Week 34

A-Level Physics

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

A-Level Physics Name: _____ Score: _____

1. One mole of a substance contains:

   ☐ 6.02×10^{23} particles

   ☐ 1000 particles

   ☐ 6.02×10^{-23} particles

   ☐ exactly 1 particle

2. A two-mark definition of an ideal gas needs which elements? Select **all** that apply.

   ☐ it obeys pV proportional to T (or pV = nRT)

   ☐ T is the thermodynamic temperature

   ☐ this holds at all values of p, V and T

   ☐ the gas is monatomic

   Tick every correct option.

3. Select **all** the assumptions of the kinetic theory of an ideal gas.

   ☐ molecules are in continuous random motion

   ☐ the molecules' own volume is negligible

   ☐ collisions are elastic

   ☐ molecules attract each other strongly at all times

   Tick every correct option.

4. The mole is one of the SI base units.

   ☐ True ☐ False

5. The equation of state of an ideal gas is:

   ☐ $pV = nRT$

   ☐ $pV = nRT^2$

   ☐ $\frac{p}{V} = nRT$

   ☐ $pVT = nR$

6. A gas exerts pressure on its container because the molecules:

   ☐ collide with the walls

   ☐ attract the walls

☐ slowly expand

☐ lose energy over time

7. A sample contains 3.01×10^{23} molecules. How many moles is this?

   _____ mol

8. You may use temperature in °C in the ideal gas equation.

   ☐ True ☐ False

9. The r.m.s. speed is the square root of the _____ speed.

   _____

10. How many atoms are there in one mole of oxygen gas, O2?

    ☐ 1.20 x 10^24, twice the Avogadro constant

    ☐ 6.02 x 10^23

    ☐ 3.01 x 10^23

    ☐ it depends on the pressure

15. Ideal gases

Answers · 答案

A-Level Physics

1. **6.02×10^{23} particles**

   A mole always contains the Avogadro number, $N_A = 6.02 \times 10^{23}$, of particles.

2. **it obeys pV proportional to T (or pV = nRT); T is the thermodynamic temperature; this holds at all values of p, V and T**

   The relationship alone earns one mark; "at all values" earns the second. Being monatomic is not part of the definition.

3. **molecules are in continuous random motion; the molecules' own volume is negligible; collisions are elastic**

   Forces between molecules are ignored *except* during (elastic) collisions — they do not attract strongly at all times.

4. **True**

   Yes — amount of substance (mole) is a base quantity, alongside the kilogram, metre, second, ampere and kelvin.

5. **$pV = nRT$**

   $pV = nRT$ (moles) or $pV = NkT$ (molecules) — with T in kelvin.

6. **collide with the walls**

   Each collision pushes on the wall; the combined effect of countless collisions is the pressure.

7. **0.5 mol**

   $n = \dfrac{N}{N_A} = \dfrac{3.01 \times 10^{23}}{6.02 \times 10^{23}} = 0.50$ mol.

8. **False**

   No — $pV \propto T$ only with T in kelvin. Convert °C to K first.

9. **mean-square**

   Take the mean of the squared speeds, then square-root it: $c_{\text{r.m.s.}} = \sqrt{\langle c^2 \rangle}$.

10. **1.20 x 10^24, twice the Avogadro constant**

    One mole is 6.02 x 10^23 molecules, and each O2 molecule holds two atoms. Saying which particle you are counting is worth a mark on its own.

A-LEVEL PHYSICS • EXERCISE SHEET

15.1 The mole

Name: _____ Class: _____ Date: _____ **Total: 21 marks**

KEY WORDS mole 摩尔 • amount of substance 物质的量 • avogadro constant 阿伏伽德罗常量 • molecule 分子 • molar mass 摩尔质量

Objective

Build the skills to answer exam questions on **the mole** and the Avogadro constant.

You must be able to:

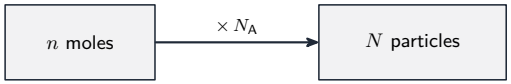
- use the **mole** as the SI unit of amount of substance
- use $N = nN_A$ and find masses of moles and single particles

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

The mole

Amount of substance is an SI base quantity; its unit is the **mole** (mol). One mole contains N_A particles:



$$N = nN_A, \quad N_A = 6.02 \times 10^{23}\ \text{mol}^{-1}$$

$$N = nN_A, \qquad N_A = 6.02 \times 10^{23}\ \text{mol}^{-1}.$$

The **molar mass** M_m is the mass of one mole; one particle has mass $m_0 = M_m/N_A$. Always say **what** you are counting (atoms or molecules).

2 Practice

Now apply the methods above.

2.1 State the SI **unit** of amount of substance. [1]

2.2 State the value of the **Avogadro constant**. [1]

2.3 How many molecules are there in 2.0 mol of a gas? [2]

3 Exam-style questions

3.1 One mole of oxygen gas (O_2) contains: [1]

- **A** 6.02×10^{23} atoms
- **B** 6.02×10^{23} molecules
- **C** 32 molecules
- **D** 16 molecules

3.2 A gas sample contains 3.0×10^{24} molecules. Find the number of **moles**. [2]

3.3 The molar mass of water is 18 g mol^{-1}. Calculate the mass of **one** water molecule. [2]

3.4 A sealed cylinder holds an ideal gas. The Avogadro constant is 6.02×10^{23} mol^{-1}, the molar gas constant is 8.31 J mol^{-1} K^{-1} and the Boltzmann constant is 1.38×10^{-23} J K^{-1}.

(a) **State** what is meant by the **Avogadro constant**. [1]

(b) **State** the relationship between the Avogadro constant N_A, the molar gas constant R and the Boltzmann constant k. [1]

(c) The cylinder holds 0.32 mol of the gas. **Determine** the number of molecules in it. [2]

(d) A second, identical cylinder holds **twice** as many molecules of the same gas at the same temperature T. **Complete** the table, giving each entry in terms of some or all of N, m, T, V, k and N_A, where sample X contains N molecules each of mass m in volume V. [4]

quantity	**sample X**	**sample Y**
pressure		
amount of substance		
mean kinetic energy per molecule		
internal energy		

(e) **Explain** why the mean kinetic energy per molecule is the same in both samples, while the internal energies differ. [2]

(f) The temperature of sample X is now varied. In the space below, **sketch** how the root-mean-square speed of its molecules varies with thermodynamic temperature. **Label** both axes. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the **mole** (mol)

2.2

- $N_\mathrm{A} = 6.02 \times 10^{23}\ \mathrm{mol}^{-1}$

2.3

- $N = nN_\mathrm{A}$

$$N = (2.0\ \mathrm{mol})(6.02 \times 10^{23}\ \mathrm{mol}^{-1}) = \mathbf{1.2 \times 10^{24}}$$

3.1 B

- one mole contains 6.02×10^{23} molecules
- **A** is $N_\mathrm{A}/10$; **C** is $2N_\mathrm{A}$; **D** is a mass, not a number

3.2

- $n = N/N_\mathrm{A}$

$$n = \frac{3.0 \times 10^{24}}{6.02 \times 10^{23}\ \mathrm{mol}^{-1}} = \mathbf{5.0}\ \mathrm{mol}$$

3.3

- $m_0 = M_\mathrm{m}/N_\mathrm{A}$

$$m_0 = \frac{0.018\ \mathrm{kg\ mol}^{-1}}{6.02 \times 10^{23}\ \mathrm{mol}^{-1}} = \mathbf{3.0 \times 10^{-26}}\ \mathrm{kg}$$

3.4

(a) the number of particles per unit amount of substance — the number of particles in one mole

(b) $N_\mathrm{A} = R/k$

(c) $N = nN_\mathrm{A}$

$$N = (0.32\ \mathrm{mol})(6.02 \times 10^{23}\ \mathrm{mol}^{-1}) = \mathbf{1.9 \times 10^{23}}$$

molecules

(d) sample Y has $2N$ molecules in the same volume at the same temperature

quantity	**sample X**	**sample Y**
pressure	NkT/V	$2NkT/V$
amount of substance	N/N_A	$2N/N_\mathrm{A}$
mean kinetic energy per molecule	$\frac{3}{2}kT$	$\frac{3}{2}kT$
internal energy	$\frac{3}{2}NkT$	$3NkT$

- pressures; amounts; both mean kinetic energies the same; internal energies

(e) the mean kinetic energy of a molecule is $\frac{3}{2}kT$, which depends **only on the temperature** — and both samples are at T

- internal energy is the **total** for all the molecules, so twice as many molecules at the same energy each gives twice the internal energy

(f) axes labelled *root-mean-square speed* and *thermodynamic temperature / K*

- a curve from the **origin** rising with a **decreasing** gradient, since $c_\mathrm{r.m.s.} \propto \sqrt{T}$ — quadrupling T only doubles the speed

3 **(a)** **(i)** State what is meant by the Avogadro constant.

.....

.....

..... [1]

(ii) State the relationship between the Avogadro constant N_A, the molar gas constant R and the Boltzmann constant k.

[1]

(b) Two samples X and Y of ideal gases are both at thermodynamic temperature T.

Sample X has volume V and consists of N molecules, each of mass m.
Sample Y has volume $2V$ and consists of $2N$ molecules, each of mass $2m$.

(i) Complete Table 3.1 by giving expressions, in terms of some or all of N, m, T, V and the constants in **(a)(ii)**, for the quantities indicated.

Table 3.1

	sample X	sample Y
pressure		
amount of substance		
mean-square speed of molecules		
internal energy		

[4]

(ii) The temperature of sample X is now varied.

On Fig. 3.1, sketch the variation with thermodynamic temperature of the root-mean-square (r.m.s.) speed of the molecules of the gas.

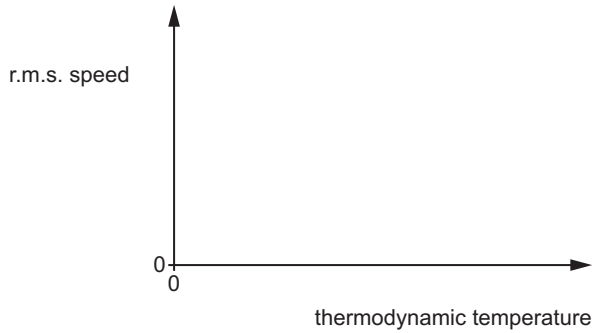


Fig. 3.1

[2]

[Total: 8]

3 **(a)** Two metal cuboids P and Q are in thermal contact with each other.

(i) P and Q are in thermal equilibrium.

State what is meant by the term thermal equilibrium.

.....

.....

..... [2]

(ii) Data for P and Q are given in Table 3.1.

Table 3.1

	P	Q
specific heat capacity / J kg^{-1} K^{-1}	390	910
mass / kg	0.54	0.37

P and Q are initially both at the same temperature.

P is supplied with 24 kJ of thermal energy. After some time, P and Q are once again both at the same temperature as each other.

P and Q are perfectly insulated from the surroundings.

Determine the change in temperature ΔT of Q.

ΔT = ... K [3]

(b) Nitrogen may be assumed to be an ideal gas. A fixed amount of nitrogen gas is contained at a constant pressure of 1.6×10^5 Pa.

The variation of the volume V of the gas with the temperature θ of the gas is shown in Fig. 3.1.

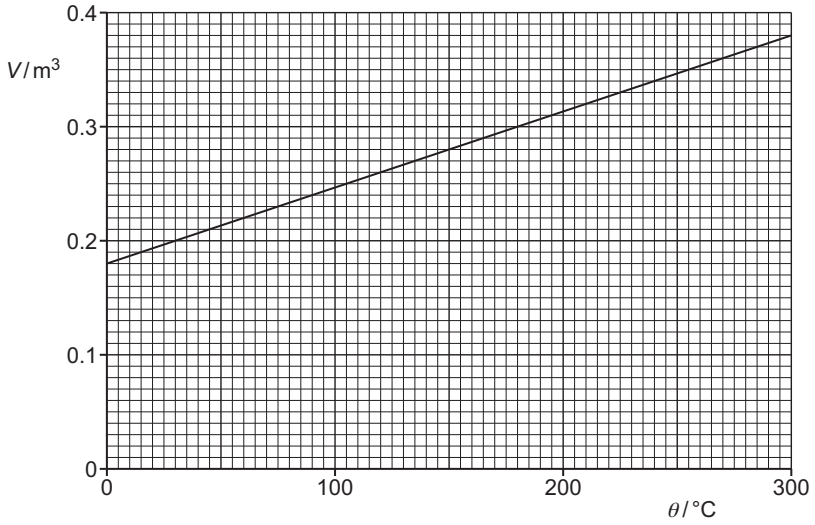


Fig. 3.1

(i) The temperature of the nitrogen gas is increased from 0 °C to 210 °C.
Determine the work done on the gas.

work done = ... J [3]

(ii) Determine the number N of molecules of nitrogen gas.

N = ... [2]

(iii) The mass of a nitrogen molecule is 4.7×10^{-26} kg.

Calculate the root-mean-square (r.m.s.) speed of a nitrogen molecule at 210 °C.

r.m.s. speed = ... $m\,s^{-1}$ [2]

[Total: 12]

8 Plutonium-238 ($^{238}_{94}Pu$) is unstable and undergoes alpha decay.

(a) Complete the equation to show the decay of plutonium-238.

$$^{238}_{94}Pu \rightarrow \,^{\cdots\cdots}_{\cdots\cdots}U + \,^{\cdots\cdots}_{\cdots\cdots}\alpha$$

[2]

(b) The power source in a space probe contains 0.874 kg of plutonium-238. Each nucleus of plutonium-238 that decays emits 5.59 MeV of energy. The half-life of plutonium-238 is 87.7 years.

(i) Calculate the initial number N_o of nuclei of plutonium-238 in the power source.

N_o = ... [1]

(ii) Determine the initial activity of the source. Give a unit with your answer.

activity = ... unit ..... [2]

(iii) Use your answer in **(b)(ii)** to determine the initial power output from the source due to the decay of plutonium-238.

power output = ... W [2]

(iv) The space probe will continue to function until the power output from the plutonium in the source decreases to 65.3% of its initial value.

Calculate the time, in years, for which the space probe will function.

time = ... years [2]

(c) An alternative power source uses energy generated from the radioactive decay of polonium-210. This isotope has a half-life of 0.378 years. The mass of the isotope needed for the same initial power output as in **(b)** is 3.37 g.

Suggest **one** advantage and **one** disadvantage of using polonium-210 as the source of energy.

advantage .....

.....

disadvantage .....

.....

[2]

[Total: 11]

A-LEVEL PHYSICS • EXERCISE SHEET

15.2 Equation of state

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS ideal gas 理想气体 • boltzmann constant 玻尔兹曼常量 • equation of state 状态方程 • molar gas constant 摩尔气体常量 • pressure 压强

Objective

Build the skills to answer exam questions on the **equation of state** of an ideal gas.

You must be able to:

- use $pV = nRT$ and $pV = NkT$ (with T in kelvin)
- recall R, k and that $k = R/N_A$
- read a pV–T **graph**: gradient $= Nk$, so $N =$ gradient$/k$

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

Equation of state

An **ideal gas** obeys $pV \propto T$. Two equal forms:

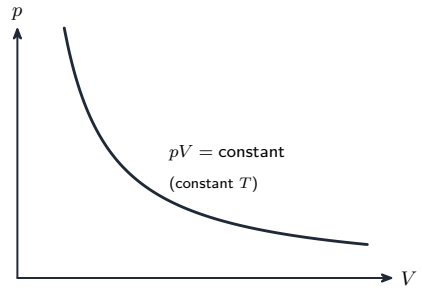
$$pV = nRT \quad (n = \text{moles}), \qquad pV = NkT \quad (N = \text{molecules}),$$

with $R = 8.31$ J mol^{-1} K^{-1}, $k = 1.38 \times 10^{-23}$ J K^{-1}, and $k = R/N_A$. **Always use Pa, m^3, K.**

Worked example. 0.50 mol at 300 K in 0.020 m^3: $p = \dfrac{nRT}{V} = \dfrac{0.50 \times 8.31 \times 300}{0.020} = 6.2 \times 10^4$ Pa.

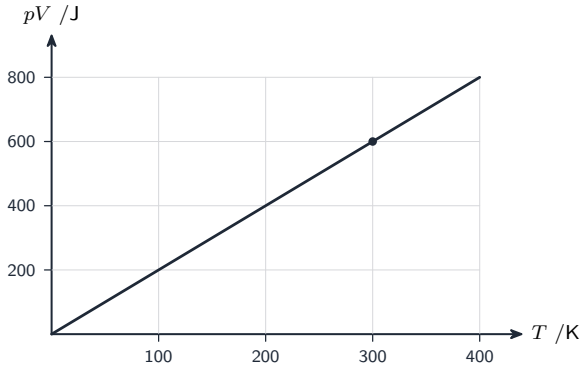
Boyle's law

At constant T (and amount), $p \propto 1/V$, so $pV =$ constant:



■ **Reading a pV–T graph**

For a fixed amount of gas, $pV = NkT$, so a graph of pV against T (in kelvin) is a **straight line through the origin** with **gradient** Nk:



A straight line **through the origin** confirms the gas is **ideal** ($pV \propto T$). To find the number of molecules, read the **gradient** and divide by k: $N = \dfrac{\text{gradient}}{k}$.

Worked example. The line above passes through (300 K, 600 J), so gradient $= 600/300 = 2.0$ J K$^{-1} = Nk$; thus $N = \dfrac{2.0}{1.38 \times 10^{-23}} = 1.4 \times 10^{23}$.

2 Practice

Now apply the methods above.

2.1 State what is meant by an **ideal gas**. [1]

2.2 Write the **two** forms of the ideal-gas equation of state. [2]

2.3 State the value of the **molar gas constant** R. [1]

2.4 A graph of pV against T for a fixed mass of ideal gas is a straight line through the origin. State what its **gradient** is equal to. [1]

3 Exam-style questions

3.1 In $pV = nRT$, the temperature T must be in: [1]

- **A** °C
- **B** K
- **C** J
- **D** mol

3.2 0.50 mol of an ideal gas at 300 K occupies 0.020 m^3. Calculate the **pressure**. ($R = 8.31$.) [3]

3.3 For a fixed mass of gas, the graph of pV against T is a straight line through the origin passing through $(300\ \text{K}, 600\ \text{J})$.

(a) State what this tells you about the gas. [1]

(b) Determine the number of molecules N in the sample. ($k = 1.38 \times 10^{-23}$.) [3]

3.4 State **Boyle's law**. [1]

3.5 For an ideal gas, two equations are used together:

$$pV = NkT \qquad \text{and} \qquad pV = \tfrac{1}{3}Nm\langle c^2 \rangle$$

(a) **State** the meaning of the symbol T in the first equation, and **identify** the constant k. [2]

(b) **State** the meanings of the symbols m and $\langle c^2 \rangle$ in the second equation. [2]

(c) Use the two equations to **derive** an expression, in terms of k and T only, for the mean kinetic energy E_K of one molecule of the gas. [3]

(d) Hence **state and explain** how the root-mean-square speed of the molecules changes when the thermodynamic temperature of the gas is **quadrupled**. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- a gas that obeys $pV \propto T$ (i.e. $pV = nRT$) exactly, at all p and T

2.2

- $pV = nRT$
- $pV = NkT$

2.3

- $R = 8.31\ \text{J mol}^{-1}\,\text{K}^{-1}$

2.4

- Nk (the number of molecules × the Boltzmann constant)

3.1 B

- T in $pV = nRT$ must be in **kelvin**
- **A** is Celsius; **C** is energy; **D** is amount of substance

3.2

- $p = nRT/V$

$$p = \frac{(0.50\ \text{mol})(8.31\ \text{J mol}^{-1}\,\text{K}^{-1})(300\ \text{K})}{0.020\ \text{m}^3} = \mathbf{6.2 \times 10^4}\ \text{Pa}$$

3.3

(a) it is an **ideal gas**: $pV \propto T$ (the line is straight and passes through the origin)

(b) gradient = 600 J/300 K = 2.0 J K^{-1}

- this equals Nk

$$N = \frac{2.0\ \text{J K}^{-1}}{1.38 \times 10^{-23}\ \text{J K}^{-1}} = \mathbf{1.4 \times 10^{23}}$$

3.4

- at constant temperature (and amount of gas), the pressure is **inversely proportional** to the volume (pV = constant)

3.5

(a) T is the **thermodynamic (absolute) temperature**, measured in kelvin

- k is the **Boltzmann constant**

(b) m is the mass of **one molecule**

- $\langle c^2 \rangle$ is the **mean square speed** — the average of the squares of the molecular speeds, not the square of the average

(c) set the two right-hand sides equal: $NkT = \frac{1}{3}Nm\langle c^2 \rangle$

- so $\frac{1}{3}m\langle c^2 \rangle = kT$
- the mean kinetic energy of one molecule is $E_\text{K} = \frac{1}{2}m\langle c^2 \rangle$
- which is $\frac{3}{2}$ times the previous line, so $E_\text{K} = \frac{3}{2}kT$

(d) $\frac{1}{2}m\langle c^2 \rangle \propto T$, so $\langle c^2 \rangle \propto T$ and $c_\text{r.m.s.} \propto \sqrt{T}$

- quadrupling T therefore **doubles** the r.m.s. speed

4 **(a)** The equation of state for an ideal gas may be written as

$$pVA = NBT$$

where p is the pressure of the gas, V is the volume of the gas, A is the Avogadro constant, B is another constant and N is the number of molecules of the gas.

(i) State the meaning, in the equation, of the symbol T.

..... [1]

(ii) Identify the constant B.

..... [1]

(b) The product pV for an ideal gas is also given by

$$pV = \tfrac{1}{3}Nm\langle c^2\rangle.$$

(i) State the meanings, in this equation, of the symbols m and $\langle c^2\rangle$.

m: .....

$\langle c^2\rangle$: .....
[2]

(ii) Use the equations in **(a)** and **(b)** to derive an expression, in terms of A, B and T, for the mean kinetic energy E_K of a molecule of the gas.

E_K = ... [2]

(c) On Fig. 4.1, sketch the variation with T of the root-mean-square (r.m.s.) speed of the molecules of an ideal gas.

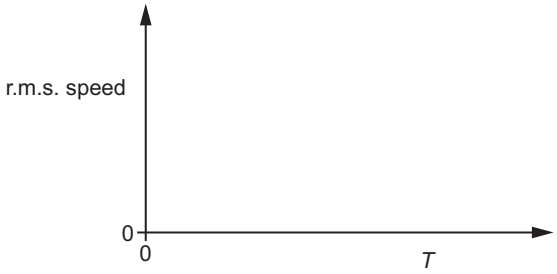


Fig. 4.1

[2]

[Total: 8]

3 **(a)** State what is meant by an ideal gas.

.....

.....

..... [2]

(b) An ideal gas at a pressure of 1.6×10^5 Pa has a density of $1.9\,\text{kg}\,\text{m}^{-3}$.

(i) Show that the root-mean-square (r.m.s.) speed of molecules of this gas is approximately $500\,\text{m}\,\text{s}^{-1}$.

[3]

(ii) One molecule of the gas has a mass of 4.7×10^{-26} kg.

Determine the thermodynamic temperature of the gas.

temperature = ... K [2]

(c) Calculate the internal energy U of 6.0 mol of the gas in **(b)**. Explain your reasoning.

U = ... J [3]

[Total: 10]

4 **(a)** State **two** of the basic assumptions of the kinetic theory of gases.

1 .....

.....

2 .....

.....

[2]

(b) An ideal gas has amount of substance n.
The gas is initially in state X, with pressure $2p$ and volume V.
The gas is cooled at constant volume to state Y, with pressure p.
The gas is then heated at constant pressure to state Z, with volume $2V$.
Finally, the gas returns at constant temperature to state X.

(i) Determine an expression for the temperature T of the gas in state X, in terms of n, p and V.
Identify any other symbols that you use.

[2]

(ii) On Fig. 4.1, sketch the variation with volume of pressure for the gas as the gas undergoes the three changes. The state X is labelled. Label states Y and Z.

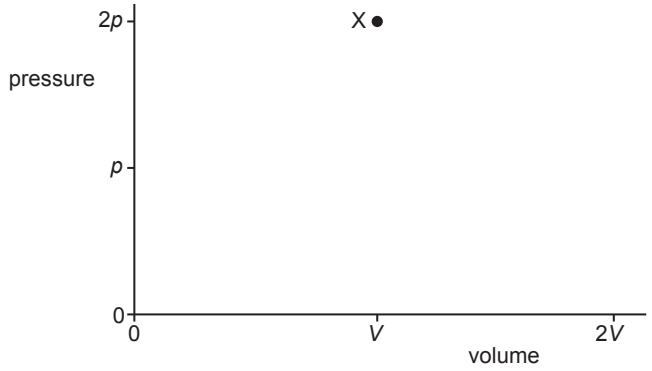


Fig. 4.1

[3]

(iii) During the change of state from Y to Z, the increase in internal energy of the gas is *U*. During the change of state from Z to X, the work done on the gas is *W*.

Complete Table 4.1 to indicate, for each of the three changes of state, the increase in internal energy of the gas, the thermal energy transferred to the gas and the work done on the gas, in terms of *p*, *V*, *U* and *W*.

Table 4.1

change	increase in internal energy of gas	thermal energy transferred to gas	work done on gas
X to Y			
Y to Z	+*U*		
Z to X			+*W*

[5]

[Total: 12]

A-LEVEL PHYSICS • EXERCISE SHEET

15.3 Kinetic theory of gases

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS root-mean-square 均方根 • kinetic theory 分子动理论 • kinetic energy 动能 • translational kinetic energy 平动动能 • mean-square speed 均方速率

Objective

Build the skills to answer exam questions on the **kinetic theory of gases**.

You must be able to:

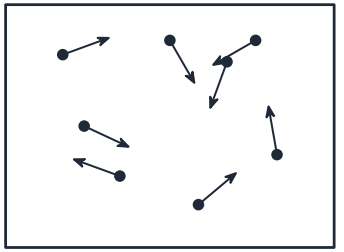
- state the **assumptions** of the kinetic theory and explain gas pressure
- use $pV = \frac{1}{3}Nm\langle c^2\rangle$ and the **r.m.s. speed**
- use mean translational KE $= \frac{3}{2}kT$

1 Worked examples

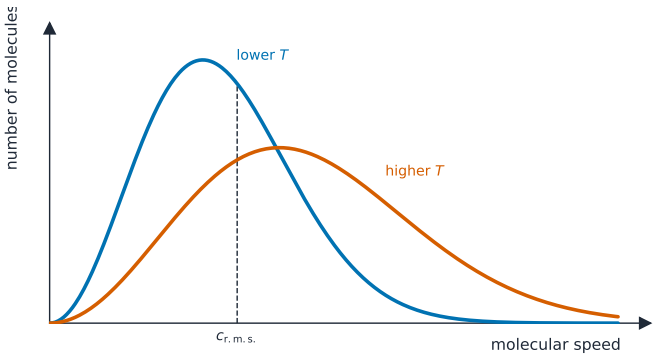
Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Kinetic model

A gas is many identical molecules in **continuous random motion**; their collisions with the walls cause **pressure**:



random motion — wall collisions cause pressure



higher T: peak → higher speeds, flatter curve

The Maxwell-Boltzmann speed distribution

Assumptions: many molecules, random motion; their volume is negligible; collision time is negligible; no forces except in collisions; collisions are **elastic**; Newton's laws hold.

■ **Key results**

$$pV = \tfrac{1}{3}Nm\langle c^2\rangle, \qquad c_{\text{r.m.s.}} = \sqrt{\langle c^2\rangle}.$$

Comparing with $pV = NkT$ gives the **mean translational KE** of a molecule:

$$\tfrac{1}{2}m\langle c^2\rangle = \tfrac{3}{2}kT.$$

So molecular KE depends only on **temperature**.

2 Practice

Now apply the methods above.

2.1 State **two** assumptions of the kinetic theory of gases. [2]

2.2 Explain how gas molecules cause **pressure** on the walls of a container. [2]

2.3 State what is meant by the **root-mean-square (r.m.s.) speed**. [1]

2.4 Write the relationship $pV = \tfrac{1}{3}Nm\langle c^2\rangle$ in words. [1]

3 Exam-style questions

3.1 The **mean translational kinetic energy** of a gas molecule is: [1]

- **A** kT
- **B** $\tfrac{3}{2}kT$
- **C** $\tfrac{1}{2}mv^2$ only
- **D** RT

3.2 Calculate the mean translational kinetic energy of a gas molecule at 300 K. ($k = 1.38 \times 10^{-23}$.) [2]

3.3 A gas at 300 K has molecules of mass 5.3×10^{-26} kg. Calculate the **r.m.s. speed**. ($k = 1.38 \times 10^{-23}$.) [3]

3.4 Explain, using the kinetic theory, why the pressure of a fixed mass of gas **increases** when it is heated at **constant volume**. [2]

3.5 A fixed sample of an ideal gas at thermodynamic temperature T has internal energy U.

(a) **State** what is meant by the **internal energy** of a system. [2]

(b) **Explain** why the internal energy of an **ideal** gas is directly proportional to its thermodynamic temperature. [2]

(c) The gas is first **compressed**, which raises its temperature to $3T$; work W is done on it during this compression. It is then **cooled at constant volume** back to $2T$. **Complete** the table, giving each entry in terms of some or all of W and U. [4]

process	**work done on the gas**	**thermal energy supplied to the gas**	**increase in internal energy**
compression to $3T$	W	_____	_____
cooling to $2T$	_____	_____	_____

(d) **Explain** why the work done on the gas during the second process is what it is. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- any two: many identical molecules in **random motion**; molecular **volume negligible**; **no forces** except during collisions; collisions are **elastic**; collision time negligible

2.2

- molecules collide with the walls and **change momentum** (rebound)
- this exerts a **force** on the wall; force over the wall area is the **pressure**

2.3

- the **square root of the mean of the squared speeds** of the molecules ($\sqrt{\langle c^2 \rangle}$)

2.4

- the product of pressure and volume equals **one third** of (the number of molecules × molecular mass × mean-square speed)

3.1 B

- mean kinetic energy of a molecule is $\frac{3}{2}kT$
- **A** is kT; **C** is a speed formula, not the mean KE; **D** is RT, which is for a mole, not one molecule

3.2

- $E = \frac{3}{2}kT$

$$E = \tfrac{3}{2}(1.38 \times 10^{-23}\ \text{J K}^{-1})(300\ \text{K}) = \mathbf{6.2 \times 10^{-21}}\ \text{J}$$

3.3

- $\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT \Rightarrow \langle c^2 \rangle = 3kT/m$

$$c_{\text{r.m.s.}} = \sqrt{\frac{3(1.38 \times 10^{-23}\ \text{J K}^{-1})(300\ \text{K})}{5.3 \times 10^{-26}\ \text{kg}}} = \mathbf{4.8 \times 10^{2}}\ \text{m s}^{-1}$$

3.4

- heating raises the **mean KE / mean speed** of the molecules
- they hit the walls **harder and more often**, so the pressure rises (volume fixed)

3.5

(a) the **sum of the random kinetic and potential energies** of all the molecules

- "random" means the motion has no overall direction — the whole container moving does not count

(b) in an ideal gas the molecules exert **no forces** on one another except during collisions, so the total **potential** energy is zero and the internal energy is entirely kinetic

- the mean kinetic energy of a molecule is $\frac{3}{2}kT$, i.e. proportional to T, so the total is proportional to T as well

(c) internal energy is proportional to T, so at T it is U, at $3T$ it is $3U$ and at $2T$ it is $2U$

process	**work done on the gas**	**thermal energy supplied**	**increase in internal energy**
compression to $3T$	W	$2U - W$	$+2U$
cooling to $2T$	0	$-U$	$-U$

- $+2U$; $-U$; the zero work; both thermal-energy entries, from $\Delta U = q + w$

(d) at **constant volume** the gas neither expands nor is compressed, so no work is done on or by it

4 **(a)** **(i)** State what is meant by the internal energy of a system.

.....

.....

..... [2]

(ii) Explain why the internal energy of an ideal gas is directly proportional to the thermodynamic temperature of the gas.

.....

.....

.....

..... [2]

(b) A sample of an ideal gas at thermodynamic temperature *T* has internal energy *U*.

The gas is compressed so that its temperature increases to 3*T*.
During this compression, work *W* is done on the gas.

The gas is then cooled at constant volume so that its temperature decreases to 2*T*.

Complete Table 4.1 to show, in terms of some or all of *W*, *T* and *U*, the work done on the gas, the thermal energy supplied to the gas and the increase in internal energy of the gas for each of the two processes.

Table 4.1

	work done on gas	thermal energy supplied to gas	increase in internal energy of gas
compression	+*W*		
cooling			

[4]

[Total: 8]

4 **(a)** **(i)** State what is meant by the internal energy of a system.

.....

.....

..... [2]

(ii) Explain why the internal energy of an ideal gas is directly proportional to the thermodynamic temperature of the gas.

.....

.....

.....

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(b) A sample of an ideal gas at thermodynamic temperature T has internal energy U.

The gas is compressed so that its temperature increases to $3T$.
During this compression, work W is done on the gas.

The gas is then cooled at constant volume so that its temperature decreases to $2T$.

Complete Table 4.1 to show, in terms of some or all of W, T and U, the work done on the gas, the thermal energy supplied to the gas and the increase in internal energy of the gas for each of the two processes.

Table 4.1

	work done on gas	thermal energy supplied to gas	increase in internal energy of gas
compression	$+W$		
cooling			

[4]

[Total: 8]

3 **(a)** The product pV for an ideal gas is given by

$$pV = \frac{1}{3}Nm\langle c^2\rangle$$

where p is the pressure of the gas and V is the volume of the gas.

(i) State the meaning of the symbols N, m and $\langle c^2\rangle$ in this equation.

N: .....

m: .....

$\langle c^2\rangle$: .....

[3]

(ii) Use the equation of state for an ideal gas to show that the average translational kinetic energy E_K of a molecule of the gas at thermodynamic temperature T is given by

$$E_K = \frac{3}{2}kT.$$

[2]

(b) The surface of a star consists mainly of a gas that may be assumed to be ideal. The molecules of the gas have a root-mean-square (r.m.s.) speed of $9300\,\text{m s}^{-1}$.

The mass of a molecule of the gas is 3.34×10^{-27} kg.

Determine, to three significant figures, the temperature of the surface of the star.

temperature = ... K [2]

(c) The radiant flux intensity of the radiation from the star in **(b)** is $2.52 \times 10^{-8}\,W\,m^{-2}$ when observed at a distance of $4.16 \times 10^{16}\,m$ from the star.

(i) Calculate the luminosity of the star. Give a unit with your answer.

luminosity = ... unit ..... [2]

(ii) Determine the radius of the star.

radius = ... m [2]

(d) The gas at the surface of a star has a very high pressure.

Use the basic assumptions of the kinetic theory to suggest why, in practice, a gas at the surface of a star is unlikely to behave as an ideal gas.

.....

.....

.....

..... [2]

[Total: 13]

4 **(a)** State **two** of the basic assumptions of the kinetic theory of gases.

1 .....

.....

2 .....

.....

[2]

(b) An ideal gas has amount of substance n.
The gas is initially in state X, with pressure $2p$ and volume V.
The gas is cooled at constant volume to state Y, with pressure p.
The gas is then heated at constant pressure to state Z, with volume $2V$.
Finally, the gas returns at constant temperature to state X.

(i) Determine an expression for the temperature T of the gas in state X, in terms of n, p and V.
Identify any other symbols that you use.

[2]

(ii) On Fig. 4.1, sketch the variation with volume of pressure for the gas as the gas undergoes the three changes. The state X is labelled. Label states Y and Z.

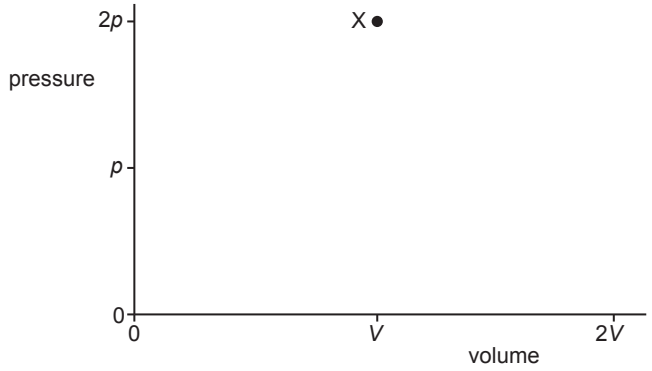

Fig. 4.1

[3]

(iii) During the change of state from Y to Z, the increase in internal energy of the gas is U. During the change of state from Z to X, the work done on the gas is W.

Complete Table 4.1 to indicate, for each of the three changes of state, the increase in internal energy of the gas, the thermal energy transferred to the gas and the work done on the gas, in terms of p, V, U and W.

Table 4.1

change	increase in internal energy of gas	thermal energy transferred to gas	work done on gas
X to Y			
Y to Z	$+U$		
Z to X			$+W$

[5]

[Total: 12]