Week 42

A-Level Physics

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

19. Capacitance

Starter Quiz · 课前小测

A-Level Physics Name: ______________________ Score: ______

1. Capacitance is the charge stored per unit:

   ☐ potential difference

   ☐ current

   ☐ resistance

   ☐ time

2. The energy stored in a capacitor is:

   ☐ $\frac{1}{2}CV^2$

   ☐ CV^2

   ☐ $\frac{1}{2}CV$

   ☐ $\frac{1}{2}QV^2$

3. During discharge through a resistor, the charge on a capacitor:

   ☐ decays exponentially

   ☐ falls in a straight line

   ☐ stays constant

   ☐ rises steadily

4. A capacitor holds 0.012 C at 4.0 V. What is its capacitance, in mF?

   ________________ mF

5. A 2.0 μF capacitor is charged to 1000 V. How much energy does it store?

   ________________ J

6. Why is a capacitor's discharge exponential?

   ☐ the rate of loss of charge is proportional to the charge remaining

   ☐ the resistance increases as the capacitor empties

   ☐ the capacitance falls with time

   ☐ the current is constant

7. A charged capacitor holds +Q on one plate and -Q on the other, so the capacitor as a whole is neutral.

   ☐ True ☐ False

8. Why is there a factor of ½ in the stored-en[illegible]ormula?

- ☐ the average p.d. during charging is only V/2
- ☐ half the charge leaks away
- ☐ the capacitor is only half full
- ☐ energy is always halved

9. Doubling the charge on a capacitor doubles the voltage, so its capacitance stays the same.

   ☐ True ☐ False

10. The energy stored equals the ______ under the V–Q graph.

    ____

A-Level Physics

1. **potential difference**

   $C = \dfrac{Q}{V}$ — charge per volt, in farads.

2. $\frac{1}{2}CV^2$

   Equivalent forms: $W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \dfrac{Q^2}{2C}$.

3. **decays exponentially**

   The bigger the charge, the bigger the current — giving an exponential decay.

4. **3 mF**

   $C = \dfrac{Q}{V} = \dfrac{0.012}{4.0} = 0.0030\ \text{F} = 3.0\ \text{mF}$.

5. **1 J**

   $W = \frac{1}{2}CV^2 = \frac{1}{2} \times 2.0 \times 10^{-6} \times (1000)^2 = 1.0\ \text{J}$.

6. **the rate of loss of charge is proportional to the charge remaining**

   Q/C = -R dQ/dt says exactly that. Every quantity with this property, radioactive decay included, gives the same curve.

7. **True**

   Electrons move from one plate to the other through the battery, so the charges are always equal and opposite. The stored charge Q means the charge on one plate.

8. **the average p.d. during charging is only V/2**

   The p.d. rises from 0 to V as it charges, so the average is V/2 — hence the triangle area $\frac{1}{2}QV$.

9. **True**

   $C = \dfrac{Q}{V}$ is fixed by the capacitor's size and dielectric — Q and V rise together.

10. **area**

    The triangle under the line has area $\frac{1}{2}QV$ — the stored energy.

19.1 Capacitors and capacitance

Name: ______________ Class: __________ Date: ____________ **Total: 24 marks**

KEY WORDS capacitance 电容 • capacitor 电容器 • potential difference 电势差 • resistor 电阻器 • farad 法拉

Objective

Build the skills to answer exam questions on **capacitance** — $C = Q/V$ and combining capacitors.

You must be able to:

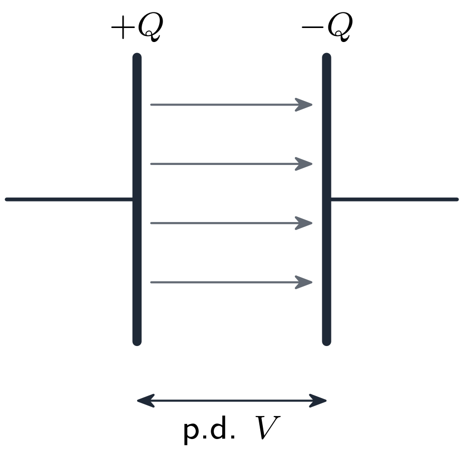
- define **capacitance** and use $C = Q/V$
- combine capacitors in **series** and **parallel**

1 Worked examples

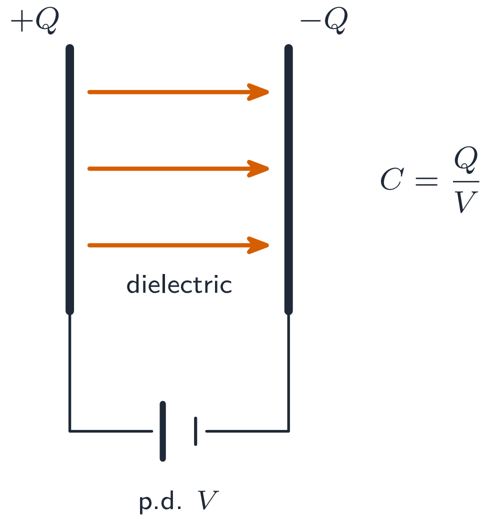
Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

Capacitance

A **capacitor** stores charge: $+Q$ on one plate, $-Q$ on the other, with p.d. V across the gap:



$$C = Q/V$$



A parallel-plate capacitor

$$C = \frac{Q}{V} \quad \text{(farad, F} = \text{C V}^{-1}\text{).}$$

■ **Combining capacitors (opposite to resistors)**

- **parallel:** $C = C_1 + C_2 + \dots$
- **series:** $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dots$

2 Practice

Now apply the methods above.

2.1 Define **capacitance**. [1]

2.2 State the **unit** of capacitance. [1]

2.3 A capacitor holds 60 μC of charge at 12 V. Find its **capacitance**. [2]

2.4 Write the formula for capacitors in **parallel**. [1]

3 Exam-style questions

3.1 The unit of capacitance is the: [1]

- **A** coulomb
- **B** farad
- **C** volt
- **D** ohm

3.2 A 100 μF capacitor is charged to 9.0 V. Calculate the **charge** stored. [2]

3.3 A 4.0 μF and a 2.0 μF capacitor are connected in **series**. Calculate the combined capacitance. [3]

3.4 Write the formula for capacitors in **series**. [1]

3.5 Three capacitors are available: $C_1 = 120\ \mu\text{F}$, $C_2 = 180\ \mu\text{F}$ and $C_3 = 220\ \mu\text{F}$.

(a) C_1 and C_2 are connected in **series** across a supply. **Complete** the table to show how the charge on each and the p.d. across each relate to the total charge Q_S and the supply p.d. V_S. [2]

quantity	**relationship**
charges Q_1, Q_2 and Q_S	______________
p.d.s V_1, V_2 and V_S	______________

(b) **Determine** the combined capacitance of C_1 and C_2 in series. [2]

(c) The supply provides 12 V. **Calculate** the charge stored by that series combination, and the p.d. across C_1. [3]

(d) C_3 is now connected in **parallel** with the series pair. **Determine** the total capacitance of the whole network, and the total charge it now stores at 12 V. [3]

(e) **State and explain** which single capacitor in the network stores the **most** charge. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the **charge stored per unit potential difference** ($C = Q/V$)

2.2

- the **farad** (F)

2.3

- $C = Q/V$

$$C = \frac{60 \times 10^{-6}\ \text{C}}{12\ \text{V}} = \mathbf{5.0}\ \mu\text{F}$$

2.4

- $C = C_1 + C_2 + \ldots$

3.1 B

- the SI unit of capacitance is the **farad**

3.2

- $Q = CV$

$$Q = (100 \times 10^{-6}\ \text{F})(9.0\ \text{V}) = \mathbf{9.0 \times 10^{-4}}\ \text{C}$$

3.3

- series: $1/C = 1/C_1 + 1/C_2$

$$\frac{1}{C} = \frac{1}{4.0\ \mu\text{F}} + \frac{1}{2.0\ \mu\text{F}} = \frac{3}{4.0\ \mu\text{F}}$$

- $C = \mathbf{1.3}\ \mu\text{F}$

3.4

- $1/C = 1/C_1 + 1/C_2 + \ldots$

3.5

(a) charges: $Q_1 = Q_2 = Q_\text{S}$ — the same charge passes through every capacitor in a series chain

- p.d.s: $V_1 + V_2 = V_\text{S}$

(b) $1/C = 1/120 + 1/180$

$$C = \frac{(120\ \mu\text{F})(180\ \mu\text{F})}{300\ \mu\text{F}} = \mathbf{72}\ \mu\text{F}$$

(c) $Q = CV$

$$Q = (72 \times 10^{-6}\ \text{F})(12\ \text{V}) = \mathbf{8.6 \times 10^{-4}}\ \text{C}$$

$$V_1 = \frac{Q}{C_1} = \frac{8.64 \times 10^{-4}\ \mathrm{C}}{120 \times 10^{-6}\ \mathrm{F}} = \mathbf{7.2}\ \mathrm{V}$$

- the **smaller** capacitor takes the larger share of the p.d.

(d) capacitors in parallel add: $C_{\text{total}} = 72\ \mu\mathrm{F} + 220\ \mu\mathrm{F} = \mathbf{292}\ \mu\mathrm{F}$

$$Q_{\text{total}} = (292 \times 10^{-6}\ \mathrm{F})(12\ \mathrm{V}) = \mathbf{3.5 \times 10^{-3}}\ \mathrm{C}$$

(e) C_3

- it has the full 12 V across it and the largest capacitance, so $Q = CV$ is largest; the series pair shares the same p.d. and behaves as only 72 μF

6 **(a)** Two parallel plate capacitors C_1 and C_2 are connected to a supply that has a potential difference (p.d.) V_S. The capacitors may be connected in series or in parallel.

The supply provides charge Q_S and the plates of the two capacitors acquire charges Q_1 and Q_2 respectively. The p.d.s across the plates of the capacitors are V_1 and V_2 respectively.

Complete Table 6.1 to indicate how Q_S, Q_1 and Q_2 relate to each other, and how V_S, V_1 and V_2 relate to each other, for series and parallel connections of the capacitors to the supply.

Table 6.1

	relationship between charges	relationship between p.d.s
series		
parallel		

[4]

(b) An isolated capacitor of capacitance 470 μF stores 19 mJ of energy.

(i) Calculate the p.d. across the capacitor.

p.d. = ... V [2]

(ii) Calculate the charge on the capacitor.

charge = ... C [2]

(iii) The capacitor is now connected in parallel with a capacitor of capacitance 180 μF that is initially uncharged.

Determine the total energy, in mJ, now stored in the two capacitors.

energy = ... mJ [3]

[Total: 11]

5 **(a)** A capacitor of capacitance C_1 is connected in series with a second capacitor of capacitance C_2.

Show that the combined capacitance C of the two capacitors is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}.$$

[2]

(b) Three identical capacitors, each of capacitance C, are connected in a network as shown in Fig. 5.1.

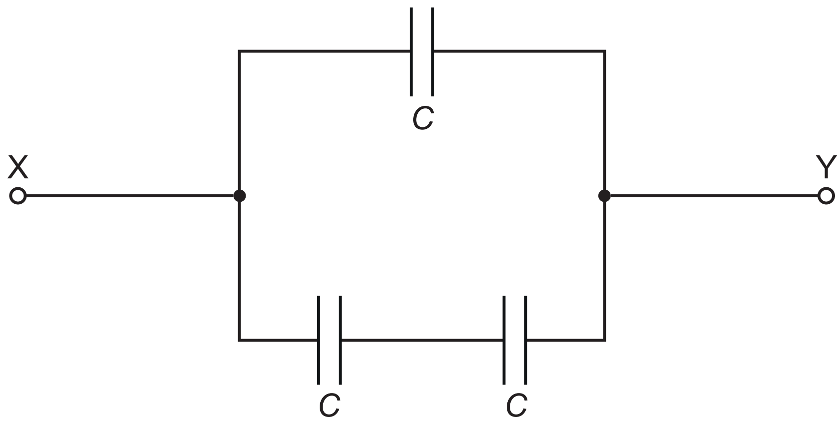


Fig. 5.1

The variation of the charge Q with the potential difference (p.d.) V between the terminals X and Y is shown in Fig. 5.2.

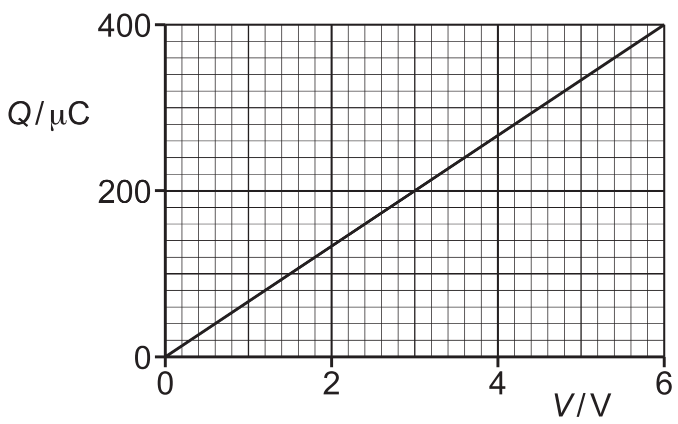


Fig. 5.2

Show that C is equal to $44\,\mu F$.

[3]

(c) The capacitor network in Fig. 5.1 is charged and then connected to a resistor of resistance $54\,k\Omega$. The capacitor network discharges through the resistor.

(i) Determine the time constant τ of the circuit. Give a unit with your answer.

τ = ... unit .... [2]

(ii) Determine the time taken for the discharge current to reduce to 15% of the initial discharge current.

time = ... s [2]

[Total: 9]

7 **(a)** Define the capacitance of a parallel-plate capacitor.

....

....

.... [2]

(b) An initially uncharged capacitor X, of capacitance *C*, is gradually charged so that the final potential difference (p.d.) between its plates is *V* and the final charge is *Q*.

(i) On Fig. 7.1, sketch the variation of charge with p.d. for capacitor X as the p.d. increases from 0 to *V*.

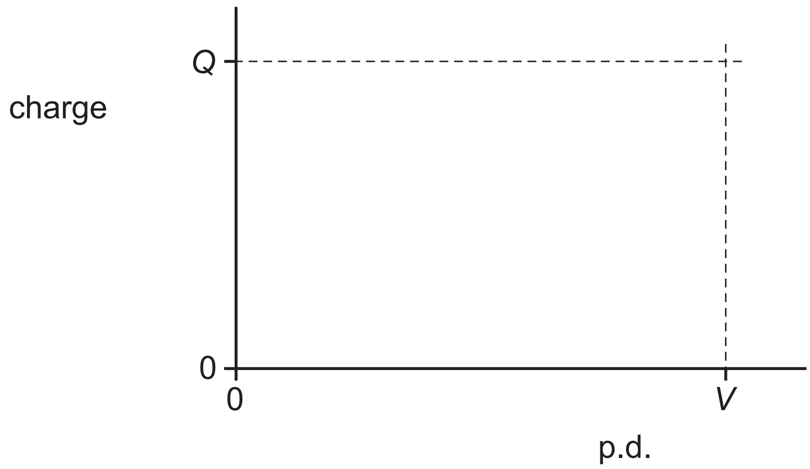


Fig. 7.1

[2]

(ii) Determine an expression, in terms of *Q* and *V*, for the work *W* done on capacitor X during the charging process. Explain your reasoning.

W = ... [2]

(c) Another capacitor Y is initially uncharged. The fully charged capacitor X in **(b)** is now connected to capacitor Y, as shown in Fig. 7.2.

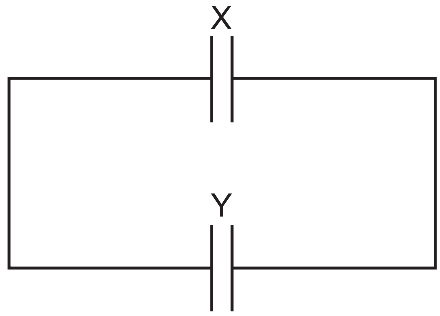


Fig. 7.2

The capacitance of capacitor Y is 3C.

(i) Complete Table 7.1 to show expressions, in terms of Q and V, for the final p.d.s across, and the final charges on, the two capacitors.
Use the space below for any working that you need.

Table 7.1

	X	Y
final p.d.		
final charge		

[3]

(ii) State whether the total energy stored in the two capacitors is less than, the same as, or greater than the energy initially stored in capacitor X.

.... [1]

[Total: 10]

6 **(a)** Two capacitors X and Y are connected in series to a power supply of voltage V, as shown in Fig. 6.1.

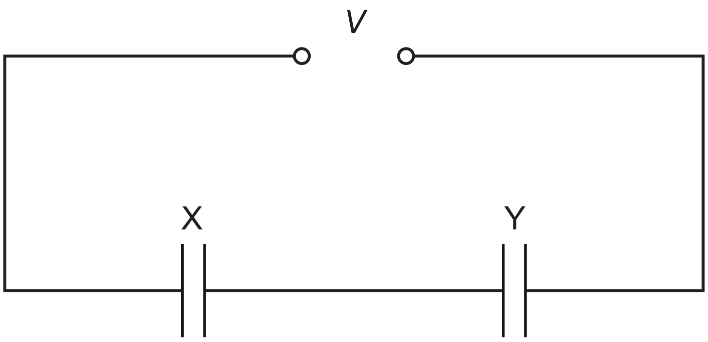


Fig. 6.1

The capacitance of X is C_X and the capacitance of Y is C_Y.

Derive an expression, in terms of C_X and C_Y, for the combined capacitance C_T of the capacitors in this circuit.
Explain your reasoning.

[3]

(b) Two capacitors P and Q are connected in parallel to a power supply of voltage V.
The capacitance of P is 200 μF. The capacitance C_Q of Q can be varied between 0 and 400 μF.
When $C_Q = 0$, the total energy stored in the capacitors is 2.5 mJ.

(i) Show that the supply voltage V is 5.0 V.

[2]

(ii) Calculate the total energy, in mJ, stored in the capacitors when C_Q has its maximum value.

total energy = ... mJ [3]

(iii) On Fig. 6.2, sketch the variation of the total energy E stored in the capacitors with C_Q, as C_Q varies from 0 to 400 μF.

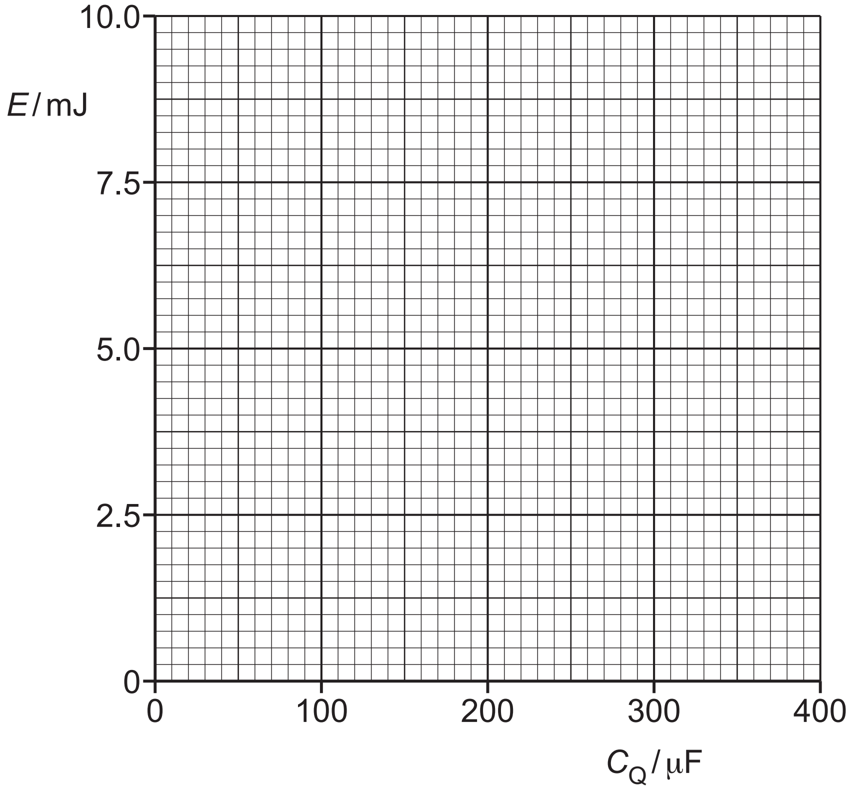


Fig. 6.2

[2]

[Total: 10]

19.2 Energy stored in a capacitor

Name: ______________ Class: __________ Date: ____________ **Total: 23 marks**

KEY WORDS energy 能量 • capacitor 电容器

Objective

Build the skills to answer exam questions on the **energy stored in a capacitor**.

You must be able to:

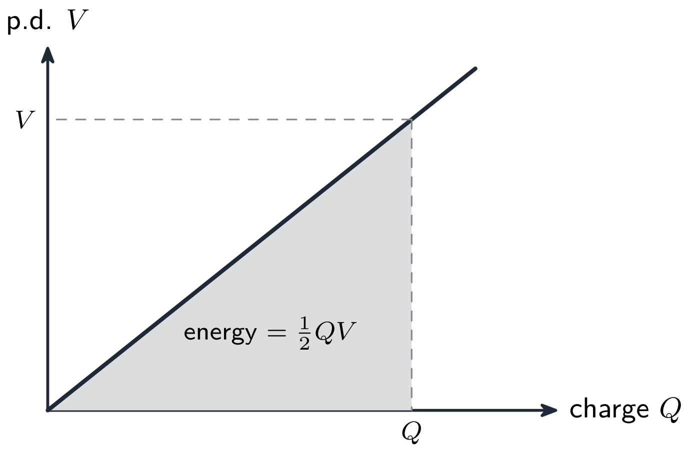
- find energy from the **area under the potential–charge graph**
- use $W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \dfrac{Q^2}{2C}$

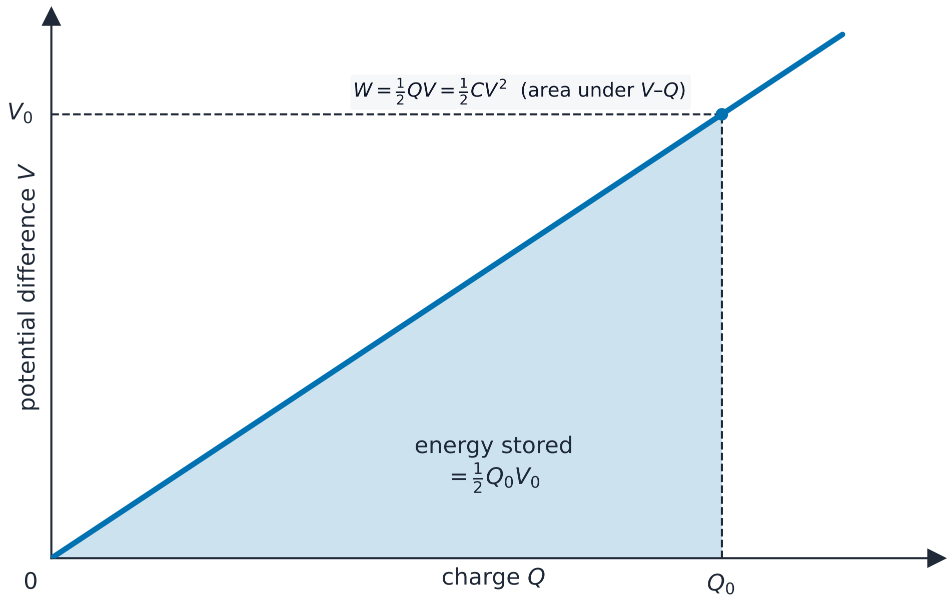
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

Energy stored

The energy is the **area under the V–Q graph** — a triangle:





Energy stored is the area under the Q-V graph

$$W = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}.$$

The factor $\frac{1}{2}$ appears because the **average** p.d. during charging is $V/2$ (it grows from 0 to V).

Worked example. 100 μF at 12 V: $W = \frac{1}{2}CV^2 = \frac{1}{2}(100\times10^{-6})(12)^2 = 7.2\times10^{-3}$ J.

2 Practice

Now apply the methods above.

2.1 Write the **three** forms of the capacitor energy formula. [1]

2.2 Find the energy stored in a 100 μF capacitor charged to 12 V. [2]

2.3 State what the **area** under a potential–charge graph represents. [1]

3 Exam-style questions

3.1 The energy stored in a capacitor is: [1]

- **A** QV
- **B** $\frac{1}{2}QV$
- **C** CV
- **D** Q^2C

3.2 A 220 μF capacitor stores 4.0 mJ of energy. Calculate the **voltage** across it. [3]

3.3 Explain why the energy stored is $\frac{1}{2}QV$ and **not** QV. [2]

3.4 A defibrillator stores energy in a 32 μF capacitor charged to 2.5 kV, and delivers it to a patient in a pulse lasting 8.0 ms.

(a) **Determine** the energy stored by the capacitor when fully charged. [2]

(b) **Determine** the charge stored. [2]

(c) In the pulse, 75% of the stored energy is delivered to the patient. **Determine** the average power delivered. [3]

(d) The capacitor is charged instead to 1.8 kV. **Determine** the percentage by which the stored energy falls, and **explain** why it is not the same percentage as the fall in p.d. [3]

(e) On the same axes, **sketch** how the energy stored varies with the p.d. across the capacitor, and how the **charge** stored varies with it. **Label** each line. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $W = \frac{1}{2}QV = \frac{1}{2}CV^2 = Q^2/(2C)$

2.2

- $W = \frac{1}{2}CV^2$

$$W = \tfrac{1}{2}(100 \times 10^{-6}\ \text{F})(12\ \text{V})^2 = \mathbf{7.2 \times 10^{-3}}\ \text{J}$$

2.3

- the **energy stored** in the capacitor

3.1 B

- $W = \frac{1}{2}QV$

3.2

- $W = \frac{1}{2}CV^2 \Rightarrow V = \sqrt{2W/C}$

$$V = \sqrt{\frac{2 \times 4.0 \times 10^{-3}\ \text{J}}{220 \times 10^{-6}\ \text{F}}} = \mathbf{6.0}\ \text{V}$$

3.3

- as the capacitor charges, the p.d. rises from 0 to V
- the **average** p.d. is $V/2$, so the work done is $Q \times (V/2) = \frac{1}{2}QV$, not QV

3.4

(a) $E = \frac{1}{2}CV^2$

$$E = \tfrac{1}{2}(32 \times 10^{-6}\ \text{F})(2500\ \text{V})^2 = \mathbf{100}\ \text{J}$$

(b) $Q = CV$

$$Q = (32 \times 10^{-6}\ \text{F})(2500\ \text{V}) = \mathbf{8.0 \times 10^{-2}}\ \text{C}$$

(c) energy delivered $= 0.75 \times 100\ \text{J} = 75\ \text{J}$

$$P = \frac{E}{t} = \frac{75\ \text{J}}{8.0 \times 10^{-3}\ \text{s}} = \mathbf{9.4 \times 10^{3}}\ \text{W}$$

(d) new energy $= \frac{1}{2}(32 \times 10^{-6}\ \text{F})(1800\ \text{V})^2 = 51.8\ \text{J}$

$$\frac{100\ \text{J} - 51.8\ \text{J}}{100\ \text{J}} = \mathbf{48\%}$$

- the p.d. fell by only 28%, because $E \propto V^2$, so the fractional change in energy is roughly **twice** the fractional change in p.d.

(e) charge: a **straight line through the origin**, since $Q = CV$

- energy: a **curve through the origin** getting steeper, since $E \propto V^2$
- both labelled, with axes labelled

6 **(a)** Two parallel plate capacitors C_1 and C_2 are connected to a supply that has a potential difference (p.d.) V_S. The capacitors may be connected in series or in parallel.

The supply provides charge Q_S and the plates of the two capacitors acquire charges Q_1 and Q_2 respectively. The p.d.s across the plates of the capacitors are V_1 and V_2 respectively.

Complete Table 6.1 to indicate how Q_S, Q_1 and Q_2 relate to each other, and how V_S, V_1 and V_2 relate to each other, for series and parallel connections of the capacitors to the supply.

Table 6.1

	relationship between charges	relationship between p.d.s
series		
parallel		

[4]

(b) An isolated capacitor of capacitance 470 μF stores 19 mJ of energy.

(i) Calculate the p.d. across the capacitor.

p.d. = ...V [2]

(ii) Calculate the charge on the capacitor.

charge = ... C [2]

(iii) The capacitor is now connected in parallel with a capacitor of capacitance 180 μF that is initially uncharged.

Determine the total energy, in mJ, now stored in the two capacitors.

energy = ... mJ [3]

[Total: 11]

4 **(a)** Three capacitors are connected as shown in Fig. 4.1.

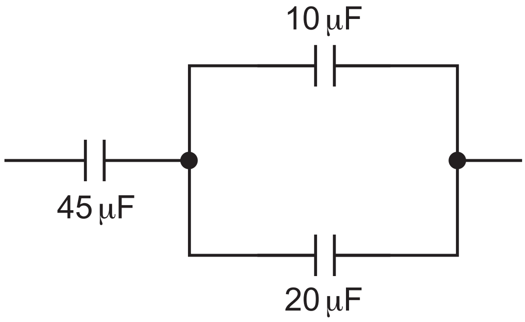


Fig. 4.1

Determine the total capacitance, in μF, of the network of three capacitors.

capacitance = ... μF [2]

(b) A capacitor of capacitance 45 μF is connected to a variable power supply initially set at 8.0 V. The output of the power supply increases so that the potential difference (p.d.) across the capacitor increases to 9.6 V.

Calculate the increase in energy ΔE stored in the capacitor.

ΔE = ... J [2]

(c) A sinusoidal a.c. power supply is connected to the input of a bridge rectifier. The output of the rectifier is connected to a load resistor.

(i) Complete the circuit in Fig. 4.2 by adding a capacitor to smooth the p.d. across the load resistor.

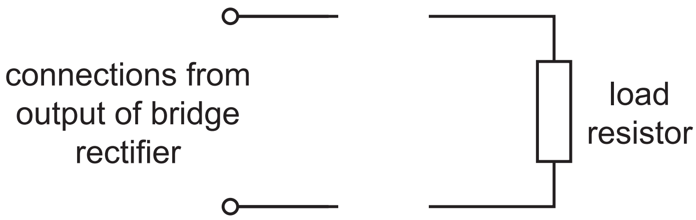


Fig. 4.2

[1]

(ii) The variation with time t of the p.d. V of the smoothed output is shown in Fig. 4.3.

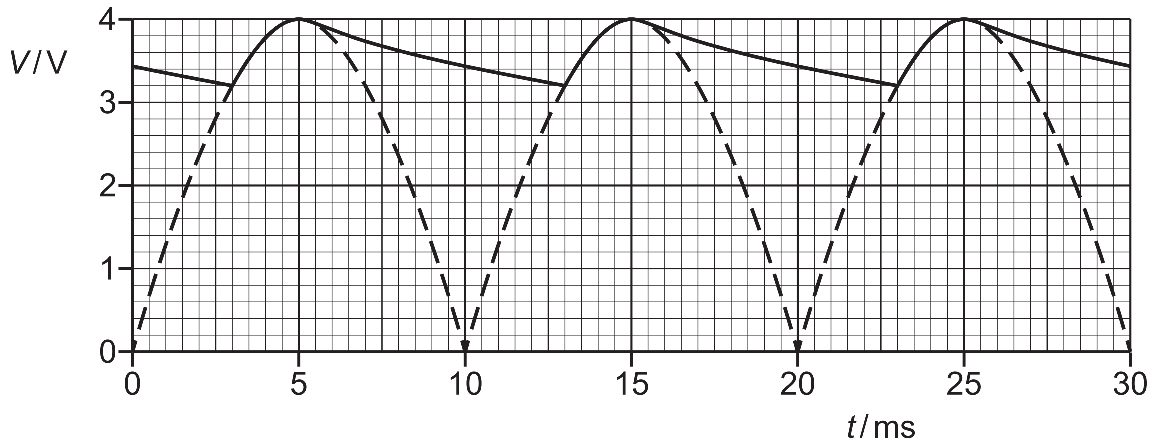


Fig. 4.3

Determine the time constant, in ms, of the smoothing circuit.

time constant = ... ms [3]

(d) A sinusoidal a.c. power supply has a maximum power of 16 W.

State the value of the mean power when the output of the power supply is:

(i) full-wave rectified

mean power = ... W [1]

(ii) half-wave rectified.

mean power = ... W [1]

[Total: 10]

5 A capacitor, a battery of electromotive force (e.m.f.) 12 V, a resistor R and a two-way switch are connected in the circuit shown in Fig. 5.1.

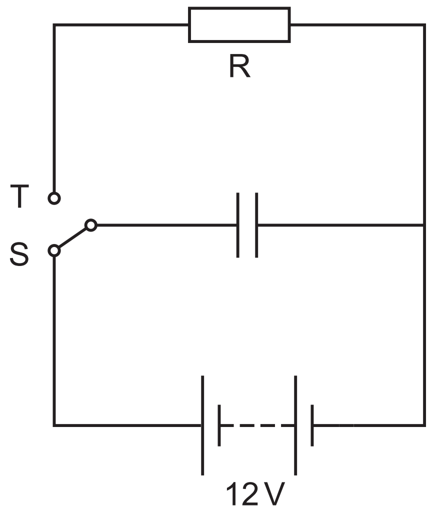


Fig. 5.1

The switch is initially in position S. When the capacitor is fully charged, the switch is moved to position T so that the capacitor discharges. At time t after the switch is moved the charge on the capacitor is Q.

The variation with t of $\ln(Q/\mu C)$ is shown in Fig. 5.2.

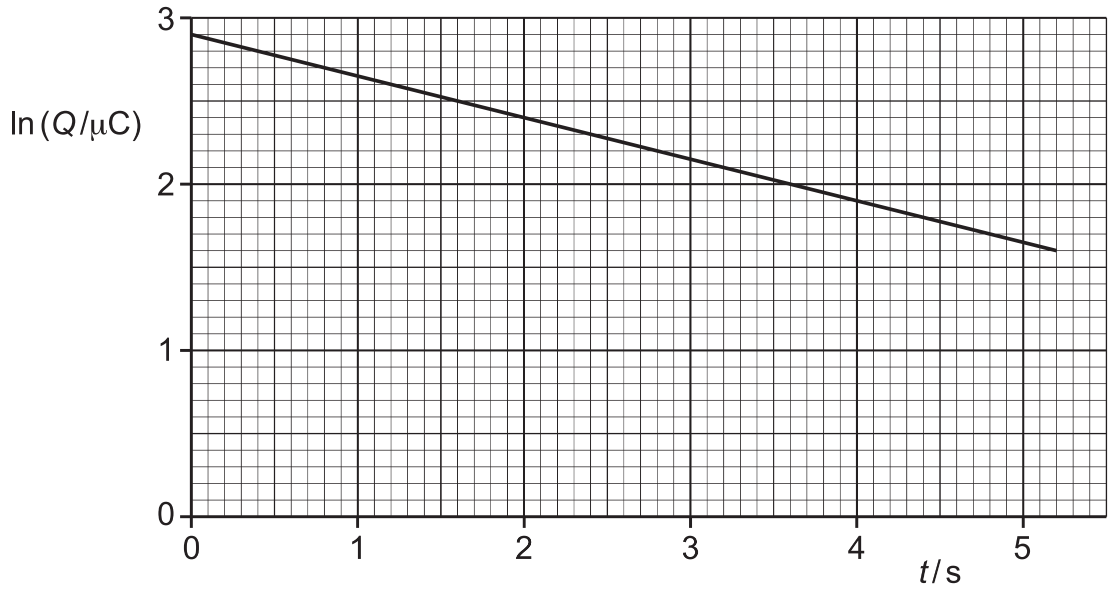


Fig. 5.2

(a) Show that the capacitance of the capacitor is 1.5 μF.

[3]

(b) Determine the resistance of R.

resistance = ... Ω [3]

(c) Calculate the energy stored in the capacitor at time $t = 0$.

energy = ... J [2]

(d) A second identical resistor is now connected in parallel with R.

The switch is initially in position S. When the capacitor is fully charged, the switch is moved to position T so that the capacitor discharges. At time t after the switch is moved the charge on the capacitor is Q.

On Fig. 5.2, sketch a line to show the variation of $\ln(Q/\mu C)$ with t between time $t = 0$ and time $t = 5.0$ s. [2]

[Total: 10]

A-LEVEL PHYSICS • EXERCISE SHEET

19.3 Discharging a capacitor

Name: ______________ Class: __________ Date: ____________ **Total: 29 marks**

KEY WORDS time constant 时间常数 • exponential decay 指数衰减 • resistor 电阻器 • resistance 电阻 • capacitor 电容器

Objective

Build the skills to answer exam questions on a **capacitor discharging** through a resistor.

You must be able to:

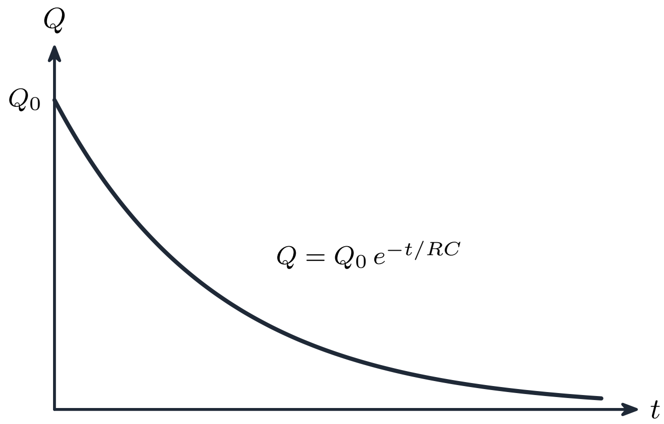
- use the **time constant** $\tau = RC$
- use $x = x_0 e^{-t/RC}$ for charge, p.d. or current
- interpret the discharge graphs

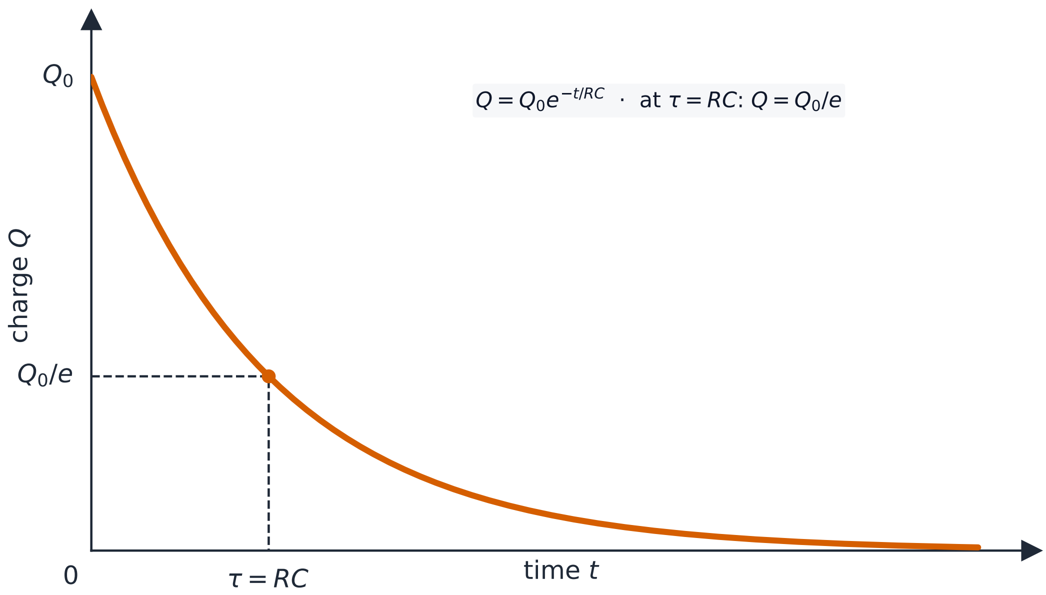
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Exponential decay

When a charged capacitor discharges through a resistor, the charge (and V, I) decays **exponentially**:





Exponential discharge of a capacitor

$$Q = Q_0 e^{-t/RC}, \quad V = V_0 e^{-t/RC}, \quad I = I_0 e^{-t/RC}.$$

The **time constant** $\tau = RC$ is the time for the charge to fall to $\frac{1}{e} \approx 37\%$ of its initial value.

Worked example. 470 μF through 2.0 kΩ: $\tau = RC = (2000)(470 \times 10^{-6}) = 0.94$ s.

■ **Finding the time — take logs**

To find the **time** for the charge (or V, or I) to fall to a given value, take the **natural log** of both sides of $V = V_0 e^{-t/RC}$:

$$\ln \frac{V}{V_0} = -\frac{t}{RC} \quad \Longrightarrow \quad t = -RC \ln \frac{V}{V_0}.$$

Worked example. For the capacitor above (charged to 6.0 V, $\tau = 0.94$ s), the time for the p.d. to fall to 2.0 V is $t = -0.94 \ln \dfrac{2.0}{6.0} = -0.94 \times (-1.10) = 1.0$ s.

2 Practice

Now apply the methods above.

2.1 Write the formula for the **time constant**. [1]

2.2 A 100 μF capacitor discharges through a 10 kΩ resistor. Find the **time constant**. [2]

2.3 State what happens to the **charge** as a capacitor discharges. [1]

2.4 Write the discharge equation for **charge**. [1]

3 Exam-style questions

3.1 The time constant RC is the time for the charge to fall to: [1]

- **A** zero
- **B** half its initial value
- **C** $1/e$ (about 37%) of its initial value
- **D** $1/2e$ of its initial value

3.2 A 470 μF capacitor charged to 6.0 V discharges through a 2.0 kΩ resistor. Calculate (a) the time constant, (b) the p.d. after 1.0 s. [4]

3.3 Sketch a graph of **charge against time** for a discharging capacitor. [2]

3.4 State what happens to the time constant if the **resistance** is doubled. [1]

3.5 A 220 μF capacitor charged to 9.0 V discharges through a 15 kΩ resistor. Calculate the **time** for the p.d. to fall to 3.0 V. [4]

3.6 A capacitor C is charged and then discharged through a resistor R. A data logger records the p.d. V_R across the resistor and the p.d. V_C across the capacitor. At $t = 0$ the p.d. across the capacitor is 9.0 V, and the initial current is 1.8 mA. The p.d. across the capacitor falls to 3.3 V after 12 s.

(a) **State** the relationship between V_C and V_R during the discharge, and give a reason. [2]

(b) **Determine** the resistance R. [2]

(c) **Determine** the time constant τ of the circuit. [3]

(d) Hence **determine** the capacitance C, in μF. [2]

(e) **Explain**, in terms of the current in the circuit, why the decay of the charge on the capacitor is **exponential**. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $\tau = RC$

2.2

- $\tau = RC$

$$\tau = (10 \times 10^3\ \Omega)(100 \times 10^{-6}\ \text{F}) = \mathbf{1.0}\ \text{s}$$

2.3

- it **decreases exponentially** towards zero

2.4

- $Q = Q_0 e^{-t/RC}$

3.1 C

- after one time constant, $Q = Q_0/e$ (about 37%)

3.2

(a) $\tau = RC$

$$\tau = (2.0 \times 10^3\ \Omega)(470 \times 10^{-6}\ \text{F}) = \mathbf{0.94}\ \text{s}$$

(b) $V = V_0 e^{-t/RC}$

$$V = (6.0\ \text{V})\, e^{-1.0/0.94} = (6.0\ \text{V})(0.345) = \mathbf{2.1}\ \text{V}$$

3.3

- a curve starting at the initial charge and **decaying exponentially** towards zero
- with the same shape (never quite reaching zero)

3.4

- it **doubles** ($\tau = RC$)

3.5

- $\tau = RC$

$$\tau = (15 \times 10^3\ \Omega)(220 \times 10^{-6}\ \text{F}) = 3.3\ \text{s}$$

- $t = -RC \ln(V/V_0)$

$$t = -3.3\ \text{s} \times \ln \frac{3.0\ \text{V}}{9.0\ \text{V}} = -3.3\ \text{s} \times (-1.10) = \mathbf{3.6}\ \text{s}$$

3.6

(a) they are **equal** at every instant, $V_C = V_R$

- the capacitor and resistor form a single loop with no other source, so the p.d. driving the current is the capacitor's own p.d.

(b) $R = V/I$

$$R = \frac{9.0\ \text{V}}{1.8 \times 10^{-3}\ \text{A}} = \mathbf{5.0 \times 10^3}$$

(c) $V = V_0 e^{-t/\tau}$, so $3.3/9.0 = e^{-12/\tau}$

$$\ln(0.367) = -\frac{12\ \text{s}}{\tau} \Rightarrow \tau = \mathbf{12}\ \text{s}$$

- the p.d. has fallen to $1/e$ of its start, which is what the time constant means

(d) $\tau = RC$, so $C = \tau/R$

$$C = \frac{12\ \text{s}}{5.0 \times 10^3\ \Omega} = 2.4 \times 10^{-3}\ \text{F} = \mathbf{2.4 \times 10^3}\ \mu\text{F}$$

(e) the current is $I = V_C/R$, and $V_C \propto Q$, so the current — the rate at which charge leaves — is **proportional to the charge still on the capacitor**

- a quantity whose rate of decrease is proportional to itself decays exponentially
- so as the charge falls the current falls with it, the discharge slows, and Q never quite reaches zero

7 Fig. 7.1 shows a circuit containing a capacitor of capacitance *C* and a resistor of resistance *R*.

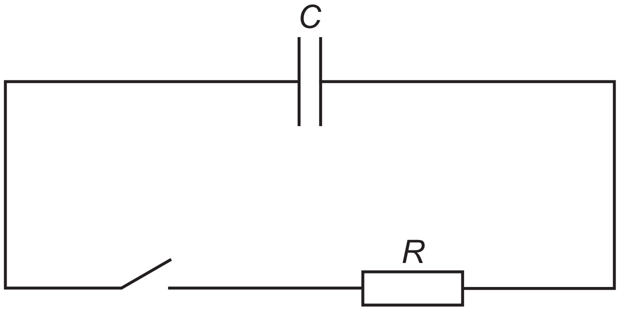


Fig. 7.1

Initially, the switch is open and the potential difference (p.d.) across the capacitor is 12 V.

The switch is closed at time $t = 0$ and the capacitor discharges through the resistor.

Fig. 7.2 shows the variation of the charge *Q* on the capacitor with the p.d. V_C across the capacitor as the capacitor discharges. Fig. 7.3 shows the variation of the current *I* in the resistor with the p.d. V_R across the resistor as the capacitor discharges.

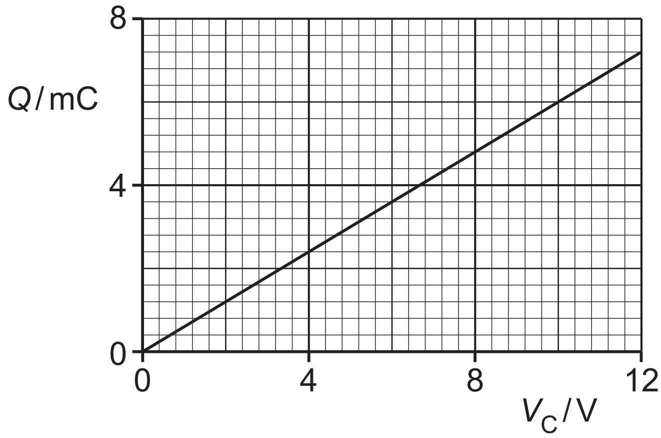


Fig. 7.2

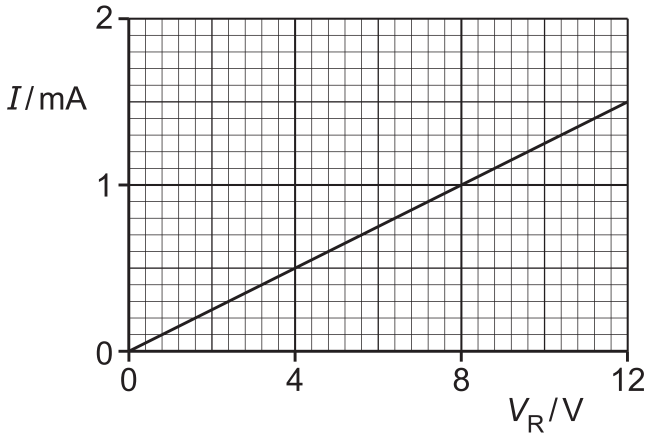


Fig. 7.3

(a) State the relationship between V_C and V_R.

.... [1]

(b) Determine:

(i) the capacitance *C*, in μF

C = ... μF [2]

(ii) the resistance R, in kΩ

R = ... kΩ [2]

(iii) the time constant τ of the circuit.

τ = ... s [2]

(c) Use Fig. 7.2, Fig. 7.3 and your answer in **(a)** to explain why the variation of Q with t is exponential in nature.

....

....

....

....

.... [3]

[Total: 10]

6 **(a)** **(i)** State what is meant by rectification of an alternating voltage.

....

.... [1]

(ii) State the difference between half-wave rectification and full-wave rectification.

....

....

.... [2]

(b) **(i)** Complete Fig. 6.1 to show a circuit that produces half-wave rectification of an alternating input voltage V_{IN} to produce output voltage V_{OUT} across the resistor R.

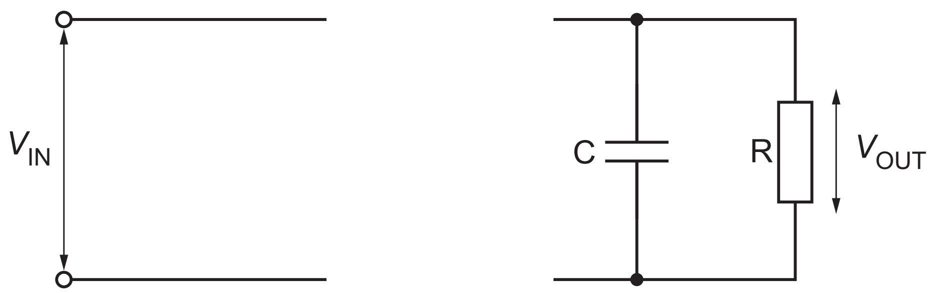


Fig. 6.1

[2]

(ii) State the purpose of the capacitor C in the circuit of Fig. 6.1.

....

.... [1]

(c) The input voltage V_{IN} in Fig. 6.1 is a square wave. Fig. 6.2 shows the variation of V_{IN} with time t.

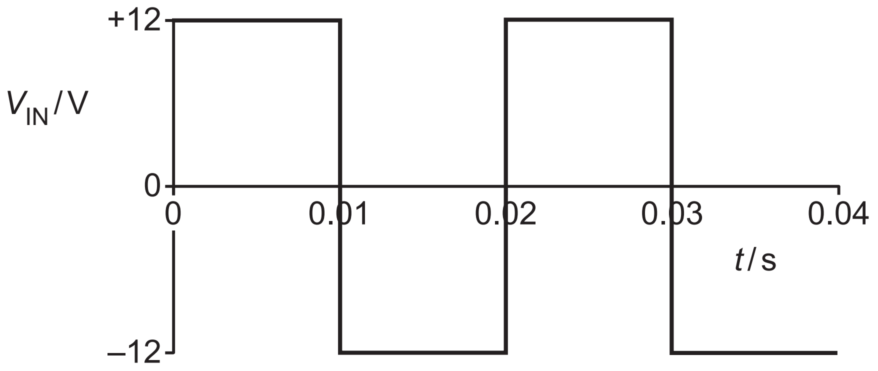


Fig 6.2

Fig. 6.3 shows the variation of V_{OUT} with t.

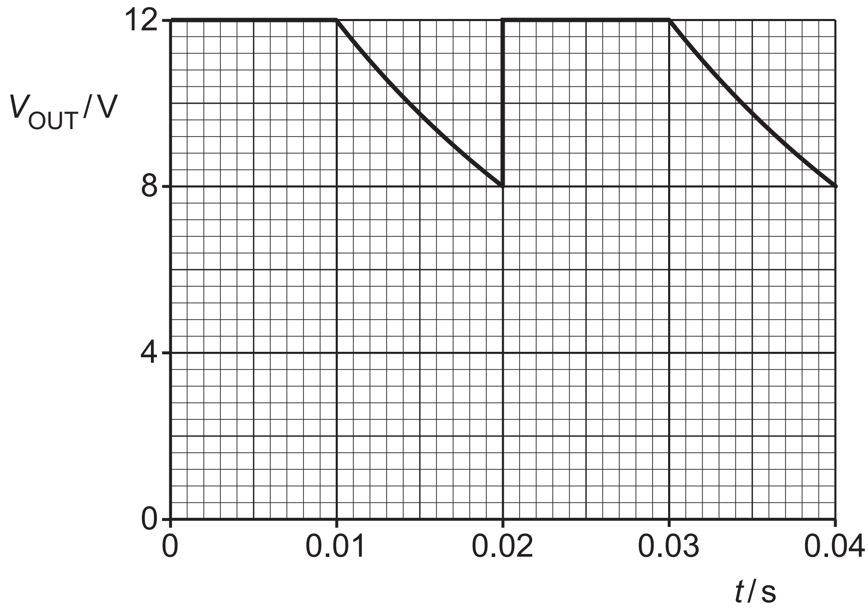


Fig. 6.3

The maximum energy stored in the capacitor is 0.041 J.

(i) Show that the capacitance of C is 570 μF.

[2]

(ii) Determine the resistance of R.

resistance = ... Ω [3]

[Total: 11]

6 Fig. 6.1 shows a capacitor of capacitance *C* connected in series with a resistor of resistance *R*.

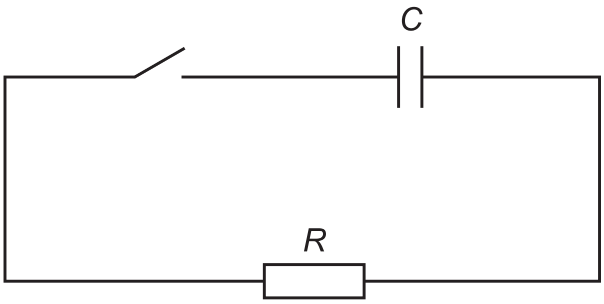


Fig. 6.1

Initially the switch is open and there is a p.d. of 12 V across the capacitor.

At time $t = 0$, the switch is closed so that there is a current I in the resistor.

Fig. 6.2 shows the variation of I with t.

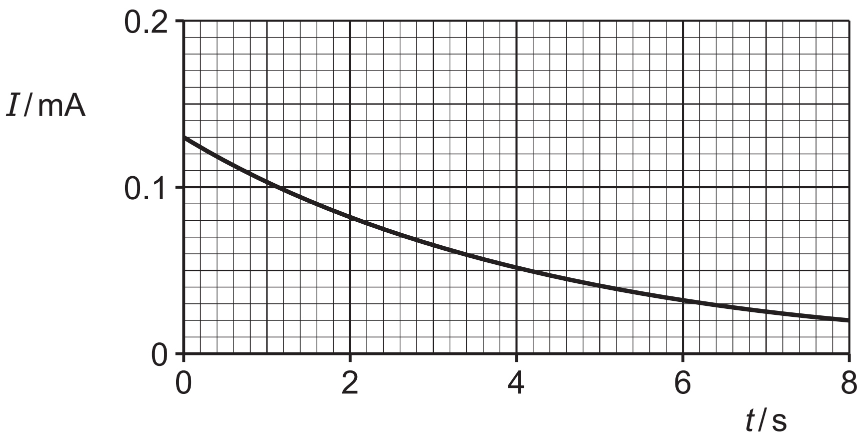


Fig. 6.2

(a) Explain the shape of the line in Fig. 6.2.

....

....

....

....

.... [3]

(b) Use Fig. 6.2 to determine:

(i) resistance R

R = ... Ω [2]

(ii) the time constant τ of the circuit in Fig. 6.1.

τ = ... s [3]

(c) Use your answers in **(b)** to determine capacitance C.

C = ... F [2]

[Total: 10]

5 Part of an electric circuit is shown in Fig. 5.1.

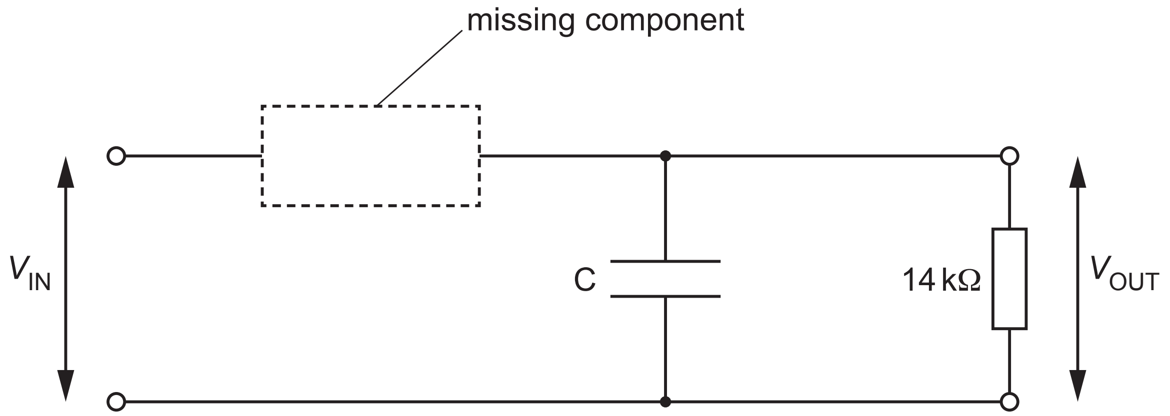


Fig. 5.1

The circuit is used to produce half-wave rectification of an alternating voltage of potential difference (p.d.) V_{IN}.

The output p.d. across the 14 kΩ resistor is V_{OUT}.

(a) (i) A component is missing from the circuit of Fig. 5.1.

Complete the circuit diagram in Fig. 5.1 by adding the circuit symbol for the missing component, correctly connected. [1]

(ii) A capacitor C is shown in the circuit of Fig. 5.1.

State the effect on V_{OUT} of including the capacitor in the circuit.

.... [1]

(b) Fig. 5.2 shows the variation with time t of V_{IN}.

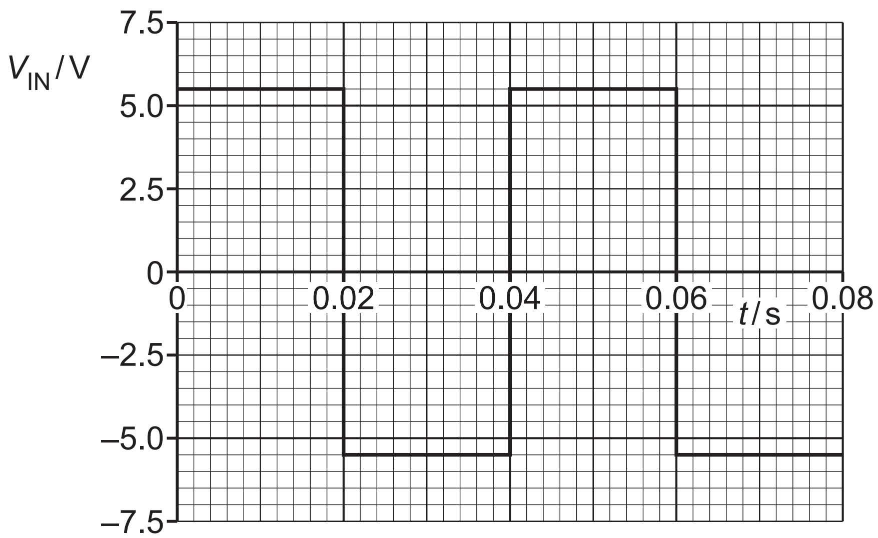


Fig. 5.2

Fig. 5.3 shows the variation with t of V_{OUT}.

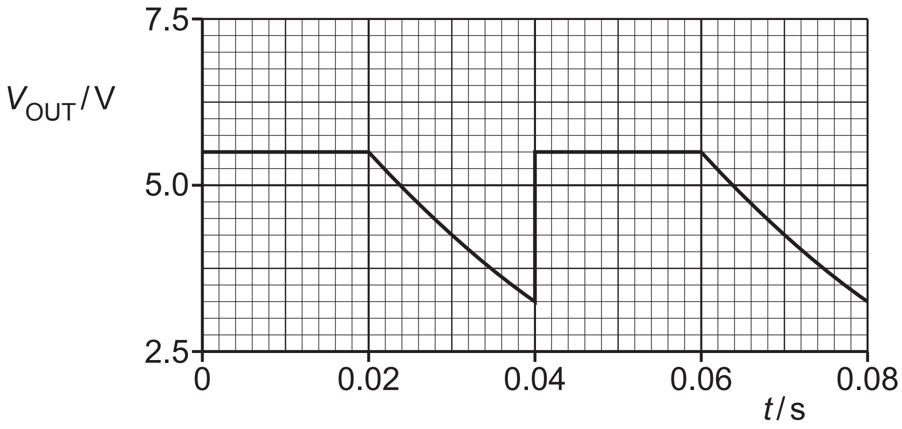


Fig. 5.3

(i) Determine the frequency of V_{IN}.

frequency = ... Hz [1]

(ii) Show that the time constant τ for the discharge of the capacitor through the resistor is 0.038 s.

[2]

(iii) Calculate the capacitance of C. Give a unit with your answer.

capacitance = ... unit .... [2]

(c) The circuit of Fig. 5.1 is modified so that it produces full-wave rectification of an input voltage.

Suggest, with a reason, how V_{OUT} now varies with time when V_{IN} is as shown in Fig. 5.2.

....

....

.... [2]

[Total: 9]