Week 41

A-Level Physics

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

19. Capacitance

Starter Quiz · 课前小测

A-Level Physics　　Name: ____________________　　Score: ______

1. Capacitance is the charge stored per unit:

   ☐ potential difference

   ☐ current

   ☐ resistance

   ☐ time

2. The energy stored in a capacitor is:

   ☐ $\frac{1}{2}CV^2$

   ☐ CV^2

   ☐ $\frac{1}{2}CV$

   ☐ $\frac{1}{2}QV^2$

3. During discharge through a resistor, the charge on a capacitor:

   ☐ decays exponentially

   ☐ falls in a straight line

   ☐ stays constant

   ☐ rises steadily

4. A capacitor holds 0.012 C at 4.0 V. What is its capacitance, in mF?

   ______________ mF

5. A 2.0 μF capacitor is charged to 1000 V. How much energy does it store?

   ______________ J

6. The charge during discharge is given by:

   ☐ $Q_0e^{-t/RC}$

   ☐ $Q_0e^{t/RC}$

   ☐ $Q_0(1 - e^{-t/RC})$

   ☐ $\dfrac{Q_0}{t}$

7. Doubling the charge on a capacitor doubles the voltage, so its capacitance stays the same.

   ☐ True　　☐ False

8. Why is there a factor of ½ in the stored-energy formula?

☐ the average p.d. during charging is only V/2

☐ half the charge leaks away

☐ the capacitor is only half full

☐ energy is always halved

9. A capacitor of 500 μF discharges through a 2000 Ω resistor. What is the time constant?

_________________ s

10. A 2.0 μF and a 3.0 μF capacitor are in parallel. What is the total capacitance, in µF?

_________________ µF

A-Level Physics

1. **potential difference**

   $C = \dfrac{Q}{V}$ — charge per volt, in farads.

2. $\frac{1}{2}CV^2$

   Equivalent forms: $W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \dfrac{Q^2}{2C}$.

3. **decays exponentially**

   The bigger the charge, the bigger the current — giving an exponential decay.

4. **3 mF**

   $C = \dfrac{Q}{V} = \dfrac{0.012}{4.0} = 0.0030\ \text{F} = 3.0\ \text{mF}$.

5. **1 J**

   $W = \frac{1}{2}CV^2 = \frac{1}{2} \times 2.0 \times 10^{-6} \times (1000)^2 = 1.0\ \text{J}$.

6. $Q_0e^{-t/RC}$

   Discharge falls toward zero: $Q = Q_0e^{-t/RC}$ (the $1 - e^{-t/RC}$ form is for *charging*).

7. **True**

   $C = \dfrac{Q}{V}$ is fixed by the capacitor's size and dielectric — Q and V rise together.

8. **the average p.d. during charging is only V/2**

   The p.d. rises from 0 to V as it charges, so the average is V/2 — hence the triangle area $\frac{1}{2}QV$.

9. **1 s**

   $\tau = RC = 2000 \times 500 \times 10^{-6} = 1.0$ s.

10. **5 µF**

    In parallel capacitances add: $2.0 + 3.0 = 5.0\ \mu\text{F}$.

A-LEVEL PHYSICS • EXERCISE SHEET

19.1 Capacitors and capacitance

Name: ____________ Class: ________ Date: __________ **Total: 24 marks**

KEY WORDS capacitance 电容 • capacitor 电容器 • potential difference 电势差 • resistor 电阻器 • farad 法拉

Objective

Build the skills to answer exam questions on **capacitance** — $C = Q/V$ and combining capacitors.

You must be able to:

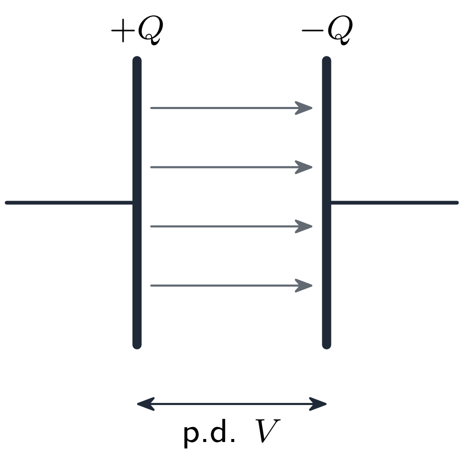
- define **capacitance** and use $C = Q/V$
- combine capacitors in **series** and **parallel**

1 Worked examples

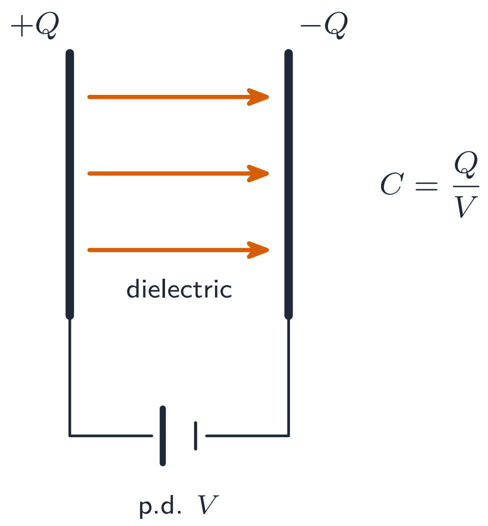
Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Capacitance

A **capacitor** stores charge: $+Q$ on one plate, $-Q$ on the other, with p.d. V across the gap:



$$C = Q/V$$



A parallel-plate capacitor

$$C = \frac{Q}{V} \quad \text{(farad, F} = \text{C V}^{-1}\text{).}$$

■ **Combining capacitors (opposite to resistors)**

- **parallel:** $C = C_1 + C_2 + \dots$
- **series:** $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dots$

2 Practice

Now apply the methods above.

2.1 Define **capacitance**. [1]

2.2 State the **unit** of capacitance. [1]

2.3 A capacitor holds 60 μC of charge at 12 V. Find its **capacitance**. [2]

2.4 Write the formula for capacitors in **parallel**. [1]

3 Exam-style questions

3.1 The unit of capacitance is the: [1]

- **A** coulomb
- **B** farad
- **C** volt
- **D** ohm

3.2 A 100 μF capacitor is charged to 9.0 V. Calculate the **charge** stored. [2]

3.3 A 4.0 μF and a 2.0 μF capacitor are connected in **series**. Calculate the combined capacitance. [3]

3.4 Write the formula for capacitors in **series**. [1]

3.5 Three capacitors are available: $C_1 = 120$ μF, $C_2 = 180$ μF and $C_3 = 220$ μF.

(a) C_1 and C_2 are connected in **series** across a supply. **Complete** the table to show how the charge on each and the p.d. across each relate to the total charge Q_S and the supply p.d. V_S. [2]

quantity	relationship
charges Q_1, Q_2 and Q_S	______________________
p.d.s V_1, V_2 and V_S	______________________

(b) **Determine** the combined capacitance of C_1 and C_2 in series. [2]

(c) The supply provides 12 V. **Calculate** the charge stored by that series combination, and the p.d. across C_1. [3]

(d) C_3 is now connected in **parallel** with the series pair. **Determine** the total capacitance of the whole network, and the total charge it now stores at 12 V. [3]

(e) **State and explain** which single capacitor in the network stores the **most** charge. [2]

Solutions

2.1 The **charge stored per unit potential difference** ($C = Q/V$).

2.2 The **farad** (F).

2.3 $C = Q/V = (60 \times 10^{-6})/12 = 5.0\ \mu$F.

2.4 $C = C_1 + C_2 + \ldots$.

3.1 **B** — farad.

3.2 $Q = CV = (100 \times 10^{-6})(9.0) = 9.0 \times 10^{-4}$ C.

3.3 $\dfrac{1}{C} = \dfrac{1}{4.0} + \dfrac{1}{2.0} = \dfrac{3}{4.0}$; $C = 1.3\ \mu$F.

3.4 $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots$.

3.5 (a) Charges: $Q_1 = Q_2 = Q_S$ — the same charge passes through every capacitor in a series chain. P.d.s: $V_1 + V_2 = V_S$.
(b) $\dfrac{1}{C} = \dfrac{1}{120} + \dfrac{1}{180}$, so $C = \dfrac{120 \times 180}{300} = 72\ \mu$F.
(c) $Q = CV = 72 \times 10^{-6} \times 12 = 8.6 \times 10^{-4}$ C; $V_1 = \dfrac{Q}{C_1} = \dfrac{8.64 \times 10^{-4}}{120 \times 10^{-6}} = 7.2$ V. (The **smaller** capacitor takes the larger share of the p.d.)
(d) Capacitors in parallel add: $C_{\text{total}} = 72 + 220 = 292\ \mu$F; $Q_{\text{total}} = 292 \times 10^{-6} \times 12 = 3.5 \times 10^{-3}$ C.
(e) C_3 — it has the full 12 V across it and the largest capacitance, so $Q = CV$ is largest; the series pair shares the same p.d. between two capacitors and behaves as only 72 μF.

6 **(a)** Two parallel plate capacitors C_1 and C_2 are connected to a supply that has a potential difference (p.d.) V_S. The capacitors may be connected in series or in parallel.

The supply provides charge Q_S and the plates of the two capacitors acquire charges Q_1 and Q_2 respectively. The p.d.s across the plates of the capacitors are V_1 and V_2 respectively.

Complete Table 6.1 to indicate how Q_S, Q_1 and Q_2 relate to each other, and how V_S, V_1 and V_2 relate to each other, for series and parallel connections of the capacitors to the supply.

Table 6.1

	relationship between charges	relationship between p.d.s
series		
parallel		

[4]

(b) An isolated capacitor of capacitance 470 μF stores 19 mJ of energy.

(i) Calculate the p.d. across the capacitor.

p.d. = ... V [2]

(ii) Calculate the charge on the capacitor.

charge = ... C [2]

(iii) The capacitor is now connected in parallel with a capacitor of capacitance 180 μF that is initially uncharged.

Determine the total energy, in mJ, now stored in the two capacitors.

energy = ... mJ [3]

[Total: 11]

5 **(a)** A capacitor of capacitance C_1 is connected in series with a second capacitor of capacitance C_2.

Show that the combined capacitance C of the two capacitors is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}.$$

[2]

(b) Three identical capacitors, each of capacitance C, are connected in a network as shown in Fig. 5.1.

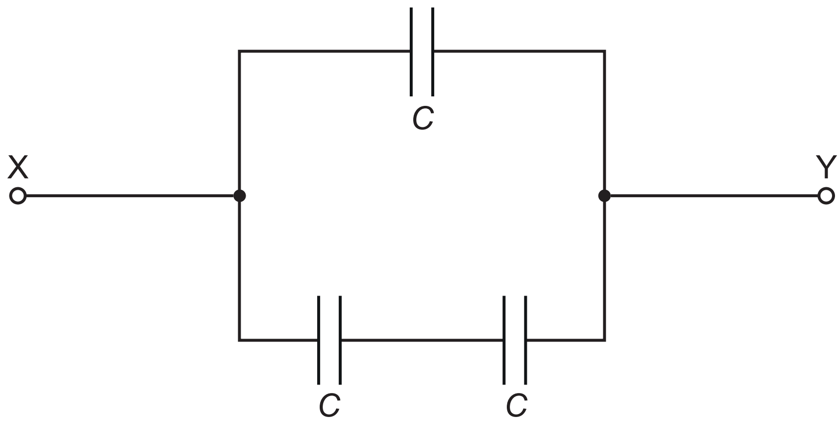


Fig. 5.1

The variation of the charge Q with the potential difference (p.d.) V between the terminals X and Y is shown in Fig. 5.2.

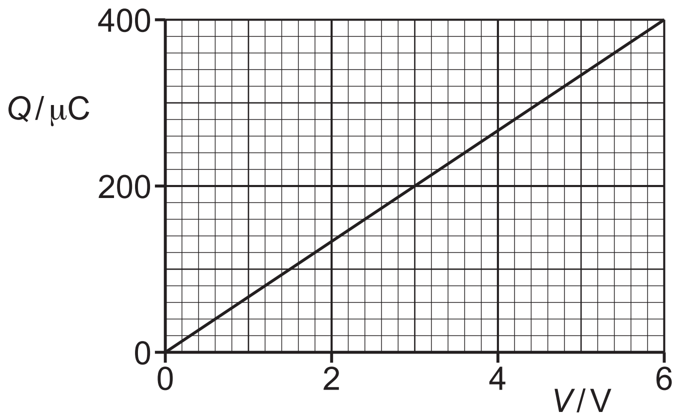


Fig. 5.2

Show that C is equal to $44\,\mu F$.

[3]

(c) The capacitor network in Fig. 5.1 is charged and then connected to a resistor of resistance $54\,k\Omega$. The capacitor network discharges through the resistor.

(i) Determine the time constant τ of the circuit. Give a unit with your answer.

τ = ... unit .... [2]

(ii) Determine the time taken for the discharge current to reduce to 15% of the initial discharge current.

time = ... s [2]

[Total: 9]

19.2 Energy stored in a capacitor

Name: ______________ Class: __________ Date: ____________ **Total: 23 marks**

KEY WORDS energy 能量 • capacitor 电容器

Objective

Build the skills to answer exam questions on the **energy stored in a capacitor**.

You must be able to:

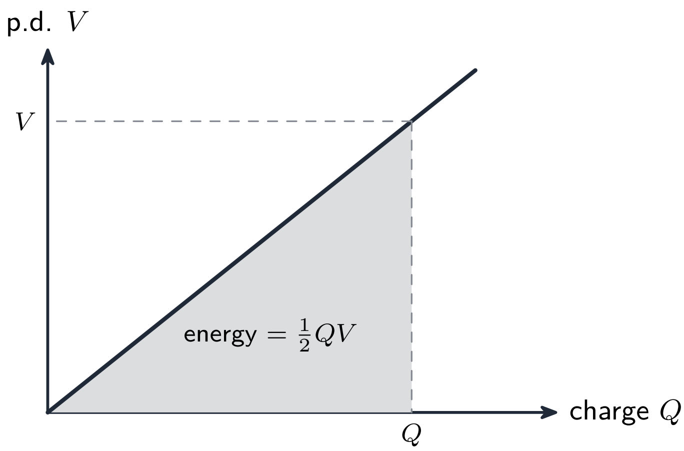
- find energy from the **area under the potential–charge graph**
- use $W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \dfrac{Q^2}{2C}$

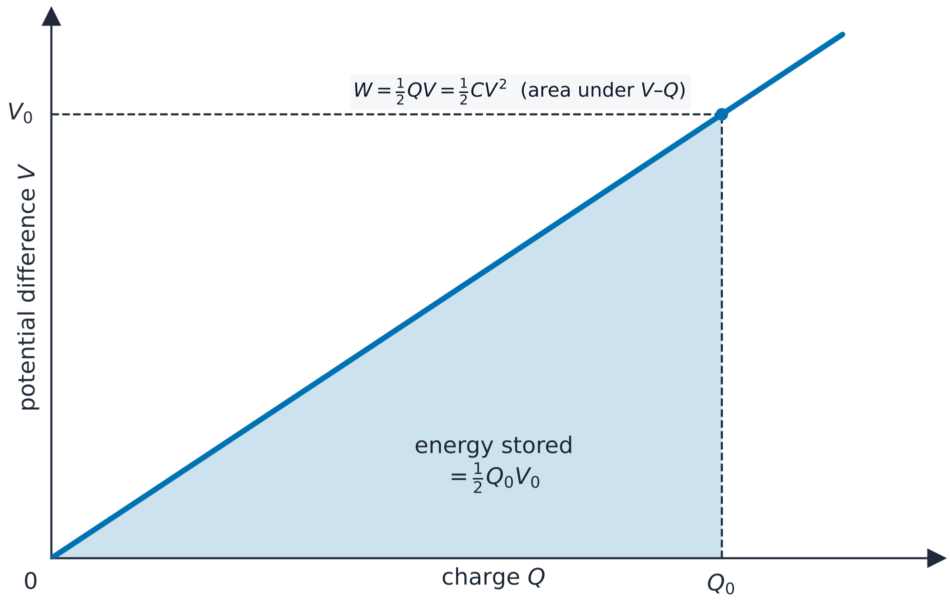
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Energy stored

The energy is the **area under the V–Q graph** — a triangle:





Energy stored is the area under the Q-V graph

$$W = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}.$$

The factor $\frac{1}{2}$ appears because the **average** p.d. during charging is $V/2$ (it grows from 0 to V).

Worked example. 100 μF at 12 V: $W = \frac{1}{2}CV^2 = \frac{1}{2}(100\times10^{-6})(12)^2 = 7.2\times10^{-3}$ J.

2 Practice

Now apply the methods above.

2.1 Write the **three** forms of the capacitor energy formula. [1]

2.2 Find the energy stored in a 100 μF capacitor charged to 12 V. [2]

2.3 State what the **area** under a potential–charge graph represents. [1]

3 Exam-style questions

3.1 The energy stored in a capacitor is: [1]

- **A** QV
- **B** $\frac{1}{2}QV$
- **C** CV
- **D** Q^2C

3.2 A 220 μF capacitor stores 4.0 mJ of energy. Calculate the **voltage** across it. [3]

3.3 Explain why the energy stored is $\frac{1}{2}QV$ and **not** QV. [2]

3.5 A defibrillator stores energy in a 32 μF capacitor charged to 2.5 kV, and delivers it to a patient in a pulse lasting 8.0 ms.

(a) **Determine** the energy stored by the capacitor when fully charged. [2]

(b) **Determine** the charge stored. [2]

(c) In the pulse, 75% of the stored energy is delivered to the patient. **Determine** the average power delivered. [3]

(d) The capacitor is charged instead to 1.8 kV. **Determine** the percentage by which the stored energy falls, and **explain** why it is not the same percentage as the fall in p.d. [3]

(e) On the same axes, **sketch** how the energy stored varies with the p.d. across the capacitor, and how the **charge** stored varies with it. **Label** each line. [3]

Solutions

2.1 $W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \dfrac{Q^2}{2C}$.

2.2 $W = \frac{1}{2}CV^2 = \frac{1}{2}(100 \times 10^{-6})(12)^2 = 7.2 \times 10^{-3}$ J.

2.3 The **energy stored** in the capacitor.

3.1 **B** — $\frac{1}{2}QV$.

3.2 $W = \frac{1}{2}CV^2 \Rightarrow V = \sqrt{\dfrac{2W}{C}} = \sqrt{\dfrac{2(4.0 \times 10^{-3})}{220 \times 10^{-6}}} = 6.0$ V.

3.3 As the capacitor charges, the p.d. rises from 0 to V; the **average** p.d. is $V/2$, so the work done is $Q \times (V/2) = \frac{1}{2}QV$, not QV.

3.5 (a) $E = \frac{1}{2}CV^2 = \frac{1}{2}(32 \times 10^{-6})(2500)^2 = 100$ J.
(b) $Q = CV = 32 \times 10^{-6} \times 2500 = 8.0 \times 10^{-2}$ C.
(c) Energy delivered $= 0.75 \times 100 = 75$ J; $P = \dfrac{E}{t} = \dfrac{75}{8.0 \times 10^{-3}} = 9.4 \times 10^3$ W. (About nine kilowatts, for eight milliseconds.)
(d) New energy $= \frac{1}{2}(32 \times 10^{-6})(1800)^2 = 51.8$ J, a fall of $\dfrac{100 - 51.8}{100} = 48\%$ — while the p.d. fell by only 28%, because $E \propto V^2$, so the fractional change in energy is roughly **twice** the fractional change in p.d..
(e) Charge: a **straight line through the origin**, since $Q = CV$. Energy: a **curve through the origin** getting steeper, since $E \propto V^2$. Both labelled, with axes labelled.

6 **(a)** Two parallel plate capacitors C_1 and C_2 are connected to a supply that has a potential difference (p.d.) V_S. The capacitors may be connected in series or in parallel.

The supply provides charge Q_S and the plates of the two capacitors acquire charges Q_1 and Q_2 respectively. The p.d.s across the plates of the capacitors are V_1 and V_2 respectively.

Complete Table 6.1 to indicate how Q_S, Q_1 and Q_2 relate to each other, and how V_S, V_1 and V_2 relate to each other, for series and parallel connections of the capacitors to the supply.

Table 6.1

	relationship between charges	relationship between p.d.s
series		
parallel		

[4]

(b) An isolated capacitor of capacitance 470 μF stores 19 mJ of energy.

(i) Calculate the p.d. across the capacitor.

p.d. = ...V [2]

(ii) Calculate the charge on the capacitor.

charge = ... C [2]

(iii) The capacitor is now connected in parallel with a capacitor of capacitance 180 μF that is initially uncharged.

Determine the total energy, in mJ, now stored in the two capacitors.

energy = ... mJ [3]

[Total: 11]

19.3 Discharging a capacitor

Name: ______________ Class: __________ Date: ____________ **Total: 29 marks**

KEY WORDS time constant 时间常数 • exponential decay 指数衰减 • resistor 电阻器 • resistance 电阻 • capacitor 电容器

Objective

Build the skills to answer exam questions on a **capacitor discharging** through a resistor.

You must be able to:

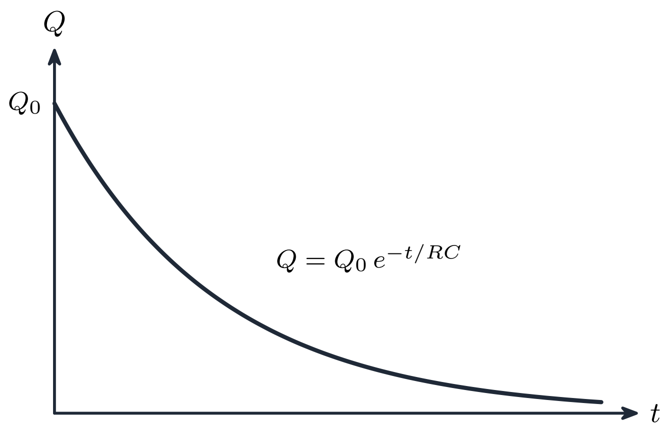
- use the **time constant** $\tau = RC$
- use $x = x_0 e^{-t/RC}$ for charge, p.d. or current
- interpret the discharge graphs

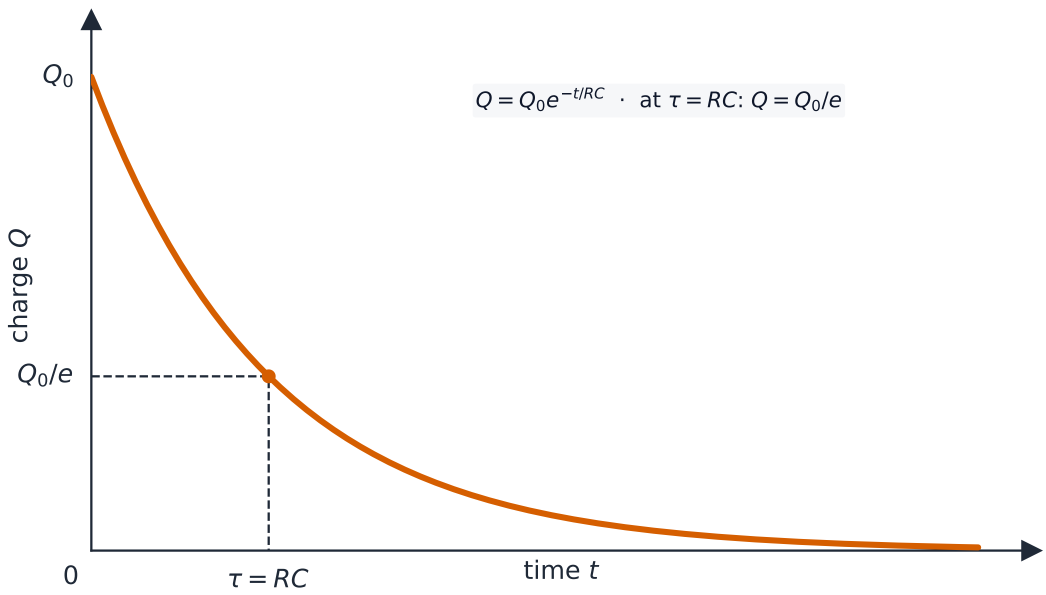
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Exponential decay

When a charged capacitor discharges through a resistor, the charge (and V, I) decays **exponentially**:





Exponential discharge of a capacitor

$$Q = Q_0 e^{-t/RC}, \quad V = V_0 e^{-t/RC}, \quad I = I_0 e^{-t/RC}.$$

The **time constant** $\tau = RC$ is the time for the charge to fall to $\frac{1}{e} \approx 37\%$ of its initial value.

Worked example. 470 μF through 2.0 kΩ: $\tau = RC = (2000)(470 \times 10^{-6}) = 0.94$ s.

■ Finding the time — take logs

To find the **time** for the charge (or V, or I) to fall to a given value, take the **natural log** of both sides of $V = V_0 e^{-t/RC}$:

$$\ln\frac{V}{V_0} = -\frac{t}{RC} \quad \Longrightarrow \quad t = -RC\,\ln\frac{V}{V_0}.$$

Worked example. For the capacitor above (charged to 6.0 V, $\tau = 0.94$ s), the time for the p.d. to fall to 2.0 V is $t = -0.94\ln\dfrac{2.0}{6.0} = -0.94 \times (-1.10) = 1.0$ s.

2 Practice

Now apply the methods above.

2.1 Write the formula for the **time constant**. [1]

2.2 A 100 μF capacitor discharges through a 10 kΩ resistor. Find the **time constant**. [2]

2.3 State what happens to the **charge** as a capacitor discharges. [1]

2.4 Write the discharge equation for **charge**. [1]

3 Exam-style questions

3.1 The time constant RC is the time for the charge to fall to: [1]

- **A** zero
- **B** half its initial value
- **C** $1/e$ (about 37%) of its initial value
- **D** $1/2e$ of its initial value

3.2 A 470 μF capacitor charged to 6.0 V discharges through a 2.0 kΩ resistor. Calculate (a) the time constant, (b) the p.d. after 1.0 s. [4]

3.3 Sketch a graph of **charge against time** for a discharging capacitor. [2]

3.4 State what happens to the time constant if the **resistance** is doubled. [1]

3.5 A 220 μF capacitor charged to 9.0 V discharges through a 15 kΩ resistor. Calculate the **time** for the p.d. to fall to 3.0 V. [4]

3.6 A capacitor C is charged and then discharged through a resistor R. A data logger records the p.d. V_R across the resistor and the p.d. V_C across the capacitor. At $t = 0$ the p.d. across the capacitor is 9.0 V, and the initial current is 1.8 mA. The p.d. across the capacitor falls to 3.3 V after 12 s.

(a) **State** the relationship between V_C and V_R during the discharge, and give a reason. [2]

(b) **Determine** the resistance R. [2]

(c) **Determine** the time constant τ of the circuit. [3]

(d) Hence **determine** the capacitance C, in μF. [2]

(e) **Explain**, in terms of the current in the circuit, why the decay of the charge on the capacitor is **exponential**. [3]

Solutions

2.1 $\tau = RC$.

2.2 $\tau = RC = (10 \times 10^3)(100 \times 10^{-6}) = 1.0$ s.

2.3 It **decreases exponentially** towards zero.

2.4 $Q = Q_0 e^{-t/RC}$.

3.1 **C** — $1/e$ (about 37%).

3.2 (a) $\tau = RC = (2.0 \times 10^3)(470 \times 10^{-6}) = 0.94$ s. (b) $V = V_0 e^{-t/RC} = 6.0\, e^{-1.0/0.94}$ $= 6.0 \times 0.345 = 2.1$ V.

3.3 A curve starting at the initial charge and **decaying exponentially** towards zero, with the same shape (never quite reaching zero).

3.4 It **doubles** ($\tau = RC$).

3.5 $\tau = RC = (15 \times 10^3)(220 \times 10^{-6}) = 3.3$ s; $t = -RC \ln \dfrac{V}{V_0} = -3.3 \ln \dfrac{3.0}{9.0} = -3.3 \times (-1.10) = 3.6$ s.

3.6 (a) They are **equal** at every instant, $V_C = V_R$, because the capacitor and resistor form a single loop with no other source, so the p.d. driving the current is the capacitor's own p.d..
(b) $R = \dfrac{V}{I} = \dfrac{9.0}{1.8 \times 10^{-3}} = 5.0 \times 10^3\ \Omega$.
(c) $V = V_0 e^{-t/\tau}$, so $\dfrac{3.3}{9.0} = e^{-12/\tau}$; $\ln(0.367) = -\dfrac{12}{\tau}$, giving $-1.003 = -\dfrac{12}{\tau}$ and $\tau = 12$ s. (The p.d. has fallen to $1/e$ of its start, which is what the time constant means.)
(d) $\tau = RC$, so $C = \dfrac{12}{5.0 \times 10^3} = 2.4 \times 10^{-3}$ F $= 2.4 \times 10^3$ μF.
(e) The current is $I = \dfrac{V_C}{R}$, and $V_C \propto Q$, so the current — the rate at which charge leaves — is **proportional to the charge still on the capacitor**. A quantity whose rate of decrease is proportional to itself decays exponentially; so as the charge falls the current falls with it, the discharge slows, and Q never quite reaches zero.

7 Fig. 7.1 shows a circuit containing a capacitor of capacitance *C* and a resistor of resistance *R*.

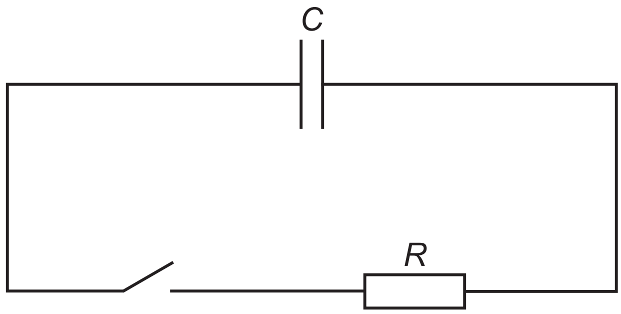


Fig. 7.1

Initially, the switch is open and the potential difference (p.d.) across the capacitor is 12 V.

The switch is closed at time $t = 0$ and the capacitor discharges through the resistor.

Fig. 7.2 shows the variation of the charge Q on the capacitor with the p.d. V_C across the capacitor as the capacitor discharges. Fig. 7.3 shows the variation of the current I in the resistor with the p.d. V_R across the resistor as the capacitor discharges.

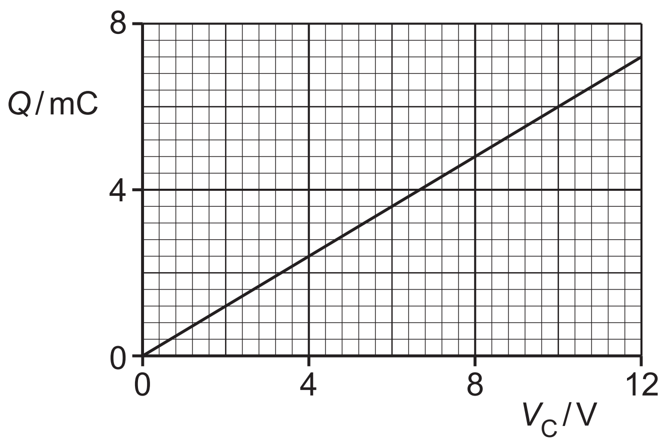


Fig. 7.2

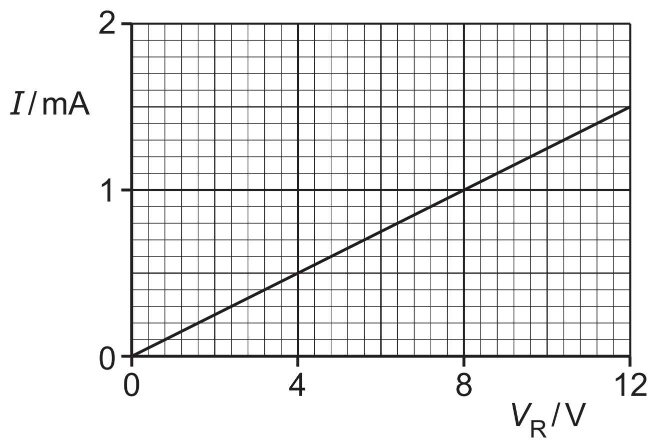


Fig. 7.3

(a) State the relationship between V_C and V_R.

.... [1]

(b) Determine:

(i) the capacitance C, in μF

C = ... μF [2]

(ii) the resistance R, in kΩ

R = ... kΩ [2]

(iii) the time constant τ of the circuit.

τ = ... s [2]

(c) Use Fig. 7.2, Fig. 7.3 and your answer in **(a)** to explain why the variation of Q with t is exponential in nature.

....

....

....

....

.... [3]

[Total: 10]