Electricity

A-Level Physics

Electric current

An **electric current** 电流 is a flow of **charge carriers** 载流子. In a metal the carriers are negative conduction **electrons** 电子; in an **electrolyte** 电解质 they are positive and negative **ions** 离子; in a **semiconductor** 半导体 they may be electrons or "**holes**" 空穴. The **conventional current** 常规电流 direction is the way **positive** charge would flow — opposite to the real flow of electrons in a wire.

Charge

Charge is **quantised** 量子化: the smallest free unit of charge is the **elementary charge** 基本电荷

$$e = 1.60 \times 10^{-19}\ \text{C}.$$

Every free charge in this syllabus is a whole-number multiple of e. The unit of charge is the **coulomb** 库仑, C.

So a particle can carry 4.8×10^{-19} C (three electrons' worth) or -2.4×10^{-18} C (fifteen), but never 1.1×10^{-19} C or 6.4×10^{-20} C; a set of measurements that gives a charge carrier 2.5×10^{-19} C cannot be right, because that is not a multiple of e. For the one-mark definitions: an **electric current is a flow of charge carriers**; the **coulomb** is the charge that passes a point in one second when the current is one ampere (an ampere multiplied by a second); a **charge carrier** can be an electron, a proton, an ion or an α-particle, but never a neutron.

Current as the rate of flow of charge

If charge Q passes a point in time t, the current is

$$I = \frac{Q}{t}, \qquad Q = It.$$

Unit of current: ampere, A ($=$ C s^{-1}). For a changing current, the charge that has flowed in a time is the **area under** an I–t graph.

Worked example. A current of 0.50 A flows for 2.0 minutes. Find the charge that passes, and how many electrons this represents. ($e = 1.60 \times 10^{-19}$ C.)

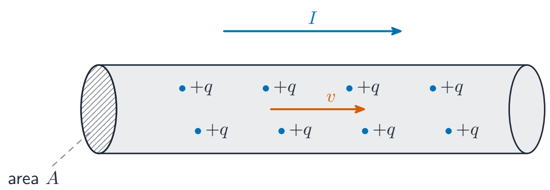
$$Q = It = 0.50 \times 120 = 60\ \text{C}, \qquad N = \frac{Q}{e} = \frac{60}{1.60 \times 10^{-19}} \approx 3.8 \times 10^{20}.$$

The same two lines answer "how many electrons pass in 10 hours at 4.0 mA" (convert the time to seconds: $N = It/e = 4.0 \times 10^{-3} \times 36\,000/(1.60 \times 10^{-19}) = 9.0 \times 10^{20}$) and, the other way round, "the average current when 6.0×10^{23} electrons pass in 24 hours" ($I = Ne/t = 1.1$ A). A beam of α-particles carries a current too: each carries $2e$, so a beam current of 6.9×10^{-9} A is $6.9 \times 10^{-9}/(2 \times 1.60 \times 10^{-19}) = 2.2 \times 10^{10}$ particles per second. A lightning strike that transfers 1×10^{20} electrons in 30 μs is a current of $Ne/t = 5.3 \times 10^5$ A.

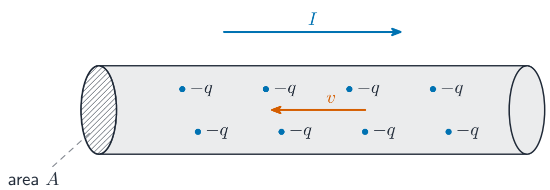
Drift velocity equation

For a uniform conductor of cross-section area A, with n charge carriers per unit volume (the **number density** 数密度), each carrying charge q, moving with average **drift velocity** 漂移速度 v:

$$I = Anvq.$$



(a) positive carriers drift with I



(b) electrons drift against I

Charge carriers drifting inside a conductor: positive carriers drift with I, electrons against it

Worked example. A copper wire of cross-sectional area 1.0×10^{-6} m^2 carries a current of 5.0 A. Copper has $n = 8.5 \times 10^{28}$ free electrons per m^3. Find the drift velocity. ($e = 1.60 \times 10^{-19}$ C.)

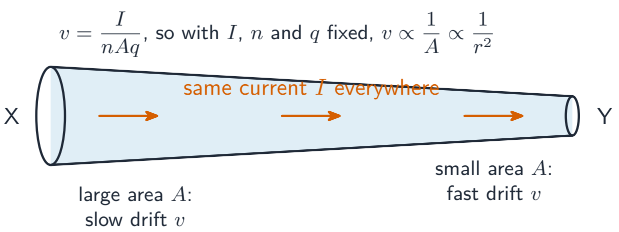
Rearranging $I = Anvq$ gives $v = \dfrac{I}{Anq}$:

$$v = \frac{5.0}{(1.0 \times 10^{-6})(8.5 \times 10^{28})(1.60 \times 10^{-19})} \approx 3.7 \times 10^{-4}\ \text{m s}^{-1}.$$

The electrons drift very slowly — less than a millimetre per second.

Two "show that" steps that often precede this calculation. The **number density** of a metal with one free electron per atom is $n = \dfrac{\text{density} \times N_{\text{A}}}{\text{molar mass}}$: for copper, $8.9 \times 10^3 \times 6.02 \times 10^{23} / 0.0635 = 8.5 \times 10^{28}\ \text{m}^{-3}$. And the **time** for an electron to drift the length of a wire is $t = L/v$: at $3.7 \times 10^{-4}\ \text{m s}^{-1}$ a 2.0 m wire takes 5 400 s, an hour and a half, even though the current is established almost instantly. Between them, the equation $I = Anvq$ is used as a ratio far more often than as a calculation:

- **two wires in series** carry the same current; if wire Y has twice the diameter of X (four times the area) and the same metal, its electrons drift at a quarter of the speed. Two wires of the same length and diameter but different metals have drift speeds in the inverse ratio of their number densities.
- **a wire that narrows** (a wedge, a tapered rod, a cable of thick and thin strands) carries the same current at every cross-section, so the drift speed rises where the area falls: $v \propto 1/A \propto 1/r^2$. A sketch of v against distance along a tapering wire is a curve that rises more and more steeply towards the narrow end, not a straight line.
- the p.d. across a uniform wire is proportional to its length at fixed current, because $V = IR = I\rho L/A$ with I, ρ and A constant.



The same current everywhere, so the electrons drift faster where the wire is thinner

Use this to compare currents:

- a thinner wire (smaller A) at the same I needs a faster drift v.

- a semiconductor has far fewer free carriers than a metal (smaller n), so for the same I the drift velocity is much larger.
- in **series** 串联 components, I is the same everywhere, so if A stays the same but the material changes, nv changes the other way.

Potential difference

The **potential difference** 电势差 (p.d.) across a component is the **energy** 能量 transferred per unit charge as that charge passes through it:

$$V = \frac{W}{Q}.$$

Unit: **volt** 伏特, V ($= \text{J C}^{-1}$).

If 1 J of electrical energy changes into other forms (thermal, light, kinetic, …) when 1 C of charge passes through a component, the p.d. across it is 1 V.

The **electromotive force** 电动势 (e.m.f.) of a source is the energy given **per unit charge by the source**. The formula is the same as for p.d.; the difference is direction: e.m.f. is energy given **to** the charge by the change; p.d. is energy given **up** by the charge to the component.

The definitions the examiner accepts: the **potential difference** across a component is the energy transferred (from electrical to other forms) per unit charge passing through it; the **e.m.f.** of a source is the energy transferred from other forms (chemical, in a cell) to electrical energy per unit charge in driving the charge round a complete circuit; one **volt** is one joule per coulomb. A battery "marked 9.0 V" therefore gives each coulomb 9.0 J of electrical energy for the whole circuit, and the product of charge and p.d. is the energy transferred.

Electrical power

High-voltage power lines carry electrical energy across the country.

Combining $V = W/Q$ and $I = Q/t$:

$$P = \frac{W}{t} = VI.$$

Using Ohm's law $V = IR$:

$$P = VI = I^2R = \frac{V^2}{R}.$$

Pick the form with the quantities you know. Examples:

- two heaters of equal resistance — the one with the larger current gives more **power** 功率 ($P = I^2R$).
- two resistors in **parallel** 并联 across the same **voltage** 电压 — the one with smaller R gives more power ($P = V^2/R$).
- a kettle marked "2.4 kW, 240 V" draws $I = P/V = 10$ A and has resistance $R = V^2/P = 24\ \Omega$.

Energy transferred in time t is $E = Pt$.

Worked example. Two lamps, P rated 250 V, 50 W and Q rated 250 V, 200 W, are connected in series to a 250 V supply. Which is brighter?

Their resistances at the rated p.d. are $R = V^2/P$: 1250 Ω for P and 313 Ω for Q. In series they carry the same current, so $P = I^2R$ makes the larger resistance, lamp P, dissipate more power: the "weaker" lamp is the brighter one. In **parallel** the p.d. is the same across both and $P = V^2/R$ favours the smaller resistance: Q. "State and explain which resistor dissipates more power" is answered by naming the quantity the two share (current in series, p.d. in parallel) and the form of the power equation that uses it.

Worked example. A supply delivers 2.4 kW at 240 V to a kettle through two cables of resistance 0.10 Ω each. Find the power lost in the cables.

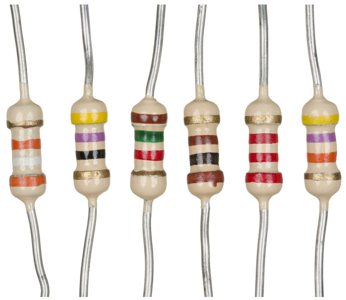
The current is $I = P/V = 10$ A, so the cables dissipate $I^2R = 10^2 \times 0.20 = 20$ W, and the kettle receives 2380 W. The **efficiency** of a circuit that exists to power one component is that component's power divided by the total power from the supply: with 6.0 V across a 0.86 Ω series resistor and 4.5 V across the component at the same current, the efficiency is $4.5/(6.0 + 4.5) = 43\%$. A thermistor across a fixed p.d. dissipates $P = V^2/R$, so as its temperature rises and R falls, the power rises. A kettle draws about 10 A from a 250 V supply; a mobile phone charger a fraction of an ampere.

Resistance and Ohm's law

The **resistance** 电阻 R of a component is

$$R = \frac{V}{I}.$$

Unit: **ohm** 欧姆, Ω (= V A^{-1}). Resistance depends on the conditions (such as temperature) when it is measured.



Real fixed resistors — the coloured bands code the resistance in ohms

Ohm's law

A conductor obeys **Ohm's law** 欧姆定律 when the current through it is **proportional to the p.d.** across it, as long as the conditions (especially temperature) stay constant. For such a conductor R is constant and the I–V graph is a straight line through the origin.

Ohm's law is an experimental result, not a definition. The definition $R = V/I$ works for **any** component; only ohmic ones have constant R.

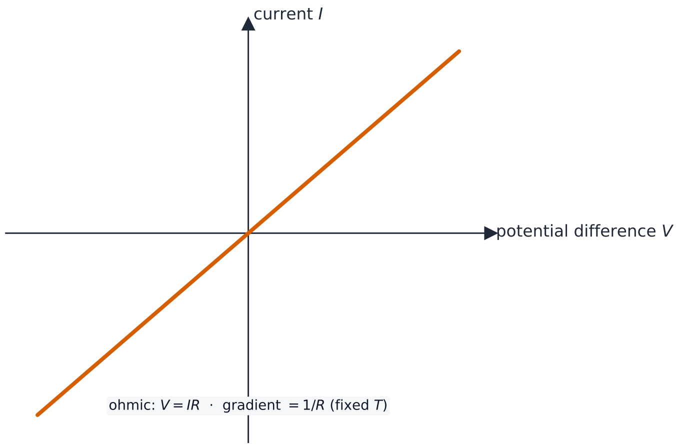
For the marks: **resistance** is the ratio of the potential difference across a component to the current in it; the **ohm** is the resistance of a component in which a p.d. of one volt produces a current of one ampere (a volt per ampere); **Ohm's law** states that the current in a metallic conductor is directly proportional to the potential difference across it, provided that its temperature (and other physical conditions) remains constant. Of several I–V graphs, only a straight line through the origin obeys Ohm's law; a straight line that misses the origin, or any curve, does not.

I–V characteristics

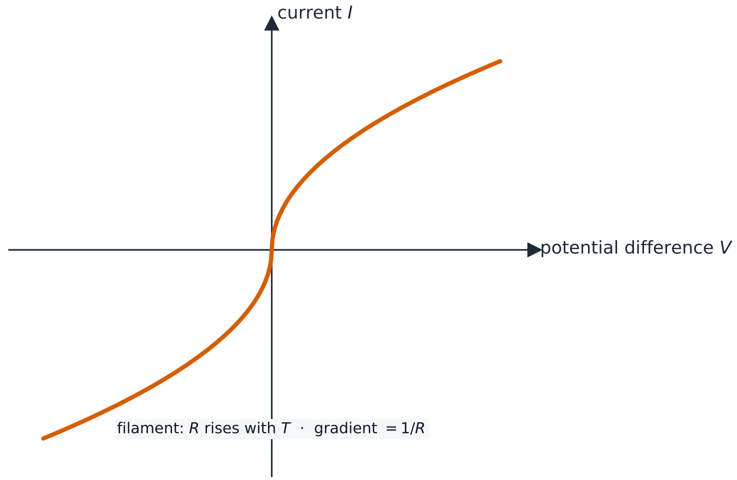
You should be able to sketch these:

- **metal wire at constant temperature** — a straight line through the origin (constant R). Reversing the p.d. drives the current the other way, giving a straight line in both directions.
- **filament lamp** 灯丝灯泡 — through the origin, steep at first, then flatter as V (and I) grow. Reason: more current heats the filament, so its resistance rises and the gradient $1/R$ falls.
- **semiconductor diode** 二极管 — almost no current for negative V or small positive V. Above a "switch-on" voltage (about 0.7 V for silicon), the current rises sharply.

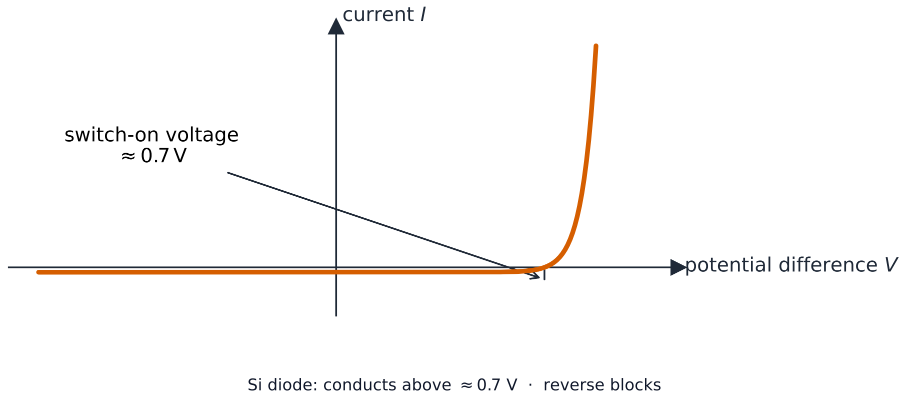
Explain the shape of the filament lamp's line (three marks): as the current increases the filament's temperature rises; the lattice ions vibrate with larger amplitude, so the free electrons collide with them more often; each collision takes energy from the electrons, so the resistance increases; on the graph the ratio V/I grows and the gradient falls. Run backwards for "the current decreases": the temperature falls, so the resistance falls. Two things to read off any characteristic. The resistance at a point is V/I for that point, never the gradient of a curve, so a diode's resistance is very large (infinite, in practice) up to the switch-on p.d. and then falls steeply as V rises further, and of four components at the same p.d. the one with the smallest current has the greatest resistance. And when a lamp (or a diode) is in series with a resistor, the two carry the same current and their p.d.s add: read the current from the component's curve at its own p.d., then use $V = IR$ for the resistor, and the supply p.d. is the sum.



I–V characteristic of an ohmic conductor (metal wire at constant temperature)



I–V characteristic of a filament lamp



I–V characteristic of a semiconductor diode

Resistivity

For a uniform conductor of length L and cross-section area A,

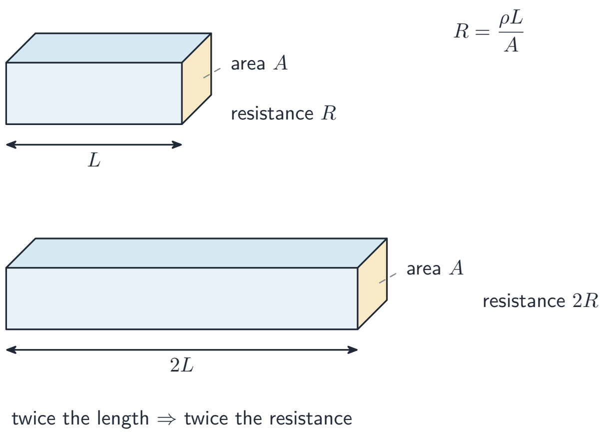
$$R = \frac{\rho L}{A}.$$

ρ is the **resistivity** 电阻率, a property of the material, with unit Ω m. Doubling the length doubles R; doubling the area halves it; halving the diameter quarters the area and so makes R four times bigger.

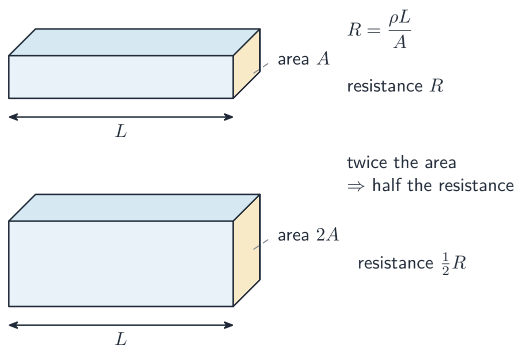
Worked example. A copper wire of length 2.0 m and cross-sectional area 1.7×10^{-7} m^2 has resistivity 1.7×10^{-8} Ω m. Find its resistance.

$$R = \frac{\rho L}{A} = \frac{(1.7 \times 10^{-8})(2.0)}{1.7 \times 10^{-7}} = 0.20\ \Omega.$$

Typical values: copper at room temperature $\rho \sim 1.7 \times 10^{-8}$ Ω m; an **insulator** 绝缘体 $\rho \sim 10^{15}$ Ω m or more.



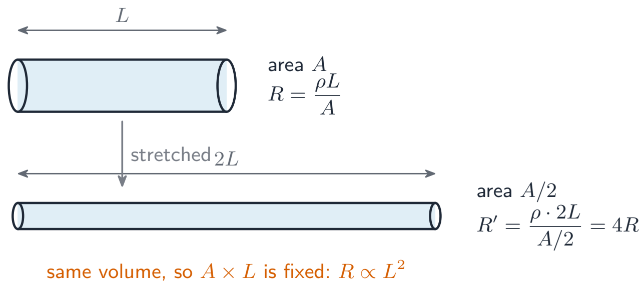
A longer conductor has more resistance — doubling L doubles R



A wider conductor has less resistance — doubling A halves R

The resistivity of a metal **rises with temperature** (more **lattice vibration** 晶格振动 **scatters** 散射 the electrons), which is why the filament lamp's I–V line curves.

Most resistivity questions are ratios. **Stretching** a wire keeps its volume ($A \times L$) constant, so if the length becomes k times longer the area becomes k times smaller and $R = \rho L/A$ becomes k^2 times larger: a wire three times as long (same mass, same metal) has nine times the resistance, and a wire whose diameter falls to 0.940 of its value has its area multiplied by 0.884, its length divided by 0.884, and its resistance multiplied by $1/0.884^2 = 1.28$.



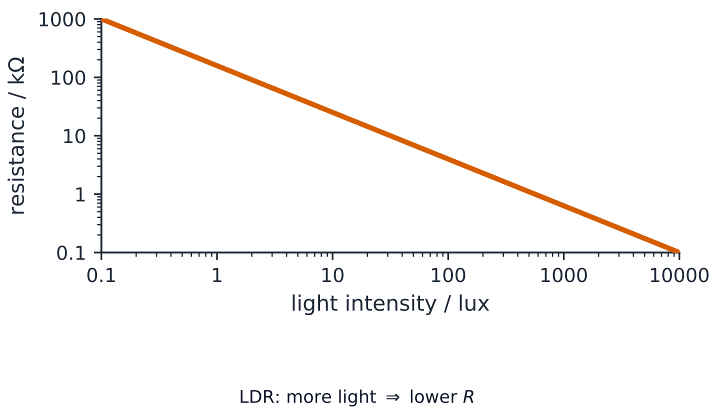
Stretched at constant volume: twice the length, half the area, four times the resistance

Worked example. A copper lightning rod of resistance 9.6 Ω and length 20 m has resistivity 1.7×10^{-8} Ω m. Find its radius.

$A = \rho L/R = 1.7 \times 10^{-8} \times 20/9.6 = 3.5 \times 10^{-8}$ m^2, and $r = \sqrt{A/\pi} = 1.1 \times 10^{-4}$ m. Doubling the radius (same length) quarters the resistance. The other standard ratios: **strands in parallel** — seven identical strands share the current, so the cable's resistance is one seventh of one strand's; two wires of the same resistance where one metal has twice the resistivity need the second wire to have twice the area (a diameter $\sqrt{2}$ times larger) for the same length; a wire's resistance per unit length, 0.92 Ω m^{-1}, multiplied by its area, 5.3×10^{-7} m^2, is its resistivity, 4.9×10^{-7} Ω m; and a cylinder of conducting putty 60 mm long and 20 mm across, or a cube of side a ($R = \rho a/a^2 = \rho/a$), uses the same $R = \rho L/A$ with the shape's own length and end area. When the wire is held under tension, $R_0 = \rho L/A$ still gives its resistance, and a stretch that lengthens it and thins it raises R for both reasons. Because $\rho = RA/L = R\pi d^2/(4L)$, the percentage uncertainty in a measured resistivity is the sum of the percentage uncertainties in R and L plus **twice** that in d.

Light-dependent resistor (LDR)

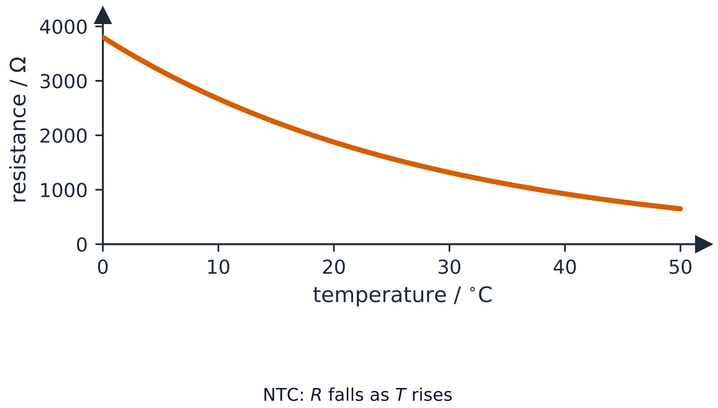
A **light-dependent resistor** 光敏电阻 (LDR) is a semiconductor whose **resistance falls as the light intensity rises**. In bright light R may be a few hundred Ω; in the dark it can be in the megaohms. LDRs are used in light-sensing circuits (street lamps, camera light meters). Here the **light intensity** 光强 controls the resistance.



Resistance of an LDR decreases as light intensity increases

Thermistor

In this syllabus a **thermistor** 热敏电阻 has a **negative temperature coefficient** 负温度系数: its **resistance falls as its temperature rises**. This is useful for sensing temperature — put it in a **potential divider** 分压器 and the output voltage changes with temperature.



Resistance of a thermistor falls as temperature rises

This is the opposite of a metal: in a semiconductor, more thermal energy frees more charge carriers, and this matters more than the extra scattering.

A sketch of a thermistor's resistance against temperature starts at R_0 at 0 °C and falls along a curve that flattens: the fall is not linear, which is the disadvantage of a thermistor as a thermometer (its scale is not uniform, so it needs calibrating). When the light on an LDR is increased, or a thermistor is warmed, its resistance falls, the current in its circuit rises, and the p.d. across it (in series with a fixed resistor) falls while the p.d. across the resistor rises; a fixed resistor and a metal wire at constant temperature keep their resistance, and a filament lamp's rises with current.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
electric current	a flow of charge carriers
coulomb	the charge passing a point in one second when the current is one ampere
potential difference	the energy transferred from electrical to other forms per unit charge passing through a component
electromotive force (e.m.f.)	the energy transferred from other forms to electrical energy per unit charge, by a source driving charge round a complete circuit
volt	one joule per coulomb

Term	Definition
resistance	the ratio of the potential difference across a component to the current in it
ohm	the resistance of a component in which a potential difference of one volt produces a current of one ampere
Ohm's law	the current in a metallic conductor is directly proportional to the potential difference across it, provided its temperature remains constant
resistivity	the constant ρ in $R = \rho L/A$, a property of the material with unit Ω m
number density	the number of charge carriers per unit volume of the conductor

Exam tips

- Use $I = Q/t$, $V = W/Q$ (energy per unit charge) and $P = VI = I^2R = V^2/R$.
- **Ohm's law** ($V = IR$) applies only to an ohmic conductor at constant temperature — a filament lamp is non-ohmic.
- Sketch and interpret the ***I*-*V* characteristics** of a resistor, filament lamp and diode.
- An **LDR's** resistance falls with light; a **thermistor's** falls as temperature rises.

Common mistakes

- Taking the resistance from the gradient of a curved I–V graph. Resistance is V/I at the point; only for a straight line through the origin is it the reciprocal of the gradient.
- Leaving a time in minutes or hours in $Q = It$. Convert to seconds first.
- Saying the filament lamp's resistance rises "because of the voltage". The chain is current, temperature, ion vibration, more collisions, more resistance.
- Forgetting that a stretched wire changes in two ways. At constant volume the area falls as the length rises, so $R \propto L^2$.
- Using the diameter as the radius in $A = \pi r^2$, or leaving mm^2 unconverted.
- Comparing powers with the wrong form. Series components share the current, so use I^2R; parallel components share the p.d., so use V^2/R.