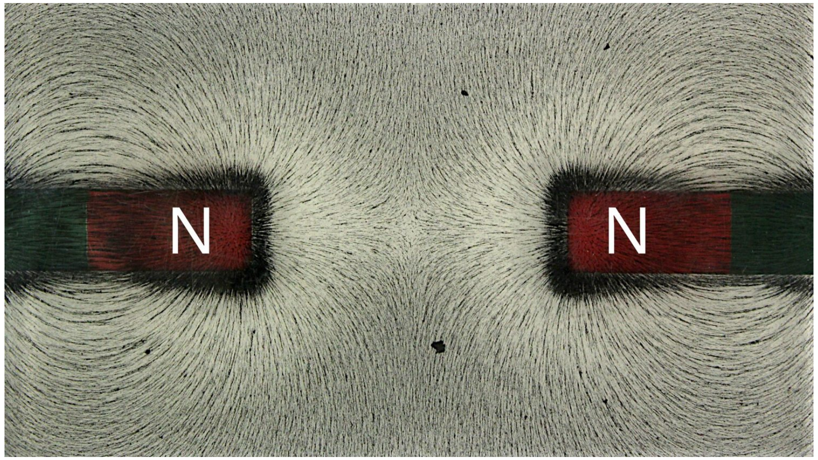
Magnetic fields

A-Level Physics

The magnetic field



Iron filings trace the field lines around bar magnets.

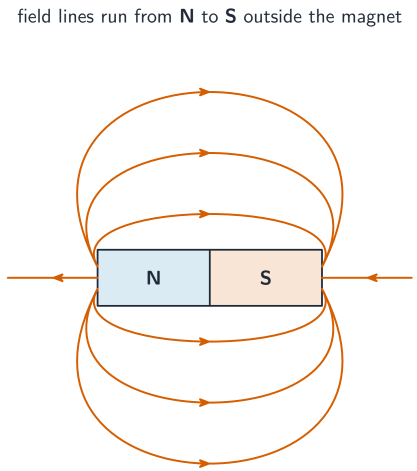
A **magnetic field** 磁场 is a region where a moving charge (or a **current** 电流) feels a **force** 力. It is made by:

- **moving charges** (usually a current in a wire), or
- **permanent magnets** 永磁体 (where it comes from tiny atomic currents).

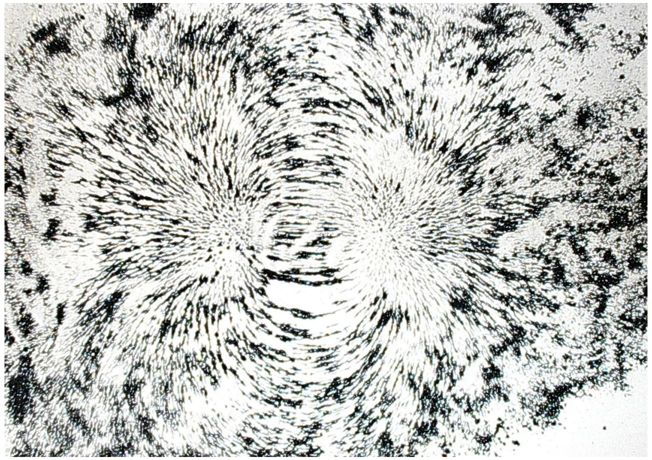
"State what is meant by a magnetic field." *A region in which a force acts on a moving charge, a current-carrying conductor or a magnetic material.* The mark scheme wants the word **force** and one of those three objects; "a region around a magnet" on its own scores nothing.

Field lines

- **field lines** 场线 point from **N** to **S** outside a magnet, and S to N inside (so they form closed loops).
- lines never cross; closer lines mean a stronger field.



The field of a bar magnet: lines run from N to S, strongest near the poles



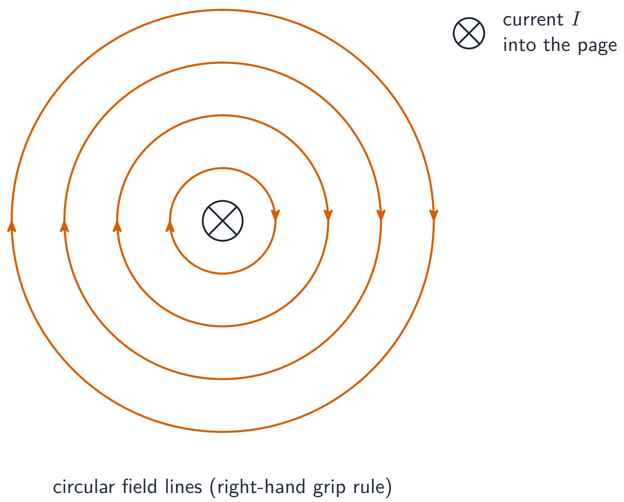
Iron filings around a bar magnet line up along the field, showing its real shape

Patterns to know:

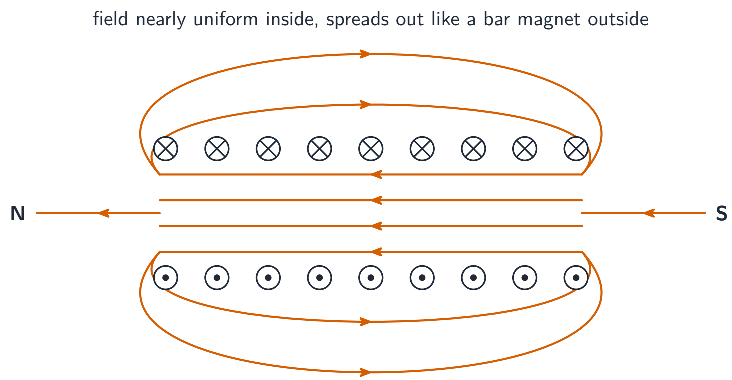
- **bar magnet** 条形磁铁 — curved lines from N to S outside, strongest near the poles.
- **long straight wire** — circles around the wire; the direction comes from the **right-hand grip rule** 右手定则 (thumb along the current, fingers curl the way the field

points).

- **flat circular coil** — the field through the centre is at right angles to the coil; the coil acts like a small bar magnet.
- **long solenoid** 螺线管 — the field inside is nearly **uniform** along the axis, like a stretched bar magnet; outside it falls off fast.

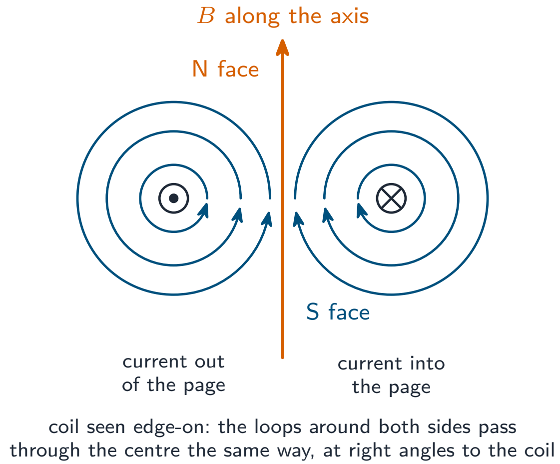


Field around a long straight wire



Field of a solenoid

Drawing the wire's field. "Draw four field lines" around a straight wire is marked on three things: the lines are **concentric circles** centred on the wire; their spacing **increases** with distance (the field weakens with distance from the wire); and every line carries an **arrow** in the direction given by the right-hand grip rule (current **into** the page: **clockwise**; current out of the page: anticlockwise). Draw them as circles, not free-hand ovals.

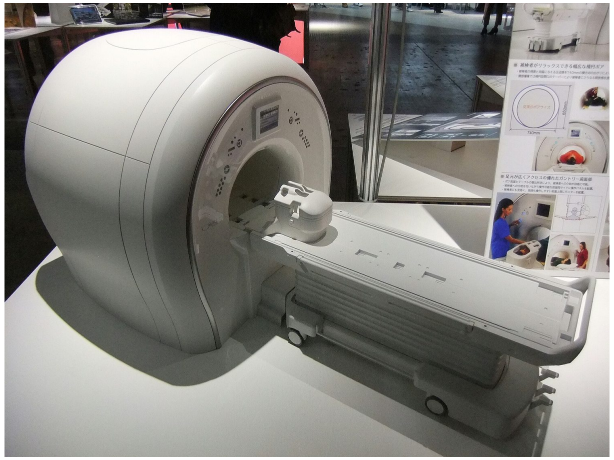


A flat circular coil seen edge-on: the field loops around each side of the coil and passes through the centre at right angles to the coil's plane

For a **flat circular coil** the loops around the two sides of the coil reinforce through the centre, so the field there is at right angles to the plane of the coil and strongest at the centre; seen from one face the coil is a north pole (the field leaves it), from the other a south pole. A long **solenoid** is many such coils in a row: the field inside is uniform and parallel to the axis, and the lines emerge from one end (its N end) and return outside to the other.

An **iron core** 铁芯 inside a solenoid greatly **increases** the field, because the iron's atomic magnets line up and add to it. This is why **electromagnets** 电磁铁 and **transformers** 变压器 have iron cores.

The syllabus calls this a **ferrous** 含铁的 core: iron, steel or another magnetic material. Asked why a core increases the field, say that the core becomes **magnetised** 被磁化 by the solenoid's field and that its own field adds to the field of the current.



An MRI scanner is built around a very strong, very uniform magnetic field — a solenoid the size of a room

Force on a current-carrying conductor

A current I in a wire of length L in a magnetic field of flux density B feels a force

$$F = BIL\sin\theta,$$

where θ is the angle between the wire and the field. The force is largest when the wire is at right angles to the field ($F = BIL$) and zero when the wire is along the field.

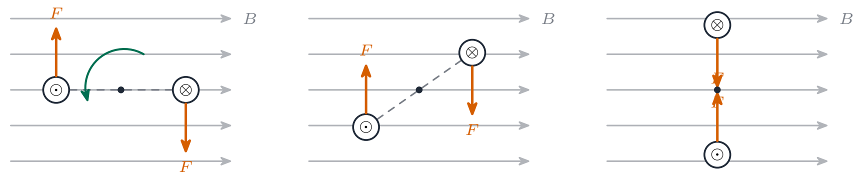
Worked example. A wire of length 0.20 m carries a current of 3.0 A at right angles to a magnetic field of flux density 0.50 T. Find the force on it.

$$F = BIL = 0.50 \times 3.0 \times 0.20 = 0.30 \text{ N}.$$

Two conditions for a force. A copper wire in a magnetic field feels a force only if (1) it carries a **current**, and (2) the current is **not parallel** to the field (some part of the wire is at right angles to B). A wire with no current, or one lying along the field lines, feels nothing: $\sin\theta = 0$.

Worked example (the current balance). A horizontal wire of length 5.0 cm lies at right angles to the field between the poles of a magnet standing on a top-pan balance. With no current the balance reads 102.30 g; with a current of 2.5 A it reads 102.86 g. Find the flux density.

By Newton's third law the force on the wire is matched by an equal and opposite force on the magnet, so the balance reading changes by the magnetic force: $F = \Delta m\, g = (0.56 \times 10^{-3})(9.81) = 5.5 \times 10^{-3}$ N. Then $B = F/(IL) = 5.5 \times 10^{-3}/(2.5 \times 0.050) = 4.4 \times 10^{-2}$ T (44 mT). The reading **rose**, so the force on the magnet is downwards and the force on the wire is upwards; reversing the current would make the reading fall by the same amount.



(a) plane parallel to the field: the couple (green) turns the coil most

(b) tilted: same forces, smaller separation

(c) plane at right angles: forces in line, no turning

A rectangular coil in a uniform field, seen end-on: the forces on the two sides that cross the field form a couple whose turning effect is largest when the coil's plane is parallel to the field and zero when it is perpendicular

Worked example (forces on a coil). A rectangular coil PQRS with sides PS and QR of length 8.0 cm carries a current of 1.2 A in a horizontal field of 0.30 T, with the plane of the coil parallel to the field. Describe the forces on the coil.

Sides PS and QR are at right angles to the field, so each feels $F = BIL = 0.30 \times 1.2 \times 0.080 = 2.9 \times 10^{-2}$ N; the currents in them run in opposite directions, so by Fleming's rule the forces are **equal and opposite**, one up and one down. Sides PQ and SR are **parallel** to the field and feel no force. The two forces form a **couple** 力偶 that turns the coil about its axis; as the coil turns the forces stay the same size but their perpendicular separation shrinks, so the turning effect falls to zero when the plane of the coil is at right angles to the field. This is the principle of the electric motor and of the moving-coil meter.

Magnetic flux density

This equation also **defines** the **magnetic flux density** 磁通密度 B:

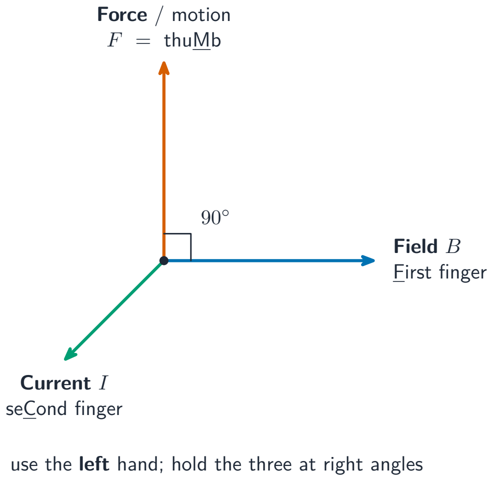
$$B = \frac{F}{IL} \quad \text{(wire at right angles to the field)}.$$

So B is the force per unit current per unit length on a wire at right angles to the field. Unit: **tesla** 特斯拉, $\mathrm{T} = \mathrm{N\,A^{-1}\,m^{-1}}$.

The two-mark definitions. *Magnetic flux density is the force per unit current per unit length acting on a (straight) conductor placed at right angles to the field.* Both "per unit current" and "per unit length" are needed, and so is "at right angles" (or "perpendicular"): without it the definition is false, because the force also depends on the angle. *The tesla is the flux density that produces a force of one newton on one metre of conductor carrying a current of one ampere at right angles to the field*: $1\ \mathrm{T} = 1\ \mathrm{N\ A^{-1}\ m^{-1}}$.

Direction — Fleming's left-hand rule

Use the **left hand** (**Fleming's left-hand rule** 弗莱明左手定则): first finger = **F**ield, second finger = **C**urrent, thumb = force (thrust). Hold the three at right angles.



Fleming's left-hand rule: thumb = force, first finger = field, second finger = current

Force on a moving charge

A charge Q moving at **velocity** 速度 v through a field feels

$$F = BQv \sin\theta,$$

with θ the angle between v and B. Same left-hand rule (the second finger is the motion of a **positive** charge — reverse it for a negative charge). The force is largest when v is at right angles to B, and zero when v is along B.

Two situations with no force. A charged particle in a magnetic field feels no force when it is **stationary** (no v) or when it moves **parallel or antiparallel** to the field ($\sin\theta = 0$). Contrast the electric field, which pushes a stationary charge just as hard: that difference is the whole point of the velocity selector below.

Circular motion in a uniform field

A charge moving at right angles to a **uniform field** 匀强场 feels a force at right angles to both v and B. This force does **no work** (always at right angles to the motion), so the **kinetic energy** 动能 and speed stay constant — the particle moves in a **circle**. Set the magnetic force equal to the **centripetal force** 向心力:

$$BQv = \frac{mv^2}{r}, \qquad r = \frac{mv}{BQ}.$$

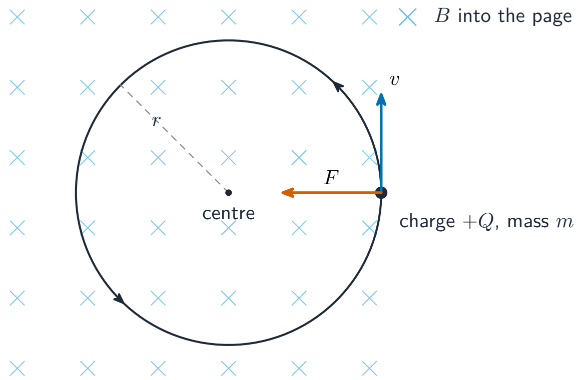
Worked example. A proton (mass 1.7×10^{-27} kg, charge 1.6×10^{-19} C) moves at 2.0×10^{6} m s^{-1} at right angles to a 0.50 T field. Find the radius of its circular path.

$$r = \frac{mv}{BQ} = \frac{(1.7 \times 10^{-27})(2.0 \times 10^{6})}{(0.50)(1.6 \times 10^{-19})} \approx 0.043\ \mathrm{m}.$$

So the radius depends on the **momentum** 动量 mv. The period is

$$T = \frac{2\pi m}{BQ},$$

which does **not** depend on the speed — a faster particle goes in a bigger circle but takes the same time per turn. If v also has a part along B, that part is unchanged, and the path is a **helix** 螺旋.



$$BQv = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{BQ}$$

Circular path of a charged particle in a magnetic field

"Explain why the path is circular." The magnetic force is always at **right angles** to the velocity, so it changes the direction of motion but not the speed, and its magnitude BQv is **constant** because B, Q and v are constant. A force of constant size that stays perpendicular to the velocity is exactly a **centripetal force**, so the path is a circle. Give all three ideas: perpendicular, constant magnitude, speed unchanged.

Worked example. Electrons moving at 1.7×10^7 m s^{-1} enter a uniform field of flux density 4.8 mT at right angles to it. Find the radius of their path. The electrons are then replaced by **positrons** 正电子 (same mass, charge $+e$) moving at 3.4×10^7 m s^{-1} along the same initial line: describe how the path changes.

$$r = \frac{mv}{BQ} = \frac{(9.11 \times 10^{-31})(1.7 \times 10^7)}{(4.8 \times 10^{-3})(1.60 \times 10^{-19})} = 2.0 \times 10^{-2} \text{ m}.$$

The positrons are deflected the **opposite** way (positive charge, so Fleming's rule with the second finger along the motion gives a force in the opposite direction), and since $r \propto v$ their radius is **twice** as large, 4.0 cm. Their period is the same ($T = 2\pi m/BQ$ has no v in it): they go round a bigger circle at twice the speed.

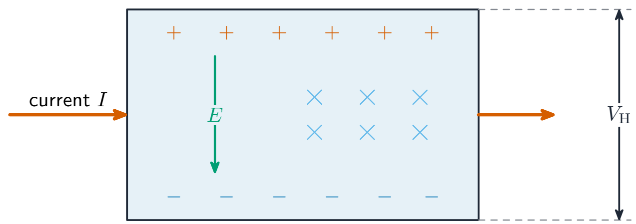
Hall effect

A slab of conductor carrying current I, in a field B at right angles to the current, develops a voltage across its faces — the **Hall voltage** 霍尔电压 V_H (the **Hall effect** 霍尔效应).

The moving charges feel a magnetic force BQv_d (v_d is the **drift velocity** 漂移速度), so they build up on one face, making an **electric field** 电场 E that opposes more build-up. At steady state $eE = Bev_d$, so $E = Bv_d$. With $V_H = Ew$ and $I = nev_d wt$:

$$V_H = \frac{BI}{ntq},$$

where q is the carrier charge. A **Hall probe** 霍尔探头 uses this to measure B: pass a known current through a thin **semiconductor** 半导体 slab and read V_H (largest when the slab is at right angles to B).



The Hall effect

Deriving $V_H = BI/(ntq)$ (the exam asks for this in full). Take a slice of width d, across which the Hall voltage appears, and thickness t along B, so the current passes through a cross-section $A = dt$. (1) At steady state the electric force on a carrier balances the magnetic force: $qE = Bqv$, so $E = Bv$. (2) The Hall field is uniform across the width, so $V_H = Ed = Bvd$. (3) The current is $I = nAvq = n\,dt\,vq$, so $v = I/(n\,dt\,q)$. Substituting: $V_H = Bd \cdot I/(n\,dt\,q) = BI/(ntq)$. Two things to notice: the voltage appears across the faces **perpendicular to the magnetic force**, not across the ends where the current enters; and it is **larger for a semiconductor**, whose number density n is millions of times smaller than a metal's, which is why probes use a semiconductor slice.

Using a Hall probe. The reading is proportional to B but also depends on the **orientation**: it is a maximum when the plane of the slice is at **right angles** to the field and zero when the field lies in the plane of the slice. So (1) rotate the probe until the reading is largest, and (2) **calibrate** 校准 it in a known field, since $V_H \propto B$ at fixed current.

Worked example. A Hall probe gives a maximum reading of 24 mV in a uniform field of 32 mT. In a second field, with the probe again turned for a maximum, the reading is 15 mV. Find the second flux density.

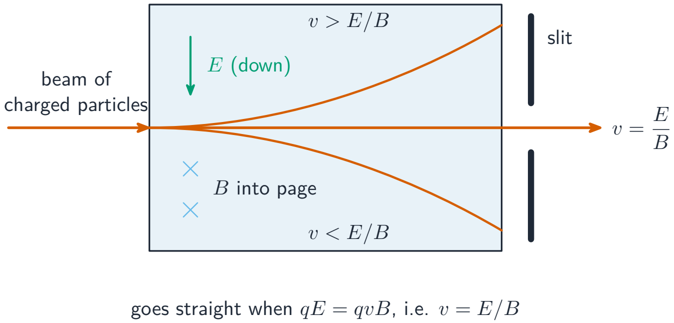
$V_H \propto B$ (same probe, same current): $B = 32 \times 15/24 = 20$ mT. If the probe had been left at some other angle the reading would be smaller and B would be underestimated, which is why "rotate for a maximum" is part of the method.

Velocity selector

A **velocity selector** 速度选择器 uses crossed electric and magnetic fields to let through only one speed. With the electric force qE and magnetic force qvB set to oppose each other, the net force is zero only when

$$qE = qvB \quad \Rightarrow \quad v = \frac{E}{B}.$$

Particles at speed E/B go straight through; faster or slower ones are deflected.

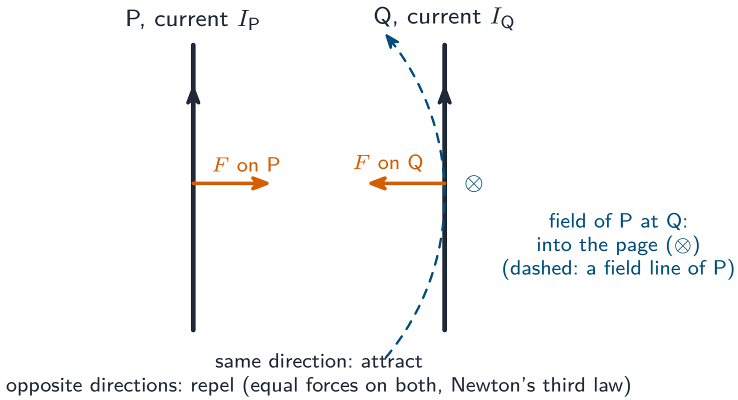


Velocity selector

Deriving and explaining the selector. The magnetic force on a positive ion moving through the crossed fields is BQv, at right angles to its motion; the electric field is applied so that the electric force QE is in the **opposite** direction (the plates must be arranged so that the field points against the magnetic force). For the ion to pass straight through, $QE = BQv$, so $v = E/B$, independent of the charge and the mass. A faster ion has a larger magnetic force and bends towards the magnetic-force side; a slower one is pushed the other way by the unchanged electric force. Asked "explain how an electric field can make the particle reach Z", say which direction the electric force must have, that it must equal the magnetic force in size, and give $E = Bv$. This is **velocity selection** 速度选择.

Force between parallel currents

Two long parallel wires each sit in the other's magnetic field. Using Fleming's left-hand rule: **parallel currents (same direction) attract; antiparallel currents (opposite directions) repel**. This is the basis of the SI definition of the ampere.



Parallel currents attract: each wire sits in the other's field, and Fleming's left-hand rule gives a force towards the other wire

Explaining the force (a standard three-marker). (1) The current in P produces a magnetic field around it (circles centred on P). (2) At Q this field is at right angles to the current in Q, so Q, a current-carrying conductor in a magnetic field, feels a force BIL. (3) Fleming's left-hand rule gives its direction: towards P for currents in the same direction, away for opposite currents. By Newton's third law the force on P is equal and opposite, so both wires attract (or both repel) with the same force. Doubling the current in **one** wire doubles the force on **both**: the force on Q rises because I_Q has doubled, the force on P because the field it sits in has doubled.

Electromagnetic induction

Magnetic flux

The **magnetic flux** 磁通量 Φ through a flat area A at right angles to B is

$$\Phi = BA.$$

If the area's normal is at angle θ to B, use $\Phi = BA\cos\theta$. Unit: **weber** 韦伯, $\mathrm{Wb} = \mathrm{T\ m^2}$. For a **coil** 线圈 of N turns, the **flux linkage** 磁链 is $N\Phi = NBA$.

The two-mark definitions. *Magnetic flux is the product of the magnetic flux density and the area (of the circuit) perpendicular to the field. Flux linkage is the product of the flux and the number of turns of the coil,* $N\Phi$, in weber-turns. Say "perpendicular" (or "normal to the field"): $\Phi = BA$ is only true for the component of the area at right angles to B.

Worked example. A small coil of 64 turns and cross-sectional area 0.71 $\mathrm{cm^2}$ is placed inside a long solenoid, on its axis, where the flux density is 2.5 mT. Find the flux linkage of the coil.

The flux through the coil uses the **coil's own** area, not the solenoid's: $\Phi = BA = (2.5\times10^{-3})(0.71\times10^{-4}) = 1.8\times10^{-7}$ Wb, so the flux linkage is $N\Phi = 64\times1.8\times10^{-7} = 1.1\times10^{-5}$ Wb. When the solenoid's current changes, the e.m.f. induced in the small coil is the rate of change of this linkage.

Faraday's and Lenz's laws

When the flux linkage through a circuit **changes**, an **electromotive force** 电动势 (e.m.f.) is induced — this is **electromagnetic induction** 电磁感应.

Faraday's law 法拉第定律: the induced e.m.f. equals the rate of change of flux linkage:

$$|\varepsilon| = N\frac{d\Phi}{dt}.$$

Worked example. A coil of 200 turns and area 0.010 $\mathrm{m^2}$ sits with its plane at right angles to a 0.50 T field. The field falls steadily to zero in 0.20 s. Find the average induced e.m.f.

The flux linkage changes from $N\Phi = NBA = 200\times0.50\times0.010 = 1.0$ Wb to zero, so

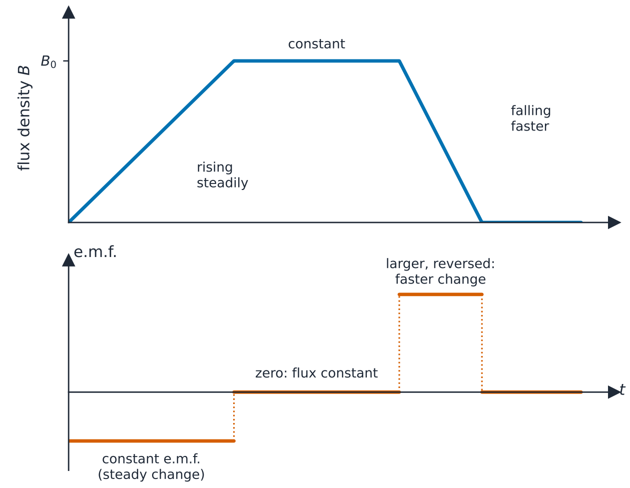
$$|\varepsilon| = \frac{\Delta(N\Phi)}{\Delta t} = \frac{1.0}{0.20} = 5.0\ \mathrm{V}.$$

Lenz's law 楞次定律: the induced e.m.f. acts to **oppose the change** that makes it. This is **conservation of energy** 能量守恒 — if it reinforced the change, energy would come from nothing. Combined:

$$\varepsilon = -\frac{d(N\Phi)}{dt}.$$

What changes the flux?

- **changing** B (moving a magnet near a coil),
- **changing area** A (a rod sliding along rails),
- **changing orientation** (a coil turning in a field — the a.c. generator, next topic).



The induced e.m.f. is minus the gradient of the flux-linkage graph: constant while the flux changes steadily, zero while it is constant, and larger and reversed when the flux falls faster

Reading an e.m.f. off a flux graph. The induced e.m.f. is the **gradient** of the flux-linkage graph (with a minus sign for Lenz). A steadily rising flux gives a **constant** e.m.f.; a constant flux gives **zero** e.m.f. however large the flux is; a faster fall gives a larger e.m.f. of the opposite sign. "Sketch the variation with time of the e.m.f." is answered by differentiating the flux graph by eye, section by section.

Worked example. The 64-turn coil above sits in a solenoid whose flux density rises uniformly from 0 to 2.5 mT in 40 ms and is then held steady. Find the e.m.f. induced in the coil during the rise, and afterwards.

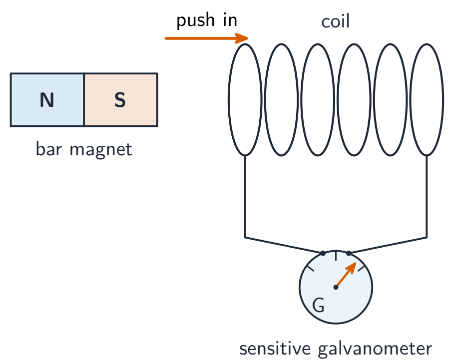
During the rise the flux linkage grows steadily to 1.1×10^{-5} Wb: $\varepsilon = \Delta(N\Phi)/\Delta t = 1.1\times10^{-5}/0.040 = 2.8\times10^{-4}$ V (0.28 mV), constant while the rise lasts. Afterwards the flux linkage is constant, so the e.m.f. is **zero**, even though the coil is still in a strong field: it is the **change** that induces.

Worked example (flux cutting). An aircraft with a **wingspan** 翼展 of 60 m flies horizontally at 250 $\mathrm{m\ s^{-1}}$ where the vertical component of the Earth's field is 45 μT. Find the e.m.f. between the wingtips.

In time Δt the wing sweeps out an area $Lv\Delta t$, cutting flux $BLv\Delta t$, so $\varepsilon = BLv = (45 \times 10^{-6})(60)(250) = 0.68$ V. Only the component of B **perpendicular** to the swept area counts (here the vertical one). There is a p.d. between the wingtips but no current, because there is no complete circuit; Fleming's rules give which tip is positive.

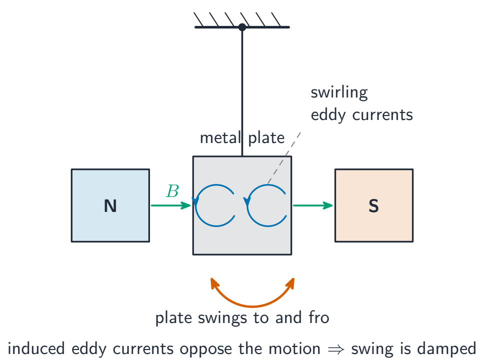
Demonstrations

- moving a bar magnet into a coil deflects a **galvanometer** 检流计; the deflection reverses when the magnet is pulled out (Lenz's law), and is larger for faster motion (Faraday's law).



Demonstrating electromagnetic induction

- a copper disc swinging into a field is quickly slowed — **eddy currents** 涡流 are induced that oppose the motion.



Eddy-current damping

Electromagnetic braking, explained with Lenz's law. A vehicle's aluminium disc rotates between the poles of an electromagnet. The disc **cuts** the field lines, so an e.m.f. is induced in it; the disc is a conductor, so **eddy currents** flow; by Lenz's law these currents produce forces that **oppose the motion** of the disc, slowing the vehicle. The kinetic energy is converted to **thermal energy** in the disc. The braking is stronger at high speed (a larger rate of flux cutting, so larger currents) and fades as the vehicle slows; it cannot hold a stationary vehicle, because no motion means no induced current.

A magnet oscillating in a coil. A bar magnet on a spring oscillates in and out of a coil connected to a resistor. The changing flux linkage induces an e.m.f.; with the circuit complete a current flows and, by Lenz's law, the coil's field opposes the magnet's motion, so the oscillation is **damped**: energy leaves the oscillation as heat in the resistor and the amplitude decays. Open the switch and the e.m.f. is still induced but no current flows, so the coil no longer damps the motion.

What makes the induced e.m.f. larger

From $\varepsilon = N\,d\Phi/dt$ with $\Phi = BA$: more turns N, a stronger B, a larger area A, or a faster change — each gives a larger induced e.m.f.

Finding the direction with Lenz's law. Push the N pole of a magnet towards a coil and the induced current makes the near face of the coil a **north** pole, to repel the approaching magnet; pull it away and the near face becomes a **south** pole, to attract it. In each case the person moving the magnet does work against that force, and that work is the source of the electrical energy: Lenz's law is conservation of energy in disguise. Two sentences to learn: *the induced e.m.f. acts in such a direction as to produce effects that oppose the change producing it* (Lenz), and *the induced e.m.f. is proportional to the rate of change of flux linkage* (Faraday).

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
magnetic field	a region in which a force acts on a moving charge, a current-carrying conductor or a magnetic material
magnetic flux density	the force per unit current per unit length on a straight conductor placed at right angles to the field
tesla	the flux density producing a force of 1 N on 1 m of conductor carrying 1 A at right angles to the field

Term	Definition
magnetic flux	the product of the flux density and the area perpendicular to the field, $\Phi = BA$
flux linkage	the product of the flux through a coil and its number of turns, $N\Phi$
weber	one tesla metre squared
Faraday's law	the induced e.m.f. is proportional to the rate of change of flux linkage
Lenz's law	the induced e.m.f. acts in such a direction as to produce effects that oppose the change producing it
Hall voltage	the p.d. that develops across a current-carrying slice in a magnetic field, $V_H = BI/(ntq)$

Exam tips

- Force on a current $F = BIL\sin\theta$; on a moving charge $F = BQv\sin\theta$; direction from **Fleming's left-hand rule** with the second finger along the **conventional** current (reverse it for an electron).
- A charge moving at right angles to B moves in a circle: $BQv = mv^2/r$, so $r = mv/(BQ)$ and $T = 2\pi m/(BQ)$, independent of speed. A component of v along B is unchanged: a helix.
- Crossed fields: $v = E/B$ passes undeflected. Hall probe: $V_H = BI/(ntq)$, largest with the slice at right angles to B; semiconductor because n is small.
- Induction: e.m.f. = rate of change of **flux linkage** $N\Phi$ (Faraday); its direction opposes the change (Lenz). On a flux–time graph the e.m.f. is the gradient: constant change gives a constant e.m.f., no change gives zero.
- Flux cutting: $\varepsilon = BLv$ for a conductor of length L moving at v at right angles to B (area swept per second times B).
- Definitions carry the words "per unit current per unit length", "at right angles", "area perpendicular to the field" and "rate of change of flux linkage": each is a mark.

Common mistakes

- Leaving "at right angles to the field" out of the definition of flux density, or defining the tesla without the 1 N, 1 m, 1 A.
- Using the right hand for the force on a current (the left-hand rule), or pointing the second finger along the electron's motion instead of the conventional current.
- Saying a stationary charge feels a magnetic force, or that a charge moving along the field lines does.

- Explaining circular motion without saying the force is perpendicular to the velocity and constant in size; or claiming the magnetic force changes the speed.
- Using the solenoid's area instead of the small coil's when finding its flux linkage.
- Saying "a large flux induces an e.m.f.": only a **changing** flux linkage does. A coil at rest in a steady field has no e.m.f.
- Quoting Lenz's law as "the current opposes the magnet" without "the change producing it", or Faraday's law without "rate of change" or without "flux linkage".
- Drawing straight-wire field lines as evenly spaced circles or without arrows; drawing a solenoid's field lines crossing inside it.
- Forgetting that doubling the current in one of two parallel wires doubles the force on both (Newton's third law).