Electric fields

A-Level Physics

Electric fields

Lightning is a giant spark driven by a huge electric field.

An **electric field** 电场 is a region where a charge feels a **force** 力 from other charges. The **electric field strength** 电场强度 E at a point is the **force per unit positive charge** on a small positive **test charge** 检验电荷 placed there:

$$E = \frac{F}{q}.$$

Unit: N C^{-1} (the same as V m^{-1}, as we will see). E is a **vector** 矢量, pointing the way the force acts on a **positive** charge. The force on a charge q is

$$F = qE,$$

opposite to the field if q is negative.

The two-mark definition. *Electric field strength at a point is the force per unit positive charge acting on a small (test) charge placed at that point.* Three words carry the marks: **force per unit charge** (not "force on a charge"), **positive** (which fixes the direction), and **small** or **test** (so the charge does not disturb the field it is measuring). The multiple-choice version offers "force per unit charge acting on a small

mass" and "force per unit mass" as distractors: a field is defined by a **charge**, never by a mass.

Worked example. A proton accelerates at 2.00 m s^{-2} in an electric field, with no other force acting. Find the field strength. ($m_p = 1.67 \times 10^{-27}$ kg, $e = 1.60 \times 10^{-19}$ C.)

$qE = ma$, so

$$E = \frac{ma}{q} = \frac{(1.67 \times 10^{-27})(2.00)}{1.60 \times 10^{-19}} = 2.09 \times 10^{-8} \text{ V m}^{-1} = 20.9 \text{ nV m}^{-1}.$$

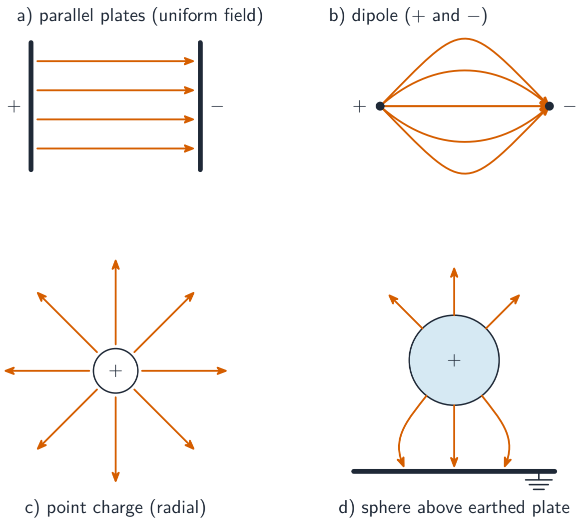
The same rearrangement the other way round gives the acceleration of an electron in a field of 1500 V m^{-1}: $a = eE/m_e = (1.60 \times 10^{-19})(1500)/(9.11 \times 10^{-31}) = 2.6 \times 10^{14}$ m s^{-2}, enormous because the electron's mass is so small. Gravity (9.81 m s^{-2}) is negligible beside it, which is why exam questions about electrons between plates ignore weight.

Field lines

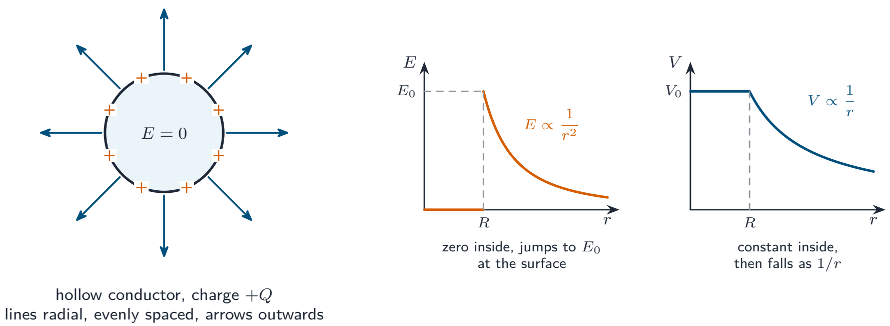
- **field lines** 场线 point the way the force acts on a positive test charge.
- lines start on **positive** charges and end on **negative** charges (or go to infinity).
- lines never cross; closer lines mean a stronger field.
- every line carries an **arrow**; lines meet the surface of a conductor at right angles.

What a field line represents. Asked "state what is represented by an electric field line", give both halves: its **direction** shows the direction of the force on a **positive** charge placed there, and the **spacing** of the lines shows the strength of the field (closer lines, stronger field). A sketch is marked on exactly these points: arrows drawn, lines not crossing, even spacing where the field is uniform, and radial lines for a point charge or a sphere.

Examples: a positive **point charge** 点电荷 has radial lines pointing out; a negative one has lines pointing in; two opposite charges (a **dipole** 偶极子) have lines curving from + to −; two parallel charged plates give a **uniform field** 匀强场 of equally spaced parallel lines.



Field-line patterns for parallel plates, a dipole, a point charge, and a charged sphere above an earthed plate



A charged hollow conductor: the field outside is that of a point charge at the centre; inside, the field is zero and the potential is constant

A charged conducting sphere

The charge on an **isolated 孤立的 conductor 导体** sits on its **outer surface**, spread evenly when the sphere is on its own. Outside the sphere the field lines are **radial**, evenly spaced and pointing outwards (for positive charge), exactly the pattern of a point charge at the centre. **Inside** a hollow (or solid) conductor the field is **zero**: the

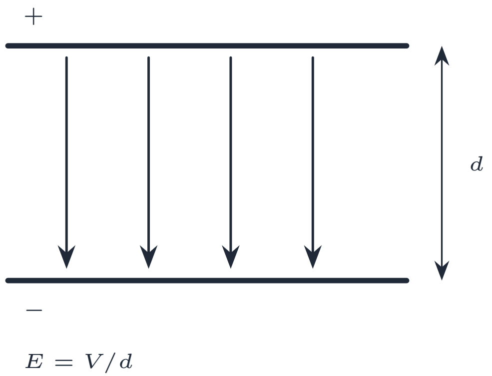
fields of all the surface charges cancel everywhere inside, so no field line enters. Two graphs follow, and both are exam favourites. The field strength E is zero out to the radius R, **jumps** to its surface value E_0 and then falls as $1/r^2$; the potential V is **constant** inside (a zero field means a zero potential gradient) and then falls as $1/r$.

Reading the graphs. Given an E–x graph for a charged sphere, the **radius** is the distance at which E jumps from zero, and the **charge** comes from any point on the curve: $Q = 4\pi\varepsilon_0 r^2 E$. Given "$E_0$ at the surface", the value at $2R$ is $E_0/4$ and at $3R$ is $E_0/9$: sketch the curve through those points, starting at E_0 on the surface line, never from the origin.

A Van de Graaff generator stores a large static charge on its metal dome, making a strong electric field around it

Uniform electric fields

Between two parallel plates a distance d apart with **potential difference 电势差** V between them, the field is uniform (apart from edge effects) with size



Between parallel plates the field is uniform, $E = V/d$

$$E = \frac{V}{d}.$$

It points from the higher-potential plate to the lower one. The unit V m^{-1} comes straight from this and equals N C^{-1}.

Worked example. Two parallel plates 5.0 mm apart have a p.d. of 200 V between them. Find the field strength, and the force on an electron in the gap. ($e = 1.6 \times 10^{-19}$ C.)

$$E = \frac{V}{d} = \frac{200}{5.0 \times 10^{-3}} = 4.0 \times 10^4 \text{ V m}^{-1}, \qquad F = qE = (1.6 \times 10^{-19})(4.0 \times 10^4) = 6.4 \times 10^{-15} \text{ N}.$$

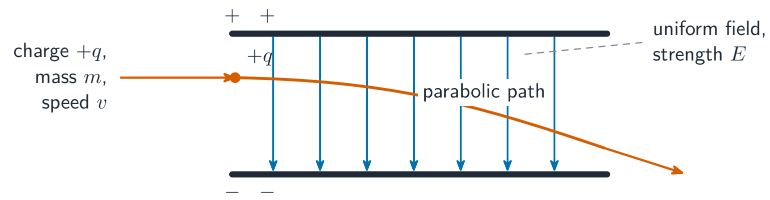
Which changes the field? Two things only: $E = V/d$ rises if the p.d. is **increased** or the plates are moved **closer**. A resistor in series with the supply changes nothing: no current flows once the plates are charged, so there is no p.d. across the resistor and the plates sit at the full supply voltage. Between plates at +800 V and +1300 V the field points from the **higher** potential to the lower; two positive potentials change nothing about the direction rule.

A charged particle in a uniform field

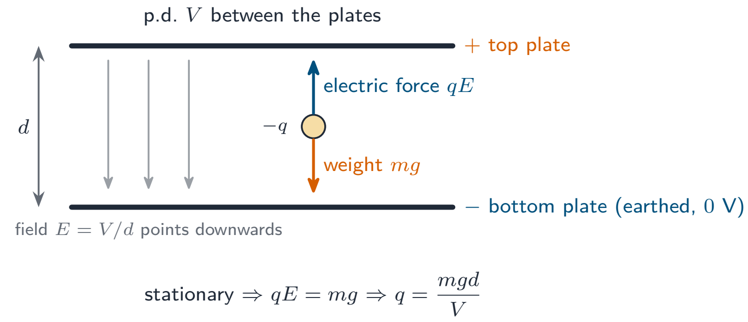
A charge q in a uniform field feels a **constant force** $F = qE$, so a **constant acceleration** $a = qE/m$ — just like a mass in a uniform gravitational field.

- released **at rest**, it speeds up along the field (positive charge) or against it (negative charge), gaining **kinetic energy** 动能.

- entering **at right angles** to the field, it follows a **parabolic** 抛物线 path — like a **projectile** 抛体 in gravity. This is how a **cathode-ray tube** 阴极射线管 used to steer its beam.



A charge entering a uniform field at right angles follows a parabolic path, like a projectile



A charged oil drop held stationary between horizontal plates: the electric force balances the weight, so $q = mgd/V$

Two forces, not one. A charged oil drop "held stationary" between horizontal plates is the exam's favourite **equilibrium** 平衡: **both** an electric force and its weight act on it (the multiple-choice distractor is "electric force only"), and they are equal and opposite. For a **negative** drop the electric force is opposite to the field, so the top plate must be **positive** to hold it up.

Worked example. An oil drop of mass 2.6×10^{-15} kg carries a charge of -4.8×10^{-19} C and is held stationary in a vacuum between horizontal plates 2.0 cm apart. Find the p.d. between the plates, and say which plate is positive.

$$qE = mg \;\Rightarrow\; \frac{qV}{d} = mg \;\Rightarrow\; V = \frac{mgd}{q} = \frac{(2.6 \times 10^{-15})(9.81)(0.020)}{4.8 \times 10^{-19}} = 1.1 \times 10^3 \text{ V}.$$

The drop is negative, so the force on it is against the field; for the force to be upwards the field must point **down**, so the **top** plate is positive. Notice that the charge is $3e$, three excess electrons, which is how Millikan showed that charge comes in multiples of e.

Worked example. Two parallel plates in a vacuum are 0.041 m apart with a p.d. of 250 V between them. An electron is released from rest at the negative plate. Find (a) the field strength, (b) the force on the electron and its acceleration, (c) the time it takes to reach the positive plate, (d) its kinetic energy on arrival. ($m_e = 9.11 \times 10^{-31}$ kg.)

(a) $E = V/d = 250/0.041 = 6.1\times10^3$ V m^{-1}. (b) $F = eE = (1.60\times10^{-19})(6.1\times10^3) = 9.8\times10^{-16}$ N; $a = F/m_e = 1.07\times10^{15}$ m s^{-2}, towards the positive plate. (c) From rest with constant acceleration, $d = \frac{1}{2}at^2$, so $t = \sqrt{2d/a} = \sqrt{2 \times 0.041/(1.07 \times 10^{15})} = 8.8 \times 10^{-9}$ s. (d) Energy method, no kinematics needed: $E_k = qV = (1.60 \times 10^{-19})(250) = 4.0 \times 10^{-17}$ J, so $v = \sqrt{2E_k/m_e} = 9.4 \times 10^6$ m s^{-1}. (Check: $v = at = (1.07 \times 10^{15})(8.8 \times 10^{-9}) = 9.4 \times 10^6$ m s^{-1}.)

Worked example (deflection). An electron travelling horizontally at 2.0×10^7 m s^{-1} enters the field between two horizontal plates 18 mm apart with a p.d. of 400 V. The plates are 30 mm long. Find the **deflection** 偏转 of the electron by the time it leaves the field.

The horizontal motion is unaffected: time in the field $t = 0.030/(2.0 \times 10^7) = 1.5 \times 10^{-9}$ s. Vertically, $a = eE/m_e = eV/(m_e d) = (1.60 \times 10^{-19})(400)/[(9.11 \times 10^{-31})(0.018)] = 3.9 \times 10^{15}$ m s^{-2}, so the deflection is $y = \frac{1}{2}at^2 = \frac{1}{2}(3.9 \times 10^{15})(1.5 \times 10^{-9})^2 = 4.4 \times 10^{-3}$ m, less than the half-gap of 9 mm, so the electron does leave the field. Its vertical velocity on exit is $at = 5.9\times10^6$ m s^{-1}, so it leaves at $\tan\theta = 5.9/20$, about 16°; the path is a parabola, and **after** the plates (no field) it travels in a straight line in the direction it had on exit.

Coulomb's law

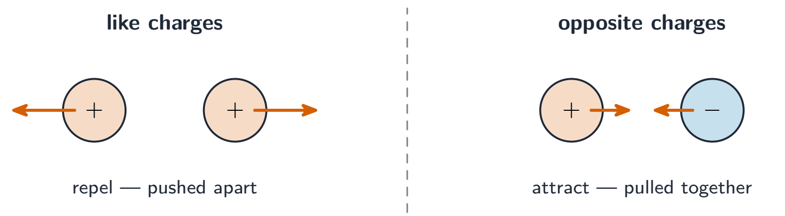
For two point charges Q_1 and Q_2 a distance r apart in free space, each feels a force of size

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}.$$

This is **Coulomb's law** 库仑定律. Here $\varepsilon_0 = 8.85\times10^{-12}$ F m^{-1} is the **permittivity of free space** 真空电容率, and $1/(4\pi\varepsilon_0) \approx 9.0\times10^9$ N m^2 C^{-2}. The force is along the line joining the charges: **repulsive** for like charges, **attractive** for opposite charges.

Stating the law in words. "State Coulomb's law" wants a sentence, not a formula: *the (electric) force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of their separation.* Both

proportionalities are needed, and the charges must be **point** charges (or spheres treated as point charges at their centres) in a vacuum.



Like charges repel; opposite charges attract — the force is along the line joining them

Worked example. Find the electrostatic force between point charges of +2.0 nC and +3.0 nC placed 4.0 cm apart. ($1/(4\pi\varepsilon_0) = 9.0 \times 10^9$ N m^2 C^{-2}.)

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2} = \frac{(9.0 \times 10^9)(2.0 \times 10^{-9})(3.0 \times 10^{-9})}{(0.040)^2} \approx 3.4 \times 10^{-5} \text{ N (repulsive).}$$

Worked example. In a hydrogen atom the proton and the electron may be treated as point charges 120 pm apart. Find the electric force between them, and compare it with their gravitational attraction. ($m_p = 1.67 \times 10^{-27}$ kg, $m_e = 9.11 \times 10^{-31}$ kg, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.)

$$F_E = \frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{(8.99 \times 10^9)(1.60 \times 10^{-19})^2}{(1.20 \times 10^{-10})^2} = 1.6 \times 10^{-8} \text{ N (attractive).}$$

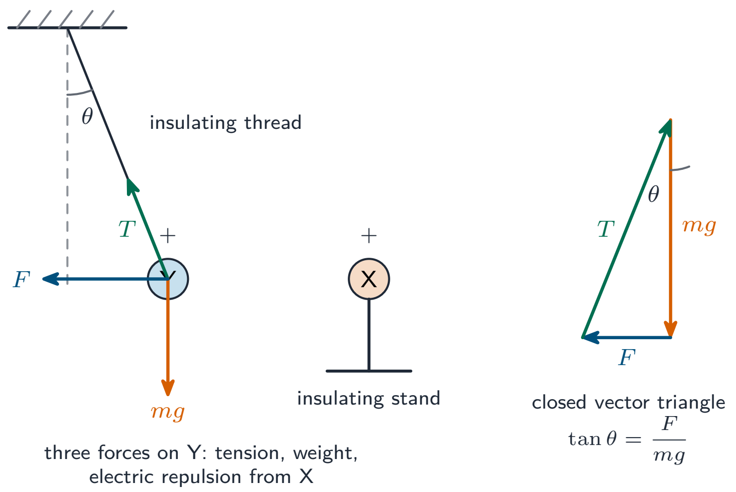
Gravity: $F_G = Gm_pm_e/r^2 = (6.67 \times 10^{-11})(1.67 \times 10^{-27})(9.11 \times 10^{-31})/(1.20 \times 10^{-10})^2 = 7.0 \times 10^{-48}$ N, about 10^{39} times smaller. Inside atoms, gravity is irrelevant.

Worked example. Two identical oil droplets in a vacuum have their centres 3.8×10^{-6} m apart. Each carries the same charge, and they repel with a force of 1.0×10^{-15} N. Find the charge on each droplet, and the number of excess electrons it carries.

$$F = \frac{Q^2}{4\pi\varepsilon_0 r^2} \Rightarrow Q = \sqrt{4\pi\varepsilon_0 F r^2} = \sqrt{\frac{(1.0 \times 10^{-15})(3.8 \times 10^{-6})^2}{8.99 \times 10^9}} = 1.3 \times 10^{-18} \text{ C,}$$

that is $1.3 \times 10^{-18}/1.60 \times 10^{-19} \approx 8$ electrons. Identical objects with the same sign of charge always **repel**; do not write "attract" for the direction.

Worked example. A charged sphere X is fixed on an insulating stand. A second sphere Y, of mass 2.0 g, hangs beside it on an insulating thread and settles in equilibrium with the thread at 12° to the vertical, its centre 5.0 cm from the centre of X on the same horizontal level. Find the electric force on Y and, if the spheres carry equal charges, the charge on each.



A charged sphere hanging near a charged sphere: three forces balance, the tension along the thread, the weight and the horizontal electric repulsion, so $\tan\theta = F/mg$

Three forces act on Y: its weight mg (down), the **tension** 张力 T (along the thread) and the electric force F (horizontal, away from X because like charges repel). Resolving, $T\cos\theta = mg$ and $T\sin\theta = F$, so

$$F = mg\tan\theta = (2.0\times10^{-3})(9.81)\tan 12° = 4.2\times10^{-3}\text{ N}.$$

Then $F = Q^2/(4\pi\varepsilon_0 r^2)$ gives $Q = \sqrt{4\pi\varepsilon_0 F r^2} = \sqrt{(4.2\times10^{-3})(0.050)^2/(8.99\times10^9)} = 3.4\times10^{-8}$ C. A closed vector triangle of the three forces is an equally good method; the exam accepts either, but it must show all **three** forces.

Worked example (helium). A helium atom may be modelled as a nucleus of charge $+2e$ with two electrons in diametrically opposite circular orbits of radius 170 pm. Find the **resultant force** 合力 on one electron, and hence its orbital speed.

Two forces act on the electron: attraction to the nucleus, at distance r, and repulsion from the other electron, at distance $2r$ on the far side:

$$F_{\text{res}} = \frac{2e^2}{4\pi\varepsilon_0 r^2} - \frac{e^2}{4\pi\varepsilon_0(2r)^2} = \frac{e^2}{4\pi\varepsilon_0 r^2}\left(2-\tfrac{1}{4}\right) = 1.75\times\frac{(8.99\times10^9)(1.60\times10^{-19})^2}{(1.70\times10^{-10})^2} = 1.4\times10^{-8}\text{ N},$$

towards the nucleus. This resultant is the centripetal force, $F = m_e v^2/r$, so $v = \sqrt{Fr/m_e} = \sqrt{(1.4\times10^{-8})(1.70\times10^{-10})/(9.11\times10^{-31})} = 1.6\times10^6\text{ m s}^{-1}$. The trap is to forget the second electron, or to put it at distance r rather than $2r$.

Spheres treated as point charges

A spherical conductor with total charge Q gives, at any point **outside**, the same field as a point charge Q at its centre (measure r from the centre). **Inside** a hollow charged conductor the field is zero, so the conductor is an **equipotential** 等势面.

Electric field due to a point charge

Coulomb's law gives the force between **two** charges. Divide it by the test charge ($E = F/q$) and you are left with the field of the source charge Q **alone**. The field at distance r from a point charge Q is

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}.$$

It points out from a positive Q, in towards a negative Q, and falls as $1/r^2$ — just like **gravitational field** 重力场 strength, except gravity is always attractive. For several charges, add the fields as a **vector sum** 矢量和.

Worked example. The Earth may be treated as a uniform conducting sphere of radius 6.37×10^6 m carrying a total charge of -4.80×10^5 C spread over its surface. Find the electric field strength at the surface, and compare it with the gravitational field strength there. ($M_E = 5.98\times10^{24}$ kg.)

$$E = \frac{Q}{4\pi\varepsilon_0 R^2} = \frac{(8.99\times10^9)(4.80\times10^5)}{(6.37\times10^6)^2} = 106\text{ V m}^{-1},$$

directed **towards** the centre (the charge is negative). Gravity: $g = GM/R^2 = 9.83\text{ N kg}^{-1}$, so $E/g \approx 11$; in symbols $E/g = Q/(4\pi\varepsilon_0 GM)$, which shows that the ratio depends only on the charge-to-mass ratio of the sphere. A charged sphere of mass m and charge q would need $qE = mg$ to float in this field: $q/m = g/E = 0.093\text{ C kg}^{-1}$.

Worked example. Point charges of +4.0 nC and +1.0 nC are 30 cm apart. Where on the line joining them is the resultant field zero?

Between the charges the two fields point in opposite directions; call the distance from the 4.0 nC charge x. Zero resultant needs equal magnitudes:

$$\frac{4.0}{x^2} = \frac{1.0}{(0.30-x)^2} \;\Rightarrow\; \frac{0.30-x}{x} = \frac{1}{2} \;\Rightarrow\; x = 0.20\ \text{m},$$

twice as far from the larger charge. For **opposite** charges the fields between them point the same way and never cancel: the zero lies **outside**, beyond the smaller charge.

Electric potential

Electric potential 电势 V at a point is the **work done per unit positive charge** in bringing a small positive test charge from **infinity** 无穷远 to that point:

$$V = \frac{W}{q}.$$

Unit: V. The potential is **zero at infinity**. For a positive source charge $V > 0$ everywhere outside; for a negative source charge $V < 0$.

The two-mark definition. *Electric potential at a point is the work done per unit positive charge in bringing a small test charge from infinity to the point.* The marks are for **work done per unit (positive) charge** and **from infinity to the point**. The potential of a positive charge is positive because work must be **done on** a positive test charge to push it in against the repulsion; near a negative charge the field does the work, so the potential is negative. "Explain why the potential near an isolated proton is positive" is answered in exactly those words.

Potential due to a point charge

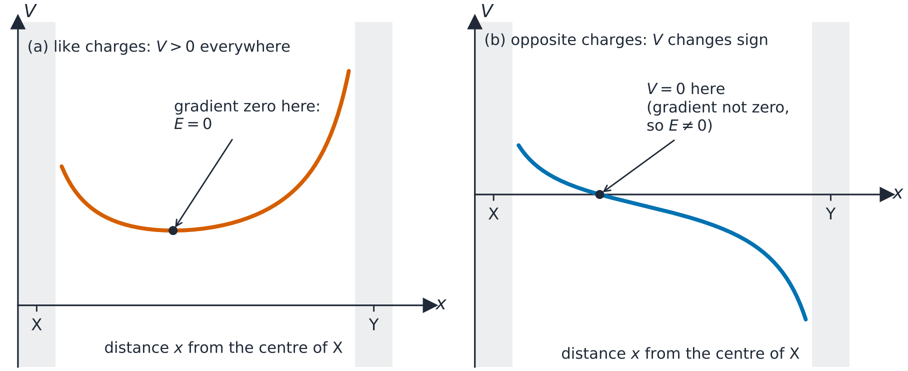
$$V = \frac{Q}{4\pi\varepsilon_0 r}.$$

Note the $1/r$ here (compared with $1/r^2$ for the field). V is a **scalar** 标量; for several charges, add the potentials (with sign).

Worked example. Find the electric potential 4.0 cm from a point charge of +3.0 nC. ($1/(4\pi\varepsilon_0) = 9.0 \times 10^9$ N m^2 C^{-2}.)

$$V = \frac{Q}{4\pi\varepsilon_0 r} = \frac{(9.0\times10^9)(3.0\times10^{-9})}{0.040} \approx 6.8\times10^2\ \text{V}.$$

Because potential is a scalar, the potential from several charges is simply their sum (each with its own sign) — no directions to resolve.



Potential along the line between two charged spheres: like charges give a minimum (field zero there); opposite charges give a zero crossing

Two charged spheres on a line

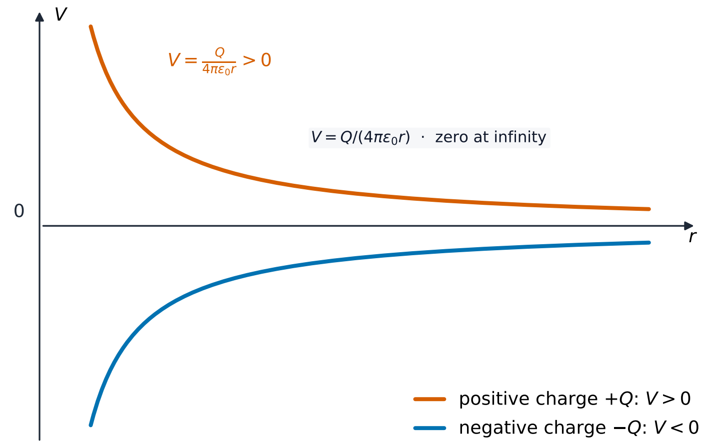
The exam's favourite structured question puts two charged spheres X and Y some distance apart and draws the potential V along the line joining their centres (or asks you to reason from it). Everything follows from two facts: potential is a **scalar** that adds with sign, and the field is the **negative gradient** of the graph.

- **Sign of the charges.** If V is positive along the whole line, both charges are positive; if V changes sign, the charges are opposite (positive near the positive sphere).
- **Where the field is zero.** Where the curve has a **minimum** (gradient zero). For like charges this lies between them, closer to the smaller charge: $Q_X/x^2 = Q_Y/(d-x)^2$. For opposite charges there is **no** such point between them: the curve crosses zero there, but its gradient is not zero, so the field is not.
- **Ratio of the charges.** Where $V = 0$ between opposite charges, $Q_X/x = Q_Y/(d-x)$: the distances give the ratio directly. For like charges use the minimum instead (the square root of the field condition).
- **Force on a charge placed at P.** Read the **gradient** at P by drawing a **tangent** 切线; then E is minus the gradient, and $F = qE$.

Worked example. Two isolated charged metal spheres X and Y have their centres 1.2 m apart in a vacuum. The potential along the line between them is positive everywhere and has a minimum at $x = 0.50$ m from the centre of X. State three conclusions, and then find the force on a proton held at $x = 0.60$ m, where the gradient of the graph is +180 V m^{-1}.

Conclusions: (1) both spheres are **positively** charged, because the potential is positive everywhere; (2) the electric field is **zero** at $x = 0.50$ m, where the gradient is zero; (3) the charge on Y is larger, because the zero-field point lies closer to X; in fact $Q_Y/Q_X = (0.70/0.50)^2 = 2.0$. At $x = 0.60$ m: $E = -180$ V m^{-1} (pointing towards X, since V rises towards Y), so $F = eE = (1.60 \times 10^{-19})(180) = 2.9 \times 10^{-17}$ N towards X. Released, the proton accelerates towards X, its acceleration **increasing** as the gradient steepens; it needs an external force to hold it still.

Describing the motion. "A positively charged particle is placed at P and released. Describe and explain its motion." The examiner wants: the direction (towards **lower** potential, down the slope of the V graph); that the force, and so the acceleration, is **not constant** (it follows the gradient, which changes along the line); and that the particle **speeds up** throughout, since the force stays in the direction of motion. If P is the zero-field point the particle stays at rest, in unstable equilibrium: a nudge either way sends it off.



The potential near a point charge varies as 1/r — positive for a positive charge, negative for a negative one

Link between field and potential

The field equals the **negative potential gradient** 电势梯度:

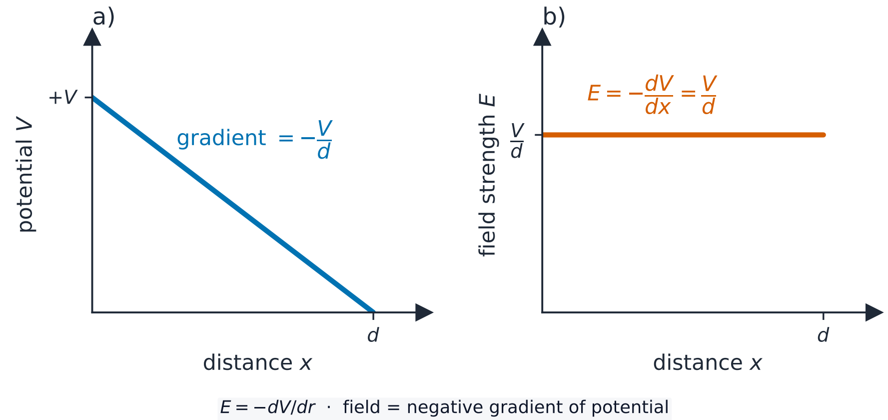
$$E = -\frac{dV}{dx}.$$

Between parallel plates V changes evenly with position, giving $E = V/d$ as before. The minus sign means the field points towards **lower** potential. For a point charge,

$$-\frac{dV}{dr} = \frac{Q}{4\pi\varepsilon_0 r^2} = E.$$

The one-line relationship. "State the relationship between electric field and electric potential": *the field strength is equal to the negative of the potential gradient*, $E = -\Delta V/\Delta x$. On a V–x graph E is minus the **gradient** of a tangent; on a V–r graph for a point charge the gradient is steepest close to the charge, where the field is strongest, and the sign of E is fixed by "the field points towards **lower** potential". The uniform-field version $E = V/d$ is the same statement with a constant gradient.

Equipotentials. An **equipotential** surface joins points of equal potential; no work is done moving a charge along it, so the field is always at **right angles** to it. Around a point charge the equipotentials are concentric spheres, growing further apart as V falls; between parallel plates they are planes parallel to the plates. The surface of a conductor is an equipotential, which is why field lines meet it at right angles.



In a uniform field the potential falls steadily with distance, so the field strength V/d is constant

Electric potential energy

A charge q at a point of potential V has **electric potential energy** 电势能 $E_P = qV$. For two point charges Q and q a distance r apart:

$$E_P = \frac{Qq}{4\pi\varepsilon_0 r}.$$

- **like** charges: $E_P > 0$ — stored energy that would be released if they flew apart.
- **opposite** charges: $E_P < 0$ — a **bound** 束缚 system; energy must be supplied to separate them.

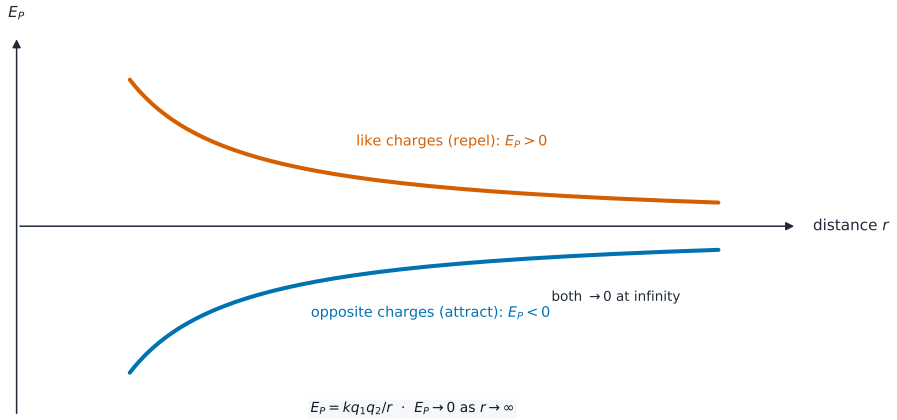
In both cases $E_P \to 0$ as $r \to \infty$.

Worked example. The proton and the electron in a hydrogen atom are 5.3×10^{-11} m apart. Find the electric potential energy of the pair.

$$E_P = \frac{Qq}{4\pi\varepsilon_0 r} = \frac{(8.99 \times 10^9)(+1.60 \times 10^{-19})(-1.60 \times 10^{-19})}{5.3 \times 10^{-11}} = -4.3 \times 10^{-18}\ \text{J} = -27\ \text{eV}.$$

The **negative** sign is part of the answer: the charges are opposite, so energy (4.3×10^{-18} J) must be **supplied** to pull them apart to infinity. (The electron also has kinetic energy of half that size, so the energy needed to **ionise** 电离 the atom is 13.6 eV; the "worked-example pattern" below shows why.)

Charge through a p.d. A charge q moved through a potential difference V changes its potential energy by qV, which is why an electron accelerated from rest through 250 V arrives with 250 eV of kinetic energy whatever the shape of the field. For an MCQ about "moving P a distance x along the field lines", the work done is qEx, and its potential energy **falls** if it moves the way the force pushes it.



Electric PE of two charges: positive for like charges, a negative well for opposite charges

Worked-example pattern

An **electron** 电子 orbits a **nucleus** 原子核 of charge $+Ze$ at distance r. The Coulomb attraction provides the **centripetal force** 向心力:

$$\frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{m_e v^2}{r}, \qquad v = \sqrt{\frac{Ze^2}{4\pi\varepsilon_0 m_e r}}.$$

The total energy is kinetic plus potential:

$$E_{\text{total}} = \tfrac{1}{2}m_e v^2 - \frac{Ze^2}{4\pi\varepsilon_0 r} = -\frac{Ze^2}{8\pi\varepsilon_0 r},$$

which is negative (a bound state).

Gravitational versus electric

The two field theories look alike:

Quantity	Gravitational	Electric
Source	mass M (always positive)	charge Q (can be $\pm$)
Field strength	$g = GM/r^2$	$E = Q/(4\pi\varepsilon_0 r^2)$
Force on test object	$F = mg$	$F = qE$
Potential	$\phi = -GM/r$	$V = Q/(4\pi\varepsilon_0 r)$
PE of two	$-GMm/r$	$Qq/(4\pi\varepsilon_0 r)$
Nature	always attractive	attractive or repulsive

The minus sign in the **gravitational potential** 引力势 reflects that gravity is always attractive; the electric potential takes the sign of the source charge.

Similarity and difference (a standard two-marker). Both potentials are proportional to $1/r$, both are zero at infinity, and both are scalars; but gravitational potential is **always negative** (the force is always attractive) whereas electric potential takes the **sign of the charge** and can be positive. For the fields: both obey an inverse-square law and both are drawn as radial lines around a point source, but gravitational field lines only ever point **inwards** (attraction), while electric field lines point outwards from a positive charge and inwards to a negative one.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
electric field	a region in which a charge experiences an (electric) force
electric field strength	the force per unit positive charge acting on a small (test) charge placed at the point

Term	Definition
electric field line	a line whose direction shows the direction of the force on a positive charge; the spacing of the lines shows the field strength
Coulomb's law	the force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of their separation
electric potential	the work done per unit positive charge in bringing a small test charge from infinity to the point
field and potential	the electric field strength is equal to the negative of the potential gradient: $E = -\Delta V/\Delta x$
electric potential energy (of two charges)	the work done in bringing the charges from infinity to their separation; $Qq/(4\pi\varepsilon_0 r)$
equipotential	a surface (or line) on which every point has the same potential; the field is at right angles to it
uniform field	a field with the same strength and direction at every point, shown by parallel, evenly spaced field lines

Exam tips

- Four formulae, two with r^2 and two with r: $F = Q_1Q_2/4\pi\varepsilon_0 r^2$ and $E = Q/4\pi\varepsilon_0 r^2$ (inverse square), $V = Q/4\pi\varepsilon_0 r$ and $E_{\mathrm{P}} = Qq/4\pi\varepsilon_0 r$. Decide which one before you write.
- Distinguish a **uniform field** ($E = V/d$, between plates) from a **radial field** ($E = Q/4\pi\varepsilon_0 r^2$, outside a point charge or a sphere); r for a sphere is measured from the **centre**, not the surface.
- The field points from high to low potential and E is minus the potential gradient: draw a tangent on a V–x graph to find it.
- Field is a vector (add components, subtract opposing ones); potential is a scalar (add with sign). Between like charges there is a zero-field point; between opposite charges a zero-potential point.
- A charged particle in a uniform field: $F = qE$, $a = qE/m$, $E_{\mathrm{k}} = qV$; along the field it is the vertical half of projectile motion, and motion at right angles to the field is unaffected.
- The data sheet gives both $\varepsilon_0 = 8.85\times10^{-12}$ F m^{-1} and $1/(4\pi\varepsilon_0) = 8.99\times10^{9}$ m F^{-1}; either works, do not mix them up.

Common mistakes

- Defining field strength as "the force on a charge" or "force per unit mass". It is the force per unit **positive charge** on a small positive charge.
- Forgetting to square r in Coulomb's law and in the field, or squaring it in the potential ($V \propto 1/r$).
- Using the diameter, or the distance from the **surface** of a sphere, instead of the centre-to-centre distance.
- Sketching E for a sphere from the origin. It is zero inside and starts at E_0 on the surface; V is constant inside, not zero.
- Field lines without arrows, crossing, or unevenly spaced in a uniform field; lines not at right angles to a conductor.
- Dropping the negative sign on the potential energy of opposite charges, or taking the direction of E from the sign of V instead of from its gradient.
- Forgetting the second electron in the helium atom, or placing it at distance r rather than $2r$.
- Using $E = V/d$ for a point charge, or $E = Q/4\pi\varepsilon_0 r^2$ between plates.
- Giving an electron weight, or using the proton's mass for an electron; in electrons-between-plates questions the weight is negligible.
- Leaving out the weight in an "oil drop held stationary" question, or getting the polarity of the top plate backwards.