Week 30

A-Level Physics

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

A-Level Physics Name: ____________ Score: ______

1. Gravitational field strength is the force per unit:
   - ☐ mass
   - ☐ charge
   - ☐ area
   - ☐ volume

2. Newton's law of gravitation gives the force between two masses as:
   - ☐ $\frac{Gm_1m_2}{r^2}$
   - ☐ $\frac{Gm_1m_2}{r}$
   - ☐ $Gm_1m_2r^2$
   - ☐ $\frac{m_1m_2}{Gr^2}$

3. The gravitational field strength at distance r from a point mass M is:
   - ☐ $\frac{GM}{r^2}$
   - ☐ $\frac{GM}{r}$
   - ☐ $\frac{GMm}{r^2}$
   - ☐ GMr^2

4. Gravitational potential is:
   - ☐ negative everywhere (zero at infinity)
   - ☐ positive everywhere
   - ☐ zero everywhere
   - ☐ always increasing with depth

5. A 2.0 kg mass feels a gravitational force of 20 N. What is g there?

   ____________ N/kg

6. Gravity is always attractive.

   ☐ True ☐ False

7. Climbing a tall building changes the value of g by a large amount.

☐ True ☐ False

8. Gravitational potential is taken to be zero at ______.

____

9. The unit N/kg is the same as m/s^2.

☐ True ☐ False

10. Two masses attract with 40 N at separation r. What is the force at separation $2r$?

_________________ N

13. Gravitational fields

Answers · 答案

A-Level Physics

1. **mass**

   $g = \frac{F}{m}$ — the gravitational force on each kilogram of a small test mass.

2. $\frac{Gm_1m_2}{r^2}$

   The pull is proportional to each mass and inversely proportional to the distance squared.

3. $\frac{GM}{r^2}$

   From $g = \frac{F}{m}$ with $F = \frac{GMm}{r^2}$, the test mass cancels: $g = \frac{GM}{r^2}$.

4. **negative everywhere (zero at infinity)**

   Gravity does the work as a mass falls in, so $\phi = -\frac{GM}{r}$ is negative, reaching zero only at infinity.

5. **10 N/kg**

   $g = \frac{F}{m} = \frac{20}{2.0} = 10\ \frac{\text{N}}{\text{kg}}$.

6. **True**

   Yes — masses only ever pull together; there is no gravitational repulsion.

7. **False**

   Earth's radius is ~6400 km, so a few metres of height barely changes r — g is effectively constant.

8. **infinity**

   That choice makes the potential negative everywhere a mass actually is.

9. **True**

   Yes — g is also the acceleration of free fall, so its units are equivalent.

10. **10 N**

    Inverse-square: $\frac{40}{2^2} = \frac{40}{4} = 10$ N.

A-LEVEL PHYSICS • EXERCISE SHEET

13.1 Gravitational field

Name: ____________ Class: ________ Date: __________ **Total: 20 marks**

KEY WORDS gravitational field 重力场 • field line 场线 • gravitational field strength 重力场强度 • force 力 • mass 质量

Objective

Build the skills to answer exam questions on the **gravitational field** — its definition and field lines.

You must be able to:

- define **gravitational field strength** as force per unit mass
- represent a gravitational field with **field lines**

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Field strength

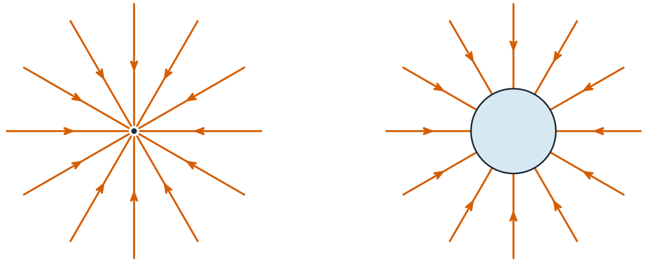
A **gravitational field** is a region where a mass feels a force. The **field strength** is the force per unit mass on a small test mass:

$$g = \frac{F}{m} \quad (\mathrm{N\ kg^{-1}}).$$

It is a **vector**, pointing towards the source mass (the same value as the free-fall acceleration).

■ Field lines

(a) radial field (b) uniform field



point mass uniform sphere

The radial gravitational field of a sphere

- around a **point mass / sphere**: **radial**, pointing **inwards**;
- near a surface (small region): **uniform** — parallel, equally spaced, pointing down.

Closer lines → stronger field.

2 Practice

Now apply the methods above.

2.1 Define **gravitational field strength**. [1]

2.2 State the unit of gravitational field strength. [1]

2.3 State what a gravitational **field line** represents. [1]

2.4 State the shape of the gravitational field around a **point mass**. [1]

3 Exam-style questions

3.1 Gravitational field strength is equal to: [1]

- **A** force × mass
- **B** force ÷ mass
- **C** mass ÷ force
- **D** mass × acceleration

3.2 A mass of 2.0 kg experiences a gravitational force of 19.6 N. Find the **gravitational field strength**. [2]

3.3 Explain what **closer** field lines indicate. [1]

3.4 Sketch the gravitational field lines (a) around a point mass and (b) near a flat surface. [2]

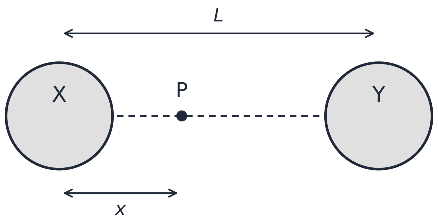
3.5 A point mass M sits at the origin. Point P is a distance x from it.

(a) **Define** the **gravitational field** at a point. [1]

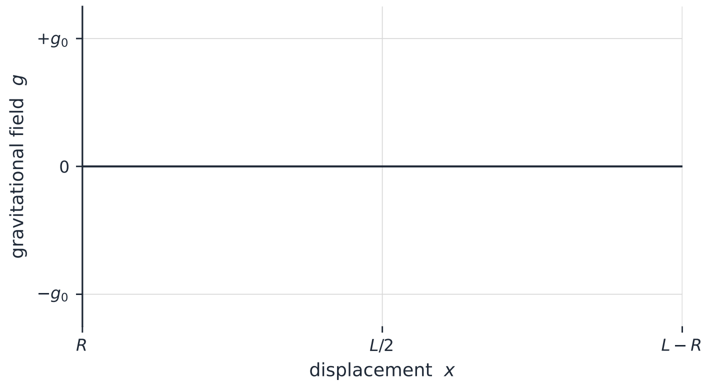
(b) By considering the force on a test mass m placed at P, **derive** an expression for the gravitational field strength g at P in terms of M and x. State the meaning of any other symbol you use. [2]

(c) Point Q lies on the **opposite** side of the mass from P, at a distance $\frac{x}{2}$ from it. **Compare** the gravitational field at Q with the field at P. [2]

(d) Two identical uniform spheres X and Y, each of radius R, have their centres a distance L apart. Point P now lies on the line joining their centres, a displacement x from the centre of X. The gravitational field strength at the surface of each sphere is g_0.



On the axes below, **sketch** how the field g at P varies with x, from $x = R$ to $x = L - R$. Take the direction from X towards Y as positive. [3]



(e) **Explain** why the field is zero at the midpoint, even though neither sphere has been removed. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the **gravitational force per unit mass** on a small test mass at that point ($g = F/m$)

2.2

- N kg^{-1}

2.3

- the **direction of the gravitational force** on a (test) mass at that point

2.4

- **radial**, pointing **inwards** towards the mass

3.1 B

- $g = F/m$, so field strength is **force ÷ mass**
- **A** is force; **C** is GMm/r^2 without dividing by m; **D** is potential

3.2

- $F = 19.6$ N, $m = 2.0$ kg; $g = F/m$

$$g = \frac{19.6\ \text{N}}{2.0\ \text{kg}} = \mathbf{9.8}\ \text{N kg}^{-1}$$

3.3

- a **stronger** gravitational field

3.4

(a) **radial** lines pointing inward to the mass

(b) **parallel, equally spaced** lines pointing down towards the surface

3.5

(a) the gravitational **force per unit mass** acting on a small test mass placed at that point

(b) the force on the test mass is $F = GMm/x^2$, where G is the gravitational constant

- field strength is the force per unit mass

$$g = \frac{F}{m} = \frac{GM}{x^2}$$

(c) the fields point in **opposite directions**, since each points towards the mass

- $g \propto 1/x^2$, so halving the distance makes the field at Q **four times** the field at P

(d) the curve starts at $(R,\ -g_0)$, since at X's surface the field points back towards X, i.e. negative

- it passes through zero at the midpoint $x = L/2$
- it ends at $(L - R,\ +g_0)$, becoming shallower up to the midpoint and steeper after it

(e) the two spheres are identical and the midpoint is equidistant from both, so their fields there are **equal in magnitude**

- and **opposite in direction**, so as a vector the two cancel to zero
- *(the gravitational potential there is not zero — only the field is)*

3 **(a)** Define gravitational field at a point.

....

.... [1]

(b) Fig. 3.1 shows an isolated point mass of mass M.

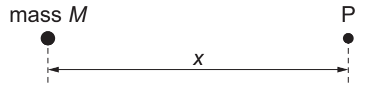


Fig. 3.1

Point P is at distance x from the point mass.

(i) By considering the force exerted by the point mass on a test mass of mass m placed at P, derive an equation for the gravitational field strength g at P, in terms of M and x. Identify any other symbols you use.

[2]

(ii) On Fig. 3.1, draw an arrow to indicate the direction of the gravitational field at P. [1]

(iii) Point Q is at distance $\frac{x}{2}$ from the point mass, on the opposite side of the mass from P, as shown in Fig. 3.2.

Fig. 3.2

Compare the gravitational field at Q with that at P.

....

....

.... [2]

(c) Two identical isolated uniform spheres X and Y each have radius R. The centres of the spheres are separated by distance L, as shown in Fig. 3.3.

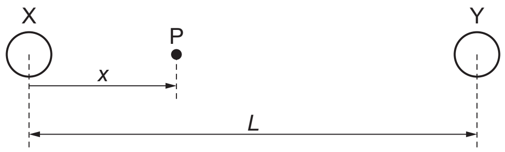


Fig. 3.3

Point P lies on the line joining the centres of X and Y, and is at a variable displacement x from the centre of sphere X.
The gravitational field strength at the surface of each sphere is g_0.

On Fig. 3.4, sketch the variation with x of the gravitational field g at point P between $x = R$ and $x = L - R$.

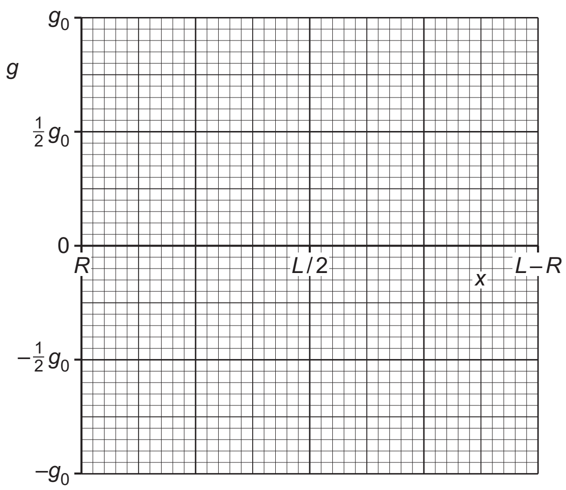


Fig. 3.4

[3]

[Total: 9]

1 **(a)** Define gravitational field.

....

.... [1]

(b) The gravitational field strength g at a distance x from the centre of a uniform spherical planet of mass M is given by the expression

$$g = \frac{GM}{x^2}$$

where G is the gravitational constant and distance x is greater than the radius of the planet.

(i) Describe the pattern of the field lines outside the planet that represent the gravitational field due to the planet.

....

....

.... [2]

(ii) Explain why, for small changes in vertical height near the surface of the planet, g may be assumed to be constant.

....

....

.... [2]

(c) Assume that the Earth is a uniform sphere. For the Earth, the product GM is equal to $3.99 \times 10^{14}\,\mathrm{m^3\,s^{-2}}$.

(i) Determine a value, to three significant figures, for the radius R of the Earth.

R = ... m [2]

(ii) Calculate the gravitational potential at the Earth's surface. Give a unit with your answer.

gravitational potential = ... unit .... [2]

(d) Explain why the gravitational potential energy of two point masses is always negative.

....

....

.... [2]

[Total: 11]

1 **(a)** **(i)** State what is indicated by the direction of the gravitational field line at a point in a gravitational field.

....

.... [1]

(ii) Explain, with reference to gravitational field lines, why the gravitational field near the surface of the Earth is approximately constant for small changes in height.

....

....

.... [2]

(b) A large isolated uniform sphere has mass M and radius R.

Point P lies on a straight line passing through the centre of the sphere, at a variable displacement x from the centre, as shown in Fig. 1.1.

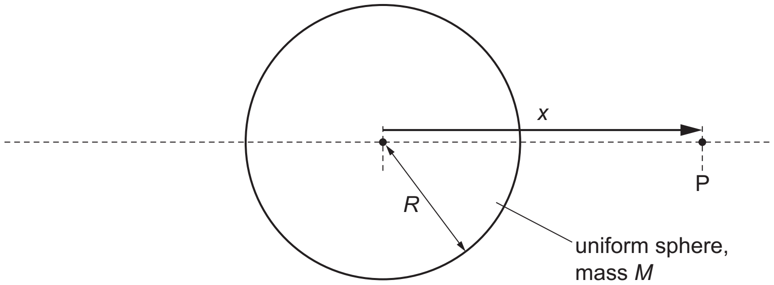


Fig. 1.1

Fig. 1.2 shows the variation with x of the gravitational field g at point P due to the sphere for the values of x for which P is inside the sphere.

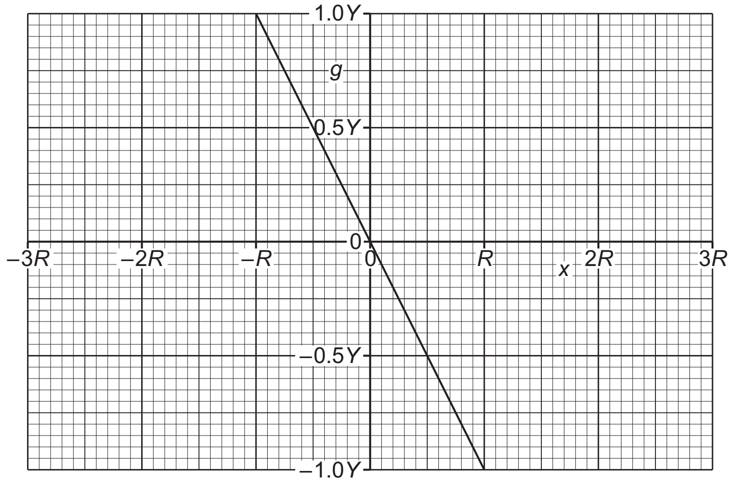


Fig. 1.2

The magnitude of the gravitational field at the surface of the sphere is Y.

(i) Determine an expression for Y in terms of M and R. Identify any other symbols that you use.

[2]

(ii) Explain why, at the surface of the sphere, g always has the opposite sign to x.

....

....

.... [2]

(iii) Complete Fig. 1.2 to show the variation of g with x for values of x, up to $\pm 3R$, for which point P is outside the sphere. [3]

[Total: 10]

A-LEVEL PHYSICS • EXERCISE SHEET

13.2 Gravitational force between point masses

Name: ____________ Class: ________ Date: __________ **Total: 16 marks**

KEY WORDS orbit 轨道 • period 周期 • satellite 卫星 • centripetal force 向心力 • geostationary 地球同步

Objective

Build the skills to answer exam questions on **Newton's law of gravitation** and orbits.

You must be able to:

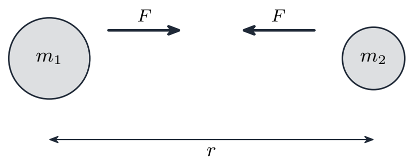
- use $F = \dfrac{Gm_1m_2}{r^2}$ and treat a sphere as a point mass
- analyse **circular orbits** (gravity provides the centripetal force)
- describe a **geostationary orbit**

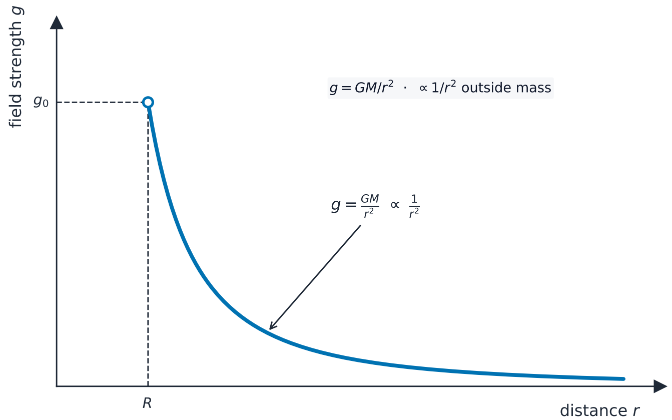
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ **Newton's law of gravitation**

Two point masses attract with **equal, opposite** forces along the line joining them:





Gravitational field strength against distance

$$F = \frac{Gm_1m_2}{r^2}, \qquad G = 6.67 \times 10^{-11}\ \mathrm{N\ m^2\ kg^{-2}}.$$

A uniform sphere acts as a **point mass at its centre** for points outside it.

■ Circular orbits

For a satellite, gravity **is** the centripetal force:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} = mr\omega^2 \Rightarrow T^2 = \frac{4\pi^2 r^3}{GM}.$$

A **geostationary** satellite: period **24 h**, orbits **west→east** above the **equator**, staying over one point on Earth.

2 Practice

Now apply the methods above.

2.1 State **Newton's law of gravitation.** [2]

2.2 State the value and unit of the gravitational constant G. [1]

2.3 Two 1000 kg masses are 2.0 m apart. Find the gravitational force between them. [2]

2.4 State **two** features of a **geostationary** orbit. [2]

3 Exam-style questions

3.1 If the distance between two masses is **doubled**, the gravitational force becomes: [1]

- **A** twice as large
- **B** half as large
- **C** one quarter as large
- **D** four times as large

3.2 Calculate the gravitational field strength at the Earth's surface. ($M = 6.0 \times 10^{24}$ kg, $R = 6.4 \times 10^6$ m, $G = 6.67 \times 10^{-11}$.) [3]

3.3 A satellite moves in a circular orbit of radius r around a planet of mass M. Show that gravity provides the centripetal force, and derive an expression linking the orbital **period** T to r. [4]

3.4 State why a **geostationary** satellite must orbit directly above the **equator**. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the gravitational force between two **point masses** is **proportional to the product of the masses**
- and **inversely proportional to the square of their separation**

2.2

- $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

2.3

- $F = Gm_1m_2/r^2$

$$F = \frac{(6.67 \times 10^{-11}\ \text{N m}^2\,\text{kg}^{-2})(1000\ \text{kg})(1000\ \text{kg})}{(2.0\ \text{m})^2} = \mathbf{1.67 \times 10^{-5}}\ \text{N}$$

2.4

- any two: period of **24 h**; orbits **west to east**; stays **above the equator**; appears fixed over one point on Earth

3.1 C

- $F \propto 1/r^2$, so doubling r makes F **one quarter**
- **A** would be $1/r$; **B** is twice as far so half the force; **D** is four times larger

3.2

- $g = GM/R^2$

$$g = \frac{(6.67 \times 10^{-11}\ \text{N m}^2\,\text{kg}^{-2})(6.0 \times 10^{24}\ \text{kg})}{(6.4 \times 10^6\ \text{m})^2} = \mathbf{9.8}\ \text{N kg}^{\mathbf{-1}}$$

3.3

- gravity provides the centripetal force: $GMm/r^2 = mv^2/r$
- $v = 2\pi r/T$

$$\frac{GM}{r^2} = \frac{4\pi^2 r}{T^2}$$

- $T^2 = 4\pi^2 r^3/(GM)$

3.4

- its orbital plane must contain the **Earth's centre**; only an equatorial orbit keeps it over a **fixed point** as the Earth spins

1 **(a)** Define gravitational potential at a point.

....

....

.... [2]

(b) Mars is a planet that may be considered to be an isolated uniform sphere of radius 3.4×10^6 m.

A satellite of mass 122 kg is in orbit around Mars at a constant height of 1.7×10^6 m above the surface of the planet.

The height of the orbit is increased to 6.8×10^6 m above the surface. This increases the gravitational potential energy of the satellite by 5.1×10^8 J.

(i) Show that the mass of Mars is 6.4×10^{23} kg.

[3]

(ii) Calculate the gravitational potential ϕ at the surface of Mars. Give a unit with your answer.

ϕ = ... unit .... [2]

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(c) The satellite in **(b)** is moved to an orbit in which the satellite remains at the same point above the surface of Mars.

(i) The orbit has a period of 25 hours.

State what can be deduced from this about the rotation of Mars on its axis.

....

.... [1]

(ii) State **one** other feature of this orbit.

....

.... [1]

[Total: 9]

1 **(a)** Define gravitational potential at a point.

....

....

.... [2]

(b) Mars is a planet that may be considered to be an isolated uniform sphere of radius 3.4×10^6 m.

A satellite of mass 122 kg is in orbit around Mars at a constant height of 1.7×10^6 m above the surface of the planet.

The height of the orbit is increased to 6.8×10^6 m above the surface. This increases the gravitational potential energy of the satellite by 5.1×10^8 J.

(i) Show that the mass of Mars is 6.4×10^{23} kg.

[3]

(ii) Calculate the gravitational potential ϕ at the surface of Mars. Give a unit with your answer.

ϕ = ... unit .... [2]

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(c) The satellite in **(b)** is moved to an orbit in which the satellite remains at the same point above the surface of Mars.

(i) The orbit has a period of 25 hours.

State what can be deduced from this about the rotation of Mars on its axis.

....

.... [1]

(ii) State **one** other feature of this orbit.

....

.... [1]

[Total: 9]

1 (a) State Newton's law of gravitation.

....

....

.... [2]

(b) A planet may be considered as a uniform sphere.

A satellite is in circular orbit of period T around the planet at a height h above the surface. The height of the orbit can be adjusted by use of the satellite's rocket engines.

Fig. 1.1 shows the variation with h of $T^{\frac{2}{3}}$.

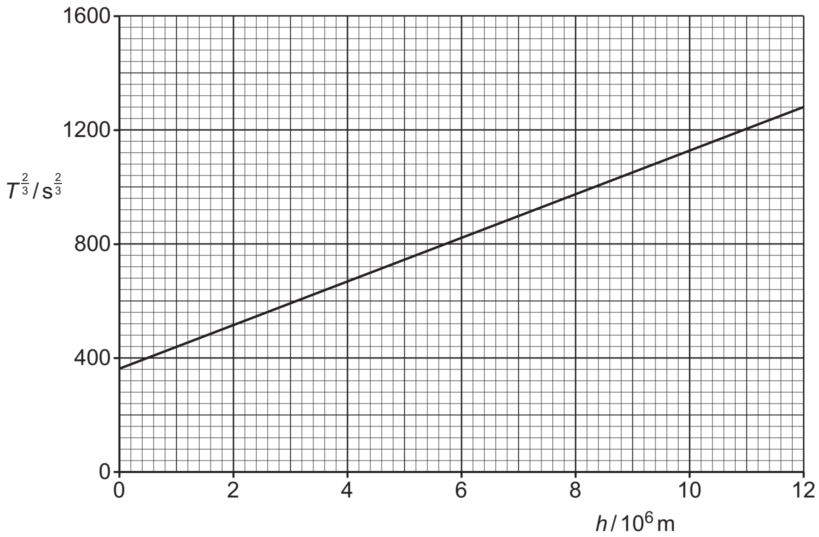


Fig. 1.1

(i) By reference to forces, explain why the orbit of the satellite is circular.

....

....

.... [2]

(ii) Use Newton's law of gravitation to show that h and T are related by

$$(h + B)^3 = \frac{GA}{4\pi^2} T^2$$

where G is the gravitational constant and A and B are constants that depend on the properties of the planet.

[3]

(iii) Use the gradient and intercept of the line in Fig. 1.1 to determine values for A and B. Give units with your answers.

A = ... unit ....

B = ... unit ....

[5]

[Total: 12]

1 **(a)** State Newton's law of gravitation.

....

....

.... [2]

(b) A satellite is in a circular orbit around a planet. The radius of the orbit is R and the period of the orbit is T. The planet is a uniform sphere.

Use Newton's law of gravitation to show that R and T are related by

$$4\pi^2 R^3 = GMT^2$$

where M is the mass of the planet and G is the gravitational constant.

[2]

(c) The Earth may be considered to be a uniform sphere of mass 5.98×10^{24} kg and radius 6.37×10^6 m.

A geostationary satellite is in orbit around the Earth.

Use the expression in **(b)** to determine the height of the satellite above the Earth's surface.

height = ... m [3]

(d) Another satellite is in a circular orbit around the Earth with the same orbital radius and period as the satellite in **(c)**.

(i) Calculate the angular speed of the satellite in this orbit. Give a unit with your answer.

angular speed = ... unit .... [2]

(ii) Despite having the same orbital period, the orbit of this satellite is not geostationary.

Suggest **two** ways in which the orbit of this satellite could be different from the orbit of the satellite in **(c)**.

1 ....

....

2 ....

....
[2]

[Total: 11]

A-LEVEL PHYSICS • EXERCISE SHEET

13.3 Gravitational field of a point mass

Name: ____________ Class: ________ Date: __________ **Total: 26 marks**

KEY WORDS gravitational field 重力场 • gravitational field strength 重力场强度 • field line 场线 • newton's law of gravitation 万有引力定律 • mass 质量

Objective

Build the skills to answer exam questions on the **gravitational field of a point mass** — the equation $g = GM/r^2$.

You must be able to:

- derive and use $g = \dfrac{GM}{r^2}$
- describe how g varies with distance
- explain why g is nearly constant near the Earth's surface

1 Worked examples

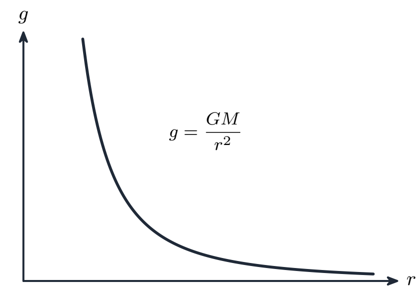
Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

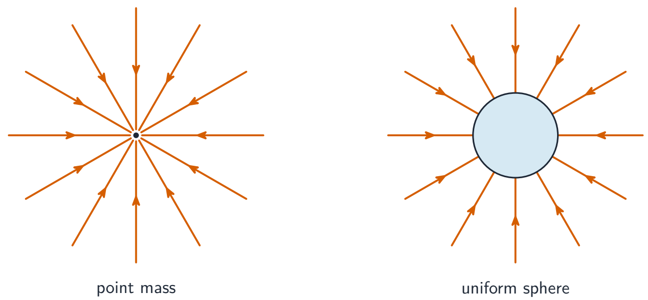
■ Field of a point mass

Put $F = \dfrac{GMm}{r^2}$ into $g = F/m$:

$$g = \frac{GM}{r^2}.$$

So g falls off as an **inverse square** of distance:





Field lines around a point mass

■ Nearly constant near the surface

Rising a height $h \ll R$ changes r from R to $R + h$, but $(R + h)/R \approx 1$, so g barely changes — it is effectively constant in a lab.

2 Practice

Now apply the methods above.

2.1 Write the equation for the gravitational field strength g due to a point mass. [1]

2.2 State how g varies with distance r from a point mass. [1]

2.3 At the Earth's surface $g = 9.8$ N kg^{-1}. Find g at a distance of $2R$ from the centre (a height R above the surface). [2]

2.4 Explain why g is **nearly constant** near the Earth's surface. [1]

3 Exam-style questions

3.1 At **twice** the distance from a point mass, g becomes: [1]

- **A** twice as large
- **B** half as large
- **C** one quarter as large
- **D** four times as large

3.2 The Moon has mass 7.3×10^{22} kg and radius 1.7×10^{6} m. Calculate g at its surface. ($G = 6.67 \times 10^{-11}$.) [3]

3.3 Derive $g = \dfrac{GM}{r^2}$ from Newton's law of gravitation and $g = F/m$. [2]

3.4 A planet has the same mass as Earth but **twice** the radius. State its surface g as a fraction of Earth's. [1]

3.5 On the axes, **sketch** how the gravitational field strength g of a planet varies with distance r from its centre, for r from the surface ($r = R$) out to $r = 4R$. Mark the surface value g_0. [2]

3.6 A planet of mass M and radius R has a gravitational field strength of g_0 at its surface. A satellite of mass m orbits it in a circle of radius r, where $r > R$.

(a) **State** why the gravitational field strength inside a **uniform** sphere behaves differently from the field outside it. [1]

(b) **Show that** the orbital speed of the satellite is $v = \sqrt{\dfrac{GM}{r}}$, and hence that it does **not** depend on the mass of the satellite. [3]

(c) **Derive** an expression for the period T of the orbit in terms of G, M and r, and state the name of the law it expresses. [3]

(d) A geostationary satellite orbits the Earth once every 24 hours. Taking $GM = 4.00 \times 10^{14}\ \mathrm{m^3\,s^{-2}}$, **determine** the radius of its orbit. [3]

(e) **State** two further conditions a geostationary orbit must satisfy, beyond having the right period. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $g = GM/r^2$

2.2

- $g \propto 1/r^2$ (inverse-square)

2.3

- at $2R$, g is $1/2^2 = 1/4$ of the surface value

$$g = \frac{9.8\ \mathrm{N\ kg^{-1}}}{4} = \mathbf{2.45}\ \mathrm{N\ kg^{-1}}$$

2.4

- a small height change is **tiny compared with the Earth's radius**, so r (and hence $g \propto 1/r^2$) barely changes

3.1 C

- $g \propto 1/r^2$, so at $2R$ the field is **one quarter**

3.2

- $g = GM/R^2$

$$g = \frac{(6.67 \times 10^{-11}\ \mathrm{N\ m^2\,kg^{-2}})(7.3 \times 10^{22}\ \mathrm{kg})}{(1.7 \times 10^6\ \mathrm{m})^2} = \mathbf{1.7}\ \mathrm{N\ kg^{-1}}$$

3.3

- force on a test mass m: $F = GMm/r^2$
- field strength $g = F/m = GM/r^2$

3.4

- $g \propto 1/R^2$, so 1/4 of Earth's

3.5

- a curve **starting at** g_0 at $r = R$ and **falling** as an inverse square
- to $g_0/4$ at $2R$, $g_0/9$ at $3R$, $g_0/16$ at $4R$ — approaching (but never reaching) zero

3.6

(a) outside the sphere all its mass may be treated as concentrated at the centre, giving $g \propto 1/x^2$; inside, only the mass **closer to the centre** than the point contributes, so the field falls towards zero at the centre instead of rising

(b) the gravitational force provides the centripetal force: $GMm/r^2 = mv^2/r$

- the satellite mass m appears on both sides and **cancels**
- $v^2 = GM/r$, so $v = \sqrt{GM/r}$

(c) $T = 2\pi r/v = 2\pi r\sqrt{r/(GM)}$

- $T^2 = 4\pi^2 r^3/(GM)$
- that is $T^2 \propto r^3$, **Kepler's third law**

(d) $T = 24 \times 3600\ \text{s} = 86\,400\ \text{s}$

$$r^3 = \frac{GMT^2}{4\pi^2} = \frac{(4.00 \times 10^{14}\ \text{m}^3\,\text{s}^{-2})(86\,400\ \text{s})^2}{4\pi^2} = 7.56 \times 10^{22}\ \text{m}^3$$

- $r = \mathbf{4.2 \times 10^7}$ m

(e) it must orbit **above the equator**, in the plane of the equator

- and travel **west to east**, the same direction as the Earth's rotation

3 **(a)** Define gravitational field at a point.

....

.... [1]

(b) Fig. 3.1 shows an isolated point mass of mass *M*.

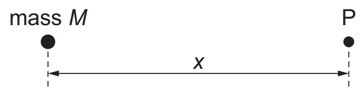


Fig. 3.1

Point P is at distance *x* from the point mass.

(i) By considering the force exerted by the point mass on a test mass of mass *m* placed at P, derive an equation for the gravitational field strength *g* at P, in terms of *M* and *x*. Identify any other symbols you use.

[2]

(ii) On Fig. 3.1, draw an arrow to indicate the direction of the gravitational field at P. [1]

(iii) Point Q is at distance $\frac{x}{2}$ from the point mass, on the opposite side of the mass from P, as shown in Fig. 3.2.

Fig. 3.2

Compare the gravitational field at Q with that at P.

....

....

.... [2]

(c) Two identical isolated uniform spheres X and Y each have radius R. The centres of the spheres are separated by distance L, as shown in Fig. 3.3.

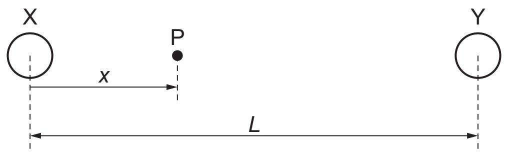


Fig. 3.3

Point P lies on the line joining the centres of X and Y, and is at a variable displacement x from the centre of sphere X.
The gravitational field strength at the surface of each sphere is g_0.

On Fig. 3.4, sketch the variation with x of the gravitational field g at point P between $x = R$ and $x = L - R$.

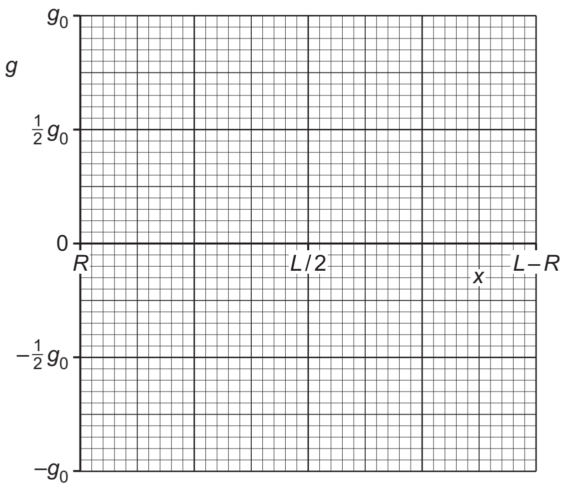


Fig. 3.4

[3]

[Total: 9]

1 **(a)** Define gravitational potential at a point.

....

....

.... [2]

(b) Mars is a planet that may be considered to be an isolated uniform sphere of radius 3.4×10^6 m.

A satellite of mass 122 kg is in orbit around Mars at a constant height of 1.7×10^6 m above the surface of the planet.

The height of the orbit is increased to 6.8×10^6 m above the surface. This increases the gravitational potential energy of the satellite by 5.1×10^8 J.

(i) Show that the mass of Mars is 6.4×10^{23} kg.

[3]

(ii) Calculate the gravitational potential ϕ at the surface of Mars. Give a unit with your answer.

ϕ = ... unit .... [2]

(c) The satellite in **(b)** is moved to an orbit in which the satellite remains at the same point above the surface of Mars.

(i) The orbit has a period of 25 hours.

State what can be deduced from this about the rotation of Mars on its axis.

....

.... [1]

(ii) State **one** other feature of this orbit.

....

.... [1]

[Total: 9]

1 **(a)** Define gravitational field.

....

.... [1]

(b) The gravitational field strength g at a distance x from the centre of a uniform spherical planet of mass M is given by the expression

$$g = \frac{GM}{x^2}$$

where G is the gravitational constant and distance x is greater than the radius of the planet.

(i) Describe the pattern of the field lines outside the planet that represent the gravitational field due to the planet.

....

....

.... [2]

(ii) Explain why, for small changes in vertical height near the surface of the planet, g may be assumed to be constant.

....

....

.... [2]

(c) Assume that the Earth is a uniform sphere. For the Earth, the product GM is equal to $3.99 \times 10^{14}\,\text{m}^3\,\text{s}^{-2}$.

(i) Determine a value, to three significant figures, for the radius R of the Earth.

R = ... m [2]

(ii) Calculate the gravitational potential at the Earth's surface. Give a unit with your answer.

gravitational potential = ... unit .... [2]

(d) Explain why the gravitational potential energy of two point masses is always negative.

....

....

.... [2]

[Total: 11]

A-LEVEL PHYSICS • EXERCISE SHEET

13.4 Gravitational potential

Name: ____________ Class: ________ Date: __________ **Total: 16 marks**

KEY WORDS gravitational potential 引力势 • gravitational potential energy 重力势能 • mass 质量 • point mass 质点 • test mass 检验质量

Objective

Build the skills to answer exam questions on **gravitational potential** — $\phi = -GM/r$ and gravitational potential energy.

You must be able to:

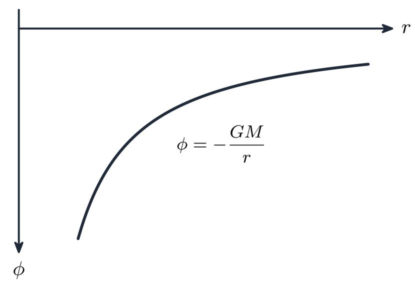
- define **gravitational potential** and use $\phi = -\dfrac{GM}{r}$
- use $E_P = -\dfrac{GMm}{r}$ for two point masses
- explain why potential and E_P are negative

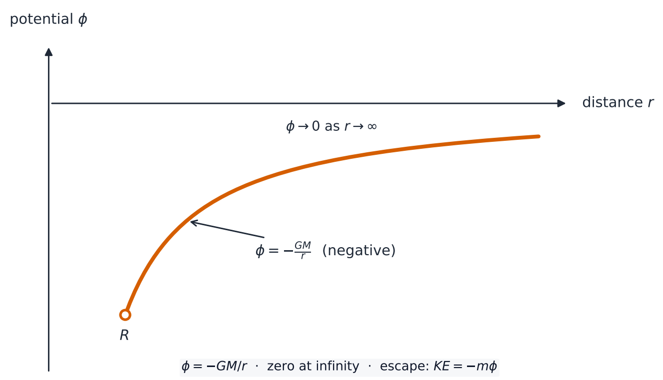
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

Gravitational potential

Gravitational potential ϕ at a point is the **work done per unit mass** to bring a small test mass **from infinity** to that point. Potential is **zero at infinity** and **negative** everywhere else:





The gravitational potential well

$$\phi = -\frac{GM}{r} \quad (\mathrm{J\ kg^{-1}}), \qquad E_P = m\phi = -\frac{GMm}{r}.$$

For two radii, $\Delta E_P = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$.

2 Practice

Now apply the methods above.

2.1 Define **gravitational potential**. [1]

2.2 State **why** gravitational potential is **negative**. [1]

2.3 State the unit of gravitational potential. [1]

2.4 Write the equation for the **gravitational potential energy** of two point masses. [1]

3 Exam-style questions

3.1 The gravitational potential at **infinity** is: [1]

- **A** a maximum positive value
- **B** zero
- **C** a large negative value
- **D** infinite

3.2 Calculate the gravitational potential at the Earth's surface. ($M = 6.0 \times 10^{24}$ kg, $R = 6.4 \times 10^6$ m, $G = 6.67 \times 10^{-11}$.) [3]

3.3 Calculate the gravitational potential energy of a 1200 kg satellite at $r = 7.0 \times 10^6$ m from the Earth's centre. [2]

3.4 Explain why **two closer** masses have **more negative** gravitational potential energy. [2]

3.5 A 1000 kg satellite orbiting a planet is moved from an orbit of radius 7.0×10^6 m to one of radius 1.4×10^7 m. Its gravitational potential energy **increases** by 2.86×10^{10} J. Calculate the **mass of the planet**, and give a **unit**. ($G = 6.67 \times 10^{-11}$.) [4]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the **work done per unit mass** in bringing a small test mass from **infinity** to that point

2.2

- gravity is **attractive**, so it does the work as the mass moves in from infinity (where $\phi = 0$); the potential therefore becomes **negative**

2.3

- $\mathrm{J\ kg^{-1}}$

2.4

- $E_\mathrm{P} = -GMm/r$

3.1 B

- gravitational potential is defined as **zero** at infinity
- **A** would be a surface value; **C** is a typical Earth-surface ϕ; **D** is not the definition

3.2

- $\phi = -GM/R$

$$\phi = -\frac{(6.67 \times 10^{-11}\ \mathrm{N\ m^2\ kg^{-2}})(6.0 \times 10^{24}\ \mathrm{kg})}{6.4 \times 10^6\ \mathrm{m}} = \mathbf{-6.3 \times 10^7}\ \mathrm{J\ kg^{-1}}$$

3.3

- $E_\mathrm{P} = -GMm/r$

$$E_\mathrm{P} = -\frac{(6.67 \times 10^{-11}\ \mathrm{N\ m^2\ kg^{-2}})(6.0 \times 10^{24}\ \mathrm{kg})(1200\ \mathrm{kg})}{7.0 \times 10^6\ \mathrm{m}} = \mathbf{-6.9 \times 10^{10}}\ \mathrm{J}$$

3.4

- as r decreases, $-GMm/r$ becomes **more negative**
- the masses are more **tightly bound** — more work would be needed to separate them to infinity

3.5

- $\Delta E_\mathrm{P} = GMm(1/r_1 - 1/r_2)$

$$\frac{1}{r_1} - \frac{1}{r_2} = \frac{1}{7.0 \times 10^6\ \mathrm{m}} - \frac{1}{1.4 \times 10^7\ \mathrm{m}} = 7.14 \times 10^{-8}\ \mathrm{m^{-1}}$$

$$M = \frac{2.86 \times 10^{10}\ \mathrm{J}}{(6.67 \times 10^{-11}\ \mathrm{N\ m^2\ kg^{-2}})(1000\ \mathrm{kg})(7.14 \times 10^{-8}\ \mathrm{m^{-1}})} = \mathbf{6.0 \times 10^{24}}\ \mathrm{kg}$$

1 (a) Define gravitational potential at a point.

....

....

.... [2]

(b) Mars is a planet that may be considered to be an isolated uniform sphere of radius 3.4×10^6 m.

A satellite of mass 122 kg is in orbit around Mars at a constant height of 1.7×10^6 m above the surface of the planet.

The height of the orbit is increased to 6.8×10^6 m above the surface. This increases the gravitational potential energy of the satellite by 5.1×10^8 J.

(i) Show that the mass of Mars is 6.4×10^{23} kg.

[3]

(ii) Calculate the gravitational potential ϕ at the surface of Mars. Give a unit with your answer.

ϕ = ... unit .... [2]

(c) The satellite in **(b)** is moved to an orbit in which the satellite remains at the same point above the surface of Mars.

(i) The orbit has a period of 25 hours.

State what can be deduced from this about the rotation of Mars on its axis.

....

.... [1]

(ii) State **one** other feature of this orbit.

....

.... [1]

[Total: 9]

1 **(a)** Define gravitational potential at a point.

....

....

.... [2]

(b) Mars is a planet that may be considered to be an isolated uniform sphere of radius 3.4×10^6 m.

A satellite of mass 122 kg is in orbit around Mars at a constant height of 1.7×10^6 m above the surface of the planet.

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State what can be deduced from this about the rotation of Mars on its axis.

....

.... [1]

(ii) State **one** other feature of this orbit.

....

.... [1]

[Total: 9]

2 **(a)** The magnitude of the gravitational potential on the surface of a planet of radius R is ϕ. The planet can be considered to be an isolated sphere.

On Fig. 2.1, sketch the variation of the gravitational potential with distance x from the centre of the planet for values of x between R and $4R$.

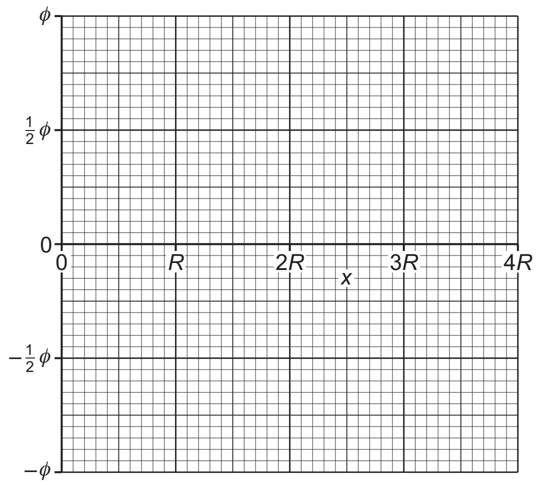

Fig. 2.1

[3]

(b) A satellite is in a geostationary orbit above the Earth. At time t = 0, the magnitude of the gravitational potential due to the Earth at the location of the satellite is ϕ.

On Fig. 2.2, sketch the variation of the gravitational potential due to the Earth at the location of the satellite for values of t between t = 0 and t = 24 hours.

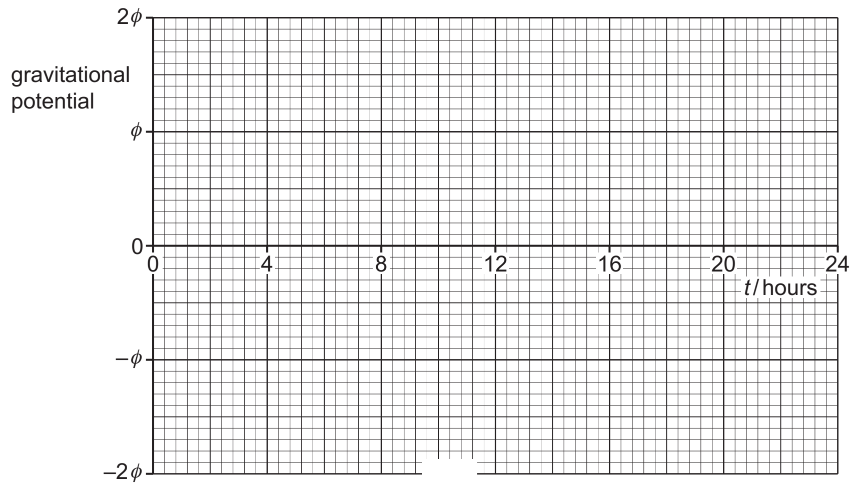

Fig. 2.2

[2]

(c) The electric potential difference (p.d.) between two parallel plates is V, as shown in Fig. 2.3.

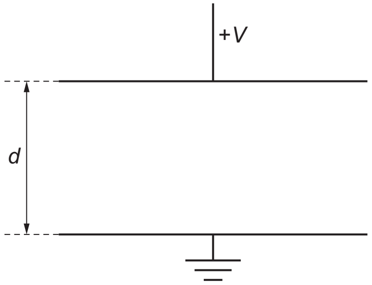

Fig. 2.3

The distance between the plates is d. The region between the plates is a vacuum.

On Fig. 2.4, sketch the variation of the electric potential with distance from the positive plate.

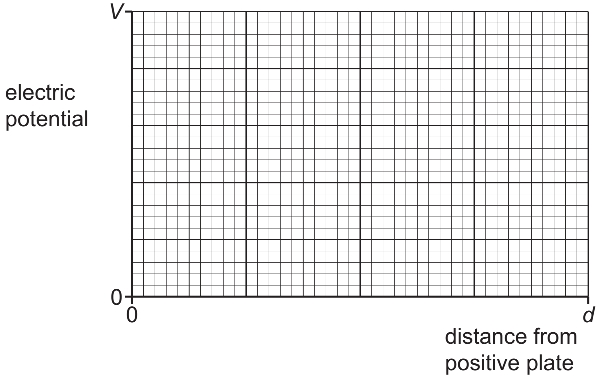

Fig. 2.4

[2]

[Total: 7]