Week 38

A-Level Physics

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

Topic 17 quiz 2

Exercise sheet 17.1 5

Past-paper questions 17.1 13

Exercise sheet 17.2 25

Past-paper questions 17.2 32

Exercise sheet 17.3 38

Past-paper questions 17.3 44

A-Level Physics Name: ____________ Score: ______

1. In simple harmonic motion, the acceleration is:
   - ☐ proportional to displacement and directed back toward equilibrium
   - ☐ constant in size and direction
   - ☐ proportional to the speed
   - ☐ always zero
2. The kinetic energy of an oscillator is greatest:
   - ☐ at the equilibrium position
   - ☐ at the extreme positions
   - ☐ everywhere equally
   - ☐ never
3. Damping of an oscillation is caused by:
   - ☐ a resistive force removing energy
   - ☐ a driving force adding energy
   - ☐ increasing the mass
   - ☐ resonance
4. A full definition of simple harmonic motion needs which statements? Select **all** that apply.
   - ☐ the acceleration is proportional to the displacement from a fixed point
   - ☐ the acceleration is always directed towards that fixed point
   - ☐ the motion repeats with a constant period
   - ☐ the speed is constant

   Tick every correct option.
5. At the extreme positions of the motion, all the energy is potential.

   ☐ True ☐ False
6. Damping reduces the frequency of an oscillation rather than its amplitude.

   ☐ True ☐ False
7. An oscillation has a period of 2.0 s. What is its angular frequency ω?

   ____________ rad/s

8. With no damping, the total energy of an oscillator stays constant.

☐ True ☐ False

9. A car suspension returns the car to its normal height quickly without bouncing. Which damping is this, and why?

☐ critical damping: it returns to equilibrium in the shortest time without oscillating

☐ light damping: it oscillates only a little

☐ heavy damping: it does not oscillate

☐ no damping is needed

10. In SHM the speed is greatest at the ______ position.

____

17. Oscillations

Answers · 答案

A-Level Physics

1. **proportional to displacement and directed back toward equilibrium**

   That is exactly $a = -\omega^2 x$ — bigger displacement, bigger pull back; always toward the middle.

2. **at the equilibrium position**

   Speed is greatest at the middle, so KE peaks there while PE is at its lowest.

3. **a resistive force removing energy**

   Friction or drag carries energy away as heat, so each swing is smaller than the last.

4. **the acceleration is proportional to the displacement from a fixed point; the acceleration is always directed towards that fixed point**

   Those two statements are the definition and carry both marks. The constant period is a consequence of them, and the speed is certainly not constant.

5. **True**

   The oscillator is momentarily at rest there, so KE = 0 and all the energy is potential.

6. **False**

   The amplitude falls as energy becomes thermal. For light damping the frequency is essentially unchanged, which is why a bell keeps its note as it fades.

7. **3.14 rad/s**

   $\omega = \frac{2\pi}{T} = \frac{2\pi}{2.0} = \pi \approx 3.14\ \frac{\text{rad}}{\text{s}}$.

8. **True**

   Energy just swaps between KE and PE; their sum is conserved.

9. **critical damping: it returns to equilibrium in the shortest time without oscillating**

   Heavy damping also avoids oscillating but takes longer, so the car would sag back slowly. Give both properties: shortest time and no oscillation.

10. **equilibrium**

    At the middle all the energy is kinetic, so the speed is maximum ($v = \omega x_0$).

A-LEVEL PHYSICS • EXERCISE SHEET

17.1 Simple harmonic oscillations

Name: ______________ Class: __________ Date: ____________ **Total: 34 marks**

KEY WORDS simple harmonic motion 简谐运动 • acceleration 加速度 • displacement 位移 • period 周期 • frequency 频率

Objective

Build the skills to answer exam questions on **simple harmonic motion** — the defining equation, the solutions, and the graphs.

You must be able to:

- use the SHM condition $a = -\omega^2 x$ and $x = x_0 \sin \omega t$
- use $v = \pm\omega\sqrt{x_0^2 - x^2}$ and $v_{\max} = \omega x_0$
- interpret the displacement, velocity and acceleration graphs

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

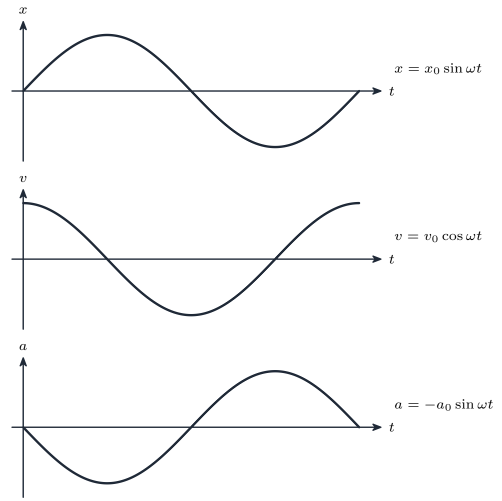
What SHM is

A particle moves with **SHM** when its acceleration is **proportional to displacement** and **directed back** to equilibrium:

$$a = -\omega^2 x, \qquad x = x_0 \sin \omega t, \qquad T = \frac{2\pi}{\omega}.$$

Velocity and the graphs

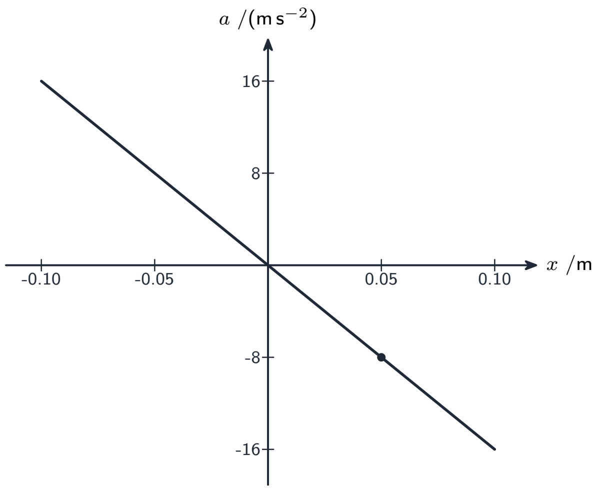
$$v = v_0 \cos \omega t, \qquad v = \pm\omega\sqrt{x_0^2 - x^2}, \qquad v_{\max} = \omega x_0, \qquad a_{\max} = \omega^2 x_0.$$



Displacement and acceleration are **antiphase**; velocity leads displacement by a **quarter cycle**.

The acceleration–displacement graph

Because $a = -\omega^2 x$, a graph of a against x is a **straight line through the origin** with a **negative gradient** of magnitude ω^2:



This is the **test for SHM**: $a \propto x$ (straight line through the origin) and **opposite in direction** (negative gradient). Read the gradient to find ω: gradient $= -\omega^2$, so $\omega = \sqrt{|\text{gradient}|}$.

Worked example. The line above passes through (0.05 m, -8 m s^{-2}), so gradient $= -8/0.05 = -160\,\text{s}^{-2} = -\omega^2$; thus $\omega = \sqrt{160} = 12.6$ rad s^{-1} and $T = 2\pi/\omega = 0.50$ s.

2 Practice

Now apply the methods above.

2.1 State the **two** conditions for simple harmonic motion. [2]

2.2 Define the **amplitude** of an oscillation. [1]

2.3 A particle has $\omega = 5.0$ rad s^{-1} and amplitude 0.040 m. Find its **maximum speed**. [2]

2.4 State the **phase relationship** between displacement and acceleration in SHM. [1]

3 Exam-style questions

3.1 In simple harmonic motion, the acceleration is: [1]

- **A** proportional to displacement and in the same direction
- **B** proportional to displacement and in the opposite direction
- **C** constant
- **D** always zero

3.2 A mass on a spring oscillates with period 0.50 s and amplitude 0.030 m. Find (a) the angular frequency, (b) the maximum speed, (c) the maximum acceleration. [4]

3.3 A particle in SHM has amplitude 0.10 m and $\omega = 8.0$ rad s^{-1}. Find its **speed** when $x = 0.060$ m. [3]

3.4 Sketch the **displacement–time** and **acceleration–time** graphs for SHM, and state their phase relationship. [3]

3.5 An object oscillates. A graph of its acceleration a against displacement x is a straight line through the origin passing through (0.05 m, −8.0 m s^{-2}).

(a) Explain how this graph shows the motion is **simple harmonic**. [2]

(b) Use the **gradient** to find the angular frequency ω and the period T. [3]

3.6 A small ball hangs on a spring and is set oscillating vertically. A motion sensor records its displacement, and the graph obtained is a smooth curve whose successive maximum displacements are equal, with a maximum displacement of x_0 and 8 complete oscillations in 6.4 s.

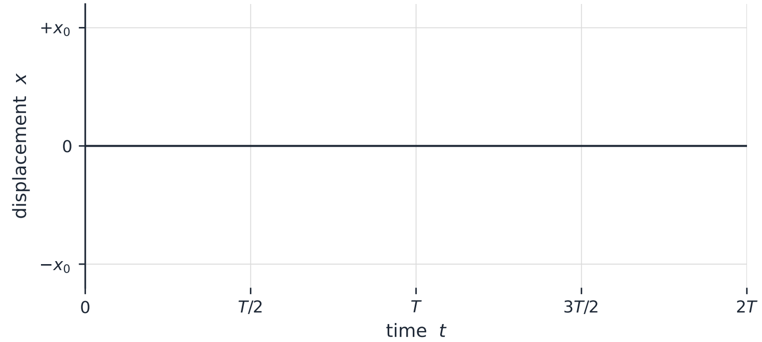
(a) A second graph plots the ball's **acceleration** against its **displacement** and gives a **straight line through the origin with a negative gradient**. **Explain** how this shows that the motion is simple harmonic. [2]

(b) **Determine** the period T and the angular frequency ω of the oscillations. [3]

(c) The magnitude of the gradient of that acceleration-displacement line is 61 s^{-2}. **Show that** this is consistent with your value of ω. [2]

(d) **State** what is meant by **damping**. [2]

(e) The ball is now made to oscillate in a beaker of oil, and is released from the same displacement x_0. On the axes below, **sketch** a possible variation of its displacement with time from $t = 0$ to $t = 2T$. [3]



Solutions

Each answer shows the steps, then the **result**.

2.1

- acceleration is **proportional to displacement** from a fixed point
- and **directed towards** that point (opposite to displacement)

2.2

- the **maximum displacement** from the equilibrium position

2.3

- $v_{\max} = \omega x_0$

$$v_{\max} = (5.0 \text{ rad s}^{-1})(0.040 \text{ m}) = \mathbf{0.20} \text{ m s}^{-1}$$

2.4

- they are **antiphase** (phase difference π / $180°$)

3.1 B

- SHM: acceleration is **proportional to displacement and opposite in direction**

3.2

(a) $\omega = 2\pi/T$

$$\omega = \frac{2\pi}{0.50 \text{ s}} = \mathbf{12.6} \text{ rad s}^{-1}$$

(b) $v_{\max} = \omega x_0$

$$v_{\max} = (12.6 \text{ rad s}^{-1})(0.030 \text{ m}) = \mathbf{0.38} \text{ m s}^{-1}$$

(c) $a_{\max} = \omega^2 x_0$

$$a_{\max} = (12.6 \text{ rad s}^{-1})^2(0.030 \text{ m}) = \mathbf{4.7} \text{ m s}^{-2}$$

3.3

- $v = \omega\sqrt{x_0^2 - x^2}$

$$v = (8.0 \text{ rad s}^{-1})\sqrt{(0.10 \text{ m})^2 - (0.060 \text{ m})^2} = (8.0 \text{ rad s}^{-1})\sqrt{0.0064 \text{ m}^2}$$

$$v = (8.0 \text{ rad s}^{-1})(0.080 \text{ m}) = \mathbf{0.64} \text{ m s}^{-1}$$

3.4

- displacement is a **sine** (or cosine) curve
- acceleration is the **same shape inverted** (negative)
- they are **antiphase** ($180°$ out of phase)

3.5

(a) the line is **straight and through the origin**, so a is **proportional to** x

- its **negative gradient** shows a is in the **opposite direction** to x — i.e. $a = -\omega^2 x$, the SHM condition

(b) gradient $= -8.0 \text{ m s}^{-2}/0.05 \text{ m} = -160 \text{ s}^{-2} = -\omega^2$

$$\omega = \sqrt{160 \text{ s}^{-2}} = 12.6 \text{ rad s}^{-1}$$

$$T = \frac{2\pi}{\omega} = \mathbf{0.50} \text{ s}$$

3.6

(a) a straight line through the origin shows $a \propto x$

- the negative gradient shows the acceleration is always directed **opposite** to the displacement, i.e. towards the equilibrium position — which is the definition of SHM

(b) $T = 6.4 \text{ s}/8 = 0.80 \text{ s}$

$$\omega = \frac{2\pi}{0.80 \text{ s}} = \mathbf{7.9} \text{ rad s}^{-1}$$

(c) for SHM $a = -\omega^2 x$, so the magnitude of the gradient is ω^2

$$\omega = \sqrt{61 \text{ s}^{-2}} = 7.8 \text{ rad s}^{-1}$$

which agrees with (b)

(d) damping is the loss of energy from an oscillating system to its surroundings, usually through **resistive forces**

- so the **amplitude decreases** with time

(e) a curve starting at $+x_0$

- oscillating with the **same period** T so that it still completes two cycles by $2T$
- with each maximum displacement **smaller** than the one before, decaying smoothly

5 A steel ball on the end of a thin string oscillates with small oscillations, as shown in Fig. 5.1.

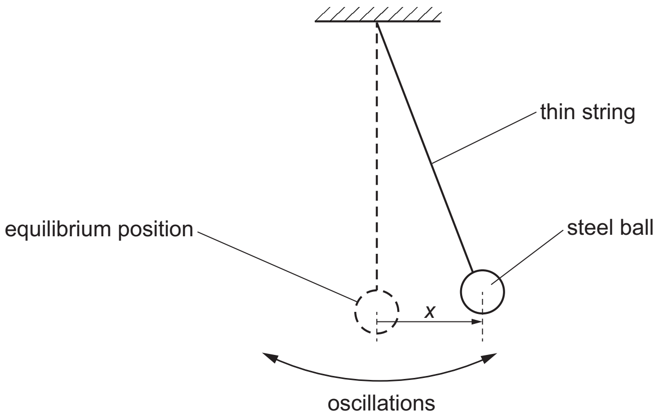


Fig. 5.1 (not to scale)

The displacement of the centre of the ball from its equilibrium position is x.

(a) Fig. 5.2 shows the variation with x of the acceleration a of the ball.

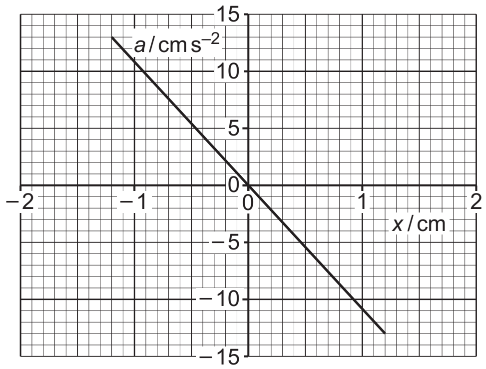


Fig. 5.2

(i) Explain how Fig. 5.2 shows that the oscillations of the ball are simple harmonic.

....

....

.... [2]

(ii) Determine the period T of the oscillations.

T = ... s [3]

(b) At time $t = 0$, when the displacement of the ball has its maximum value, the ball is immersed in a trough containing thick oil so that the ball is just below the surface of the oil. This results in the subsequent motion of the ball being heavily damped.

(i) State what is meant by damping.

....

....

.... [2]

(ii) On Fig. 5.3, sketch a possible variation of the displacement x of the ball with t between $t = 0$ and $t = 2T$.

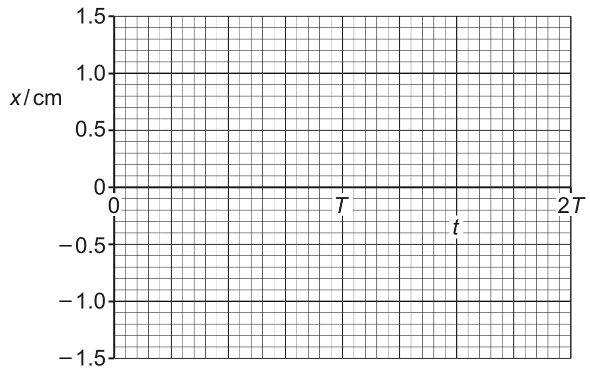


Fig. 5.3

[3]

[Total: 10]

4 **(a)** State what is meant by the frequency of the oscillations of an oscillating object.

....

.... [1]

(b) An object is oscillating.

Fig. 4.1 shows the variation of the acceleration a of the object with its displacement x from the equilibrium position.

Fig. 4.2 shows the variation of the kinetic energy E_K of the object with time t.

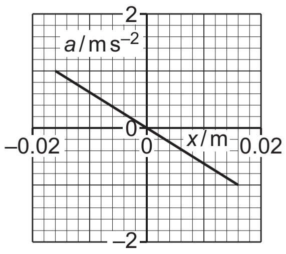


Fig. 4.1

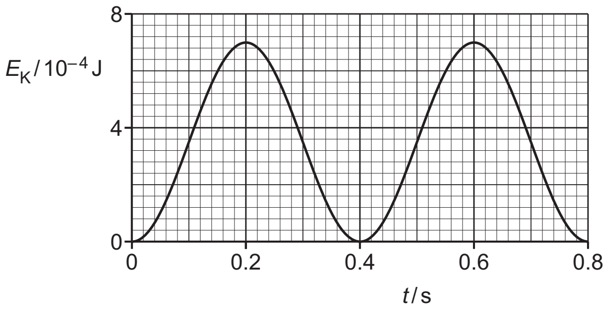


Fig. 4.2

(i) Explain how Fig. 4.2 shows that the period of the oscillations is 0.80 s.

....

....

.... [1]

(ii) Calculate the angular frequency ω of the oscillations.

ω = ... rad s^{-1} [2]

(iii) Apart from the period, frequency and angular frequency of the oscillations, determine **three** other conclusions about the object and its oscillations that may be drawn from Fig. 4.1 and Fig. 4.2. The conclusions may be qualitative or quantitative. Use the space below for any working.

1 ....

....

2 ....

....

3 ....

....
[3]

(iv) Describe the interchange between kinetic energy and potential energy during the oscillations. Numerical values are **not** required.

....

....

....

....

.... [3]

[Total: 10]

5 **(a)** State what is meant by simple harmonic motion.

....

....

.... [2]

(b) A block is suspended by a spring. The block oscillates vertically with simple harmonic motion.

The velocity v of the block varies with time t according to

$$v = 0.56 \cos 16t$$

where v is in $m\,s^{-1}$ and t is in s.

(i) Calculate the period of the oscillation.

period = ... s [1]

(ii) Determine the amplitude x_0 of the oscillation.

x_0 = ... m [2]

(iii) Use your answer in **(b)(ii)** to determine the equation for v in terms of the displacement x of the block, where v is in $m\,s^{-1}$ and x is in m.

v = ... [1]

(iv) On Fig. 5.1, sketch the variation of v with x.

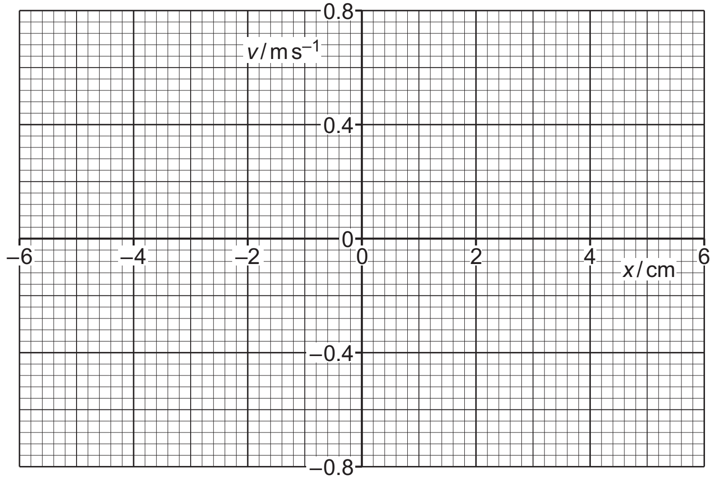


Fig. 5.1

[3]

[Total: 9]

5 A cuboidal block floats in a liquid with its base horizontal, as shown in Fig. 5.1.

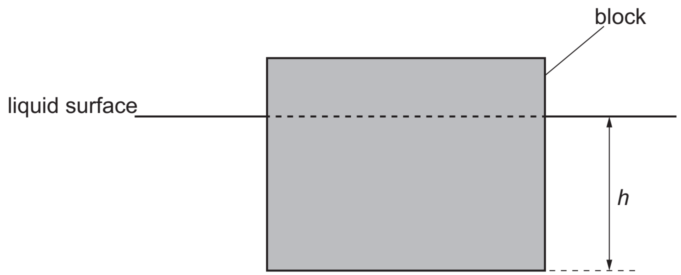


Fig. 5.1

The base of the block is at a depth *h* below the surface of the liquid.

The block is displaced downwards by a small distance and then released so that it oscillates.

Fig. 5.2 shows the variation with *h* of the acceleration *a* of the block.

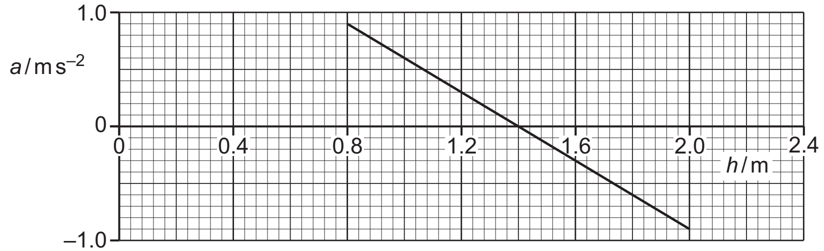


Fig. 5.2

Fig. 5.3 shows the variation with *h* of the kinetic energy E_K of the block.

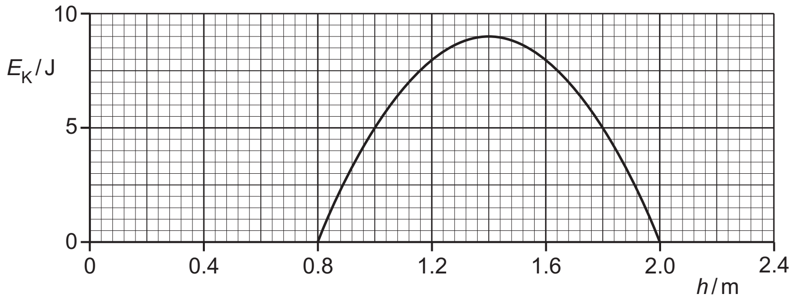


Fig. 5.3

Name: A-Level Physics: **s25 43**

(a) **(i)** Determine the amplitude of the oscillations.

amplitude = ... m [1]

(ii) State what the line in Fig. 5.2 shows about the nature of the oscillations.

.... [1]

(b) State **three** other quantitative conclusions that can be drawn from Fig. 5.2 and Fig. 5.3 about the block and its oscillations. Use the space for any working.

1 ....

....

2 ....

....

3 ....

.... [3]

(c) On Fig. 5.4, sketch the variation with *h* of the potential energy E_P of the oscillations.

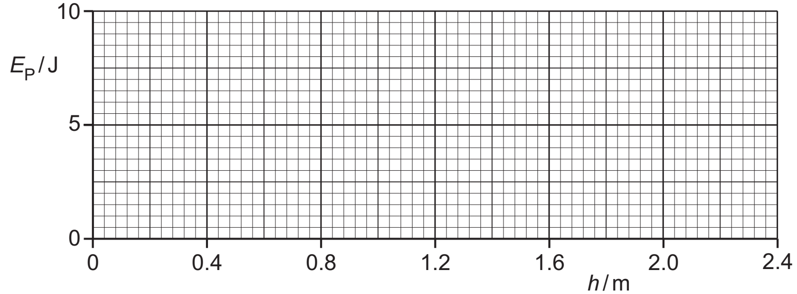


Fig. 5.4

[3]

[Total: 8]

4 **(a)** State what is meant by simple harmonic motion.

....

....

.... [2]

(b) A small sphere is suspended from a fixed point P by a string of negligible mass, as shown in Fig. 4.1.

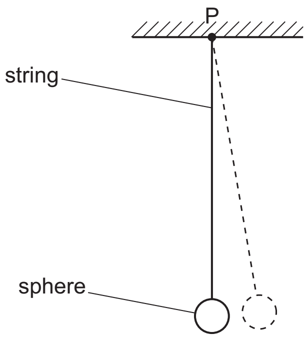


Fig. 4.1

The sphere is given a small horizontal displacement and is then released.

The variation with time of the horizontal velocity *v* of the sphere is shown in Fig. 4.2.

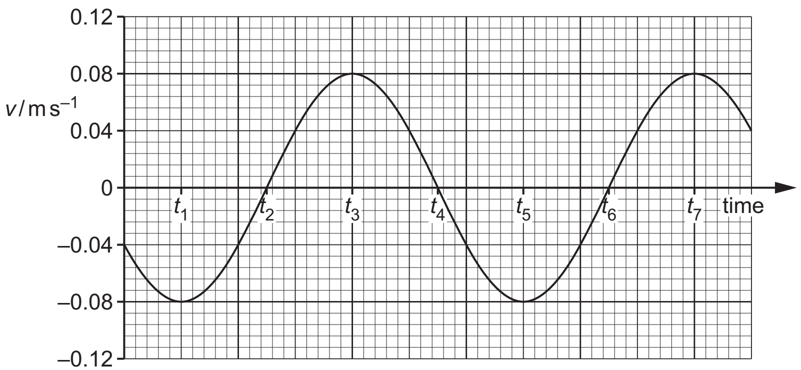


Fig. 4.2

(i) State two times at which the sphere is passing in the same direction through the equilibrium position.

time .... and time .... [1]

(ii) The time interval between t_1 and t_6 is 2.2 s.

Calculate the frequency of oscillation of the sphere.

frequency = ... Hz [2]

(c) The sphere in **(b)** is undergoing simple harmonic motion.

Use your answer in **(b)(ii)** and data from Fig. 4.2 to determine the maximum displacement of the sphere from its equilibrium position.

maximum displacement = ... m [3]

[Total: 8]

4 A small crystal is made to vibrate with simple harmonic motion. The variation with time t of the displacement x of one surface of the crystal from its equilibrium position is shown in Fig. 4.1.

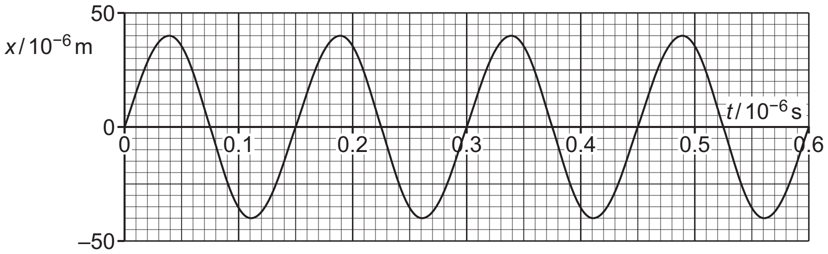


Fig. 4.1

(a) Show that the angular frequency of the vibration of the surface is $4.2 \times 10^7\,\text{rad s}^{-1}$.

[2]

(b) Determine the maximum acceleration a_0 of the vibration of the surface.

a_0 = ... m s^{-2} [2]

(c) The crystal may be modelled as a single mass of 2.4×10^{-4} kg that vibrates as shown in Fig. 4.1.

Calculate the total energy E of the vibrations.

E = ... J [3]

(d) The crystal generates ultrasound waves that are used to obtain diagnostic information about internal structures.

(i) The crystal is made from piezoelectric material.

Explain how the crystal is made to vibrate.

....

....

....

.... [2]

(ii) A parallel beam of ultrasound waves is incident on a muscle-bone boundary. Data for muscle and bone are given in Table 4.1.

Table 4.1

material	density / kg m^{-3}	speed of sound / m s^{-1}
muscle	1100	1600
bone	1900	4100

Calculate the percentage of the intensity of the ultrasound beam that is transmitted at this boundary.

percentage transmitted = ... % [3]

[Total: 12]

A-LEVEL PHYSICS • EXERCISE SHEET

17.2 Energy in simple harmonic motion

Name: ____________ Class: ________ Date: __________ **Total: 29 marks**

KEY WORDS kinetic energy 动能 • damping 阻尼 • displacement 位移 • equilibrium 平衡 • amplitude 振幅

Objective

Build the skills to answer exam questions on **energy in simple harmonic motion**.

You must be able to:

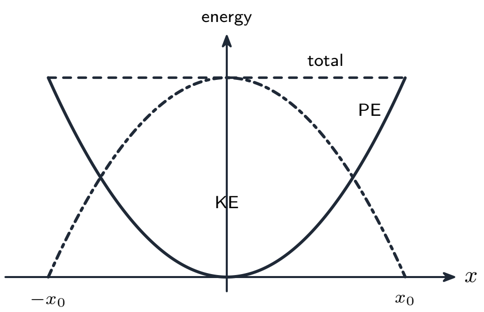
- describe the **KE** ↔ **PE** interchange during SHM
- use $E = \frac{1}{2}m\omega^2 x_0^2$ for the total energy

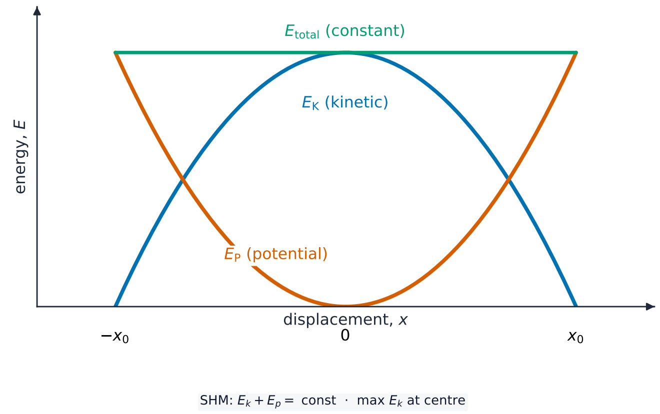
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Energy interchange

KE and PE swap as the oscillator moves; with no damping the **total energy is constant**:





Energy interchange in simple harmonic motion

- at $x = 0$: KE is **maximum**, PE minimum;
- at $x = \pm x_0$: KE $= 0$, PE **maximum**.

■ Total energy

$$E = \tfrac{1}{2}mv_{\max}^2 = \tfrac{1}{2}m\omega^2 x_0^2.$$

So $E \propto x_0^2$ (doubling the amplitude gives **four times** the energy) and $E \propto \omega^2$.

■ Energy varies at twice the frequency

In one full oscillation the object passes through the centre (KE maximum) **twice**, so the **KE and PE vary at twice** the frequency of the displacement. On a graph of KE against **time**, one repeat of the KE pattern takes **half** the oscillation period — so if the KE graph repeats every t_E, the oscillation period is $T = 2t_E$.

2 Practice

Now apply the methods above.

2.1 Describe the **energy interchange** during SHM. [2]

2.2 State where the **kinetic energy** is maximum and where the **potential energy** is maximum. [2]

2.3 Write the formula for the **total energy** of a simple harmonic oscillator. [1]

3 Exam-style questions

3.1 In SHM, the total energy is proportional to: [1]

- **A** the amplitude x_0
- **B** the amplitude squared x_0^2
- **C** $1/x_0$
- **D** the period T

3.2 A 0.20 kg mass oscillates with $\omega = 10$ rad s^{-1} and amplitude 0.050 m. Calculate the **total energy**. [3]

3.3 Sketch how the **kinetic energy** and **potential energy** vary with displacement in SHM. [2]

3.4 State what happens to the total energy if the **amplitude is doubled**. [1]

3.5 A graph of the **kinetic energy** of an oscillator against **time** repeats every 0.40 s.

(a) State the **period** of the oscillation, explaining your answer. [2]

(b) Calculate the **angular frequency** ω of the oscillation. [2]

3.6 A mass hangs on a spring and oscillates vertically with simple harmonic motion. A data logger records its **kinetic energy** against time, giving a curve that repeats every 0.40 s. The mass is 0.25 kg and the amplitude of the oscillation is 4.0 cm.

(a) **Determine** the period of the oscillation of the **mass**, and explain why it is not 0.40 s. [2]

(b) **Determine** the angular frequency of the oscillation. [2]

(c) **Determine** the total energy of the oscillation. [3]

(d) **Determine** the speed of the mass when its displacement is 2.0 cm. [3]

(e) The oscillation is **lightly damped**. **State and explain** what happens to the time between successive maxima of the kinetic-energy curve, and to the height of those maxima. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- energy is continually transferred between **kinetic** and **potential** forms
- with no damping the **total** stays constant

2.2

- KE is maximum at the **equilibrium** ($x = 0$)
- PE is maximum at the **extremes** ($x = \pm x_0$)

2.3

- $E = \frac{1}{2}m\omega^2 x_0^2$

3.1 B

- $E \propto x_0^2$, so energy is proportional to the **amplitude squared**

3.2

- $E = \frac{1}{2}m\omega^2 x_0^2$

$$E = \tfrac{1}{2}(0.20\ \text{kg})(10\ \text{rad s}^{-1})^2(0.050\ \text{m})^2 = \mathbf{0.025}\ \text{J}$$

3.3

- KE: a **downward** parabola, maximum at $x = 0$, zero at $\pm x_0$
- PE: an **upward** parabola, zero at $x = 0$, maximum at $\pm x_0$; their sum is constant

3.4

- it becomes **four times** larger ($E \propto x_0^2$)

3.5

(a) the KE varies at **twice** the oscillation frequency (the object passes the centre twice per cycle)

- so the oscillation period is **twice** the KE-graph period: $T = 2 \times 0.40\ \text{s} = \mathbf{0.80}\ \text{s}$

(b) $\omega = 2\pi/T$

$$\omega = \frac{2\pi}{0.80\ \text{s}} = \mathbf{7.9}\ \text{rad s}^{\mathbf{-1}}$$

3.6

(a) the kinetic energy is zero at **both** ends of the swing and maximum at the centre twice per cycle, so the energy curve repeats **twice** per oscillation

- the period of the mass is therefore $2 \times 0.40\ \text{s} = 0.80\ \text{s}$

(b) $\omega = 2\pi/T$

$$\omega = \frac{2\pi}{0.80\ \text{s}} = \mathbf{7.9}\ \text{rad s}^{\mathbf{-1}}$$

(c) the total energy equals the maximum kinetic energy, $E = \frac{1}{2}m\omega^2 x_0^2$

$$E = \tfrac{1}{2}(0.25\text{ kg})(7.85\text{ rad s}^{-1})^2(0.040\text{ m})^2 = \mathbf{1.2 \times 10^{-2}}\text{ J}$$

(d) $v = \omega\sqrt{x_0^2 - x^2}$

$$v = (7.85\text{ rad s}^{-1})\sqrt{(0.040\text{ m})^2 - (0.020\text{ m})^2} = (7.85\text{ rad s}^{-1})(0.0346\text{ m}) = \mathbf{0.27}\text{ m s}^{-1}$$

(e) light damping barely changes the period, so the time between maxima stays about 0.40 s

- the maxima get **lower** with time
- because the amplitude decays and the total energy is proportional to the **square** of the amplitude, so the energy falls faster than the amplitude does

5 A cuboidal block floats in a liquid with its base horizontal, as shown in Fig. 5.1.

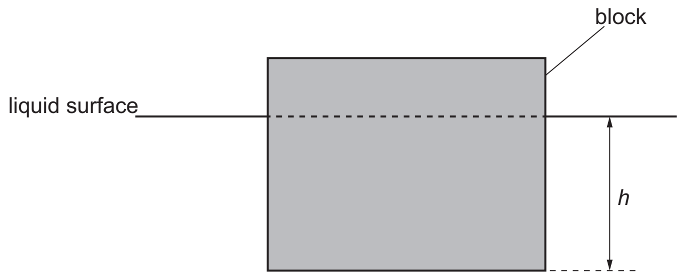


Fig. 5.1

The base of the block is at a depth *h* below the surface of the liquid.

The block is displaced downwards by a small distance and then released so that it oscillates.

Fig. 5.2 shows the variation with *h* of the acceleration *a* of the block.

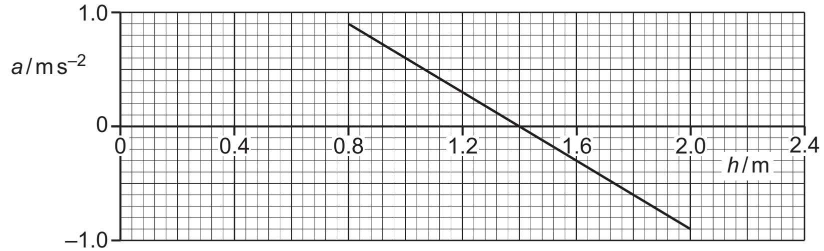


Fig. 5.2

Fig. 5.3 shows the variation with *h* of the kinetic energy E_K of the block.

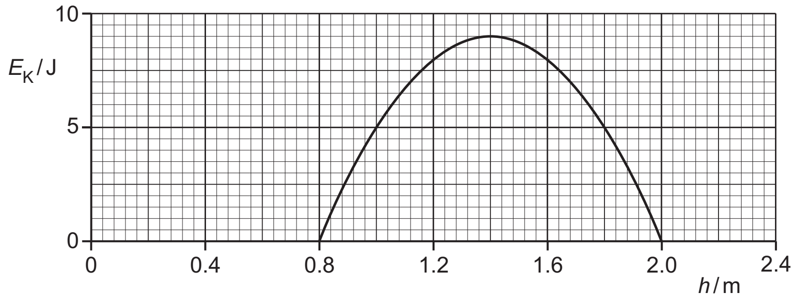


Fig. 5.3

(a) (i) Determine the amplitude of the oscillations.

amplitude = ... m [1]

(ii) State what the line in Fig. 5.2 shows about the nature of the oscillations.

.... [1]

(b) State **three** other quantitative conclusions that can be drawn from Fig. 5.2 and Fig. 5.3 about the block and its oscillations. Use the space for any working.

1 ....

....

2 ....

....

3 ....

....

[3]

(c) On Fig. 5.4, sketch the variation with h of the potential energy E_P of the oscillations.

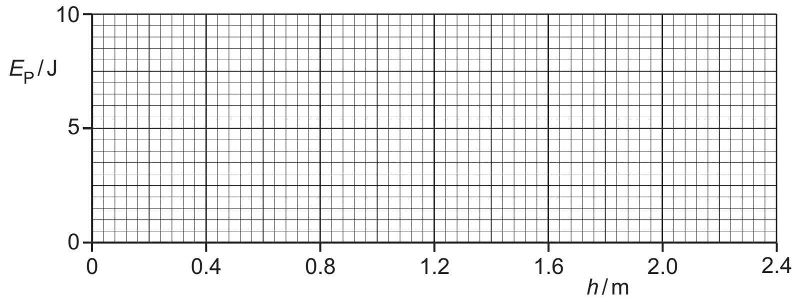


Fig. 5.4

[3]

[Total: 8]

4 A small crystal is made to vibrate with simple harmonic motion. The variation with time t of the displacement x of one surface of the crystal from its equilibrium position is shown in Fig. 4.1.

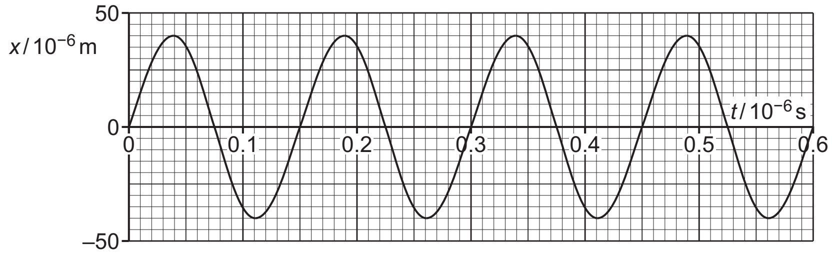


Fig. 4.1

(a) Show that the angular frequency of the vibration of the surface is 4.2×10^7 rad s^{-1}.

[2]

(b) Determine the maximum acceleration a_0 of the vibration of the surface.

a_0 = ... m s^{-2} [2]

(c) The crystal may be modelled as a single mass of 2.4×10^{-4} kg that vibrates as shown in Fig. 4.1.

Calculate the total energy E of the vibrations.

E = ... J [3]

(d) The crystal generates ultrasound waves that are used to obtain diagnostic information about internal structures.

(i) The crystal is made from piezoelectric material.

Explain how the crystal is made to vibrate.

....

....

....

.... [2]

(ii) A parallel beam of ultrasound waves is incident on a muscle-bone boundary. Data for muscle and bone are given in Table 4.1.

Table 4.1

material	density / kg m^{-3}	speed of sound / m s^{-1}
muscle	1100	1600
bone	1900	4100

Calculate the percentage of the intensity of the ultrasound beam that is transmitted at this boundary.

percentage transmitted = ... % [3]

[Total: 12]

4 A small steel sphere is oscillating vertically on the end of a spring, as shown in Fig. 4.1.

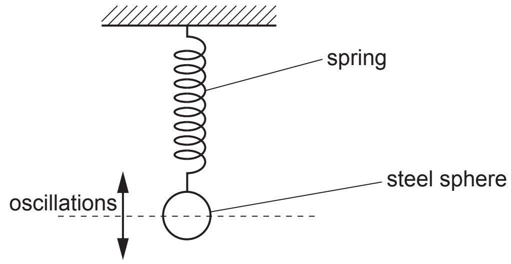


Fig. 4.1

The velocity v of the sphere varies with displacement x from its equilibrium position according to

$$v = \pm 9.7\sqrt{(11.6 - x^2)}$$

where v is in cm s^{-1} and x is in cm.

(a) (i) Calculate the frequency of the oscillations.

frequency = ... Hz [2]

(ii) Show that the amplitude of the oscillations is 3.4 cm.

[1]

(iii) Calculate the maximum acceleration a_0 of the sphere.

a_0 = ... m s^{-2} [2]

(b) On Fig. 4.2, sketch the variation with x of the acceleration a of the sphere.

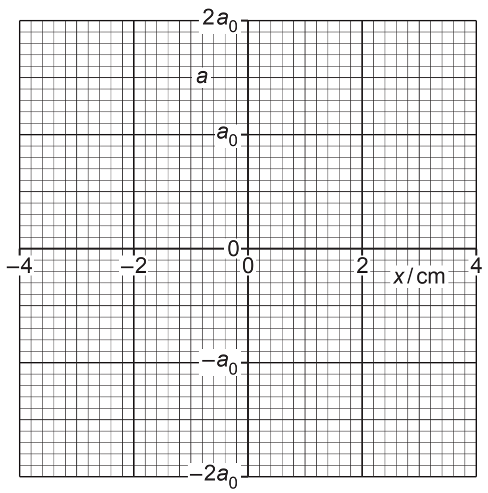


Fig. 4.2

[3]

(c) Describe, without calculation, the interchange between the potential energy and the kinetic energy of the oscillations.

....

....

....

....

.... [3]

[Total: 11]

A-LEVEL PHYSICS • EXERCISE SHEET

17.3 Damped and forced oscillations, resonance

Name: ____________ Class: ________ Date: __________ **Total: 23 marks**

KEY WORDS resonance 共振 • driving frequency 驱动频率 • natural frequency 固有频率 • forced oscillation 受迫振动 • critical damping 临界阻尼

Objective

Build the skills to answer exam questions on **damping** and **resonance**.

You must be able to:

- describe **light**, **critical** and **heavy** damping and sketch their graphs
- explain **resonance** and the condition for it

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

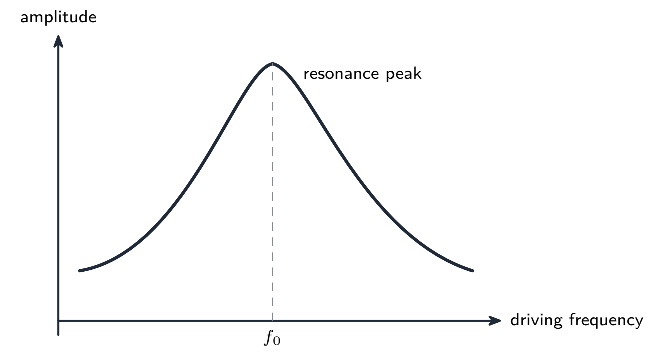
Damping

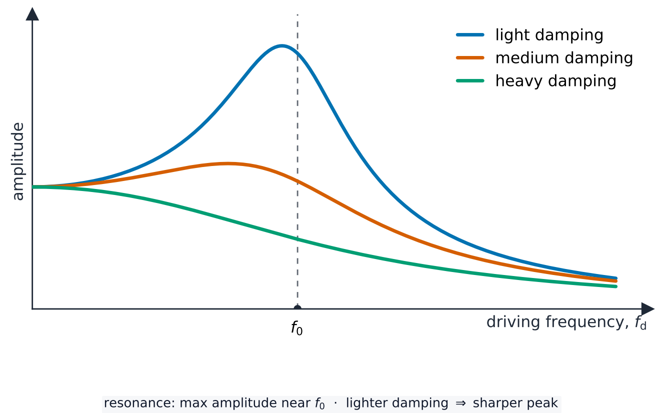
A **resistive force** removes energy, so the amplitude **shrinks** — this is **damping**:

- **light:** amplitude dies away slowly over many cycles;
- **critical:** returns to equilibrium **fastest** with **no overshoot**;
- **heavy:** returns slowly, no oscillation.

Resonance

A system **forced** to oscillate has the largest amplitude when the **driving frequency equals its natural frequency** f_0 — this is **resonance**:





The resonance curve and damping

More damping makes the resonance peak **lower and broader**.

2 Practice

Now apply the methods above.

2.1 State what causes **damping**. [1]

2.2 Name the **three** types of damping. [2]

2.3 State what is meant by **resonance**. [1]

2.4 State the condition for resonance to occur. [1]

3 Exam-style questions

3.1 Critical damping is the damping that: [1]

- **A** lets the system oscillate forever
- **B** returns the system to equilibrium fastest, without overshooting
- **C** returns the system to equilibrium most slowly
- **D** increases the amplitude

3.2 Describe how the **amplitude** of a forced oscillation changes as the driving frequency is increased through the natural frequency. [3]

3.3 Give **one** example each of where **light** damping and **critical** damping are useful. [2]

3.4 Explain what happens to the **resonance peak** when more damping is added. [2]

3.5 A machine part is mounted on rubber blocks. When the machine runs, the part is driven into oscillation, and a technician measures the amplitude of the part at each driving frequency.

(a) **State** the conditions needed for **resonance** to occur. [2]

(b) The amplitude peaks sharply at a driving frequency of 24 Hz. **State** what this tells you about the part, and **determine** its natural period. [2]

(c) The rubber blocks are replaced with stiffer ones, which **increase** the natural frequency of the part but also **increase** the damping. On one set of axes, **sketch** two curves of amplitude against driving frequency: the original mounting, labelled A, and the stiffer one, labelled B. [4]

(d) **Explain** why an engineer would prefer the stiffer, more heavily damped mounting even though the machine's own running frequency cannot be changed. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- a **resistive force** (friction / drag / air resistance) removing energy from the oscillator

2.2

- **light**, **critical** and **heavy** damping

2.3

- the condition where a forced oscillation reaches its **maximum amplitude**

2.4

- the **driving frequency equals the natural frequency** of the system

3.1 B

- critical damping **returns to equilibrium fastest without overshooting**

3.2

- the amplitude **rises** as the driving frequency approaches the natural frequency
- reaching a **maximum at** f_0 (resonance)
- then **falls** again as the frequency increases past f_0

3.3

- light damping — e.g. a pendulum/clock or musical instrument (sustained oscillation)
- critical damping — e.g. a galvanometer/voltmeter needle or car suspension/door closer (quick settle)

3.4

- the peak becomes **lower** (smaller maximum amplitude)
- and **broader** (occurs over a wider range of frequencies)

3.5

(a) the system is driven by a **periodic force** whose frequency equals the **natural frequency** of the system

- there is a **maximum transfer of energy** to it, so its amplitude becomes maximum

(b) the natural frequency of the part is 24 Hz

$$T = \frac{1}{f} = \frac{1}{24\text{ Hz}} = \mathbf{0.042}\text{ s}$$

(c) curve A: a sharp peak at 24 Hz

- curve B: its peak is at a **higher** frequency, is **lower** in amplitude, and is **broader** — heavier damping flattens and widens the resonance curve; both curves start at a non-zero amplitude at zero frequency and fall away at high frequency

(d) the machine's running frequency is now **further from** the part's natural frequency, so the part is driven well away from resonance

- the extra damping lowers the amplitude at every frequency, so even at resonance the oscillation would be smaller and less likely to cause damage

5 A steel ball on the end of a thin string oscillates with small oscillations, as shown in Fig. 5.1.

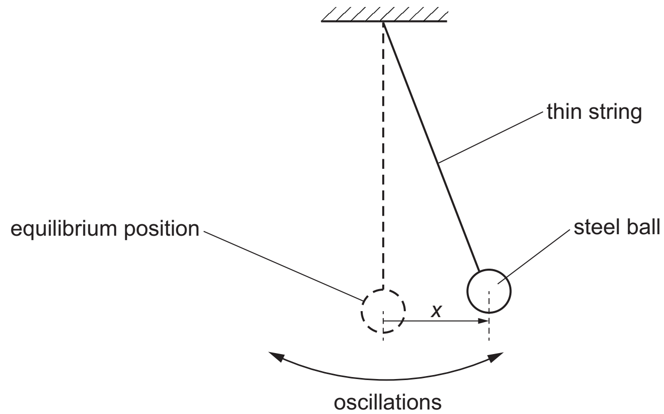


Fig. 5.1 (not to scale)

The displacement of the centre of the ball from its equilibrium position is x.

(a) Fig. 5.2 shows the variation with x of the acceleration a of the ball.

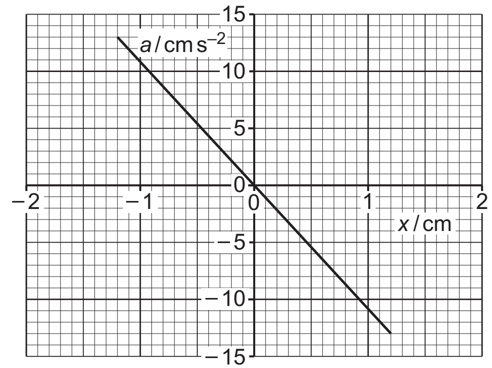


Fig. 5.2

(i) Explain how Fig. 5.2 shows that the oscillations of the ball are simple harmonic.

....

....

.... [2]

(ii) Determine the period T of the oscillations.

T = ... s [3]

(b) At time t = 0, when the displacement of the ball has its maximum value, the ball is immersed in a trough containing thick oil so that the ball is just below the surface of the oil. This results in the subsequent motion of the ball being heavily damped.

(i) State what is meant by damping.

....

....

.... [2]

(ii) On Fig. 5.3, sketch a possible variation of the displacement x of the ball with t between t = 0 and t = 2T.

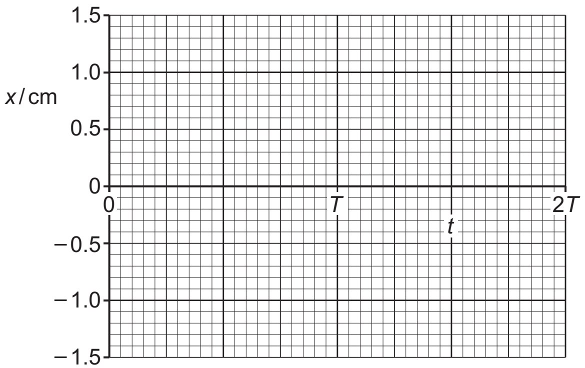


Fig. 5.3

[3]

[Total: 10]

7 A bar magnet is suspended from a spring. One pole of the magnet oscillates freely in a coil of wire, as shown in Fig. 7.1.

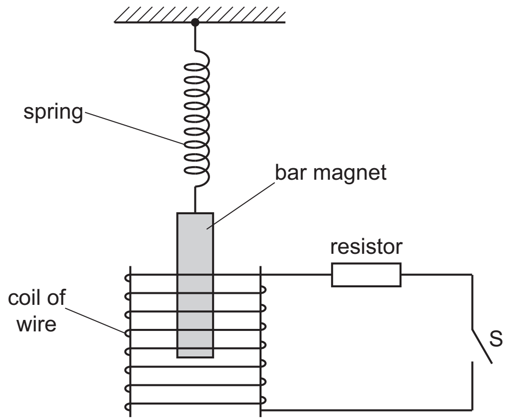


Fig. 7.1

The switch S is initially open.

(a) The switch S is now closed. As a result, the oscillations of the magnet are lightly damped.

(i) State what is meant by damping.

....

....

.... [2]

(ii) Describe what is observed to indicate that the damping is light.

....

.... [1]

(iii) By reference to electromagnetic induction and to conservation of energy, explain why the oscillations are damped.

....

....

....

.... [3]

(b) The procedure in **(a)** is repeated after replacing the resistor with one of greater resistance.

Suggest, with a reason, the effect of this change on the oscillations.

....

....

.... [2]

[Total: 8]

4 A heavy metal sphere of mass 0.81 kg is suspended from a string. The sphere is undergoing small oscillations from side to side, as shown in Fig. 4.1.

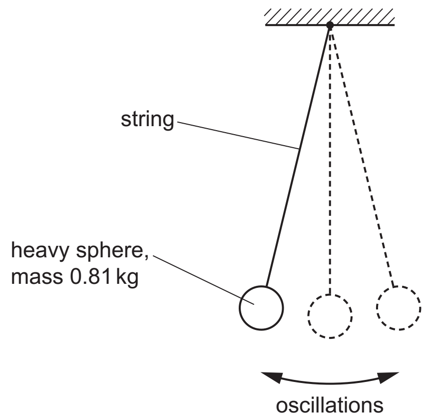


Fig. 4.1

The oscillations of the sphere may be considered to be simple harmonic with amplitude 0.036 m and period 3.0 s.

(a) State what is meant by simple harmonic motion.

....

....

.... [2]

(b) Calculate:

(i) the angular frequency of the oscillations

angular frequency = ... $rad\,s^{-1}$ [2]

(ii) the total energy of the oscillations.

total energy = ... J [2]

(c) The suspended sphere is now lowered into water. The sphere is given a sideways displacement of +0.036 m from its equilibrium position and is then released at time $t = 0$.
The water causes the motion of the sphere to be critically damped.

On Fig. 4.2, sketch the variation of the displacement x of the sphere from its equilibrium position with t from $t = 0$ to $t = 6.0$ s.

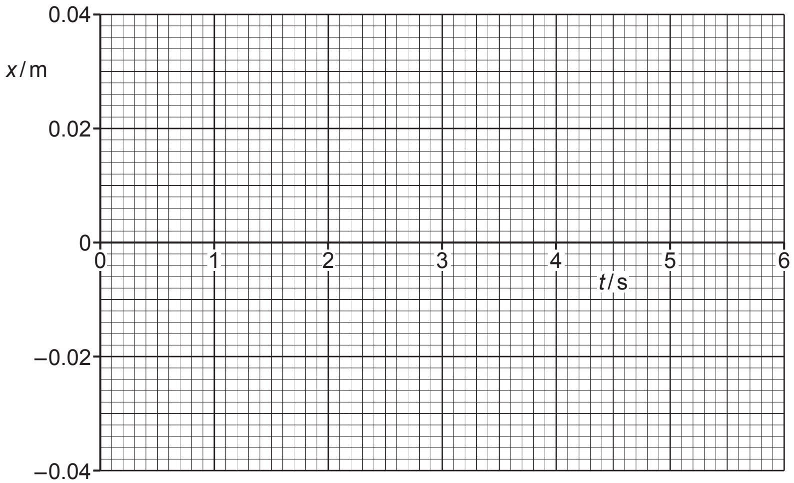


Fig. 4.2

[3]

[Total: 9]