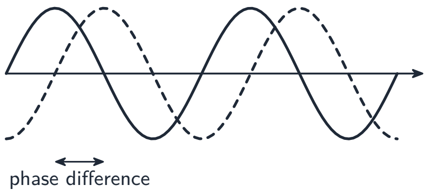
Waves

A-Level Physics

Progressive waves

Ripples spreading on water are progressive waves that carry energy outward.



Two waves of the same frequency, shifted by a phase difference

A **wave** 波 carries **energy** 能量 from one place to another without moving matter overall. The particles of the **medium** 介质 **oscillate** 振动 about fixed rest positions; only the disturbance (and its energy) **propagates** 传播. Examples: a **transverse wave** 横波 on a rope, a **longitudinal wave** 纵波 on a slinky spring, ripples on water, and sound in air. A wave that travels and carries energy is a **progressive wave** 行波.

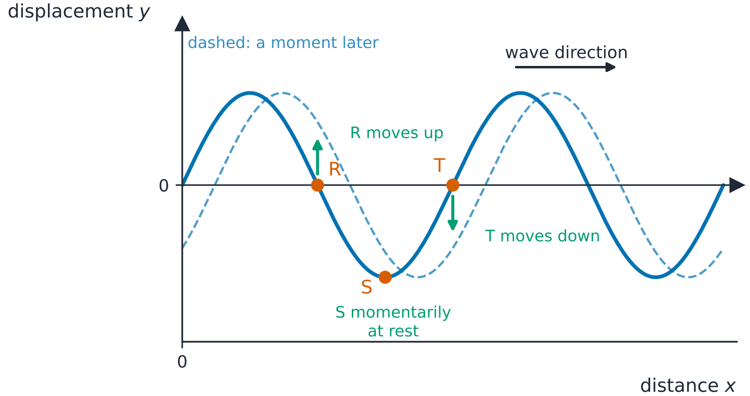
Wave motion 波动 is a series of oscillations of the particles of a medium, each passing the disturbance on to the next: shake one end of a rope, push one end of a spring, or touch the surface of the water in a **ripple tank** 水波槽, and the oscillation travels away from where it started. For the one-mark definition, a progressive wave transfers energy from one place to another without transferring matter; the particles only oscillate about their rest positions.

Key terms

- **displacement** 位移 y — how far a particle has moved from its rest position at a moment. A **vector** 矢量.
- **amplitude** 振幅 A — the largest displacement from the rest position.
- **wavelength** 波长 λ — the shortest distance along the wave between two points that move **in phase** 同相 (for example, two next-door **crests** 波峰).
- **period** 周期 T — the time for one full oscillation of a particle.
- **frequency** 频率 f — the number of full oscillations per second; $f = 1/T$. Unit: **hertz** 赫兹, Hz.
- **speed** 速率 v — how fast a crest travels along the medium.
- **phase difference** 相位差 — the fraction of a cycle by which one oscillation leads or lags another. Given in **radians** 弧度 (a full cycle is 2π) or degrees (a full cycle is 360°).

Two points one wavelength apart are in phase (phase difference 0 or 2π). Two points half a wavelength apart are exactly out of phase (phase difference π).

To find the phase difference between two points from a displacement–distance graph, divide their separation by the wavelength and multiply by 360° (or 2π): two points 0.50 m apart on a wave of wavelength 2.0 m ($v = 600$ m s^{-1}, $f = 300$ Hz) differ in phase by $0.25 \times 360° = 90°$. Only the fraction of a cycle matters, so 450° is the same as 90°. From two displacement–time graphs, read the time shift between the peaks as a fraction of the period. Which way is a point moving at the instant a graph shows? The whole profile moves along, so each point takes up the displacement that its neighbour on the side the wave comes from has now; a point at a crest or a trough is momentarily at rest.



Which way each point moves: towards the displacement its neighbour on the incoming side has now; a crest or trough is momentarily at rest

Worked example. Points R and T on a string are 0.62 cm apart and a quarter of a cycle out of phase. The wave speed is 0.27 m s^{-1}. Find the frequency.

A quarter of a cycle is a quarter of a wavelength, so $\lambda = 4 \times 0.62 = 2.5$ cm and $f = v/\lambda = 0.27/0.025 = 11$ Hz.

The wave equation

In one period T, the wave moves forward by one wavelength λ. So speed = distance/time = $\lambda/T = \lambda f$:

$$v = f\lambda.$$

This comes straight from the definitions of speed, frequency and wavelength, and works for every progressive wave. For the two-mark derivation write both steps: in one period T the wave travels one wavelength λ, so $v = \lambda/T$; and $f = 1/T$, so $v = f\lambda$. Because v is fixed by the medium, a higher frequency means a shorter wavelength. A period question is the same equation the other way round: light of wavelength 460 nm has $f = c/\lambda = 6.5 \times 10^{14}$ Hz and $T = 1/f = 1.5 \times 10^{-15}$ s.

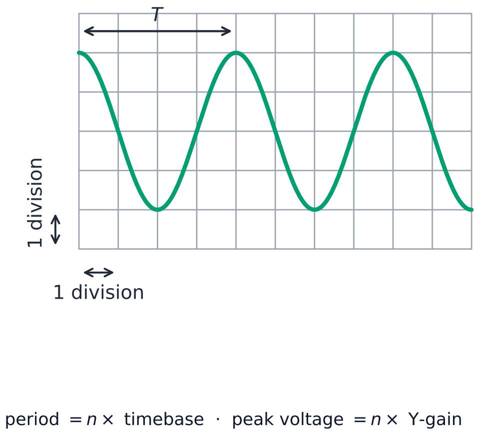
Reading a CRO trace

A **cathode-ray oscilloscope** 示波器 (CRO) draws a voltage signal — for sound, the output of a microphone — against time. Two controls matter:

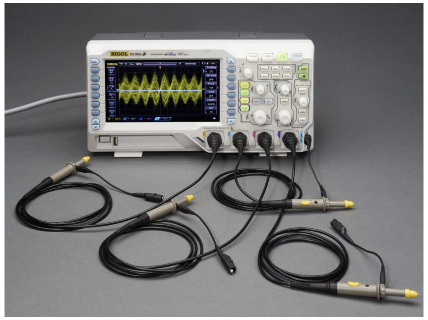
- **time-base** 时基 (seconds per division across): turns horizontal distance on the screen into time. Read the period T as the distance between two next-door peaks, then $f = 1/T$.
- **y-gain** 垂直增益 (volts per division up): turns vertical distance into voltage. The amplitude in volts is the peak height from the centre line.

If the time-base is 5 ms/div and one full cycle takes 4 divisions, then $T = 4 \times 5$ ms = 20 ms and $f = 50$ Hz.

The time-base is the time represented by one division (or one centimetre) across the screen, in s div^{-1}. To set it for a given signal, work backwards: a 2000 Hz sound has $T = 0.50$ ms, so for one cycle to span 2.5 cm the time-base must be $0.50/2.5 = 0.20$ ms cm^{-1}. For the amplitude, multiply the peak height in divisions by the y-gain: 2.0 divisions at 3.5 mV cm^{-1} is 7.0 mV. When the intensity of the sound is reduced to a quarter at the same frequency, the trace keeps its period and its peaks halve in height, because $I \propto A^2$.



Reading the period T from a CRO trace using the grid and time-base



A real oscilloscope: the grid lets you read off the period and the amplitude

Intensity of a wave

A wave carries energy. The **intensity** 强度 at a point is the **power** 功率 passing through unit area at right angles to the direction of travel:

$$I = \frac{P}{A}.$$

Unit: W m^{-2}.

Intensity is proportional to the square of the amplitude:

$$I \propto A^2.$$

For a **point source** 点源 sending out energy equally in all directions, the **wavefronts** 波前 are spheres; the surface area at distance r is $4\pi r^2$, so

$$I = \frac{P}{4\pi r^2}, \qquad I \propto \frac{1}{r^2}.$$

Doubling the distance cuts the intensity to a quarter, which means the amplitude is halved (since $I \propto A^2$).

Worked example. A lamp emits 60 W of light equally in all directions. Find the intensity of the light 2.0 m away.

$$I = \frac{P}{4\pi r^2} = \frac{60}{4\pi (2.0)^2} \approx 1.2\ \text{W m}^{-2}.$$

Worked example. Light of power 750 W falls at right angles on a square solar panel of side 1.2 m. Find the intensity.

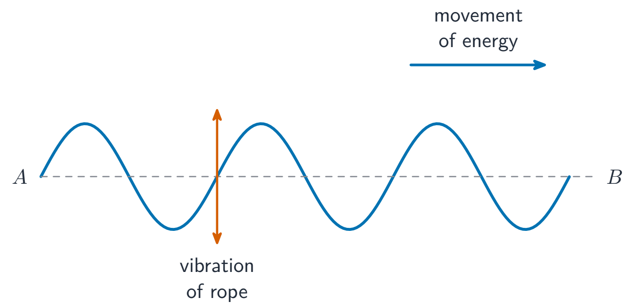
$I = P/A = 750/1.2^2 = 520\ \text{W m}^{-2}$. A smaller panel in the same light receives the same intensity but less power, in proportion to its area. The other way round, a magnifying glass of radius r collects a power $I \times \pi r^2$ and concentrates it on a small spot.

Two rules the multiple-choice questions turn on. Because $I \propto A^2$, the ratio of two intensities is the **square** of the ratio of the amplitudes: waves of amplitude 3.0 cm and 2.0 cm have intensities in the ratio 2.25, and when an amplitude falls to a half the intensity falls to a quarter. For a point source, $I \propto 1/r^2$ means $A \propto 1/r$, so a sketch of A/A_0 against d/x_0 is a curve falling as $1/d$, not a straight line. The one-line statement the scheme wants: intensity is proportional to the amplitude squared.

Transverse and longitudinal waves

Transverse waves

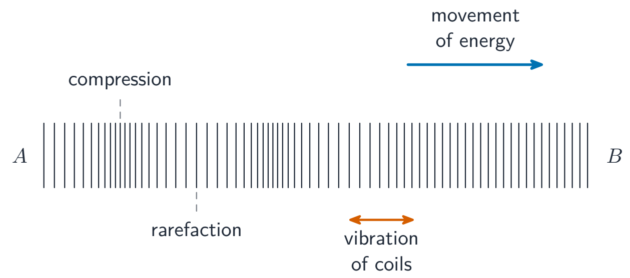
The particles oscillate **perpendicular** 垂直 to the direction the energy travels. A wave on a rope, all electromagnetic waves, and S-waves in the Earth are transverse.



Transverse wave on a rope

Longitudinal waves

The particles oscillate **parallel** to the direction the energy travels. Sound in any medium, P-waves in the Earth, and the squashes on a slinky are longitudinal. The wave is made of **compressions** 压缩 (higher pressure, particles close together) and **rarefactions** 稀疏 (lower pressure, particles spread out).



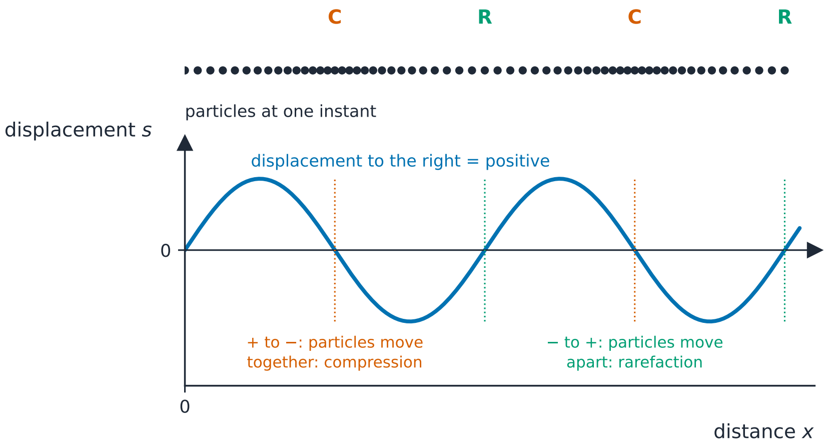
Longitudinal wave on a slinky spring

Graphs of waves

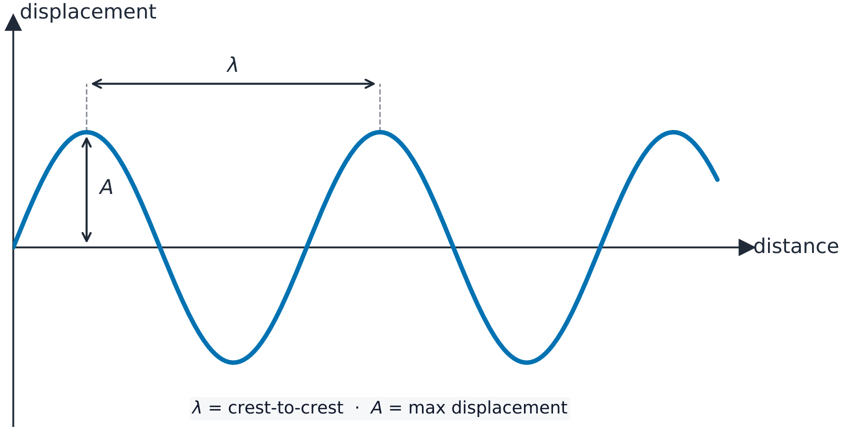
A graph of **particle displacement against position** at one moment looks like a **sine curve** 正弦曲线 for both kinds of wave. The difference: for a transverse wave the displacement axis is the real sideways displacement; for a longitudinal wave it is the small back-and-forth displacement along the direction of travel (positive one way, negative the other).

For a longitudinal wave take displacement to the **right** as positive. Where the graph crosses zero going from positive to negative, the particles on either side have moved

towards that point, so it is a compression; where it crosses from negative to positive they have moved apart, a rarefaction. Two neighbouring compressions are one wavelength apart, and a compression and the next rarefaction are half a wavelength apart. The direction of motion of a particle follows the same rule as for a transverse wave: it moves towards the displacement of its neighbour on the side the wave comes from.

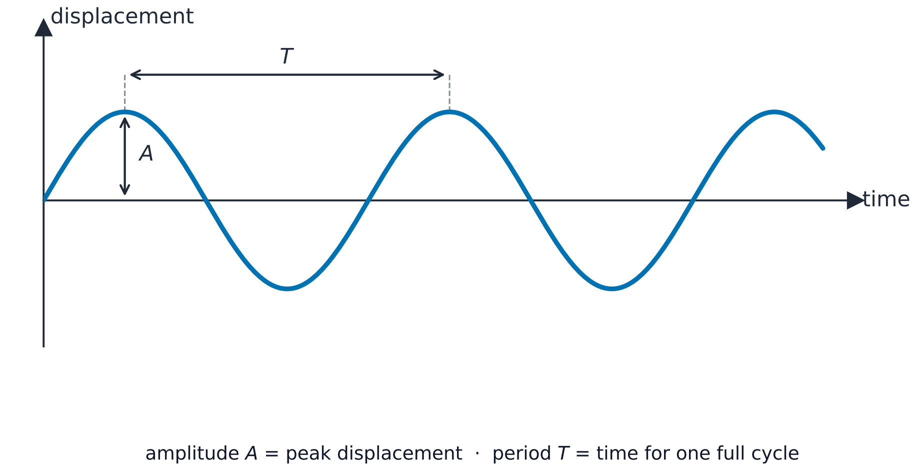


The same longitudinal wave as particles and as a graph: a compression where the displacement changes from positive to negative, a rarefaction where it changes from negative to positive



A displacement–distance graph shows the wave's amplitude and wavelength

A graph of **particle displacement against time** at one point in space is also a sine curve for both kinds. Read the period T from this graph.



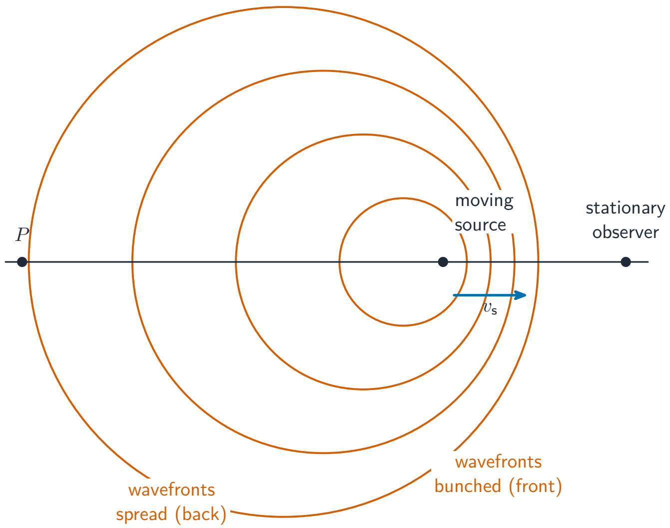
A displacement–time graph shows the wave's amplitude and period

The two-mark comparison, with reference to the direction of energy transfer: in a transverse wave the oscillations are **perpendicular** to the direction of energy transfer; in a longitudinal wave they are **parallel** to it. Both kinds transfer energy without transferring matter, both can be reflected, refracted and diffracted, and both can form stationary waves (topic 8). Only transverse waves can be polarised, and only electromagnetic waves, which are all transverse, can travel through a vacuum; sound needs a medium. Reading the two graphs together: the displacement–distance graph gives λ, the displacement–time graph gives T, and $v = \lambda/T$.

Doppler effect (moving source, stationary observer)

When the **source** 波源 of a sound moves relative to a **stationary** 静止 **observer** 观察者, the heard frequency is different from the source frequency. This is the **Doppler effect** 多普勒效应.

- source moving towards the observer: the wavefronts in front are squashed, so the wavelength is shorter and the heard frequency is **higher**.
- source moving away from the observer: the wavefronts behind are spread out, so the wavelength is longer and the heard frequency is **lower**.



A moving source squashes the wavefronts ahead of it, raising the observed frequency

The formula (source moving at speed v_s along the line to the observer; wave speed v, source frequency f_s, heard frequency f_o):

$$f_o = \frac{v \cdot f_s}{v \pm v_s}.$$

Choose the sign to match the physics:

- minus sign on the bottom when the source moves **towards** the observer ($f_o > f_s$),
- plus sign when the source moves **away** ($f_o < f_s$).

You only need the case of a stationary observer.

Worked example. A car horn at $f_s = 800$ Hz moves at 30 m s^{-1} towards a still listener. Speed of sound $v = 340$ m s^{-1}:

$$f_o = \frac{340 \times 800}{340 - 30} = \frac{272\,000}{310} \approx 877 \text{ Hz}.$$

Moving away at the same speed, the listener hears $340 \times 800/(340 + 30) = 735$ Hz: the frequency is constant while the car approaches, drops as it passes, and is constant again, lower, as it recedes. A sketch of observed frequency against time for a source passing at constant speed is two level lines joined by a fall. Working backwards, a source of 1200 Hz heard as 960 Hz is moving away ($f_o < f_s$): $960 = 1200 \times 340/(340 +$

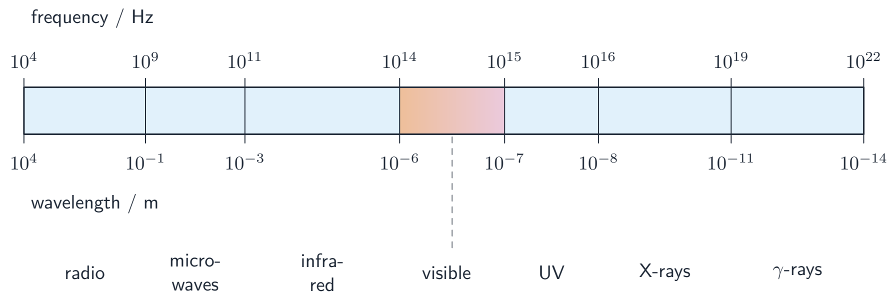
v_s) gives $v_s = 85$ m s^{-1}. A buzzer swung in a horizontal circle at 25 m s^{-1} while emitting 846 Hz is heard between $846 \times 330/355 = 786$ Hz (moving directly away) and $846 \times 330/305 = 915$ Hz (moving directly towards), once each per revolution. If the period of a CRO trace of the sound rises continuously, the observed frequency is falling: the source is moving away with increasing speed. The Doppler effect happens for all waves, light included (topic 25); only the sound formula is examined here.

Electromagnetic spectrum

All **electromagnetic waves** 电磁波 (EM waves) are **transverse** and travel in a **vacuum** 真空 at the same speed:

$$c = 3.00 \times 10^8 \text{ m s}^{-1}.$$

The **electromagnetic spectrum** 电磁波谱 includes radio waves, **microwaves** 微波, **infrared** 红外线, visible light, **ultraviolet** 紫外线, **X-rays** X 射线 and **γ-rays** γ 射线.



The electromagnetic spectrum

Approximate wavelength ranges in free space (learn the orders of magnitude):

- radio waves: $> 10^{-1}$ m (up to many km).
- microwaves: 10^{-3} m to 10^{-1} m.
- infrared: $\sim 7 \times 10^{-7}$ m to 10^{-3} m.
- visible light: 400 nm (violet) to 700 nm (red), i.e. 4×10^{-7} m to 7×10^{-7} m.
- ultraviolet: $\sim 10^{-8}$ m to 4×10^{-7} m.
- X-rays: $\sim 10^{-11}$ m to 10^{-8} m.
- γ-rays: $< 10^{-11}$ m.

The boundaries between regions are not sharp. Use $c = f\lambda$ to change between wavelength and frequency. Only light with wavelengths 400–700 nm can be seen.

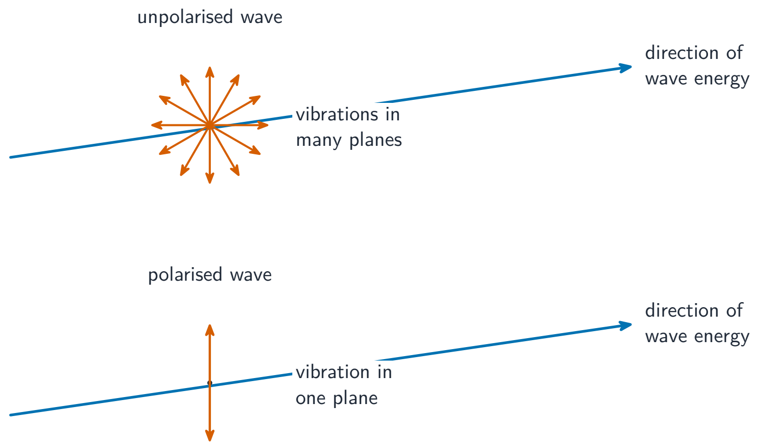
To identify a region, convert to a wavelength with $\lambda = c/f$ and compare with the ranges: 2.1 cm is a microwave; 138 pm $= 1.4 \times 10^{-10}$ m is an X-ray; 30 THz gives $\lambda = 1.0 \times 10^{-5}$ m, infrared; 3.0×10^{16} Hz gives 1.0×10^{-8} m, ultraviolet. Visible light spans frequencies of about 4.3×10^{14} to 7.5×10^{14} Hz, so a wave of 5.0×10^{14} Hz can be seen and one of wavelength 5.0×10^{-6} m (infrared) cannot. Red light has a longer wavelength and a lower frequency than green. A list "in order of increasing wavelength" runs γ-rays, X-rays, ultraviolet, visible, infrared, microwaves, radio waves. A pulse of light reflected from a wall 150 m away returns after $2 \times 150/(3.00 \times 10^8) = 1.0\ \mu$s.

Polarisation

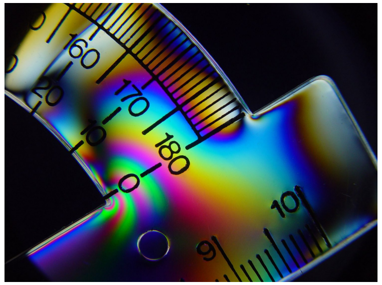
Polarisation 偏振 means making a transverse wave oscillate in one plane only.

- Only **transverse** waves can be polarised — the oscillation is perpendicular to the direction of travel, so different perpendicular planes are real choices.
- **Longitudinal** waves (sound) cannot be polarised — the oscillation is along the direction of travel, so there is no other plane.

So polarisation is a test: if a wave can be polarised, it must be transverse.



Unpolarised waves vibrate in many planes; a polarised wave vibrates in one plane



A see-through plastic protractor placed between two crossed polarising filters. Light only reaches your eye because the stressed plastic rotates its plane of polarisation — the colours map where the plastic is squeezed most. With ordinary light it would just look clear

Malus's law

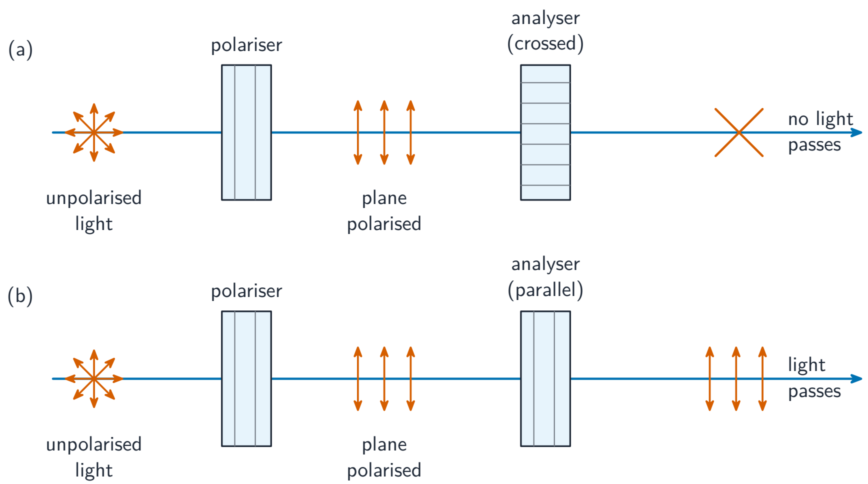
Plane-polarised 平面偏振 light of intensity I_0 passes through a **polarising filter** 偏振片 whose **transmission axis** 透光轴 is at angle θ to the plane of polarisation. The transmitted intensity is given by **Malus's law** 马吕斯定律:

$$I = I_0 \cos^2 \theta.$$

- $\theta = 0°$: filter lined up with the polarisation, $I = I_0$, all passes through.
- $\theta = 90°$: filter at right angles, $I = 0$, all blocked.
- $\theta = 60°$: $I = I_0 \cos^2 60° = I_0 \cdot 0.25 = I_0/4$.

Worked example. Plane-polarised light of intensity 12 W m^{-2} meets a polarising filter whose axis is at 30° to the plane of polarisation. Find the transmitted intensity.

$$I = I_0 \cos^2 \theta = 12 \times \cos^2 30° = 12 \times 0.75 = 9.0 \text{ W m}^{-2}.$$



Crossed filters (a) block the light; parallel filters (b) let it pass

For two filters in a row, use Malus's law twice with the angle between each pair. Be careful with the angle each time — after the first filter the polarisation is along that filter's axis, and the second filter's angle is measured from there.

(You do not need to work out the effect of a polarising filter on an unpolarised wave.)

When a filter is rotated through 360° in front of plane-polarised light, the transmitted intensity varies as $\cos^2\theta$: it is a maximum when the transmission axis is parallel to the plane of polarisation (0° and 180°), zero when perpendicular (90° and 270°), and never negative. Because $I \propto A^2$, the transmitted **amplitude** is $A_0 \cos\theta$: at 45° the intensity halves and the amplitude falls to 0.71 of its value. For two filters, the first sets the plane of polarisation along its own axis, so the second filter's angle is measured from that: vertically polarised light through filters at 50° and then 80° to the vertical keeps $\cos^2 50° \times \cos^2 30° = 0.31$ of its intensity. The one-mark definitions: polarisation is the oscillation of a transverse wave in a single plane, which contains the direction of travel; sound cannot be polarised because its oscillations are along the direction of travel, so there is no plane to select.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
progressive wave	a wave that transfers energy from one place to another without transferring matter
transverse wave	a wave whose oscillations are perpendicular to the direction of energy transfer
longitudinal wave	a wave whose oscillations are parallel to the direction of energy transfer
displacement	the distance of a particle from its equilibrium (rest) position, in a stated direction
amplitude	the maximum displacement of a particle from its equilibrium position
wavelength	the minimum distance between two points on the wave that are in phase (for example, adjacent crests)
period	the time for one complete oscillation of a particle (or for the wave to travel one wavelength)
frequency	the number of oscillations per unit time (or the number of wavefronts passing a point per unit time)
phase difference	the fraction of a cycle, as an angle, by which one oscillation leads or lags another
intensity	the power per unit area, at right angles to the direction of travel
Doppler effect	the change in the observed frequency of a wave when the source moves relative to the observer
polarisation	the oscillations of a transverse wave are in one plane only, containing the direction of travel
time-base	the time represented by one division across the screen of a CRO

Exam tips

- Define terms precisely (displacement, amplitude, wavelength, period, frequency) and use $v = f\lambda$.
- Distinguish **transverse** (vibration perpendicular to travel) from **longitudinal** (parallel); only transverse waves can be **polarised**.
- For the **Doppler effect**, the observed frequency **rises** as the source approaches and falls as it recedes.
- Learn the **electromagnetic spectrum** order; all its waves travel at c in a vacuum.

Common mistakes

- A phase difference bigger than one cycle, or degrees where radians were asked. Only the fraction of a cycle matters: 450° is 90°, and 90° is $\pi/2$.
- Reading the wavelength from a displacement–time graph. That graph gives the period; the wavelength comes from the displacement–distance graph.
- Doubling the amplitude and "doubling" the intensity. Intensity goes as the amplitude squared: four times.
- Choosing the Doppler sign by feel. Towards means minus on the bottom (frequency up); away means plus (frequency down). Check that the answer lies on the right side of f_s.
- Using the first filter's angle for the second. After a filter the light is polarised along that filter's axis, and the next angle is measured from there.
- Saying that sound can be polarised. Only transverse waves can.