Astronomy and cosmology

A-Level Physics

Luminosity and radiant flux intensity

The **luminosity** 光度 L of a star is the total **power** 功率 of radiation it gives out — the **energy** 能量 radiated per second in all directions. Unit: watt (W).

The one-mark definition. *The luminosity of a star is the total power of radiation emitted by the star* (or the total energy emitted per unit time). It is a property of the star alone; how bright it **looks** from the Earth depends also on how far away it is. Asked for **two reasons why some stars appear brighter than others**, give exactly those two: a **greater luminosity**, and a **smaller distance** (the flux falls as $1/d^2$).

At distance d, this power has spread over a sphere of area $4\pi d^2$. The **radiant flux intensity** 辐射通量密度 F (power per unit area) at distance d is

$$F = \frac{L}{4\pi d^2}.$$

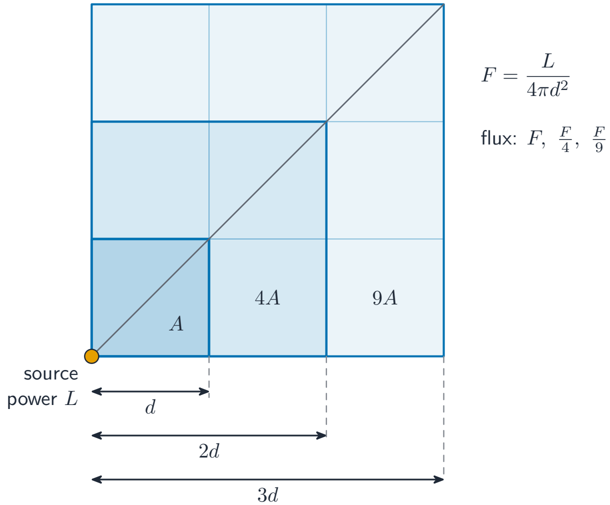
Worked example. The Sun's luminosity is $L = 3.8 \times 10^{26}$ W. Find the radiant flux intensity at the Earth, a distance $d = 1.5 \times 10^{11}$ m away.

$$F = \frac{L}{4\pi d^2} = \frac{3.8 \times 10^{26}}{4\pi(1.5 \times 10^{11})^2} \approx 1.4 \times 10^3 \text{ W m}^{-2}.$$

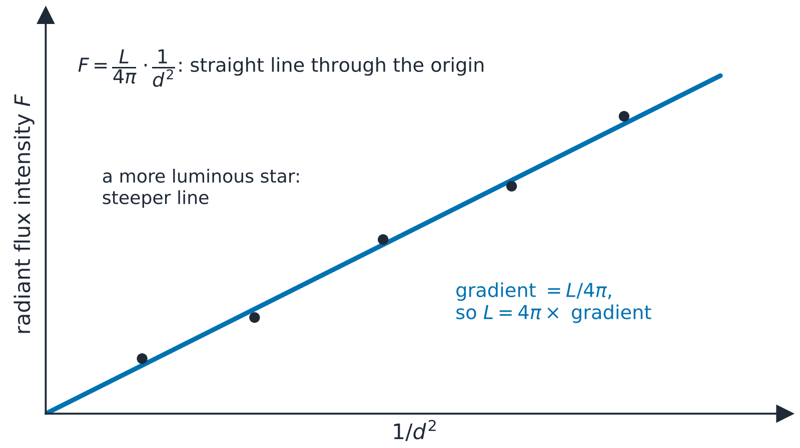
Unit: W m^{-2}. This is the **inverse-square law** 平方反比定律 for flux: doubling the distance cuts the flux to a quarter. A telescope measures F; if L is known, the distance follows:

$$d = \sqrt{\frac{L}{4\pi F}}.$$

The four units of ESO's Very Large Telescope in Chile, used to measure the flux from distant stars



The same power spreads over a larger area as distance grows, so flux falls as $1/d^2$



Plot F against $1/d^2$ and the inverse-square law becomes a straight line through the origin; its gradient is $L/4\pi$

Reading the graph. A graph of F against $1/d^2$ for one star is a straight line **through the origin** with gradient $L/4\pi$, so $L = 4\pi \times$ gradient; a more luminous star gives a steeper line. A star whose galaxy is receding still obeys the inverse-square law, but the light we receive is redshifted, so the flux at each wavelength is shifted along the spectrum, and a detector sensitive to one band may see a different fraction of it.

Worked example. The Sun has radius 6.96×10^8 m and surface temperature 5780 K. A space probe carrying a 2.0 m^2 solar panel is 4.5×10^{10} m from the Sun's centre. Find the power falling on the panel when it faces the Sun.

Luminosity: $L = 4\pi\sigma r^2 T^4 = 4\pi(5.67 \times 10^{-8})(6.96 \times 10^8)^2(5780)^4 = 3.85 \times 10^{26}$ W. Flux at the probe: $F = L/(4\pi d^2) = 3.85 \times 10^{26}/[4\pi(4.5 \times 10^{10})^2] = 1.5 \times 10^4\ \mathrm{W\ m^{-2}}$, eleven times the flux at the Earth. Power on the panel: $P = FA = 1.5 \times 10^4 \times 2.0 = 3.0 \times 10^4$ W. The flux is power per unit area **perpendicular** to the radiation; a tilted panel receives $FA\cos\theta$.

Standard candles

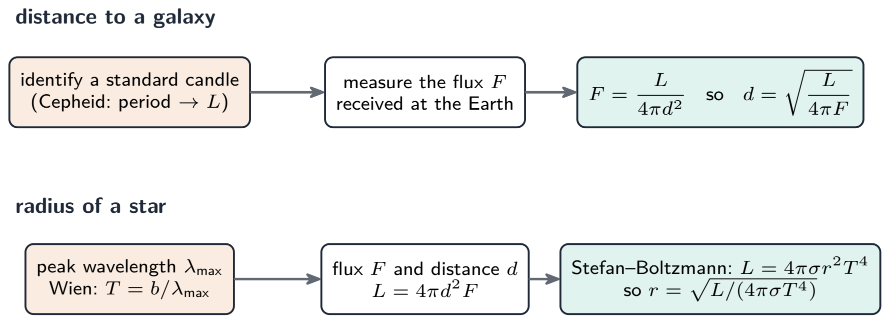
A **standard candle** 标准烛光 is an object whose **luminosity is known** from its type. Once you find one in a distant galaxy and measure the flux F from it, you get its distance from $d = \sqrt{L/(4\pi F)}$.

Examples:

- **Cepheid variables** 造父变星 (pulsating stars) — the pulsation period is tightly linked to the luminosity, so the period gives L.
- **Type Ia supernovae** 超新星 — a white dwarf reaching a critical mass and exploding always has about the same peak luminosity.

A standard candle gives L without first knowing the distance, so it reaches galaxies far beyond **parallax** 视差.

The definition. *A standard candle is an object (a star or a supernova) of known luminosity.* The luminosity is known because it is fixed by a property that can be measured from any distance: the pulsation period of a Cepheid variable, or the type of a supernova.

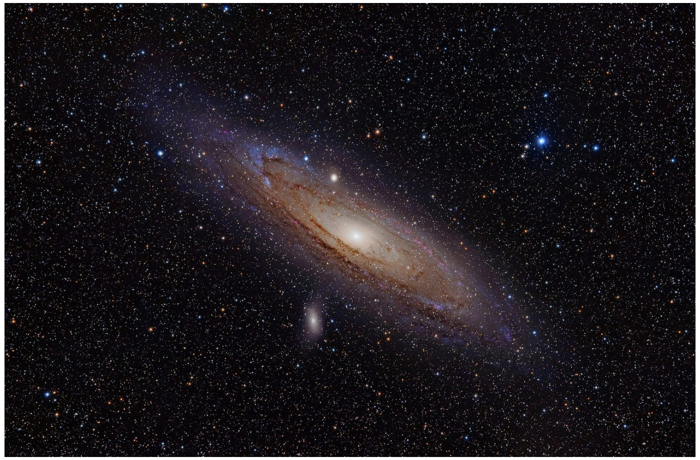


The two recipes of this topic: a distance from a standard candle, and a radius from a spectrum and a flux. Every calculation in the exam is one of them, or a step of one

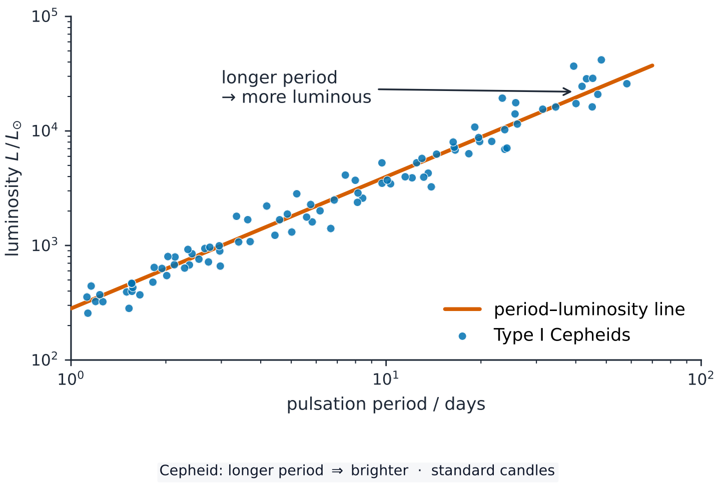
"Explain how a standard candle is used to determine the distance of a galaxy" (three marks). (1) A standard candle in the galaxy is identified (a Cepheid variable, whose **period** gives its luminosity, or a Type Ia supernova) so that its **luminosity** L is known. (2) The **radiant flux intensity** F of its light arriving at the Earth is measured. (3) The distance follows from the inverse-square law, $d = \sqrt{L/(4\pi F)}$. The order matters: the luminosity comes from the candle's type, **not** from the distance, which is the thing being found.

Worked example. A Type Ia supernova, luminosity 1.0×10^{36} W at its peak, is observed with a peak flux of $2.0 \times 10^{-14}\ \mathrm{W\ m^{-2}}$. How far away is its galaxy?

$d = \sqrt{L/(4\pi F)} = \sqrt{1.0 \times 10^{36}/(4\pi \times 2.0 \times 10^{-14})} = 6.3 \times 10^{24}$ m, about 670 million **light years** 光年 (1 ly $= 9.5 \times 10^{15}$ m). A Cepheid, with a luminosity of order 10^{30} W, could not be seen at that distance; supernovae are the candles for the far Universe.



The Andromeda Galaxy, our nearest large galaxy, about 2.5 million light-years away — Cepheids in it are standard candles



For Cepheid variables the pulsation period sets the luminosity, making them standard candles

Stellar surface temperature

Wien's displacement law

A hot body gives out a continuous (**blackbody** 黑体) spectrum with a peak at a **wavelength** 波长 $\lambda_{\max}$ set by its **temperature** 温度. **Wien's displacement law** 维恩位移定律:

$$\lambda_{\max} T = \text{constant}, \qquad b \approx 2.90 \times 10^{-3}\ \text{m K}.$$

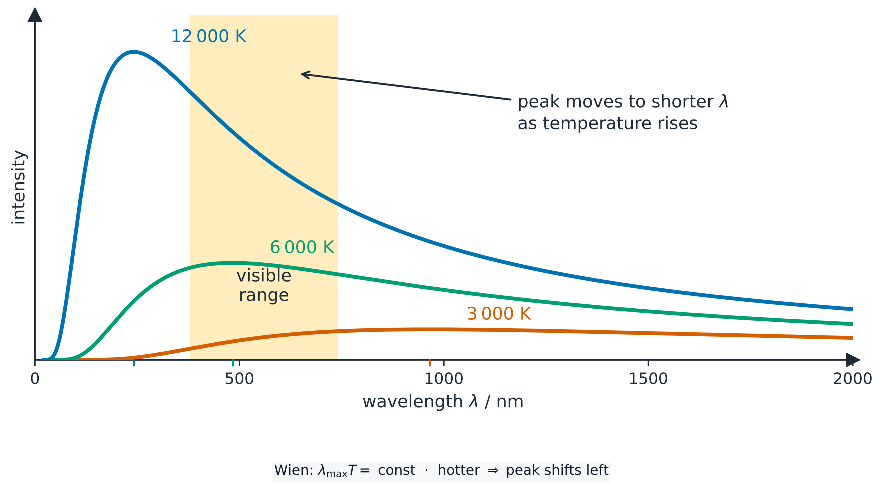
Worked example. A star's blackbody spectrum peaks at $\lambda_{\max} = 500$ nm. Find its surface temperature. ($b = 2.90 \times 10^{-3}$ m K.)

$$T = \frac{b}{\lambda_{\max}} = \frac{2.90 \times 10^{-3}}{500 \times 10^{-9}} \approx 5800\ \text{K}.$$

Hotter stars peak at **shorter** wavelengths: a cool red star (~ 3000 K) peaks in the infrared; the Sun (~ 5800 K) peaks near 500 nm; a hot blue-white star ($\sim 20{,}000$ K) peaks in the ultraviolet. Measuring $\lambda_{\max}$ gives the surface temperature.

"State Wien's displacement law" (two marks). *The wavelength at which the intensity of the radiation from a black body is a maximum is inversely proportional to its thermodynamic temperature*: $\lambda_{\max} \propto 1/T$, or $\lambda_{\max} T = \text{constant}$ (2.90×10^{-3} m K). Say "wavelength of **maximum intensity**", not just "the wavelength", and the temperature must be in **kelvin**. The law describes the peak of the continuous black-body curve, not the spectral lines.

The Pillars of Creation in the Eagle Nebula — clouds of gas and dust lit by hot, newly formed stars



A hotter black body radiates more, and its peak wavelength shifts towards the blue (Wien's law)

Stefan–Boltzmann law

A star, treated as a blackbody sphere of radius r and surface temperature T, has luminosity

$$L = 4\pi\sigma r^2 T^4,$$

where $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4} is the **Stefan–Boltzmann constant** 斯特藩-玻尔兹曼常量 (the **Stefan–Boltzmann law** 斯特藩-玻尔兹曼定律). Two strong dependences:

- $L \propto r^2$ — twice the radius, four times the luminosity (same T).
- $L \propto T^4$ — twice the temperature, sixteen times the luminosity (same r).

Worked example. A star has radius $r = 7.0 \times 10^8$ m and surface temperature $T = 5800$ K. Find its luminosity. ($\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}.)

$$L = 4\pi\sigma r^2 T^4 = 4\pi(5.67 \times 10^{-8})(7.0 \times 10^8)^2(5800)^4 \approx 3.9 \times 10^{26} \text{ W}.$$

Estimating a star's radius

Combine the two laws:

1. measure $\lambda_{max} \to$ get T from Wien's law.
2. find L (e.g. from flux F and distance d: $L = 4\pi d^2 F$).
3. solve the Stefan–Boltzmann law for r: $r = \sqrt{L/(4\pi\sigma T^4)}$.

This is how astronomers estimate radii of stars they cannot see as a disc.

Worked example (the radius of the Sun). The radiant flux intensity of sunlight at the Earth is 1370 W m^{-2} at a distance of 1.50×10^{11} m, and the Sun's spectrum peaks at 500 nm. Estimate the radius of the Sun.

Luminosity: $L = 4\pi d^2 F = 4\pi(1.50 \times 10^{11})^2(1370) = 3.87 \times 10^{26}$ W. Temperature: $T = b/\lambda_{max} = 2.90 \times 10^{-3}/(500 \times 10^{-9}) = 5800$ K. Radius: $r = \sqrt{L/(4\pi\sigma T^4)} = \sqrt{3.87 \times 10^{26}/[4\pi(5.67 \times 10^{-8})(5800)^4]} = 6.9 \times 10^8$ m. Three steps, each a one-line formula; keep the full precision of L and T until the end, because T is raised to the fourth power.

Worked example. A star in a distant galaxy has a radiant flux intensity of 2.52×10^{-8} W m^{-2} at the Earth, a distance of 4.16×10^{17} m away, and a surface temperature of 9500 K. Find its radius.

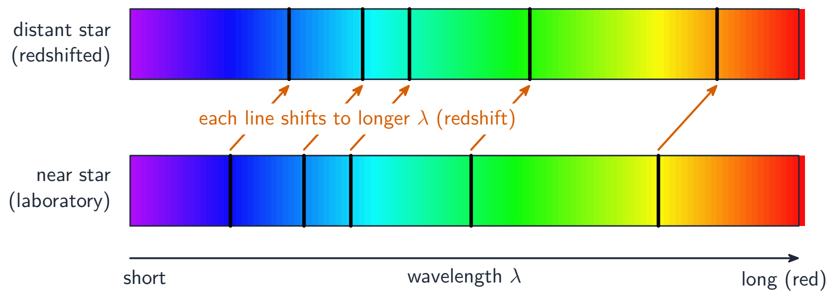
$L = 4\pi d^2 F = 4\pi(4.16 \times 10^{17})^2(2.52 \times 10^{-8}) = 5.48 \times 10^{28}$ W. Then $r = \sqrt{5.48 \times 10^{28}/[4\pi(5.67 \times 10^{-8})(9500)^4]} = 3.1 \times 10^9$ m, about four times the Sun's radius. Check the sense: a star 140 times as luminous as the Sun but only 1.6 times as hot must be considerably bigger, since $L \propto r^2 T^4$.

Why T^4 matters so much. Two stars of the same radius at 3000 K and 6000 K differ in luminosity by $2^4 = 16$ times; a small error in the temperature is a large error in the luminosity, which is why the peak wavelength must be read carefully and, for a receding galaxy, corrected for redshift (below).

Redshift, Hubble's law and the Big Bang

Cosmological redshift

The **spectral lines** 谱线 of light from distant galaxies are seen at **longer wavelengths** than their known laboratory values — the whole spectrum is stretched towards the red. This is **redshift** 红移.



The hydrogen absorption lines of a distant star are shifted to longer wavelengths — a redshift

Reading it as a Doppler shift, the galaxy is moving **away**. For $v \ll c$:

$$\frac{\Delta\lambda}{\lambda} \approx \frac{v}{c},$$

where $\Delta\lambda = \lambda_{\text{observed}} - \lambda_{\text{emitted}}$ and v is the speed of **recession** 退行. Example: light emitted at 4.62×10^{-7} m but seen at 4.91×10^{-7} m gives $\Delta\lambda = 0.29 \times 10^{-7}$ m and

$$v \approx \frac{\Delta\lambda}{\lambda_{\text{em}}} c \approx 1.9 \times 10^{7}\ \text{m s}^{-1}.$$

Why redshift means an expanding Universe

Almost every distant galaxy is redshifted (a few near ones are **blueshifted** 蓝移 by local motion). So galaxies are, on average, moving apart — not just from us but from each other. The Universe is **expanding**, with the space between galaxies stretching. More distant galaxies are redshifted more.

"State what is meant by redshift." *The observed wavelength of the radiation (its spectral lines) from a source is longer than the wavelength emitted, because the source is moving away from the observer.* It is the **Doppler effect** 多普勒效应 for light: the fractional change in wavelength equals the fractional change in frequency and, for $v \ll c$, the ratio v/c.

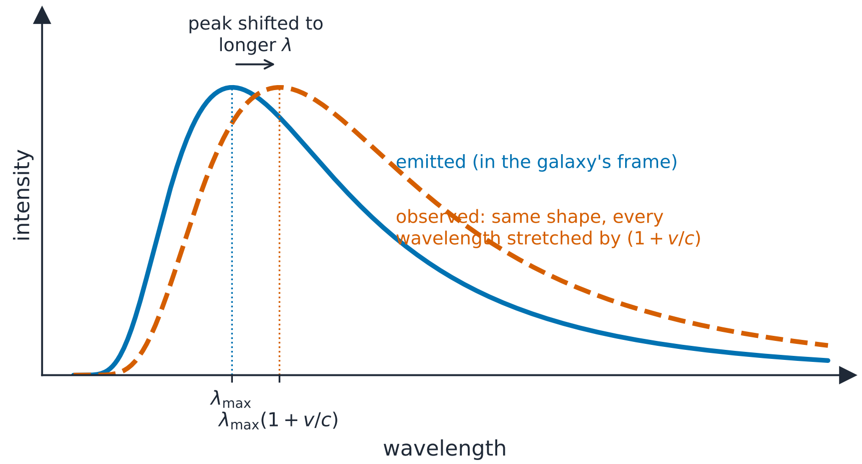
Worked example. A hydrogen line measured in the laboratory at 656.3 nm is observed in the light from a galaxy at 660.9 nm. Find the galaxy's speed of recession and, using $H_0 = 2.3 \times 10^{-18}$ s^{-1}, its distance.

$\Delta\lambda = 660.9 - 656.3 = 4.6$ nm, so $v = c\,\Delta\lambda/\lambda = (3.00 \times 10^{8})(4.6/656.3) = 2.1 \times 10^{6}$ m s^{-1}. Distance: $d = v/H_0 = 2.1 \times 10^{6}/2.3 \times 10^{-18} = 9.1 \times 10^{23}$ m (about 100

million light years). Divide by the **emitted** (laboratory) wavelength, and keep the nanometres consistent in the ratio.

Worked example (the other way round). A galaxy in Corona Borealis recedes at 21 400 km s^{-1}. At what wavelength is its 656.3 nm hydrogen line observed?

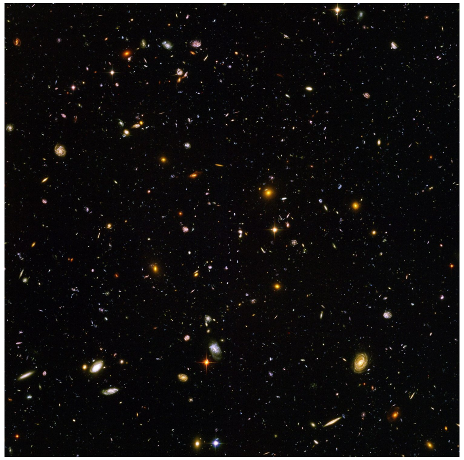
$\Delta\lambda = \lambda v/c = 656.3 \times (2.14 \times 10^{7}/3.00 \times 10^{8}) = 46.8$ nm, so the line appears at 703 nm, moved from the red almost into the infrared. Every line and the whole continuous spectrum are stretched by the same factor $(1 + v/c) = 1.071$.



A receding galaxy's continuous spectrum keeps its shape but slides to longer wavelengths. Wien's law applied to the observed peak gives a temperature that is too low

A trap the exam sets. If the peak wavelength of a **receding** galaxy's spectrum is fed straight into Wien's law, the temperature comes out **too low**, because the observed $\lambda_{\max}$ is longer than the emitted one. Correct the peak first: $\lambda_{\text{emitted}} = \lambda_{\text{observed}}/(1 + v/c)$. Asked to sketch the observed spectrum on the same axes as the emitted one, draw the same shape shifted to longer wavelengths, peak included.

"Explain how redshift leads to the idea that the Universe is expanding" (three marks). (1) The spectral lines from (almost) all distant galaxies are shifted to **longer** wavelengths, so (2) by the Doppler effect the galaxies are **moving away** from us, and (3) the further away a galaxy is, the **greater** its redshift and so its speed: this is what would be seen from **any** galaxy if the space between all galaxies were expanding, so the Universe as a whole is expanding, not just moving away from the Earth.



The Hubble Ultra Deep Field — almost every point of light is a whole galaxy, most of them redshifted and receding

Hubble's law

The link between recession speed v and distance d is **Hubble's law** 哈勃定律:

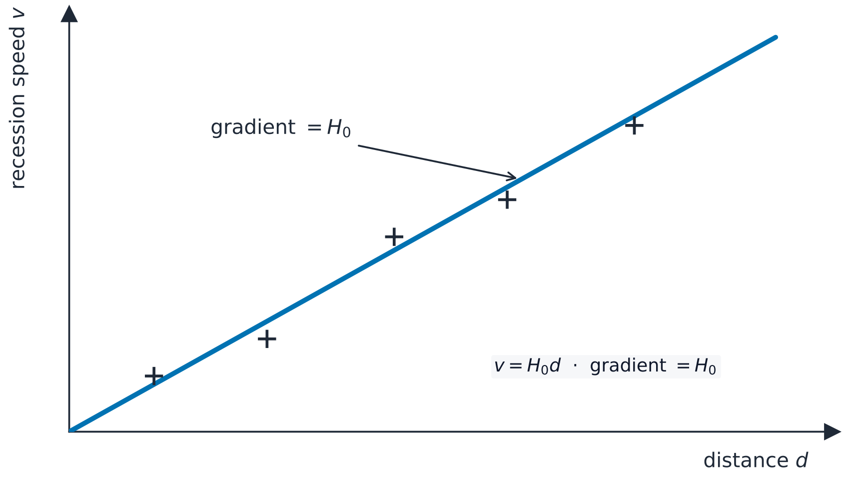
$$v \approx H_0 \cdot d,$$

where H_0 is the **Hubble constant** 哈勃常数 ($\approx 2.3 \times 10^{-18}$ s^{-1}). Always use SI units. Example: a galaxy receding at 1.9×10^7 m s^{-1} is at $d = v/H_0 \approx 8.3 \times 10^{24}$ m.

"State Hubble's law" (two marks). *The speed of recession of a galaxy is (directly) proportional to its distance from the Earth (the observer)*: $v = H_0 d$, where v is the recession speed, d the distance and H_0 the Hubble constant. Identify every symbol when asked. The value of H_0 is quoted in the exam in SI units (s^{-1}), so v must be in m s^{-1} and d in metres, never kilometres per second per megaparsec.

Worked example. A star in a distant galaxy emits radiation whose intensity peaks at 4.62×10^{-7} m; the peak in the light received at the Earth is at 4.91×10^{-7} m. Find the star's surface temperature, the galaxy's recession speed, and its distance ($H_0 = 2.3 \times 10^{-18}$ s^{-1}).

Temperature from the **emitted** peak: $T = 2.90 \times 10^{-3}/4.62 \times 10^{-7} = 6300$ K. Speed: $v = c\,\Delta\lambda/\lambda = (3.00 \times 10^8)(0.29/4.62) = 1.9 \times 10^7$ m s^{-1}, about 6% of c. Distance: $d = v/H_0 = 1.9 \times 10^7/2.3 \times 10^{-18} = 8.2 \times 10^{24}$ m (about 870 million light years). Using the observed peak for the temperature would have given 5900 K, 400 K too low.



Hubble's law: a galaxy's recession speed is proportional to its distance, $v = H_0 d$

From Hubble's law to the Big Bang

Hubble's law means the galaxies were once together. Running the expansion backwards, all distances shrink to zero at $t = -1/H_0$ — the Universe was once a tiny, hugely dense, hot point. This is the **Big Bang** 大爆炸. The age of the Universe (for steady expansion) is about

$$T_{\text{age}} \approx \frac{1}{H_0} \approx 4.3 \times 10^{17} \text{ s} \approx 14 \text{ billion years.}$$

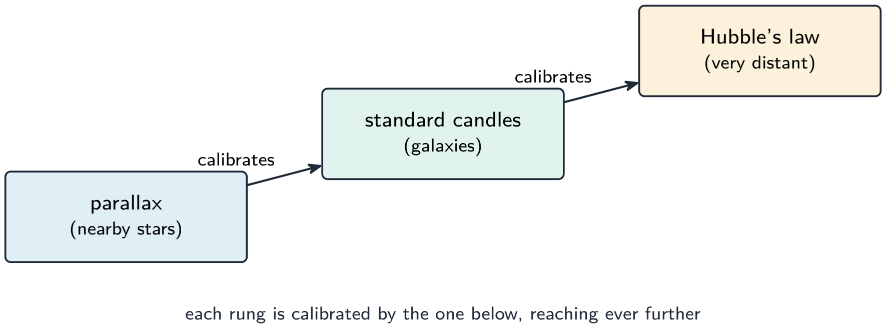
The expansion, the redshift of galaxies, the **cosmic microwave background** 宇宙微波背景, and the hydrogen/helium abundances are the main evidence for the Big Bang.

"Explain how Hubble's law leads to the Big Bang theory" (three marks). (1) Galaxies are receding with speeds **proportional to their distances**, so (2) if time is run backwards, every galaxy, at whatever distance, arrives at the **same point at the same time**, $t = d/v = 1/H_0$ ago; (3) so the Universe must have begun as a single point of enormous density and temperature that has been expanding ever since. The age estimate: $1/H_0 = 1/(2.3 \times 10^{-18}) = 4.3 \times 10^{17}$ s, and dividing by 3.16×10^7 s per year gives 1.4×10^{10} years. This assumes the expansion rate has not changed; it is an estimate, not a measurement.

Distance ladder

Astronomers combine methods, each calibrated by the one below:

1. **parallax** — for nearby stars.
2. **standard candles** (Cepheids, Type Ia supernovae) — for galaxies.
3. **Hubble's law** ($d = v/H_0$, with v from redshift) — for very distant galaxies.



The distance ladder: each method is calibrated by the one below and reaches further out

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
luminosity	the total power of radiation emitted by a star
radiant flux intensity	the power of radiation received per unit area (at right angles to the radiation), $F = L/(4\pi d^2)$
standard candle	an object of known luminosity, from which a distance can be found by measuring the flux received
Wien's displacement law	the wavelength of maximum intensity of a black body is inversely proportional to its thermodynamic temperature, $\lambda_{\max}T = \text{constant}$
Stefan–Boltzmann law	the luminosity of a black body of radius r is $L = 4\pi\sigma r^2T^4$
redshift	the increase in the observed wavelength of radiation from a source moving away from the observer
Hubble's law	the speed of recession of a galaxy is proportional to its distance from the observer, $v = H_0d$
Hubble constant	the constant of proportionality in Hubble's law, in s^{-1}; $1/H_0$ estimates the age of the Universe
Big Bang theory	the Universe began from a single point of very high density and temperature and has been expanding since

Exam tips

- Two recipes: **distance** = candle's L, measured F, $d = \sqrt{L/(4\pi F)}$; **radius** = T from $\lambda_{\max}$, L from F and d, r from $L = 4\pi\sigma r^2T^4$. Write each step as its own formula.
- F is a flux (per square metre), L a power; $L = 4\pi d^2F$ links them and 4π is part of both laws.
- Wien: $\lambda_{\max}$ in metres, T in kelvin. Stefan–Boltzmann: r^2 and T^4; a 10% error in T is a 46% error in L.
- Redshift: $\Delta\lambda/\lambda_{\text{emitted}} = v/c$; then $d = v/H_0$ in **SI** units. Correct a receding galaxy's peak wavelength before using Wien's law.
- The three "explain" answers (standard candle to distance, redshift to expansion, Hubble's law to the Big Bang) are marked point by point: three statements each, in order.
- The age of the Universe is $1/H_0$, about 14 billion years; state the assumption of a constant expansion rate.

Common mistakes

- Defining luminosity as brightness, or as power per unit area; that is the flux.
- Finding a standard candle's luminosity from its distance, which is circular; the luminosity comes from its type.
- Using degrees Celsius in Wien's or Stefan's law, or forgetting the fourth power.
- Dividing $\Delta\lambda$ by the observed wavelength instead of the emitted one, or mixing nanometres and metres in one ratio.
- Using H_0 with v in km s^{-1}; convert to m s^{-1}.
- Feeding a redshifted peak wavelength into Wien's law and reporting a temperature that is too low.
- Stating Hubble's law without "proportional" or without saying what v and d are.
- Explaining the Big Bang without the backward-in-time argument that all galaxies were together at $t = 1/H_0$ ago.