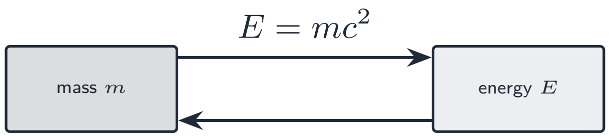
Nuclear physics

A-Level Physics

Mass-energy equivalence

Einstein's special relativity gives the famous link (**mass-energy equivalence** 质能等价):



Mass and energy are equivalent and can change into each other

$$E = mc^2,$$

where $c = 3.00 \times 10^8$ m s^{-1}. A mass m matches an **energy** 能量 E — the two can change into each other. For a mass change Δm:

Worked example. The star Sirius loses mass through nuclear fusion at 1.09×10^{11} kg s^{-1}. Find the power it radiates.

Every kilogram that disappears leaves as energy: $P = c^2 \times$ (mass lost per second) $= (3.00 \times 10^8)^2(1.09 \times 10^{11}) = 9.8 \times 10^{27}$ W. This is the star's **luminosity** 光度 (Topic 25); the Sun's is 3.8×10^{26} W, so it loses about 4 million tonnes a second. The same idea in reverse: a power station producing 1 GW for a year converts $E/c^2 = 3.2 \times 10^{16}/9.0 \times 10^{16} = 0.35$ kg of mass, which is why the fuel weighs almost the same afterwards.

$$\Delta E = c^2 \Delta m.$$

In nuclear physics the masses are tiny but c^2 is huge, so a small mass change means a large energy. A mass change of 1 u (1.661×10^{-27} kg) matches $\Delta E \approx 1.49 \times 10^{-10}$ J. This gives a conversion you will use again and again:

$$1 \text{ u} \longleftrightarrow 931 \text{ MeV}.$$

Nuclear reactions

A **nuclear reaction** 核反应 is written like

$${}^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \rightarrow {}^{17}_{8}\text{O} + {}^{1}_{1}\text{H},$$

with **nucleon number** 核子数 conserved (top numbers: $14 + 4 = 17 + 1$) and charge conserved (bottom numbers: $7 + 2 = 8 + 1$) — this is **conservation of charge** 电荷守恒. Use these to fill in an unknown: identify the species, then balance the top and bottom numbers.

The decay equations. Alpha decay removes ${}^{4}_{2}$He, so A falls by 4 and Z by 2: ${}^{211}_{84}\text{Po} \rightarrow {}^{207}_{82}\text{Pb} + {}^{4}_{2}\text{He}$. Beta-minus decay turns a neutron into a proton, emitting an electron and an antineutrino: ${}^{15}_{6}\text{C} \rightarrow {}^{15}_{7}\text{N} + {}^{0}_{-1}\text{e} + \bar{\nu}$ (A unchanged, Z up by 1). Beta-plus decay turns a proton into a neutron, emitting a **positron** 正电子 and a **neutrino** 中微子: ${}^{18}_{9}\text{F} \rightarrow {}^{18}_{8}\text{O} + {}^{0}_{+1}\text{e} + \nu$ (Z down by 1). Gamma emission changes neither number. In a chain of decays just keep the books: a nucleus W that emits β^-, then α, then β^- ends with $A - 4$ and $Z + 1 - 2 + 1 = Z$, an **isotope of W**. Check both lines of every equation you complete, and remember the neutrino: the mark scheme includes it.

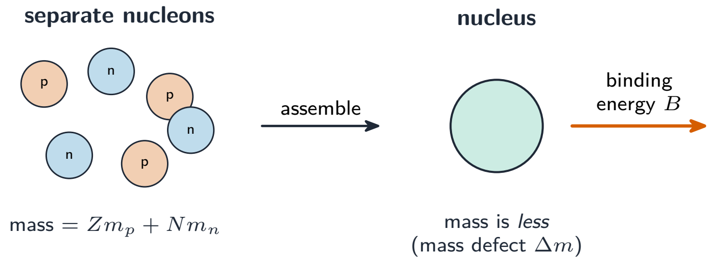
Mass defect and binding energy

The mass of a **nucleus** 原子核 is **less than** the total mass of its separate **protons** 质子 and **neutrons** 中子. The difference is the **mass defect** 质量亏损 Δm:

$$\Delta m = (Zm_{\text{p}} + Nm_{\text{n}}) - m_{\text{nucleus}}.$$

By $E = mc^2$, this "missing" mass was released as energy when the nucleus formed. To pull the nucleus fully apart you must put that energy back — the **binding energy** 结合能 B:

$$B = \Delta m \cdot c^2.$$



The assembled nucleus has less mass than its separate nucleons; the missing mass is released as binding energy

Worked example. A helium-4 nucleus has a mass defect of $\Delta m = 0.0304$ u. Find its binding energy. (1 u corresponds to 931 MeV.)

$$B = \Delta m \cdot c^2 = 0.0304 \times 931 \approx 28 \text{ MeV}.$$

A more tightly bound nucleus has a larger mass defect and larger binding energy. The **binding energy per nucleon 比结合能** is B/A (usually in MeV per nucleon) — a measure of how tightly each nucleon is held, useful for comparing nuclides.

The two-mark definitions. *The mass defect of a nucleus is the difference between the total mass of its separate nucleons and the mass of the nucleus. The binding energy is the minimum energy required to separate the nucleus into its individual nucleons* (equivalently, the energy released when the nucleus is formed from separate nucleons). Say "separate nucleons" or "individual protons and neutrons"; "the energy holding the nucleus together" scores nothing. Binding energy is **released**, not stored: the nucleus has *less* energy than its parts.

Worked example. The masses are: proton 1.007 276 u, neutron 1.008 665 u, polonium-212 nucleus 211.945 4 u. Find the mass defect and the binding energy per nucleon of $^{212}_{84}\text{Po}$.

$Z = 84$ protons and $N = 212 - 84 = 128$ neutrons: total $84 \times 1.007\,276 + 128 \times 1.008\,665 = 84.611\,2 + 129.109\,1 = 213.720\,3$ u. Mass defect $\Delta m = 213.720\,3 - 211.945\,4 = 1.774\,9$ u. Binding energy $= 1.774\,9 \times 931.5 = 1653$ MeV, so per nucleon $1653/212 = 7.80$ MeV, on the falling part of the curve. Keep every decimal place until the subtraction: the defect is a small difference of two large numbers.

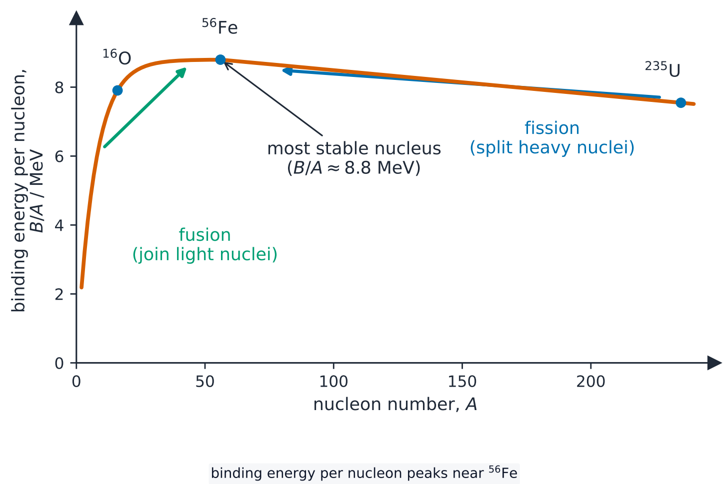
Binding energy per nucleon vs nucleon number

A graph of B/A against A has a typical shape:

- for light nuclei ($A < 20$), B/A rises quickly (with a spike at the very stable ^4_2He).
- around $A \sim 56$ (iron), B/A reaches its **maximum** of about 8.8 MeV. **Iron-56 is the most stable nucleus.**
- for heavy nuclei ($A > 100$), B/A falls slowly, to about 7.5 MeV for uranium.

So the curve is dome-shaped, rising to iron then falling.

Sketching the curve. The exam gives blank axes (A from 1 to 250, B/A up to about 9 MeV) and marks: a steep rise from near zero at $A = 1$, a **maximum** near $A = 56$ at about 8.8 MeV, then a **slow, gentle fall** to about 7.5 MeV at $A = 238$. Do not start the curve at the origin exactly (hydrogen-1 has no binding energy but is a single point), do not make the fall as steep as the rise, and do not let the curve reach zero on the right. Asked to mark a nucleus that undergoes **alpha decay**, put X on the far right, $A > 200$; a nucleus that undergoes **fusion** goes at the far left, $A < 10$; both are at low B/A, moving up the curve when they react.

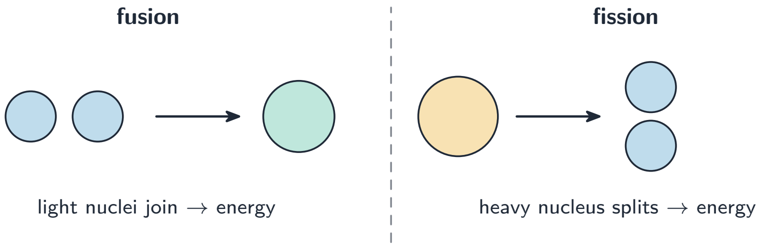


Binding energy per nucleon peaks near iron ($A \approx 56$); lighter and heavier nuclei are less tightly bound

Nuclear fusion and fission

A nuclear power station releases energy by nuclear fission.

Energy is **released** when nuclei move **towards** the iron peak — by joining light nuclei or splitting heavy ones.



Fusion joins light nuclei; fission splits a heavy nucleus — both release energy by moving towards the iron peak

Nuclear fusion

Nuclear fusion 核聚变 joins two light nuclei into one heavier nucleus:

$$ {}^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n} + \text{energy}. $$

The product has greater binding energy per nucleon than the reactants, so energy is released. Fusion powers stars. It needs very high temperatures (millions of kelvin) so the nuclei have enough **kinetic energy** 动能 to beat their **electrostatic** 静电 repulsion and get close enough for the **strong nuclear force** 强核力 to take over.

Nuclear fission

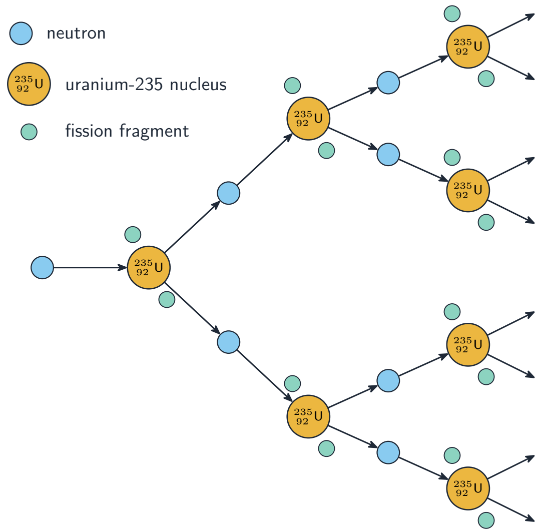
Nuclear fission 核裂变 splits a heavy nucleus into two lighter ones:

$$ {}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\,{}^{1}_{0}\mathrm{n} + \text{energy}. $$

The products have higher binding energy per nucleon than ${}^{235}\mathrm{U}$, so energy is released. The extra neutrons can cause more fissions — a **chain reaction** 链式反应 in a large enough mass of fuel (the **critical mass** 临界质量). This is the basis of nuclear power and weapons.

"Describe the differences between fission and fusion." Fission: a **heavy** nucleus (large A) splits into two **lighter** nuclei of roughly similar mass, usually after absorbing a neutron, releasing further neutrons. Fusion: two **light** nuclei (small A) join to form one heavier nucleus; it needs very high temperature and pressure to overcome the electrostatic repulsion between the nuclei. Both release energy, but per kilogram of fuel fusion releases more.

"Explain, with reference to the curve, why energy is released." In both processes the products lie **higher** on the binding-energy-per-nucleon curve than the reactants: each nucleon ends up more tightly bound, so the total binding energy **increases**. The extra binding energy is released (as the kinetic energy of the products and as photons), and the total mass of the products is **less** than that of the reactants by $\Delta E/c^2$. A nucleus near the peak (iron) can release energy by neither process, which is why the stars' fusion stops at iron.



In an uncontrolled chain reaction each fission of uranium-235 releases neutrons that cause more fissions

Calculating the energy released

1. find the total mass of the reactants.
2. find the total mass of the products.
3. mass change $\Delta m = m_{\text{reactants}} - m_{\text{products}}$ (positive when energy is released).
4. energy released $\Delta E = c^2 \Delta m$.

In kg this gives joules; in atomic mass units use ΔE (MeV) $= \Delta m$ (u) $\times 931$.

Worked example. In a nuclear reaction the total mass decreases by 0.020 u. Find the energy released.

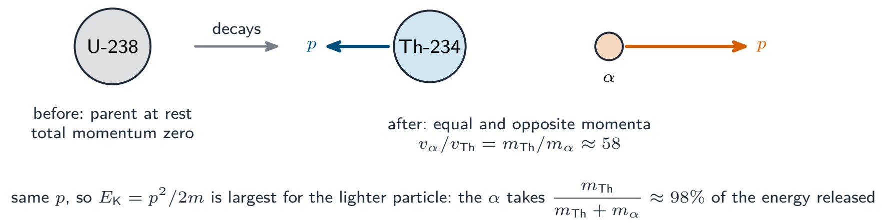
$$\Delta E = \Delta m\ (\text{u}) \times 931 = 0.020 \times 931 \approx 19\ \text{MeV}.$$

Worked example (fusion). The mass defect of deuterium $^2_1$H is 0.002 388 u and that of helium-4 is 0.030 377 u. Find the energy released when two deuterium nuclei fuse to form one helium-4 nucleus.

Energy released = (binding energy of the products) − (binding energy of the reactants) $= [0.030\,377 - 2 \times 0.002\,388] \times 931.5 = 0.025\,601 \times 931.5 = 23.8$ MeV (3.82×10^{-12} J). Mass defects can be used directly like this because the number of nucleons is the same on both sides; the difference in the defects **is** the mass converted. Per kilogram of deuterium this is 5.7×10^{14} J, about a million times a chemical fuel.

Worked example (fission). $^{235}_{92}\text{U} + {}^1_0\text{n} \rightarrow {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 3\,{}^1_0\text{n}$. Masses: U-235 235.043 9 u, n 1.008 665 u, Ba-141 140.914 4 u, Kr-92 91.926 2 u. Find the energy released.

Reactants: $235.043\,9 + 1.008\,665 = 236.052\,6$ u. Products: $140.914\,4 + 91.926\,2 + 3 \times 1.008\,665 = 235.866\,6$ u. $\Delta m = 0.186\,0$ u, so $\Delta E = 0.186\,0 \times 931.5 = 173$ MeV $= 2.8 \times 10^{-11}$ J per fission. Count the three neutrons on the right and the one on the left: forgetting one changes the answer by a whole nucleon mass.



Alpha decay from rest: equal and opposite momenta, so the light alpha particle carries about 98% of the energy released and the heavy nucleus barely recoils

Worked example (alpha decay and momentum). A stationary $^{238}_{92}$U nucleus decays to $^{234}_{90}$Th by emitting an α-particle; the total kinetic energy released is 4.27 MeV. Find the kinetic energy of the α-particle.

Momentum is conserved and the parent was at rest, so the α and the thorium nucleus have **equal and opposite** momenta p. With $E_{\text{K}} = p^2/2m$, the energies are in the inverse ratio of the masses: $E_\alpha/E_{\text{Th}} = m_{\text{Th}}/m_\alpha = 234/4$. So $E_\alpha = 4.27 \times 234/238 = 4.20$ MeV and the thorium **recoil** 反冲 takes only 0.07 MeV. The α-particles from a given decay are all emitted with this one energy, which is the sign that the energy is shared between just two bodies.

Worked example (a radioactive power source). A space probe is powered by 0.874 kg of plutonium-238, half-life 87.7 years, each decay releasing 5.59 MeV. Find the power available at launch.

Number of nuclei: $N = 0.874/(238 \times 1.661 \times 10^{-27}) = 2.21 \times 10^{24}$. Decay constant: $\lambda = 0.693/(87.7 \times 3.156 \times 10^7) = 2.50 \times 10^{-10}\ \text{s}^{-1}$. Activity: $A = \lambda N = 5.53 \times 10^{14}$ Bq. Power $= A \times E = 5.53 \times 10^{14} \times 5.59 \times 1.60 \times 10^{-13} = 490$ W, falling to half after 87.7 years. A nuclide with a **shorter** half-life would give more power per kilogram but would not last the mission; polonium-210 (138 days) would be far more powerful at first and useless within two years.

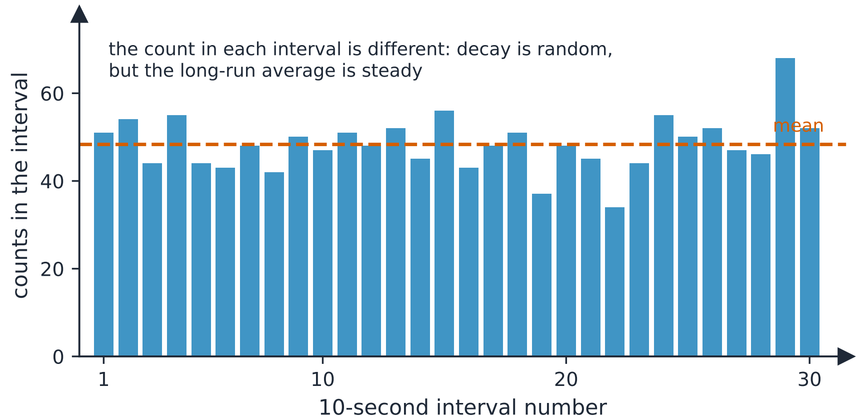
Radioactive decay

Random and spontaneous

Radioactive decay is:

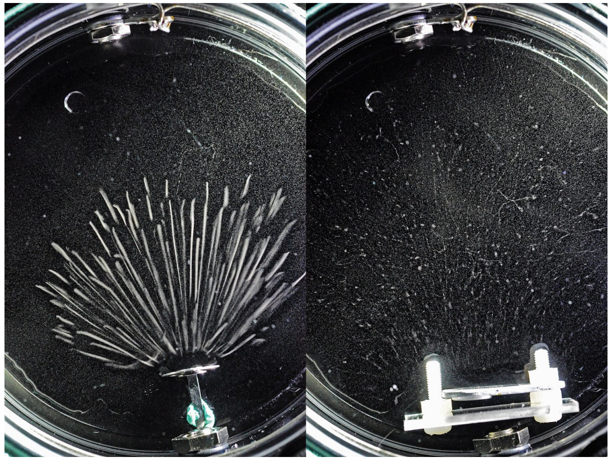
- **spontaneous** 自发 — it happens with no outside trigger, and the rate is not changed by temperature, pressure or chemical state; and
- **random** 随机 — you cannot predict when a given nucleus will decay, only the probability that it decays in a time.

Evidence for randomness: the count rate **fluctuates**. A **Geiger counter** 盖革计数器 next to a source clicks at uneven intervals — never a steady stream — although the long-run mean rate is well-defined.



Counts in equal time intervals from the same source are never the same twice: the fluctuation is the evidence that decay is random, while the steady mean shows the probability is constant

The two-mark definitions. *Radioactive decay is the spontaneous and random emission of a particle (α or β) or a photon (γ) from an unstable nucleus. Spontaneous* means the decay is **not affected by external factors**: temperature, pressure, chemical state, or the presence of other nuclei. *Random* means it is **impossible to predict which** nucleus will decay next, or **when** a given nucleus will decay; only a probability can be given. Evidence for randomness: the **count rate fluctuates** from one interval to the next, even for a source whose activity is not changing over the experiment.



Each beta particle from the source leaves a thin track in a cloud chamber – direct evidence of separate, random decays

Activity and decay constant

For N undecayed nuclei of a **radionuclide** 放射性核素, the rate of decay is

$$A = \lambda N.$$

- A is the **activity** 活度 — decays per unit time. Unit: **becquerel** 贝克勒尔 (Bq) $= \text{s}^{-1}$.
- λ is the **decay constant** 衰变常数 — the probability per unit time that a nucleus decays. Unit: s^{-1}.

λ is fixed for a nuclide; a larger sample (larger N) has proportionally larger activity.

The one-mark definitions. *Activity is the number of decays (of nuclei) per unit time*, or the rate of decay. *The decay constant is the probability per unit time that a (given) nucleus will decay.* Not "the rate of decay": that is the activity. Note the units are the same, s^{-1}, but A counts events and λ is a probability per second.

Worked example. Fluorine-18 has a half-life of 110 minutes. Show that its decay constant is $1.05 \times 10^{-4}\ \text{s}^{-1}$, and find the activity of 2.1×10^{-12} kg of fluorine-18.

$\lambda = 0.693/(110 \times 60) = 1.05 \times 10^{-4}\ \text{s}^{-1}$. The number of nuclei is the mass divided by the mass of one nucleus: $N = 2.1 \times 10^{-12}/(18 \times 1.661 \times 10^{-27}) = 7.0 \times 10^{13}$. So $A = \lambda N = 1.05 \times 10^{-4} \times 7.0 \times 10^{13} = 7.4 \times 10^{9}$ Bq. A tiny mass gives a huge activity because the half-life is short; the same mass of uranium-238 (half-life 4.5×10^{9} years) would give about 10^{-5} Bq.

Exponential decay

Since λ is the fractional decay rate, $\dfrac{dN}{dt} = -\lambda N$, whose solution is an **exponential decay** 指数衰减:

$$N = N_0 e^{-\lambda t}.$$

Because $A = \lambda N$, the activity (and any **count rate** 计数率 proportional to it) decays the same way:

$$A = A_0 e^{-\lambda t}.$$

Why exponential? For each nucleus, λ is a fixed probability per unit time, independent of the others and of the nucleus's age. So the same **fraction** decays in each time interval, which gives exponential decay.

Writing the explanation (three marks). (1) The decay constant is the **probability per unit time** of decay and is the **same for every nucleus** of the nuclide, whatever its age. (2) So the rate of decay, the activity, is **proportional to the number of undecayed nuclei** present: $A = \lambda N$. (3) A rate of change proportional to the quantity itself gives an **exponential** change; equivalently, the **same fraction** of the remaining nuclei decays in every equal time interval, so the number never reaches zero but halves in every half-life.

Half-life

The **half-life** 半衰期 $t_{1/2}$ is the time for the number of undecayed nuclei (or the activity, or the count rate) to fall to **half**. From $N = N_0 e^{-\lambda t}$ with $N = N_0/2$:

$$\ln 2 = \lambda t_{1/2}, \qquad \lambda = \frac{\ln 2}{t_{1/2}} \approx \frac{0.693}{t_{1/2}}.$$

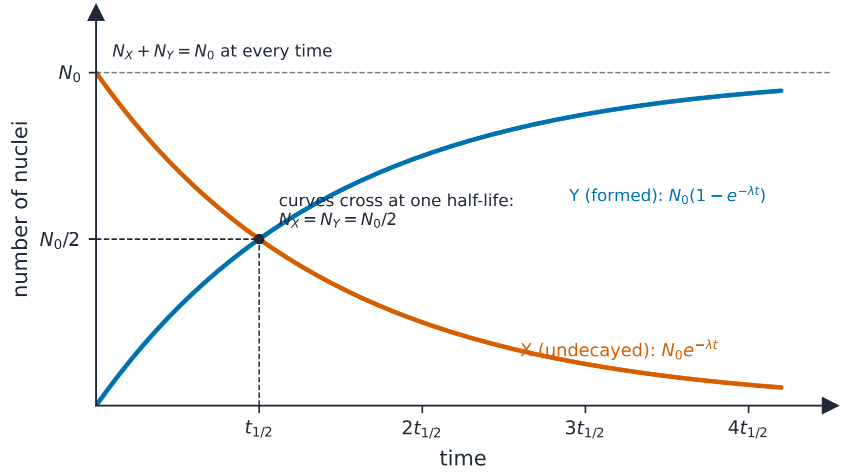
Worked example. A radioactive isotope has a half-life of 6.0 hours. Find its decay constant.

Convert the half-life to seconds: 6.0 h = 21 600 s. Then

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{21\,600} \approx 3.2 \times 10^{-5}\ \mathrm{s}^{-1}.$$

A larger decay constant means a shorter half-life. After n half-lives the surviving fraction is $(1/2)^n$; after 5 half-lives only about 3% remains.

The definition of half-life. *The time taken for the number of undecayed nuclei (or the activity) of a sample to fall to half its initial value.* "Half the atoms decay" is accepted; "half the sample disappears" is not, since the decayed nuclei are still there as the daughter product.

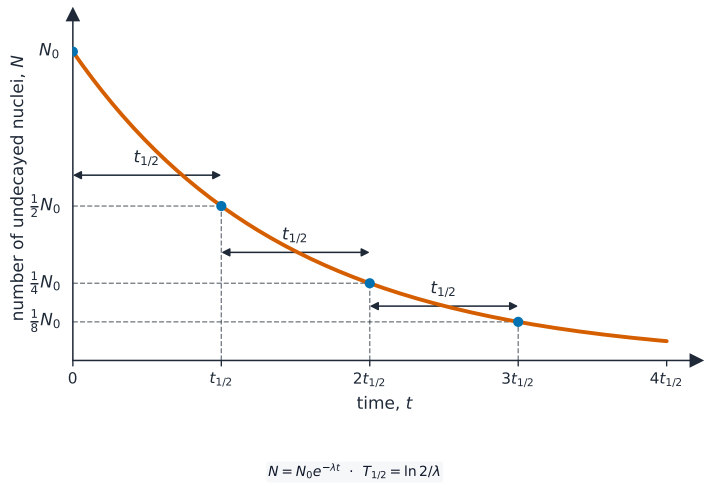


A parent decaying into a stable daughter: what X loses, Y gains, so the curves cross after one half-life and the half-life can be read off the crossing

Reading a two-isotope graph. When X decays to a **stable** Y, $N_{\mathrm{Y}} = N_0 - N_{\mathrm{X}} = N_0(1 - e^{-\lambda t})$. The half-life is the time at which the curves **cross** ($N_{\mathrm{X}} = N_{\mathrm{Y}} = N_0/2$), or the time for N_{X} to halve. If the graph gives the initial number N_0 and the initial mass m, the nucleon number follows from $m = N_0 A u$: for $m = 7.3 \times 10^{-4}$ kg and $N_0 = 2.0 \times 10^{21}$, $A = m/(N_0 u) = 7.3 \times 10^{-4}/(2.0 \times 10^{21} \times 1.661 \times 10^{-27}) = 220$.

Worked example. A sample of a single radioactive isotope has an activity of 180 Bq at $t = 0$ and 45 Bq at $t = 8.4$ minutes. Find the half-life and the decay constant, and the activity after a further 8.4 minutes.

$45/180 = 1/4 = (1/2)^2$: two half-lives in 8.4 minutes, so $t_{1/2} = 4.2$ min $= 252$ s and $\lambda = 0.693/252 = 2.8 \times 10^{-3}\ \mathrm{s}^{-1}$. After another two half-lives the activity is $45/4 = 11$ Bq. When the ratio is not a neat power of two, use $t = \ln(A_0/A)/\lambda$: for the activity to fall from 180 to 50 Bq takes $\ln(3.6)/2.75 \times 10^{-3} = 466$ s. Always convert the half-life to **seconds** before finding λ if the activity is in becquerels.



The number of undecayed nuclei falls by half in each half-life

Finding λ from data

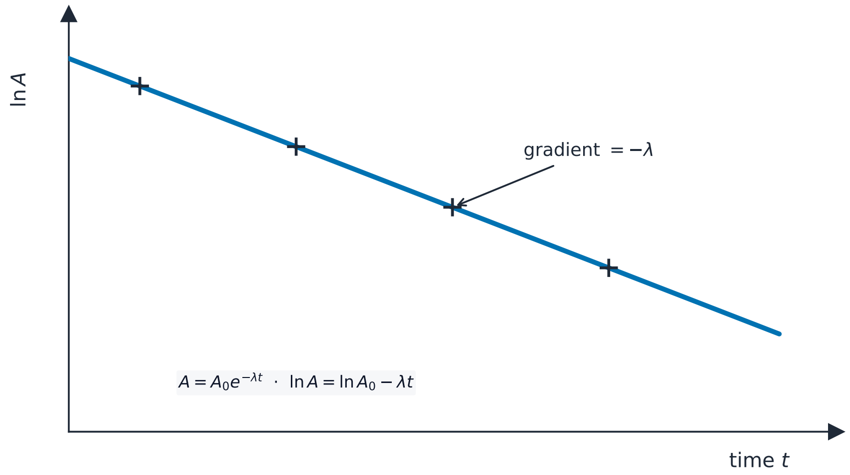
Given A_0 and A at time t:

$$\lambda = \frac{1}{t}\ln\frac{A_0}{A}, \qquad t_{1/2} = \frac{\ln 2}{\lambda}.$$

Taking logs of $A = A_0 e^{-\lambda t}$ gives $\ln A = \ln A_0 - \lambda t$, so a plot of $\ln A$ against t is a straight line with gradient $-\lambda$. Use this with several data points.

Count rate is not activity. A detector records only the radiation that reaches it and is absorbed in it: a fraction set by the solid angle it covers, by absorption in the air and in the source itself, and by its efficiency. So the measured count rate is **smaller** than the activity, but **proportional** to it, and the half-life obtained from a count-rate graph is correct. Subtract the **background radiation** 本底辐射 (measured with the source removed) from every reading before taking ratios or logarithms; an unsubtracted background makes the curve flatten and the half-life appear too long.

Tracers in medicine. Fluorine-18 and oxygen-15 are β^+ emitters used as **tracers** 示踪剂 in PET scanning (Topic 24): the positron annihilates with an electron, giving two gamma photons that leave in opposite directions and reveal where the tracer is. A **short** half-life (110 minutes, 2 minutes) is chosen so that the activity falls quickly after the scan and the dose to the patient stays small, at the price of having to make the isotope close to the hospital and use it at once.



A graph of $\ln A$ against time is a straight line of gradient $-\lambda$

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
mass defect	the difference between the total mass of the separate nucleons and the mass of the nucleus
binding energy	the minimum energy needed to separate a nucleus into its individual nucleons (the energy released when it forms from them)
nuclear fusion	two light nuclei combine to form a single heavier nucleus
nuclear fission	a heavy nucleus splits into two lighter nuclei of similar mass (usually after absorbing a neutron)
radioactive decay	the spontaneous and random emission of α, β or γ radiation from an unstable nucleus
spontaneous	not affected by external factors such as temperature, pressure or chemical state
random	it cannot be predicted which nucleus will decay, or when a given nucleus will decay
activity	the number of nuclei decaying per unit time
decay constant	the probability per unit time that a nucleus decays

Term	Definition
half-life	the time for the number of undecayed nuclei (or the activity) to fall to half its initial value

Exam tips

- $E = mc^2$ with 1 u = 931.5 MeV: work in u and MeV for reactions, then convert to joules only if asked (1 MeV = 1.60×10^{-13} J).
- Energy released = (total mass before − total mass after) $\times c^2$, or (binding energy after − binding energy before). Mass defects can be subtracted directly when the nucleon count is unchanged.
- The curve: steep rise, peak 8.8 MeV near $A = 56$, gentle fall. Products higher on the curve means energy released; fusion on the left, fission (and α-decay) on the right.
- Balance every equation twice: nucleon numbers along the top, proton numbers along the bottom; β decays carry a neutrino or antineutrino.
- $A = \lambda N$, $\lambda = 0.693/t_{1/2}$ with $t_{1/2}$ in **seconds**; N from mass: $N = m/(Au)$. Ratios of 1/2, 1/4, 1/8 mean whole half-lives; otherwise $t = \ln(x_0/x)/\lambda$.
- Explanations are marked on the words: constant probability, rate proportional to number, same fraction per interval; fluctuating count rate for randomness; unaffected by external conditions for spontaneous.

Common mistakes

- Defining binding energy as "the energy holding the nucleus together" or "the energy stored in the nucleus"; it is the energy to **separate** the nucleons.
- Subtracting masses the wrong way round and reporting a negative energy release, or forgetting the neutron(s) on one side of a fission equation.
- Drawing the binding-energy curve falling as steeply as it rises, or reaching zero at large A.
- Using a half-life in minutes or years with an activity in becquerels; convert to seconds first.
- Confusing the decay constant (a probability per second) with the activity (decays per second).
- Saying "half the sample disappears" for a half-life, or that after two half-lives nothing is left.
- Giving "random" as "it happens at any time" without "cannot predict which nucleus or when", or "spontaneous" without "unaffected by external factors".
- Treating the count rate as the activity, or forgetting to subtract background.

- Assuming the α-particle and the recoil nucleus share the energy equally; they share the **momentum** equally.