$

> **Worked example.** A star's blackbody spectrum peaks at $\lambda_{\max} = 500$ nm. Find its surface temperature. ($b = 2.90 \times 10^{-3}$ m K.)

$$T = \frac{b}{\lambda_{\max}} = \frac{2.90 \times 10^{-3}}{500 \times 10^{-9}} \approx 5800 \text{ K}.$$

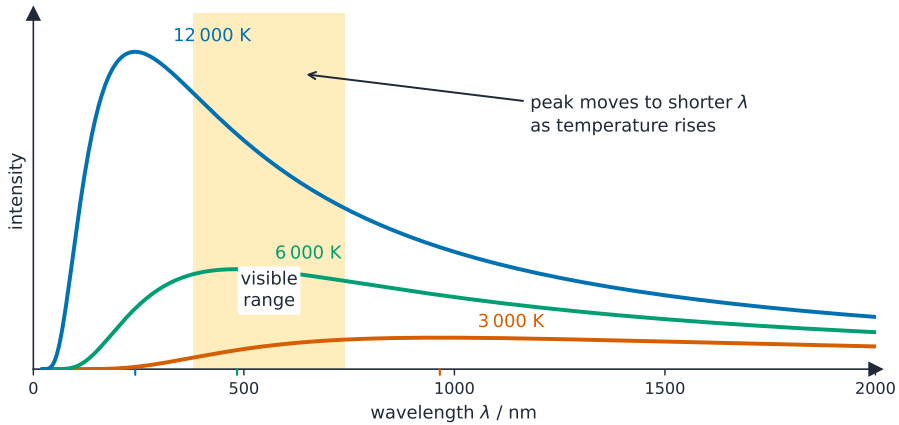
Hotter stars peak at **shorter** wavelengths: a cool red star (~ 3000 K) peaks in the infrared; the Sun (~ 5800 K) peaks near 500 nm; a hot blue-white star ($\sim 20{,}000$ K) peaks in the ultraviolet. Measuring $\lambda_{\max}$ gives the surface temperature.

"State Wien's displacement law" (two marks). *The wavelength at which the intensity of the radiation from a black body is a maximum is inversely proportional to its thermodynamic temperature*: $\lambda_{\max} \propto 1/T$, or $\lambda_{\max} T = \text{constant}$ (2.90×10^{-3} m K). Say "wavelength of **maximum intensity**", not just "the wavelength", and the temperature must be in **kelvin**. The law describes the peak of the continuous black-body curve, not the spectral lines.



The Pillars of Creation in the Eagle Nebula – clouds of gas and dust lit by hot, newly formed stars

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Wien: $\lambda_{\max} T =$ const · hotter ⇒ peak shifts left

A hotter black body radiates more, and its peak wavelength shifts towards the blue (Wien's law)

Stefan–Boltzmann law

A star, treated as a blackbody sphere of radius r and surface temperature T, has luminosity

$$L = 4\pi\sigma r^2 T^4,$$

where $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4} is the **Stefan–Boltzmann constant** 斯特藩-玻尔兹曼常量 (the **Stefan–Boltzmann law** 斯特藩-玻尔兹曼定律). Two strong dependences:

- $L \propto r^2$ – twice the radius, four times the luminosity (same T).
- $L \propto T^4$ – twice the temperature, sixteen times the luminosity (same r).

> **Worked example.** A star has radius $r = 7.0 \times 10^8$ m and surface temperature $T = 5800$ K. Find its luminosity. ($\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}.)

$$L = 4\pi\sigma r^2 T^4 = 4\pi(5.67 \times 10^{-8})(7.0 \times 10^8)^2 (5800)^4 \approx 3.9 \times 10^{26} \text{ W}.$$

Estimating a star's radius

Combine the two laws:

1. measure $\lambda_{\max} \to$ get T from Wien's law.
2. find L (e.g. from flux F and distance d: $L = 4\pi d^2 F$).
3. solve the Stefan–Boltzmann law for r: $r = \sqrt{L/(4\pi\sigma T^4)}$.

This is how astronomers estimate radii of stars they cannot see as a disc.

> **Worked example (the radius of the Sun).** The radiant flux intensity of sunlight at the Earth is 1370 W m^{-2} at a distance of 1.50×10^{11} m, and the Sun's spectrum peaks at 500 nm. Estimate the radius of the Sun.

Luminosity: $L = 4\pi d^2 F = 4\pi(1.50 \times 10^{11})^2(1370) = 3.87 \times 10^{26}$ W. Temperature: $T = b/\lambda_{\max} = 2.90 \times 10^{-3}/(500 \times 10^{-9}) = 5800$ K. Radius: $r = \sqrt{L/(4\pi\sigma T^4)} = \sqrt{3.87 \times 10^{26}/[4\pi(5.67 \times 10^{-8})(5800)^4]} = 6.9 \times 10^8$ m. Three steps, each a one-line formula; keep the full precision of L and T until the end, because T is raised to the fourth power.

> **Worked example.** A star in a distant galaxy has a radiant flux intensity of 2.52×10^{-8} W m^{-2} at the Earth, a distance of 4.16×10^{17} m away, and a surface temperature of 9500 K. Find its radius.

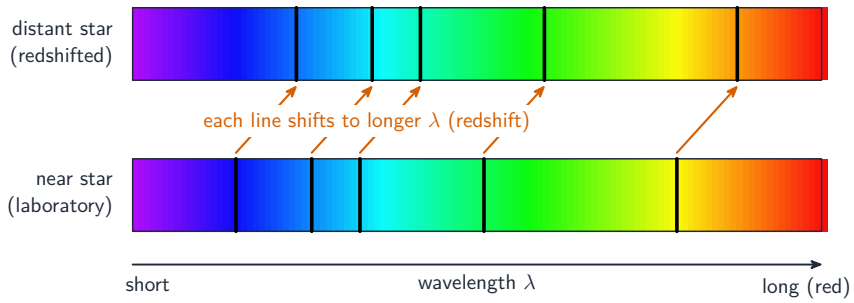
$L = 4\pi d^2 F = 4\pi(4.16 \times 10^{17})^2(2.52 \times 10^{-8}) = 5.48 \times 10^{28}$ W. Then $r = \sqrt{5.48 \times 10^{28}/[4\pi(5.67 \times 10^{-8})(9500)^4]} = 3.1 \times 10^9$ m, about four times the Sun's radius. Check the sense: a star 140 times as luminous as the Sun but only 1.6 times as hot must be considerably bigger, since $L \propto r^2 T^4$.

Why T^4 matters so much. Two stars of the same radius at 3000 K and 6000 K differ in luminosity by $2^4 = 16$ times; a small error in the temperature is a large error in the luminosity, which is why the peak wavelength must be read carefully and, for a receding galaxy, corrected for redshift (below).

Redshift, Hubble's law and the Big Bang

Cosmological redshift

The **spectral lines** 谱线 of light from distant galaxies are seen at **longer wavelengths** than their known laboratory values – the whole spectrum is stretched towards the red. This is **redshift** 红移.



The hydrogen absorption lines of a distant star are shifted to longer wavelengths – a redshift

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Reading it as a Doppler shift, the galaxy is moving **away**. For $v \ll c$:

$$\frac{\Delta\lambda}{\lambda} \approx \frac{v}{c},$$

where $\Delta\lambda = \lambda_{\text{observed}} - \lambda_{\text{emitted}}$ and v is the speed of **recession** 退行. Example: light emitted at 4.62×10^{-7} m but seen at 4.91×10^{-7} m gives $\Delta\lambda = 0.29 \times 10^{-7}$ m and

$$v \approx \frac{\Delta\lambda}{\lambda_{\text{em}}} c \approx 1.9 \times 10^7 \text{ m s}^{-1}.$$

Why redshift means an expanding Universe

Almost every distant galaxy is redshifted (a few near ones are **blueshifted** 蓝移 by local motion). So galaxies are, on average, moving apart – not just from us but from each other. The Universe is **expanding**, with the space between galaxies stretching. More distant galaxies are redshifted more.

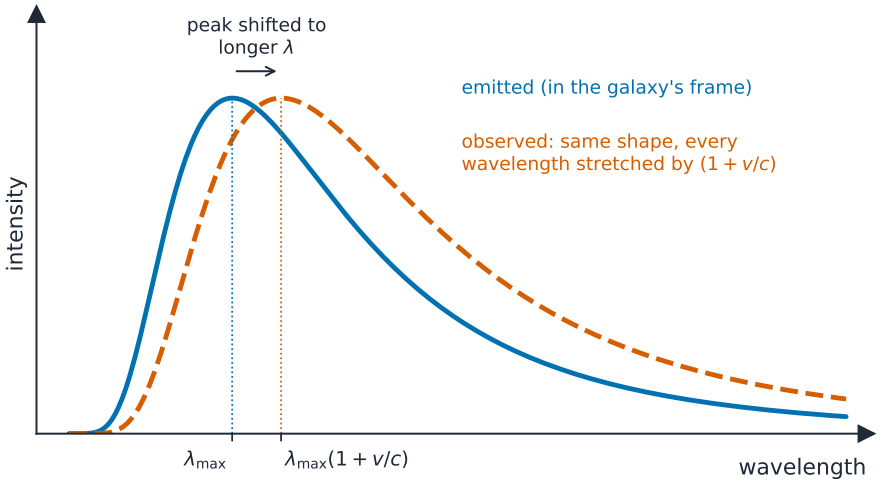
"State what is meant by redshift." *The observed wavelength of the radiation (its spectral lines) from a source is longer than the wavelength emitted, because the source is moving away from the observer.* It is the **Doppler effect** 多普勒效应 for light: the fractional change in wavelength equals the fractional change in frequency and, for $v \ll c$, the ratio v/c.

> **Worked example.** A hydrogen line measured in the laboratory at 656.3 nm is observed in the light from a galaxy at 660.9 nm. Find the galaxy's speed of recession and, using $H_0 = 2.3 \times 10^{-18}$ s^{-1}, its distance.

$\Delta\lambda = 660.9 - 656.3 = 4.6$ nm, so $v = c\Delta\lambda/\lambda = (3.00 \times 10^8)(4.6/656.3) = 2.1 \times 10^6$ m s^{-1}. Distance: $d = v/H_0 = 2.1 \times 10^6/2.3 \times 10^{-18} = 9.1 \times 10^{23}$ m (about 100 million light years). Divide by the **emitted** (laboratory) wavelength, and keep the nanometres consistent in the ratio.

> **Worked example (the other way round).** A galaxy in Corona Borealis recedes at 21 400 km s^{-1}. At what wavelength is its 656.3 nm hydrogen line observed?

$\Delta\lambda = \lambda v/c = 656.3 \times (2.14 \times 10^7/3.00 \times 10^8) = 46.8$ nm, so the line appears at 703 nm, moved from the red almost into the infrared. Every line and the whole continuous spectrum are stretched by the same factor $(1 + v/c) = 1.071$.

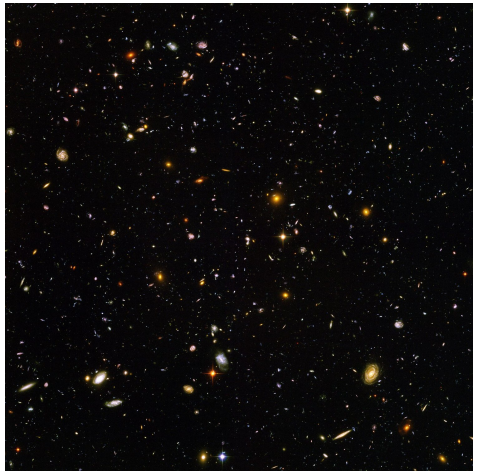


A receding galaxy's continuous spectrum keeps its shape but slides to longer wavelengths. Wien's law applied to the observed peak gives a temperature that is too low

A trap the exam sets. If the peak wavelength of a **receding** galaxy's spectrum is fed straight into Wien's law, the temperature comes out **too low**, because the observed $\lambda_{\max}$ is longer than the emitted one. Correct the peak first: $\lambda_{\text{emitted}} = \lambda_{\text{observed}}/(1 + v/c)$. Asked to sketch the observed spectrum on the same axes as the emitted one, draw the same shape shifted to longer wavelengths, peak included.

"Explain how redshift leads to the idea that the Universe is expanding" (three marks). (1) The spectral lines from (almost) all distant galaxies are shifted to **longer** wavelengths, so (2) by the Doppler effect the galaxies are **moving away** from us, and (3) the further away a galaxy is, the **greater** its redshift and so its speed: this is what would be seen from **any** galaxy if the space between all galaxies were expanding, so the Universe as a whole is expanding, not just moving away from the Earth.

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The Hubble Ultra Deep Field – almost every point of light is a whole galaxy, most of them redshifted and receding

Hubble's law

The link between recession speed v and distance d is **Hubble's law** 哈勃定律:

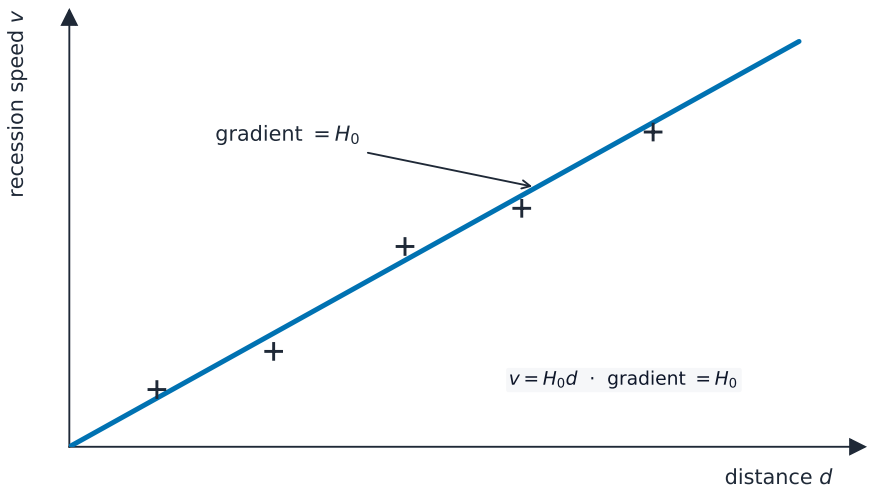
$$v \approx H_0 \cdot d,$$

where H_0 is the **Hubble constant** 哈勃常数 ($\approx 2.3 \times 10^{-18}$ s^{-1}). Always use SI units. Example: a galaxy receding at 1.9×10^7 m s^{-1} is at $d = v/H_0 \approx 8.3 \times 10^{24}$ m.

"State Hubble's law" (two marks). *The speed of recession of a galaxy is (directly) proportional to its distance from the Earth (the observer)*: $v = H_0 d$, where v is the recession speed, d the distance and H_0 the Hubble constant. Identify every symbol when asked. The value of H_0 is quoted in the exam in SI units (s^{-1}), so v must be in m s^{-1} and d in metres, never kilometres per second per megaparsec.

> **Worked example.** A star in a distant galaxy emits radiation whose intensity peaks at 4.62×10^{-7} m; the peak in the light received at the Earth is at 4.91×10^{-7} m. Find the star's surface temperature, the galaxy's recession speed, and its distance ($H_0 = 2.3 \times 10^{-18}$ s^{-1}).

Temperature from the **emitted** peak: $T = 2.90 \times 10^{-3}/4.62 \times 10^{-7} = 6300$ K. Speed: $v = c\,\Delta\lambda/\lambda = (3.00 \times 10^8)(0.29/4.62) = 1.9 \times 10^7$ m s^{-1}, about 6% of c. Distance: $d = v/H_0 = 1.9 \times 10^7/2.3 \times 10^{-18} = 8.3 \times 10^{24}$ m (about 870 million light years). Using the observed peak for the temperature would have given 5900 K, 400 K too low.



Hubble's law: a galaxy's recession speed is proportional to its distance, $v = H_0 d$

From Hubble's law to the Big Bang

Hubble's law means the galaxies were once together. Running the expansion backwards, all distances shrink to zero at $t = -1/H_0$ – the Universe was once a tiny, hugely dense, hot point. This is the **Big Bang** 大爆炸. The age of the Universe (for steady expansion) is about

$$T_{\text{age}} \approx \frac{1}{H_0} \approx 4.3 \times 10^{17}\ \text{s} \approx 14\ \text{billion years}.$$

The expansion, the redshift of galaxies, the **cosmic microwave background** 宇宙微波背景, and the hydrogen/helium abundances are the main evidence for the Big Bang.

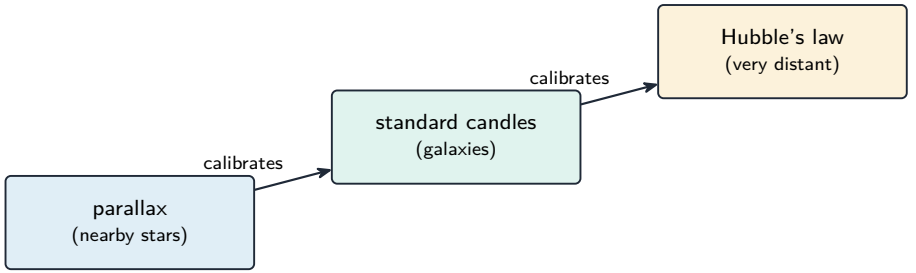
"Explain how Hubble's law leads to the Big Bang theory" (three marks). (1) Galaxies are receding with speeds **proportional to their distances**, so (2) if time is run backwards, every galaxy, at whatever distance, arrives at the **same point at the same time**, $t = d/v = 1/H_0$ ago; (3) so the Universe must have begun as a single point of enormous density and temperature that has been expanding ever since. The age estimate: $1/H_0 = 1/(2.3 \times 10^{-18}) = 4.3 \times 10^{17}$ s, and dividing by 3.16×10^7 s per year gives 1.4×10^{10} years. This assumes the expansion rate has not changed; it is an estimate, not a measurement.

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Distance ladder

Astronomers combine methods, each calibrated by the one below:

1. **parallax** – for nearby stars.
2. **standard candles** (Cepheids, Type Ia supernovae) – for galaxies.
3. **Hubble's law** ($d = v/H_0$, with v from redshift) – for very distant galaxies.



The distance ladder: each method is calibrated by the one below and reaches further out

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
luminosity	the total power of radiation emitted by a star
radiant flux intensity	the power of radiation received per unit area (at right angles to the radiation), $F = L/(4\pi d^2)$
standard candle	an object of known luminosity, from which a distance can be found by measuring the flux received
Wien's displacement law	the wavelength of maximum intensity of a black body is inversely proportional to its thermodynamic temperature, $\lambda_{\text{max}} T = \text{constant}$
Stefan–Boltzmann law	the luminosity of a black body of radius r is $L = 4\pi\sigma r^2 T^4$
redshift	the increase in the observed wavelength of radiation from a source moving away from the observer

Term	Definition
Hubble's law	the speed of recession of a galaxy is proportional to its distance from the observer, $v = H_0 d$
Hubble constant	the constant of proportionality in Hubble's law, in s^{-1}; $1/H_0$ estimates the age of the Universe
Big Bang theory	the Universe began from a single point of very high density and temperature and has been expanding since

Exam tips

- Two recipes: **distance** = candle's L, measured F, $d = \sqrt{L/(4\pi F)}$; **radius** = T from $\lambda_{\max}$, L from F and d, r from $L = 4\pi\sigma r^2 T^4$. Write each step as its own formula.
- F is a flux (per square metre), L a power; $L = 4\pi d^2 F$ links them and 4π is part of both laws.
- Wien: $\lambda_{\max}$ in metres, T in kelvin. Stefan–Boltzmann: r^2 and T^4; a 10% error in T is a 46% error in L.
- Redshift: $\Delta\lambda/\lambda_{\text{emitted}} = v/c$; then $d = v/H_0$ in **SI** units. Correct a receding galaxy's peak wavelength before using Wien's law.
- The three "explain" answers (standard candle to distance, redshift to expansion, Hubble's law to the Big Bang) are marked point by point: three statements each, in order.
- The age of the Universe is $1/H_0$, about 14 billion years; state the assumption of a constant expansion rate.

Common mistakes

- Defining luminosity as brightness, or as power per unit area; that is the flux.
- Finding a standard candle's luminosity from its distance, which is circular; the luminosity comes from its type.
- Using degrees Celsius in Wien's or Stefan's law, or forgetting the fourth power.
- Dividing $\Delta\lambda$ by the observed wavelength instead of the emitted one, or mixing nanometres and metres in one ratio.
- Using H_0 with v in km s^{-1}; convert to m s^{-1}.
- Feeding a redshifted peak wavelength into Wien's law and reporting a temperature that is too low.
- Stating Hubble's law without "proportional" or without saying what v and d are.
- Explaining the Big Bang without the backward-in-time argument that all galaxies were together at $t = 1/H_0$ ago.

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