Kinematics

A-Level Physics

Key definitions

A speedometer shows speed: the distance travelled per unit time.

These five quantities come up in almost every **kinematics** 运动学 question. Learn the exact words — the examiner gives marks for precise wording.

- **distance** 距离 — the total length of the path travelled. A **scalar** 标量.
- **displacement** 位移 — the straight-line distance from the start to the end, with a direction. A **vector** 矢量.
- **speed** 速率 — the rate of change of distance with time. A scalar.
- **velocity** 速度 — the rate of change of displacement with time. A vector.
- **acceleration** 加速度 — the rate of change of velocity with time. A vector.

The unit of speed and velocity is m s^{-1}; the unit of acceleration is m s^{-2}.

Two more phrases the examiner uses: **uniform acceleration** 匀加速 means constant acceleration (a straight line on a velocity–time graph), and the **acceleration of free fall** 自由落体加速度 g is the acceleration of an object falling freely in a **uniform gravitational field** 匀强重力场 when air resistance is **negligible** 可忽略的 (small enough to ignore).

A common mistake: **deceleration** 减速度 just means acceleration in the opposite direction to the velocity. It is not a separate quantity.

Speeds are sometimes given in km h^{-1}. Convert before you calculate: 85 km h^{-1} = $85 \times 1000/3600 = 23.6$ m s^{-1}.

A speed camera measures average speed over a known distance — displacement over time, made into a fine

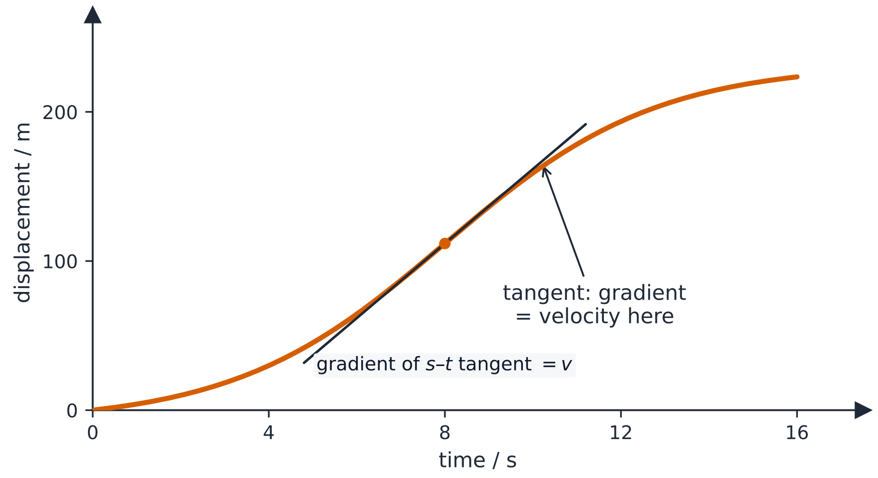
Motion graphs

Many marks come from reading or drawing motion graphs, the **graphical methods** 图像法 the syllabus names. Two graphs matter.

Displacement–time graph

The **gradient** 斜率 (steepness) of a displacement–time graph at a point gives the velocity at that moment.

- flat line → the object is at rest.
- straight sloping line → constant velocity (gradient = velocity).
- curved line → changing velocity. Draw a **tangent** 切线 at the point and find its gradient.



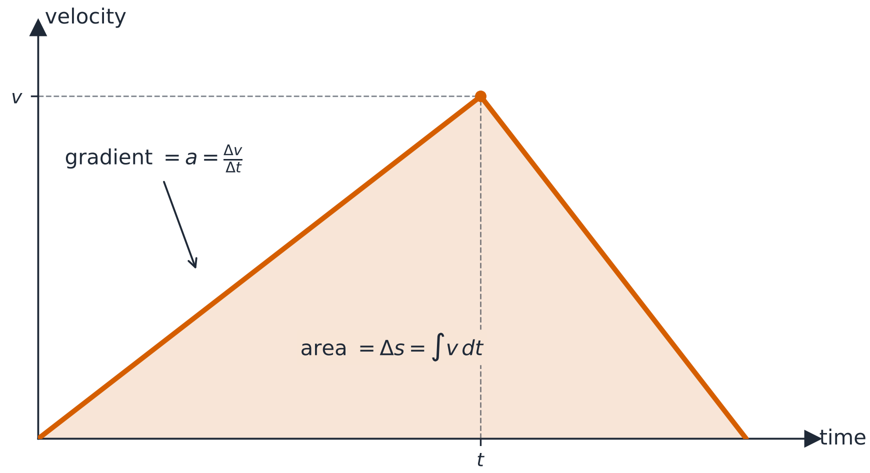
Displacement–time graph of a car on a test track

Velocity–time graph

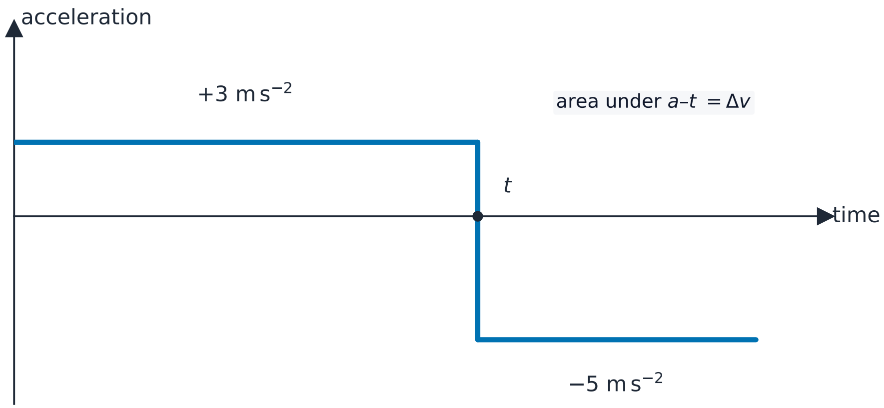
The gradient of a velocity–time graph gives the acceleration at that moment.

The **area** between the line and the time axis gives the displacement in that time.

- flat line → constant velocity (zero acceleration).
- straight sloping line → uniform acceleration (constant acceleration).
- curved line → changing acceleration.
- area above the time axis is positive displacement; area below is negative (the object moved backwards).



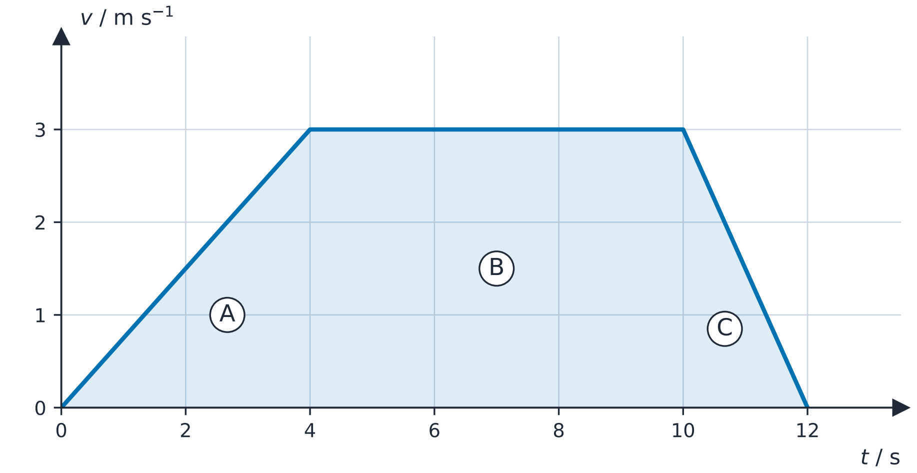
Velocity–time graph — gradient gives acceleration, area gives displacement



Acceleration–time graph derived from the same motion

To find the displacement, split the area into triangles and rectangles, or count grid squares. Area of a triangle is $\frac{1}{2}$ × base × height; area of a rectangle is base × height.

Worked example. The graph shows the velocity of a lift. Find the acceleration in each stage and the total distance travelled.



Velocity–time graph of a lift: gradient gives each acceleration, the shaded area gives the distance

- Stage A: gradient $= \dfrac{3.0 - 0}{4.0} = 0.75$ m s^{-2}.
- Stage B: the line is flat, so the acceleration is zero.
- Stage C: gradient $= \dfrac{0 - 3.0}{2.0} = -1.5$ m s^{-2} (a deceleration of 1.5 m s^{-2}).
- Distance: triangle A $= \frac{1}{2} \times 4.0 \times 3.0 = 6.0$ m, rectangle B $= 6.0 \times 3.0 = 18$ m, triangle C $= \frac{1}{2} \times 2.0 \times 3.0 = 3.0$ m; total 27 m.

When the graph is a curve, the area is still the displacement: count the squares under the curve (half a square or more counts as one) and multiply by the value of one square.

The four SUVAT equations

For motion in a straight line with uniform acceleration, we use five symbols: starting velocity u, final velocity v, acceleration a, displacement s, and time t. Four equations link them:

$$v = u + at$$

$$s = ut + \tfrac{1}{2}at^2$$

$$s = \tfrac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

Each equation uses four of the five symbols. To pick the right one: write down what you know and what you want, then choose the equation with exactly those four.

Worked example. A car accelerates uniformly from 8 m s^{-1} to 20 m s^{-1} over a distance of 56 m. Find its acceleration.

We know u, v and s and want a, so use $v^2 = u^2 + 2as$:

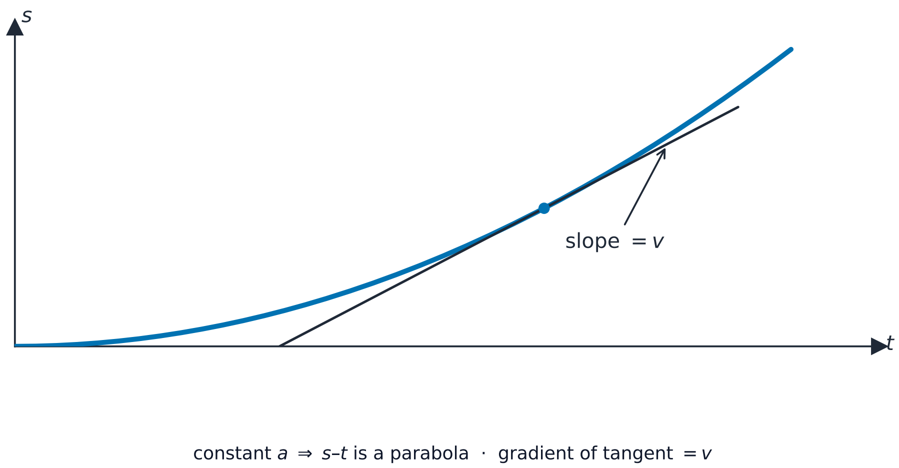
$$20^2 = 8^2 + 2a(56) \quad\Rightarrow\quad 336 = 112\,a \quad\Rightarrow\quad a = 3.0 \text{ m s}^{-2}.$$

Where the SUVAT equations come from

You should be able to get these from the definitions of velocity and acceleration. Think of the velocity–time graph of the motion: a straight line from u at $t = 0$ to v at time t.

- $v = u + at$: acceleration is the gradient, $a = (v - u)/t$, so $v = u + at$.
- $s = \frac{1}{2}(u + v)t$: displacement is the area under the line, a **trapezium** 梯形 with parallel sides u and v and width t, so $s = \frac{1}{2}(u + v)t$.
- $s = ut + \frac{1}{2}at^2$: put $v = u + at$ into the area: $s = \frac{1}{2}(u + u + at)t = ut + \frac{1}{2}at^2$.
- $v^2 = u^2 + 2as$: from the first equation $t = (v - u)/a$; put this into $s = \frac{1}{2}(u + v)t$ to get $2as = (v + u)(v - u) = v^2 - u^2$.

A “derive” question wants exactly these steps, each starting from a definition or an equation already established, with no numbers.



Displacement–time graph for uniform acceleration — the slope at any point equals the instantaneous velocity

If a question asks "which equation can be found using only the gradient of a velocity–time graph?", the answer is $v = u + at$ (the gradient is the acceleration).

"Show that" questions. When the question gives the answer ("show that the height is 3.2 km"), the marks are for the method. State the equation, substitute every value with its unit, and give the result to one more significant figure than the value quoted (3.24 km, which rounds to 3.2 km). Reaching the quoted value proves nothing on its own.

Choosing a positive direction

Pick a positive direction at the start and keep it. Anything pointing the other way gets a minus sign. For a ball thrown straight up, if "up" is positive: u is positive, $a = -g$ (**gravity** 重力 pulls down), and at the highest point the displacement is positive but the velocity is zero.

Free fall under gravity

When **air resistance** 空气阻力 can be ignored, an object in **free fall** 自由落体 has a constant acceleration $g \approx 9.81$ m s^{-2} downwards. This is the same for every mass.

For a ball dropped from rest and falling a distance h:

$$h = \tfrac{1}{2}gt^2, \qquad v = gt, \qquad v^2 = 2gh.$$

For a ball thrown straight up with speed u:

- greatest height: put $v = 0$ in $v^2 = u^2 - 2gh$, giving $h = u^2/(2g)$.
- time to reach the top: put $v = 0$ in $v = u - gt$, giving $t = u/g$.
- total time to fall back to the start height: $2u/g$ (the motion is **symmetric** 对称).

Worked example. A ball is thrown straight up at 20 m s^{-1}. Find the greatest height it reaches (take $g = 9.81$ m s^{-2}).

At the highest point $v = 0$, so from $h = u^2/(2g)$:

$$h = \frac{20^2}{2 \times 9.81} = \frac{400}{19.62} \approx 20.4 \text{ m}.$$

When air resistance is not negligible

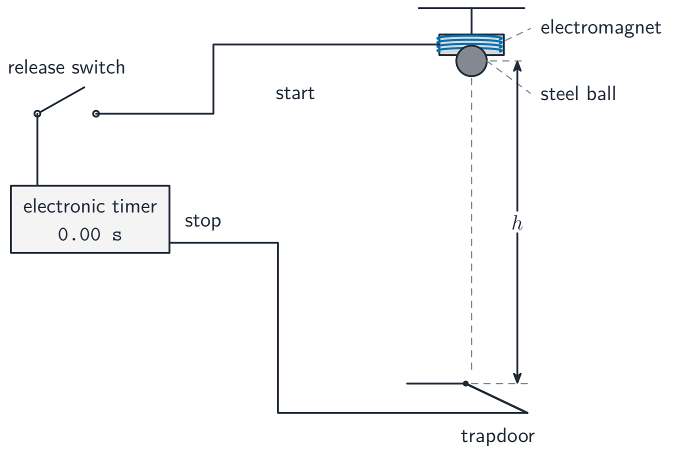
The equations above assume the only force is the weight. With air resistance, the resultant force on a falling object is smaller than its weight, so the acceleration is less than g; and because air resistance grows with speed, the acceleration keeps decreasing as the object speeds up. On a velocity–time graph the line curves, its gradient falling towards zero as the object approaches **terminal velocity** 收尾速度 (topic 3). For a projectile, air resistance shortens the range and lowers the maximum height, and the path is no longer a symmetrical **parabola** 抛物线: the object comes down more steeply than it went up. A "state and explain" question wants the force first, then its effect on the acceleration, then the effect on the motion.

Experiment to find g

A common method: drop an object from rest, then measure the distance h it falls and the time t it takes. Then

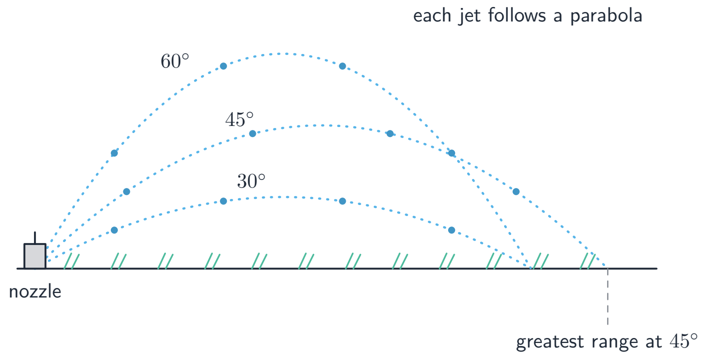
$$g = \frac{2h}{t^2}.$$

Repeat for several heights and plot h against t^2. The gradient of the best straight line is $g/2$, so g is twice the gradient. Repeating reduces **random error** 随机误差. An electronic timer — using **light gates** 光电门, or a switch the ball hits — removes **reaction-time** 反应时间 error.



Experimental set-up for measuring the acceleration due to free fall

Motion in two directions



Water jets from a sprinkler trace parabola paths — a real example of projectile motion

When an object moves at constant velocity in one direction (say **horizontal** 水平) and speeds up in a direction at right angles to it (say **vertical** 竖直, under gravity), the two motions do not affect each other. Treat each direction on its own, with its own SUVAT equation.

Horizontal throw

An object thrown horizontally with speed u_H from height h, with air resistance ignored:

- **horizontal**: constant velocity u_H. After time t, the horizontal distance is $x = u_H t$.
- **vertical**: starts from rest and speeds up downwards at g. After time t, it has fallen $y = \frac{1}{2}gt^2$ and has vertical velocity $v_V = gt$.

The time to reach the ground depends **only** on the height h, not on u_H. Solve $h = \frac{1}{2}gt^2$ for t; then the horizontal **range** 射程 is $u_H t$.

Worked example. A ball is thrown horizontally at 15 m s^{-1} from the top of a cliff 20 m high. Find the time it takes to land and how far from the base it lands (take $g = 9.81$ m s^{-2}).

Vertical motion gives the time: from $h = \frac{1}{2}gt^2$,

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 20}{9.81}} \approx 2.0 \text{ s}.$$

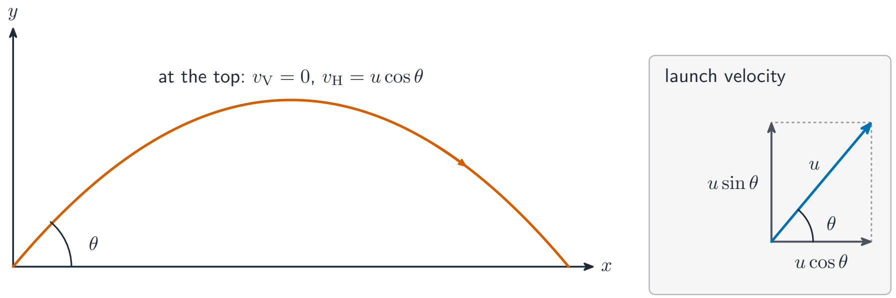
Horizontal motion then gives the range: $x = u_H t = 15 \times 2.0 \approx 30$ m.

The horizontal-velocity graph is a flat line at u_H. The vertical-velocity graph is a straight line from the origin with gradient g.

Comparing two objects. If object A is dropped and object B is thrown horizontally from the same height at the same moment, both reach the ground at the same time: their vertical motions are identical. If instead B is thrown with an upward component, it takes longer, because it must first rise and come back to the start height before it falls the same distance. Its speed on landing is still found from energy: the kinetic energy gained equals the potential energy lost, whatever the direction of the throw, so a ball launched at any angle with the same speed from the same height lands at the same speed (topic 5).

Projectile at an angle

A **projectile** 抛体 thrown at speed u at angle θ above the horizontal:

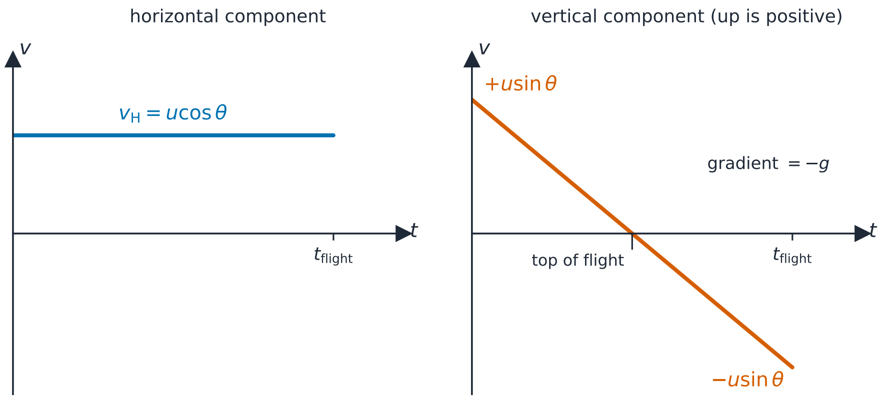


Projectile launched at angle θ — horizontal and vertical motions are independent

- horizontal **component** 分量 of the starting velocity: $u_H = u\cos\theta$ (stays constant during the flight).
- vertical component of the starting velocity: $u_V = u\sin\theta$ (gets smaller, becomes zero at the top, then grows downwards).

At the highest point, $v_V = 0$, but v_H is still $u\cos\theta$. The time to the top is $t_{up} = u\sin\theta/g$; the total flight time (back to the start height) is $2t_{up}$.

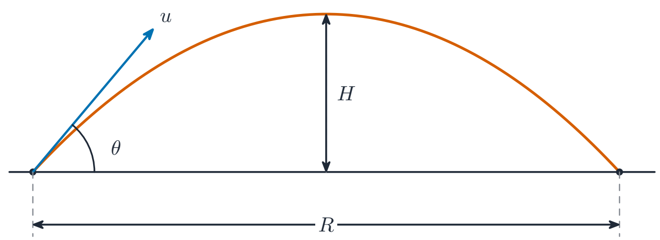
Sketching the velocity graphs. Examiners often ask you to sketch v_H and v_V against time on the same axes, taking upwards as positive. The horizontal component is a flat line at $u\cos\theta$ for the whole flight. The vertical component is a straight line of gradient $-g$: it starts at $+u\sin\theta$, crosses zero at the top of the flight, and reaches $-u\sin\theta$ on landing at the start height. Label each line, mark the times where the lines start and stop, and make the crossing sit exactly at t_{up}.



Velocity–time sketches for a projectile: the horizontal component stays constant; the vertical component falls in a straight line through zero at the top

Worked example. A ball leaves the ground at 18 m s^{-1} at 60° to the horizontal. Show that it reaches its maximum height at $t = 1.6$ s.

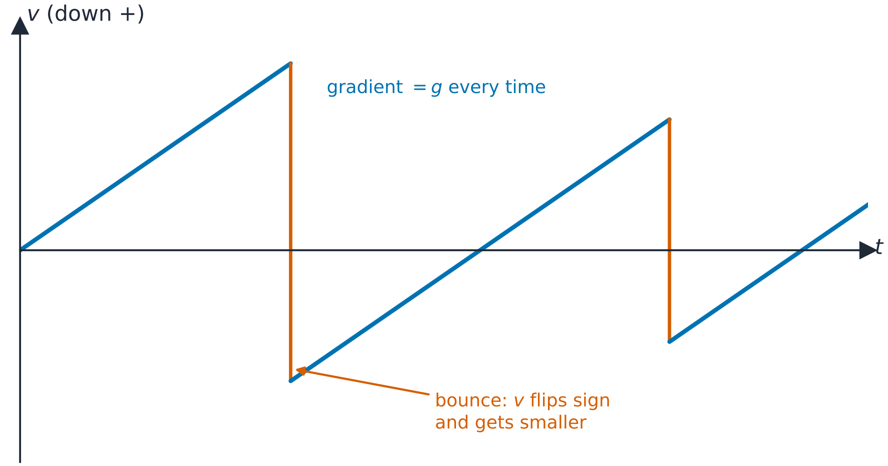
$u_V = 18\sin 60° = 15.6$ m s^{-1}. At the top $v_V = 0$, so from $v = u + at$ with $a = -9.81$ m s^{-2}: $t = 15.6/9.81 = 1.59$ s ≈ 1.6 s.



Range R of a projectile launched from and landing on level ground

Bouncing ball

When a ball bounces, its velocity–time graph is a set of straight sloping lines (constant g) with a sudden jump at each bounce (the velocity flips direction, and gets smaller if some **energy** 能量 is lost). Add up the times and the distances across the bounces.



A bouncing ball: every sloping line has gradient g; the velocity flips sign and shrinks at each bounce

Read such a graph carefully: the gradient of every sloping line is the same g; the ball is at its highest point wherever the line crosses the time axis; and the height of each bounce comes from the area of the triangle above (or below) the axis for that flight.

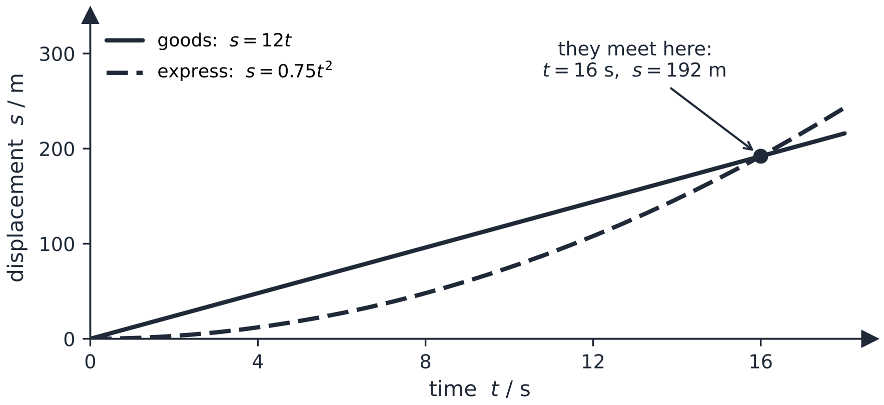
Two objects meeting

When two objects move along the same line in different ways, write a displacement equation for each. Use the same start time and the same positive direction. Then set the two displacements equal (or set their difference to a given gap).

For a goods train at constant velocity u_G and an express train starting from rest with acceleration a, both passing the same point at $t = 0$:

$$s_G = u_G t, \qquad s_E = \tfrac{1}{2}at^2.$$

They are level again when $s_G = s_E$, giving $t = 2u_G/a$.



Meet where the displacement–time graphs cross (same s at same t)

Tips for solving problems

1. **Draw a diagram** and mark the positive direction.
2. **List the SUVAT symbols** with their known and unknown values, including signs.
3. **Choose the SUVAT equation** with exactly the four symbols you have, plus the one you want.
4. For projectile motion, **split into horizontal and vertical** SUVAT problems, linked only by the time t.

5. Always **check the units** of your answer, and that its size is sensible.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
distance	the total length of the path travelled (a scalar)
displacement	the distance moved in a stated direction, from start to finish (a vector)
speed	the rate of change of distance with time
velocity	the rate of change of displacement with time
acceleration	the rate of change of velocity with time
uniform acceleration	acceleration that is constant in magnitude and direction
acceleration of free fall	the acceleration of an object falling freely under gravity alone, with air resistance negligible

Exam tips

- Use the **SUVAT** equations only for **constant acceleration**; list s, u, v, a, t and pick the equation missing your unknown.
- Choose one direction as **positive** and keep signs consistent (usually $g = -9.81$ m s^{-2}).
- On a **velocity-time graph**, gradient = acceleration and area = displacement.
- For projectiles, treat **horizontal (constant velocity) and vertical** ($a = g$) **motion separately**, linked by the same time.

Common mistakes

- Saying the heavier ball hits the ground first, or faster, when air resistance is ignored. Both fall with the same acceleration and land at the same speed.
- Taking the gradient of a v–t graph when the question wants the area, or the area when it wants the gradient. Gradient is acceleration; area is displacement.
- Drawing a curve on a v–t graph for a body under constant acceleration. Constant acceleration is a straight line; only the sign of v changes at a bounce.
- Using $g = +9.81$ for a ball thrown upwards with ”up” as positive. Once up is positive, $a = -9.81$ m s^{-2} for the whole flight, on the way down too.

- Using a SUVAT equation when the acceleration is changing (air resistance, a curved graph). Then only the graph methods work.