Week 9

A-Level Physics

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

A-Level Physics Name: ____________ Score: ______

1. A force of 20 N acts at a perpendicular distance of 0.50 m from a pivot. What is the moment?

   ____________ N · m

2. Select **both** conditions a body must meet to be in equilibrium.

   ☐ The resultant force is zero

   ☐ The resultant moment is zero

   ☐ The object is at its centre of gravity

   ☐ The object is not moving at all

   Tick every correct option.

3. A block has mass 5.0 kg and volume 0.0020 m^3. What is its density?

   ____________ kg/m^3

4. Why is a door handle placed far from the hinges?

   ☐ A larger distance gives a larger moment for the same push

   ☐ It uses a larger force

   ☐ It makes the door heavier

   ☐ It has no effect on the turning

5. A body with zero resultant force must be in equilibrium.

   ☐ True ☐ False

6. Pressure is the force per unit ______ (acting at right angles to the surface).

   ____

7. Only the part of a force perpendicular to the arm contributes to the turning.

   ☐ True ☐ False

8. At balance, the total clockwise moment equals the total ______ moment.

   ____

9. The extra pressure at a depth in a liquid depends on:

   ☐ the density of the liquid and the depth

   ☐ the shape of the container

   ☐ the area of the surface at the top

☐ the total volume of liquid

10. Select **all** the features of a couple.

    ☐ The two forces are equal in size

    ☐ They point in opposite directions

    ☐ Their lines of action are a distance apart

    ☐ The resultant force is zero

    ☐ They point in the same direction

    Tick every correct option.

4. Forces, density and pressure

Answers · 答案

A-Level Physics

1. **10 N · m**

   $M = F \times d = 20 \times 0.50 = 10\ \text{N} \cdot \text{m}$.

2. **The resultant force is zero; The resultant moment is zero**

   Equilibrium = zero resultant force **and** zero resultant moment. A body can be in equilibrium while moving at constant velocity.

3. **2500 kg/m^3**

   $\rho = \frac{m}{V} = \frac{5.0}{0.0020} = 2500\ \frac{\text{kg}}{\text{m}^3}$.

4. **A larger distance gives a larger moment for the same push**

   Moment $= F \times d$. A bigger d means a bigger turning effect for the same force, so the door opens more easily.

5. **False**

   Not necessarily — a couple gives zero resultant force but still turns the body. You also need zero resultant moment.

6. **area**

   $p = \frac{F}{A}$, measured in Pa $= \frac{\text{N}}{\text{m}^2}$.

7. **True**

   Yes — that is why $M = Fr\sin\theta$: $\sin\theta$ picks out the perpendicular part of the force.

8. **anticlockwise**

   That is the principle of moments — the two turning effects cancel about any chosen point.

9. **the density of the liquid and the depth**

   $\Delta p = \rho g \Delta h$ — only density and depth matter; the container shape does not.

10. **The two forces are equal in size; They point in opposite directions; Their lines of action are a distance apart; The resultant force is zero**

    A couple is equal, opposite forces a distance apart, giving zero resultant force but a pure turning effect. Same-direction forces are not a couple.

A-LEVEL PHYSICS • EXERCISE SHEET

4.1 Turning effects of forces

Name: ____________ Class: ________ Date: __________ **Total: 27 marks**

KEY WORDS force 力 • moment 力矩 • weight 重力 • couple 力偶 • torque 力偶矩

Objective

Build the skills to answer exam questions on the **turning effects of forces** — centre of gravity, moments, couples and torque.

You must be able to:

- use the **centre of gravity** 重心 when drawing weight
- define and calculate the **moment** 力矩 of a force
- define a **couple** and calculate its **torque** 力偶矩
- locate the **centre of gravity** of a hanging object, and use a torque to find how far it lies from the pivot

1 Worked examples

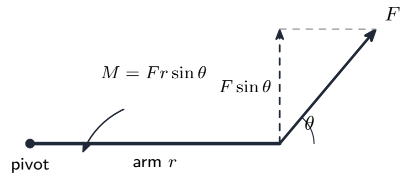
Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

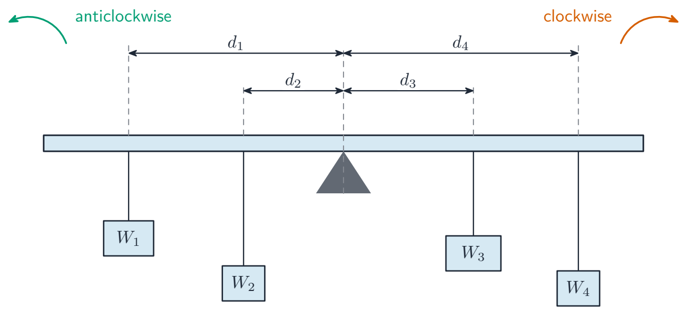
■ Centre of gravity and moment

The whole **weight** acts at the **centre of gravity** (the middle of a uniform shape). The **moment** of a force about a point is

$$M = F \times d,$$

where d is the **perpendicular distance** from the point to the force's line of action. If the force acts at angle θ to a lever arm of length r, then $M = Fr\sin\theta$ — only the part **perpendicular** to the arm turns it:





The principle of moments balances turning effects

Worked example. 50 N at the end of a 0.40 m spanner, at 30° to it:

$$M = 50 \times 0.40 \times \sin 30° = 10 \text{ N m}.$$

■ Finding a centre of gravity

For a **non-uniform** object the centre of gravity is not at the middle, and the exam asks you to find it two ways.

Hang it up. An object free to swing settles with its centre of gravity **vertically below** the point it hangs from. So the centre of gravity lies **somewhere on that vertical line** — hang it from a second point and the two lines cross at it.

Hold it still. If the object is held so that its centre of gravity is a horizontal distance d from the pivot, the weight has a moment mgd about the pivot, and the torque you must apply is equal and opposite:

$$\tau = mgd \quad \Rightarrow \quad d = \frac{\tau}{mg}$$

Worked example. Holding a 2.0 kg non-uniform plate on a rod takes a torque of 1.6 N m:

$$d = \frac{1.6}{2.0 \times 9.81} = 0.082 \text{ m}$$

The direction tells you which side: if the centre of gravity is to the **right** of the rod, the weight turns the plate **clockwise**, so the applied torque must be **anticlockwise**.

■ Couple and torque

A **couple** is two forces that are **equal, opposite**, and a perpendicular distance apart — it produces **turning only** (zero resultant force). Its **torque** is

$$\tau = F \times d,$$

where d is the distance between the two lines of action. *(Two equal forces in the **same** direction are **not** a couple.)*

2 Practice

Now apply the methods above.

2.1 Define the **moment** of a force. [1]

2.2 A 20 N force acts at right angles to a lever, 0.30 m from the pivot. Find its **moment**. [2]

2.3 State the **two** conditions a pair of forces must meet to be a **couple**. [2]

2.4 State where the **weight** of a uniform rod acts. [1]

3 Exam-style questions

3.1 The **moment** of a force is measured in: [1]

- **A** N
- **B** N m
- **C** N m^{-1}
- **D** J

3.2 A force of 50 N is applied at the end of a 0.40 m spanner, at 30° to the spanner. Calculate the **moment** about the nut. [2]

3.3 Two parallel forces, each 8.0 N, act in opposite directions 0.25 m apart, forming a couple. Calculate the **torque**. [2]

3.4 Explain why two **equal** forces acting in the **same** direction do **not** form a couple. [2]

3.5 A square aluminium sheet of **non-uniform** density has a mass of 1.6 kg and a uniform thickness of 3.0 mm. A thin rod of negligible mass is fixed through the centre of the sheet, and the sheet hangs vertically from the rod. To hold the sheet still, with one corner directly above the rod, a torque of 1.9 N m must be applied. In that position the centre of gravity of the sheet lies to the **right** of the rod.

(a) **Define** the **torque of a couple**. [2]

(b) Explain what is meant by the **centre of gravity** of the sheet. [1]

(c) **Show that** the centre of gravity of the sheet is a horizontal distance of about 0.12 m from the rod. [2]

(d) **State and explain** whether the torque applied to the rod is clockwise or anticlockwise. [2]

(e) The density of the aluminium is 2700 kg m^{-3}. **Show that** the side length of the square sheet is about 0.44 m. [3]

(f) The rod is now removed and the sheet is hung freely from one corner. **State** where its centre of gravity lies relative to that corner. [1]

(g) A second, uniform sheet of the same aluminium is 0.44 m square and 5.0 mm thick. **Determine** its weight. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- moment = force × the **perpendicular distance** from the pivot to the line of action of the force

2.2

- $M = Fd$

$$M = (20\text{ N})(0.30\text{ m}) = \mathbf{6.0}\text{ N m}$$

2.3

- the two forces are **equal** in size and **opposite** in direction
- their lines of action are a **perpendicular distance apart** (not the same line)

2.4

- at its **centre of gravity** — the midpoint of a uniform rod

3.1 B

- the SI unit of moment (and of torque) is the **newton-metre**, N m

3.2

- the perpendicular distance is $r\sin\theta$

$$M = Fr\sin\theta = (50\text{ N})(0.40\text{ m})(\sin 30°) = 50 \times 0.40 \times 0.50 = \mathbf{10}\text{ N m}$$

3.3

- $\tau = Fd$

$$\tau = (8.0\text{ N})(0.25\text{ m}) = \mathbf{2.0}\text{ N m}$$

3.4

- two equal forces in the **same** direction have a resultant $2F \neq 0$
- they cause translation, not turning only — a couple needs the forces **opposite**

3.5

(a) torque of a couple = **one of the forces** × the **perpendicular distance between the two lines of action**

(b) the single point at which the **whole weight** of the sheet may be taken to act

(c) the applied torque balances the moment of the weight about the rod: $\tau = mgd$

$$d = \frac{1.9\text{ N m}}{(1.6\text{ kg})(9.81\text{ m s}^{-2})} = \mathbf{0.12}\text{ m}$$

(d) **anticlockwise**

- the centre of gravity is to the right of the rod, so the weight turns the sheet **clockwise**; the applied torque must oppose it

(e) $V = m/\rho$

$$V = \frac{1.6 \text{ kg}}{2700 \text{ kg m}^{-3}} = 5.93 \times 10^{-4} \text{ m}^3$$

- square sheet: $V = L^2 t$

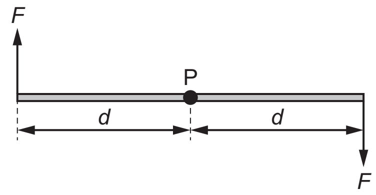
$$L^2 = \frac{5.93 \times 10^{-4} \text{ m}^3}{3.0 \times 10^{-3} \text{ m}} = 0.198 \text{ m}^2 \Rightarrow L = \mathbf{0.44} \text{ m}$$

(f) directly **below** that corner, on the vertical line through it

(g)

$$V = L^2 t = (0.44 \text{ m})^2 (5.0 \times 10^{-3} \text{ m}) = 9.68 \times 10^{-4} \text{ m}^3$$
$$m = \rho V = (2700 \text{ kg m}^{-3})(9.68 \times 10^{-4} \text{ m}^3) = 2.61 \text{ kg}$$
$$W = mg = (2.61 \text{ kg})(9.81 \text{ m s}^{-2}) = \mathbf{26} \text{ N}$$

14 Two forces, each of magnitude F, act in opposite directions on a rod.



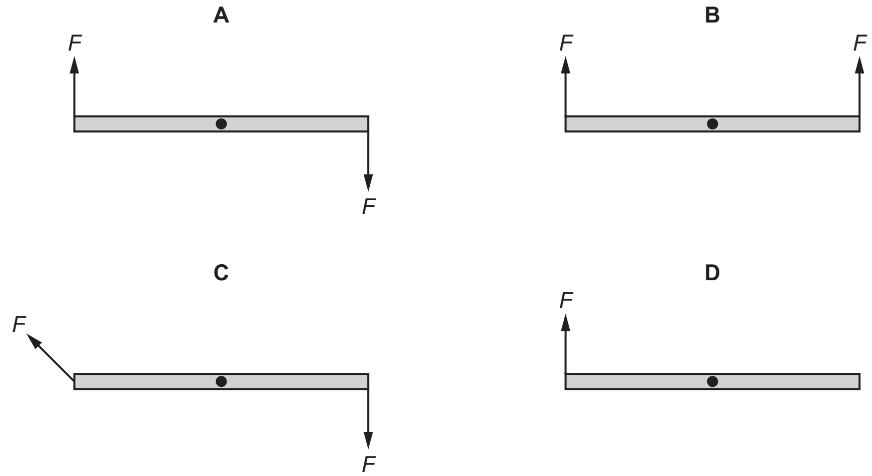
Each force acts on the rod at a distance d from the pivot P.

What is the torque of this couple about P?

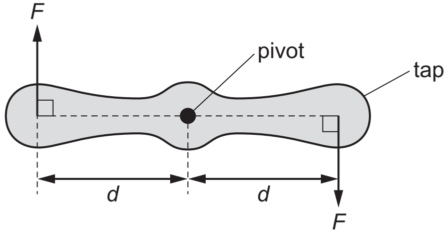
A 0 **B** $F \times d$ **C** $2F \times d$ **D** $2F \times 2d$

A-Level Physics: **w25 12**

13 Which diagram shows a couple?



14 A couple is applied to a tap, as shown.



What is the torque of the couple?

A $\dfrac{Fd}{2}$ **B** Fd **C** $2Fd$ **D** $4Fd$

2 Fig. 2.1 shows a square metal sheet of non-uniform density, with a thin wooden rod fixed at its centre. One of the corners of the sheet is labelled X.

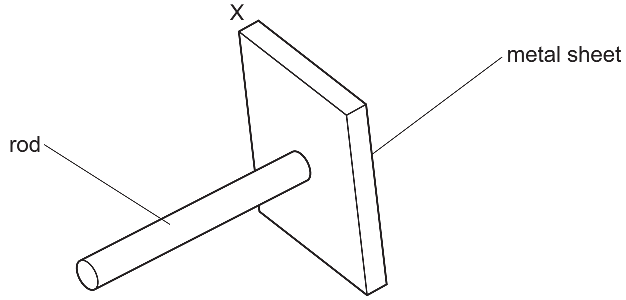


Fig. 2.1

The rod has negligible mass. The mass of the metal sheet is 2.8 kg.
The rod is supported so that the rod is horizontal and the metal sheet is vertical.

(a) Define the torque of a couple.

....

....

.... [2]

(b) When the rod is supported in such a way that it can rotate freely within its support, the sheet hangs in equilibrium with point X vertically above the rod, as shown in Fig. 2.2.

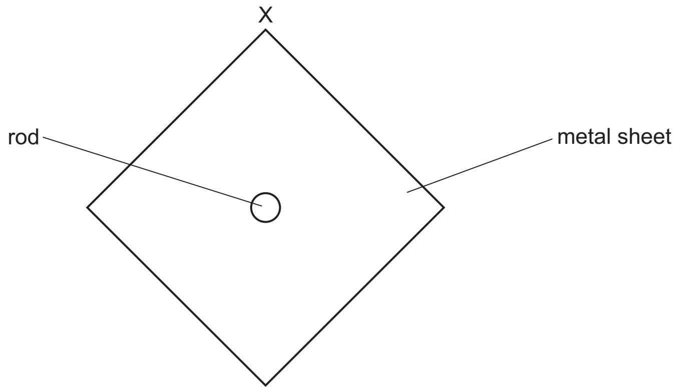


Fig. 2.2

On Fig. 2.2, draw a line to indicate the range of possible positions for the centre of gravity of the metal sheet. [1]

(c) When a torque of 3.3 N m is applied to the rod, the sheet is held in equilibrium with two of its edges horizontal, as shown in Fig. 2.3. Point X is at the top-left corner.

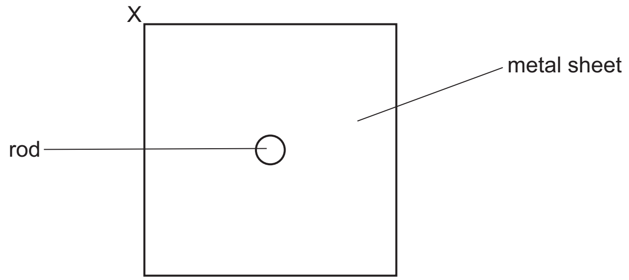


Fig. 2.3

(i) Explain whether the torque applied to the rod to hold the sheet in equilibrium is clockwise or anticlockwise.

....

.... [1]

(ii) Show that the centre of gravity of the sheet has a horizontal displacement of 0.12 m from the rod.

[1]

(d) The square metal sheet has an average density of 3000 kg m^{-3} and a uniform thickness of 4.0 mm.

Show that the side length of the sheet is 0.48 m.

[3]

(e) Use the answer in **(b)** and the information in **(c)** and **(d)** to determine the position of the centre of gravity of the sheet. Indicate this position on Fig. 2.3 with a point labelled Y. [2]

[Total: 10]

13 Which row describes a pair of forces that forms a couple?

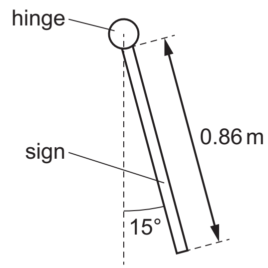
	direction of the forces	magnitude of the forces
A	opposite	different
B	opposite	equal
C	same	different
D	same	equal

A-Level Physics: **s25 12**

15 A square shop sign of uniform density has mass 2.4 kg and sides of length 0.86 m.

The sign is supported by a hinge along its top edge.

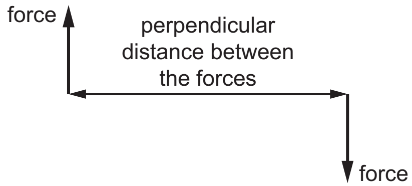
There is friction in the hinge so that the sign hangs from it in equilibrium at an angle of 15° to the vertical, as shown.



What is the moment about the hinge of the weight of the sign?

A 2.6 N m **B** 4.0 N m **C** 5.2 N m **D** 9.8 N m

17 The diagram shows a couple.



How is the torque of the couple calculated?

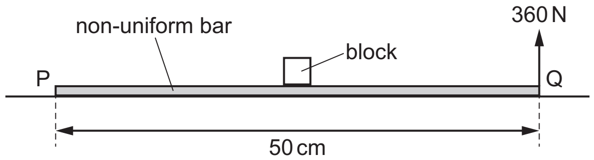
A $\frac{1}{2}$ × perpendicular distance between the forces × magnitude of one of the forces

B perpendicular distance between the forces × magnitude of one of the forces

C perpendicular distance between the forces × magnitude of the sum of the forces

D 2 × perpendicular distance between the forces × magnitude of one of the forces

12 A non-uniform bar PQ has length 50 cm and weight 200 N.



A block of weight 400 N is attached to the top of the bar at its centre. The bar rests on horizontal ground.

A vertical force of 360 N is now exerted upwards on the bar at end Q and is just sufficient to lift end Q from the ground.

What is the distance from the centre of gravity of the bar to end P?

A 0.050 m **B** 0.10 m **C** 0.25 m **D** 0.40 m

10 Which statement about a couple is correct?

A It acts to produce both translational motion and rotation.

B It acts to produce rotation only.

C It acts to produce translational motion only.

D It acts to produce neither rotation nor translational motion.

2 **(a)** Define the moment of a force about a point.

....

.... [1]

(b) A tree of mass 270 kg grows out of sloping ground and is supported by a post, as shown in Fig. 2.1.

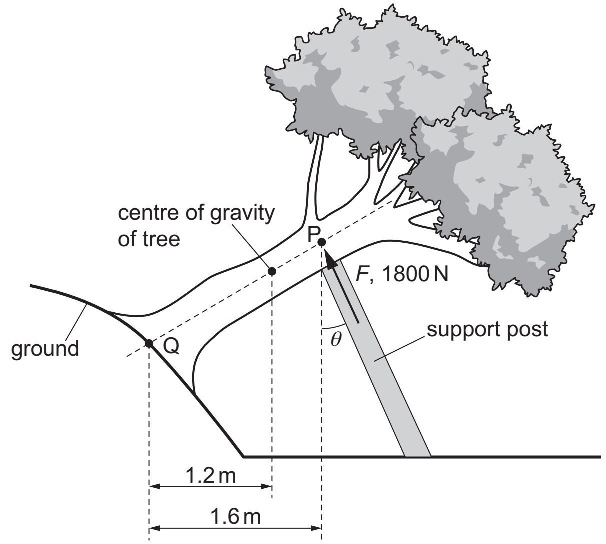


Fig. 2.1 (not to scale)

The ground applies a total force R on the tree at point Q.
The centre of gravity of the tree is a horizontal distance of 1.2 m from Q.
The post applies a force F of 1800 N perpendicular to the line PQ. The line of action of F passes through point P at an angle θ to the vertical. P is a horizontal distance of 1.6 m from Q.
The tree is in equilibrium and all forces act on the tree in the same plane.

(i) By taking moments about point Q, show that θ is 25°.

[3]

(ii) On Fig. 2.2, draw a labelled scale vector triangle to represent the forces acting on the tree. The weight of the tree has been drawn to scale.

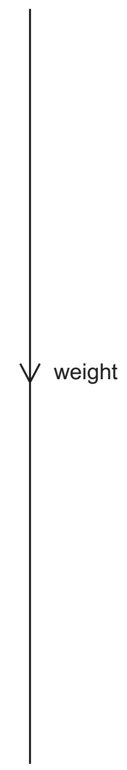


Fig. 2.2

[2]

(iii) The tree exerts a pressure of 150 kPa on the top of the post.

Determine the surface area of the tree in contact with the post.

area = ... m^2 [2]

[Total: 8]

2 **(a)** Define the moment of a force about a pivot.

....

.... [1]

(b) Three objects A, B and C are placed on a horizontal beam. The beam is in equilibrium, as shown in Fig. 2.1.

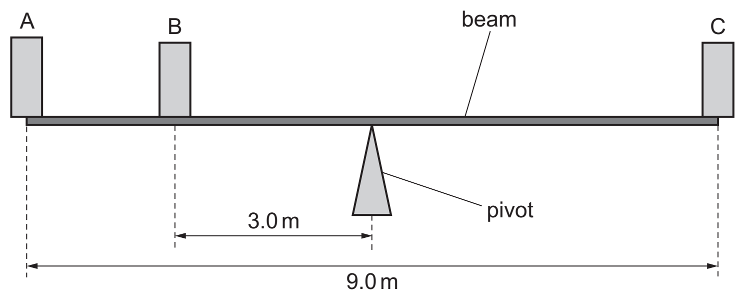


Fig. 2.1 (not to scale)

The beam is uniform and has length 9.0 m.
A pivot is at the midpoint of the beam.
Object A has mass 90 kg and is at one end of the beam.
Object B has mass *m* and is a distance of 3.0 m from the pivot.
Object C has mass 150 kg and is at the other end of the beam.

(i) Calculate *m*.

m = ... kg [3]

(ii) Object A is removed and replaced by a wire fixed to the end of the beam and to the ground, as shown in Fig. 2.2.

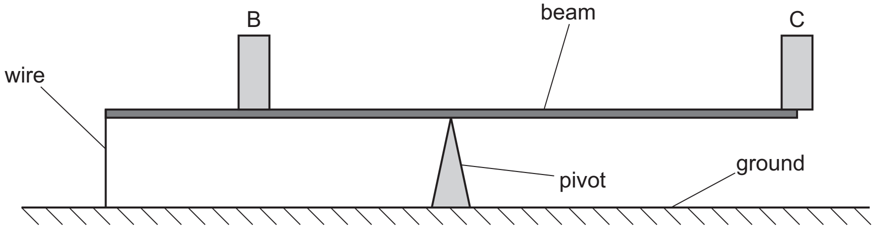


Fig. 2.2

After the change, the beam is again horizontal and in equilibrium. The positions of B and C are unchanged.

The wire has a diameter of 1.8×10^{-3} m and has a strain of 1.2×10^{-3}.
The wire is not extended beyond its limit of proportionality.

Calculate the Young modulus of the wire.

Young modulus = ... Pa [3]

(iii) Object B is now moved to a new position closer to the pivot without passing it. The beam is again horizontal and in equilibrium.

State and explain the effect, if any, that this has on the strain in the wire.

....

....

.... [2]

[Total: 9]

A-LEVEL PHYSICS • EXERCISE SHEET

4.2 Equilibrium of forces

Name: ____________ Class: ________ Date: __________ **Total: 30 marks**

KEY WORDS equilibrium 平衡 • vector triangle 矢量三角形 • force 力 • moment 力矩 • principle of moments 力矩原理

Objective

Build the skills to answer exam questions on **equilibrium 平衡** — the principle of moments, the two conditions for equilibrium, and the vector triangle.

You must be able to:

- state and apply the **principle of moments**
- use the **two conditions** for equilibrium (no resultant force, no resultant moment)
- use a closed **vector triangle** for three coplanar forces, with numbers
- **draw** labelled force arrows on a diagram before calculating anything
- take moments about a **hinge** and find the force the hinge itself exerts

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

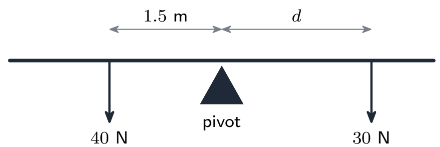
Conditions for equilibrium

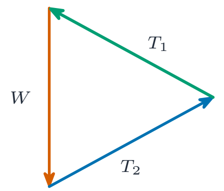
A body is in **equilibrium** when **both**:

1. the **resultant force** is zero, and
2. the **resultant moment** (about any point) is zero.

Principle of moments

For a body not turning, the **total clockwise moment = total anticlockwise moment** about any point.





A closed vector triangle shows equilibrium

Worked example. Beam pivoted at its centre (so its weight gives no moment), 40 N at 1.5 m on the left:

$$40 \times 1.5 = 30 \times d \Rightarrow d = \frac{60}{30} = 2.0 \text{ m}.$$

Tip: take moments about the point where an **unknown** force acts, so it drops out.

■ Vector triangle

Three coplanar forces in equilibrium form a **closed triangle** (tip-to-tail). Solve it with the sine/cosine rule, or resolve into components and set $\sum F_x = 0$, $\sum F_y = 0$.

Worked example. A 250 N sign hangs from a horizontal wire and a wire at 40° to the vertical. Resolving the sloping tension T:

$$\text{vertical: } T \cos 40° = 250 \Rightarrow T = \frac{250}{0.766} = 326 \text{ N}$$

$$\text{horizontal: } T \sin 40° = F \Rightarrow F = 326 \times 0.643 = 210 \text{ N}$$

The vertical equation comes first because only **one** unknown appears in it. A closed triangle drawn to scale gives the same two answers.

■ Forces on a hinged beam

A beam hinged at one end and held up somewhere else has **three** forces on it, and the hinge one is the awkward one — so:

1. **Draw the arrows first**, labelled: the weight at the **centre of gravity**, the support force, the load.
2. **Take moments about the hinge**, because its own force then has zero moment and drops out.
3. Get the hinge force **last**, from vertical equilibrium — and be ready for it to point **down**. If the support is between the hinge and the load, the hinge must pull the beam down to stop it tipping.

2 Practice

Now apply the methods above.

2.1 State the **two** conditions for a body to be in **equilibrium**. [2]

2.2 State the **principle of moments**. [1]

2.3 A 40 N weight hangs 1.5 m to the left of a pivot. How far to the **right** must a 30 N weight hang to balance it? [2]

2.4 State what a **closed vector triangle** tells you about three forces. [1]

3 Exam-style questions

3.1 Three coplanar forces act on an object that is in equilibrium. Which statement **must** be true? [1]

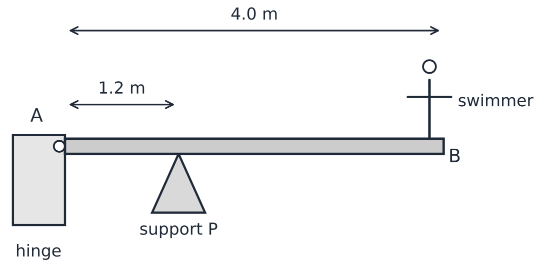
- **A** The three forces are equal in magnitude.
- **B** Drawn tip to tail, the three forces form a closed triangle.
- **C** The three forces all act through the same point and are parallel.
- **D** The resultant of two of the forces is equal in both magnitude and direction to the third.

3.2 A uniform beam of length 4.0 m is pivoted at its centre. A 40 N weight hangs 1.5 m from the pivot. Calculate the distance on the other side at which a 30 N weight balances the beam. [3]

3.3 A uniform beam of weight 60 N and length 4.0 m rests on supports **A** (left end) and **B** (right end). A 40 N load sits 1.0 m from A. **Show that** the upward force on support **B** is 40 N. [3]

3.4 Explain why, when balancing a **heavy uniform rod**, you must include its **weight** in the moments calculation. [2]

3.5 A uniform diving board AB has length 4.0 m and weight 800 N. It is hinged to a wall at end A and rests on a support P a distance 1.2 m from A. A swimmer of weight 620 N stands at end B, and the board is in equilibrium.



(a) **State** the principle of moments. [2]

(b) On the diagram above, **draw** arrows showing the **weight of the board**, the **force from the support P** and the **weight of the swimmer**. **Label** each arrow. [3]

(c) By taking moments about A, **determine** the force that the support P exerts on the board. [3]

(d) **Determine** the magnitude and the direction of the force that the hinge at A exerts on the board. [3]

(e) **Explain** why taking moments about A is a better choice than taking them about P. [1]

(f) The swimmer now walks slowly back towards A. **Give** the distance from A at which she must stand for the hinge to exert **no** force on the board. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the **resultant force** is zero
- **and** the **resultant moment** about any point is zero

2.2

- principle of moments: **total clockwise moment** about any point = **total anti-clockwise moment** about that point

2.3

- take moments about the pivot

$$(40\text{ N})(1.5\text{ m}) = (30\text{ N})d \Rightarrow d = \mathbf{2.0}\text{ m}$$

2.4

- the three forces are **in equilibrium** (resultant zero)

3.1 B

- in equilibrium the three forces have zero resultant, so tip-to-tail they close into a **triangle**
- **D** is the trap: the resultant of two must be equal and **opposite** to the third, not in the same direction

3.2

- the beam's weight acts at the pivot, so it has **no moment**

$$(40\text{ N})(1.5\text{ m}) = (30\text{ N})d$$

$$d = \frac{60\text{ N m}}{30\text{ N}} = \mathbf{2.0}\text{ m}$$

3.3

- take moments about **A** so the force at A drops out

$$B \times 4.0\text{ m} = (60\text{ N})(2.0\text{ m}) + (40\text{ N})(1.0\text{ m}) = 160\text{ N m}$$

$$B = \frac{160\text{ N m}}{4.0\text{ m}} = \mathbf{40}\text{ N}$$

3.4

- the rod's weight acts at its **centre of gravity**, a distance from the pivot
- so it contributes a **moment** that must be included

3.5

(a) for a body in **equilibrium**

- sum of **clockwise moments** about any point = sum of **anticlockwise moments** about that point

(b)

- arrow **down** at the middle of the board (2.0 m from A) labelled *weight of board / 800 N*
- arrow **up** at P labelled *support force*
- arrow **down** at B labelled *weight of swimmer / 620 N*

(c) moments about A (hinge force has no moment)

- clockwise = (800 N)(2.0 m) + (620 N)(4.0 m) = 4080 N m
- anticlockwise = $P \times 1.2$ m

$$P = \frac{4080 \text{ N m}}{1.2 \text{ m}} = \mathbf{3400} \text{ N}$$

(d) vertical equilibrium: up = down

- $3400 \text{ N} + F_{\text{hinge}} = 800 \text{ N} + 620 \text{ N}$
- $F_{\text{hinge}} = -1980$ N, so **1980** N **downwards**

(e) the hinge force is unknown; moments **about** A give it zero moment, so it drops out

(f) no hinge: take moments about **P**

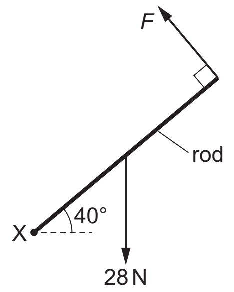
- board's weight is 2.0 m − 1.2 m = 0.80 m to the right of P
- clockwise: (800 N)(0.80 m) = 640 N m
- swimmer must be on the **other** side of P, a distance (1.2 m − x) from it

$$(620 \text{ N})(1.2 \text{ m} - x) = 640 \text{ N m} \Rightarrow x = \mathbf{0.17} \text{ m}$$

from A

13 A uniform rod of weight 28 N is supported at one end by force F.

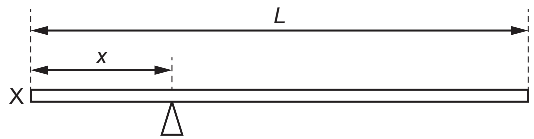
The rod is in equilibrium and attached to a frictionless hinge at end X.



What is the magnitude of F?

A 9.0 N **B** 11 N **C** 18 N **D** 21 N

14 The diagram shows a uniform bar of mass M and length L resting on a pivot at a distance x from end X.

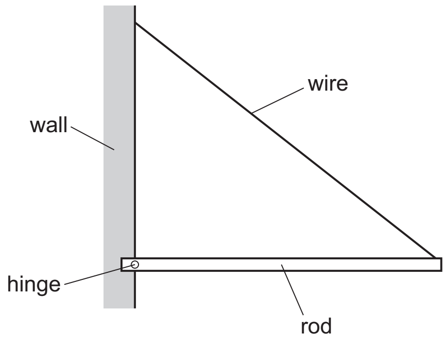


An object of mass m is placed on the bar at distance y from the pivot so that the bar is in equilibrium.

What is an expression for y?

A $\frac{xM}{m}$ **B** $\frac{M}{m}(L - x)$ **C** $\frac{M}{2m}(L - 2x)$ **D** $\frac{1}{m}(Lm - xM)$

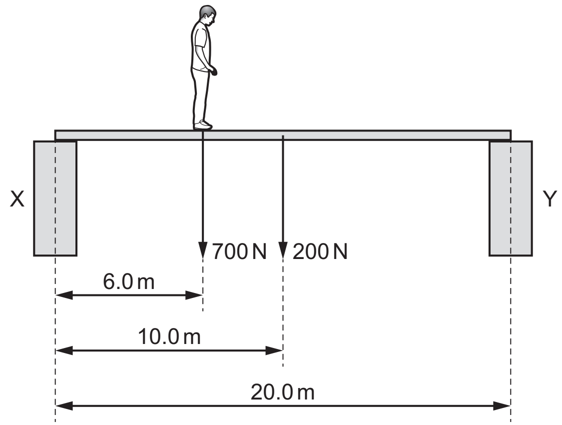
13 A uniform rod is attached by a hinge at one end to a wall. The other end of the rod is supported by a wire so that the rod is horizontal and in equilibrium.



Which arrow shows the direction of the force on the rod from the hinge?

A **B** **C** **D**

15 A uniform beam rests on two supports, X and Y, as shown.



not to scale

The beam has length 20.0 m and weight 200 N.

A man of weight 700 N stands on the beam at a distance of 6.0 m from support X.

The beam is in equilibrium.

What is the contact force of support X on the beam?

A 310 N **B** 450 N **C** 590 N **D** 900 N

3 A spring is fixed at one end and attached to the frame of a pulley at the other end. A cable is passed around the wheel of the pulley. The spring is stretched to a fixed length using the cable and pulley.

Fig. 3.1 shows the view from above of the spring, cable and pulley.

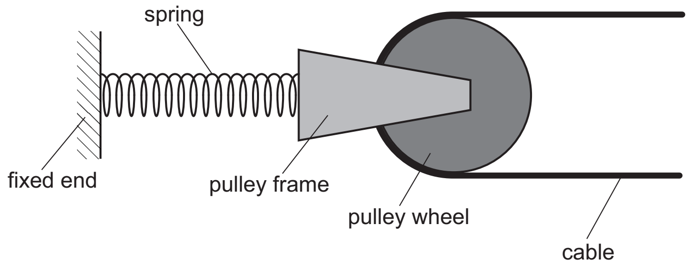


Fig. 3.1

The spring obeys Hooke's law and has a spring constant k of $250\,\mathrm{N\,m^{-1}}$. A force F acts on the spring. The tension in the cable is T. The pulley is in equilibrium.

(a) On Fig. 3.2, draw labelled arrows to show the directions of the forces acting on the pulley.

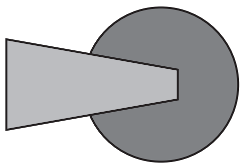


Fig. 3.2

[2]

(b) The force F is 110 N.

(i) Determine T.

T = ... N [1]

(ii) Calculate the extension of the spring.

extension = ... m [2]

(c) A second identical spring with the same spring constant of $250\,\mathrm{N\,m^{-1}}$ is now also connected to the pulley, as shown in Fig. 3.3.

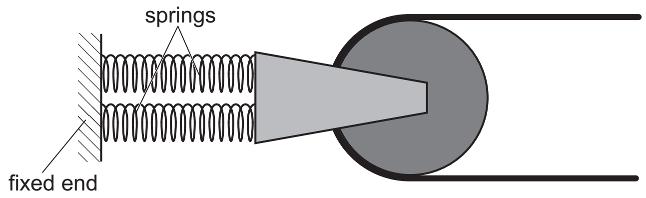


Fig. 3.3

The tension in the cable is kept the same. The pulley is again in equilibrium.

(i) Determine the extension of the springs.

extension = ... m [2]

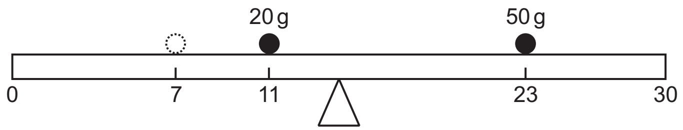
(ii) The elastic potential energy stored in the spring in Fig. 3.1 is E_1. The **total** elastic potential energy stored in the two springs in Fig. 3.3 is E_2.

Calculate the ratio $\dfrac{E_1}{E_2}$.

ratio = ... [2]

[Total: 9]

15 A student balances a 30 cm ruler on a fulcrum set at the 15 cm mark. She then places a 50 g mass on the 23 cm mark and a 20 g mass on the 11 cm mark, as shown.

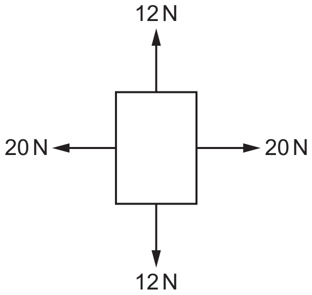


Which mass should she place on the 7 cm mark to restore the balance?

A 30 g **B** 40 g **C** 47 g **D** 133 g

A-Level Physics: **s25 13**

10 Four forces, all in the same plane, act on an object.



What could describe the motion of the object?

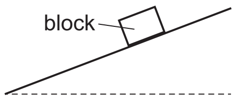
A It is accelerating.

B It is moving in a straight line.

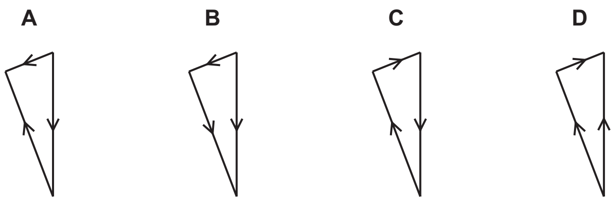
C It is moving with decreasing speed.

D It is moving with increasing momentum.

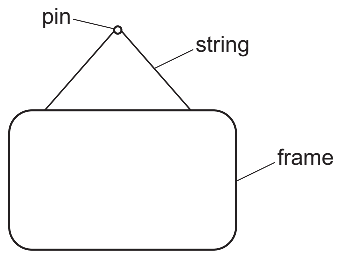
14 A block is in equilibrium on a slope.



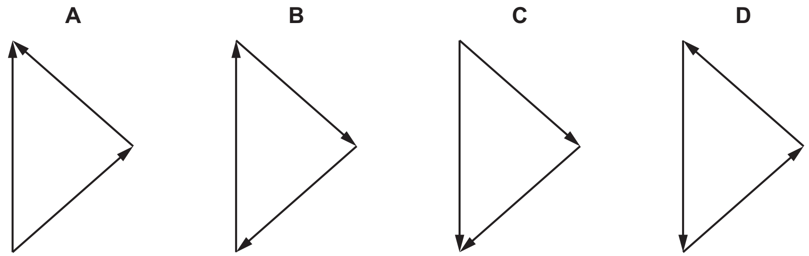
Which vector triangle represents the three forces acting on the block?



11 A picture frame hangs from a string. The string is supported by a pin. The frame is in equilibrium.

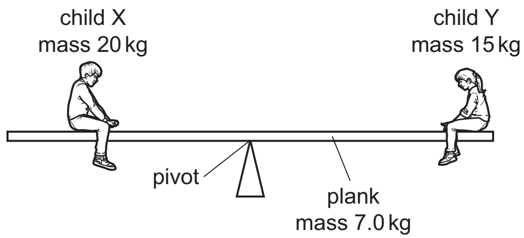


Which diagram shows the vector triangle of forces acting on the picture frame?



A-Level Physics: **m25 12**

9 The diagram shows a child X of mass 20 kg and a child Y of mass 15 kg seated on a uniform plank.



The plank has a mass of 7.0 kg and has a pivot at its midpoint. The plank is horizontal and in equilibrium.

Which statement about the weight of the plank is correct?

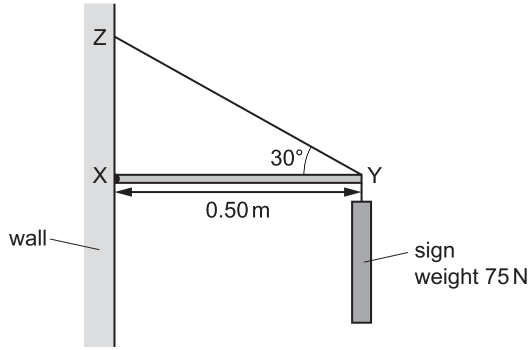
A The weight of the plank can be considered to be acting at its midpoint.

B The weight of the plank is causing an anticlockwise moment.

C The weight of the plank is causing a clockwise moment.

D The weight of the plank equals the force on the plank from the pivot.

11 A shop sign weighing 75 N hangs from a frame attached to a vertical wall.



The frame consists of a horizontal rod XY and a rod YZ that is at an angle of 30° to the horizontal. Rod XY is attached to the wall by a hinge at X and has length 0.50 m. Assume that the weights of the rods are negligible.

What is the horizontal force exerted by the wall on rod XY?

A 0 N **B** 43 N **C** 130 N **D** 150 N

2 **(a)** State the principle of moments.

....

....

.... [2]

(b) A solid plastic cylinder floats in water. It is used to support one end of a horizontal uniform beam AB as shown in Fig. 2.1.

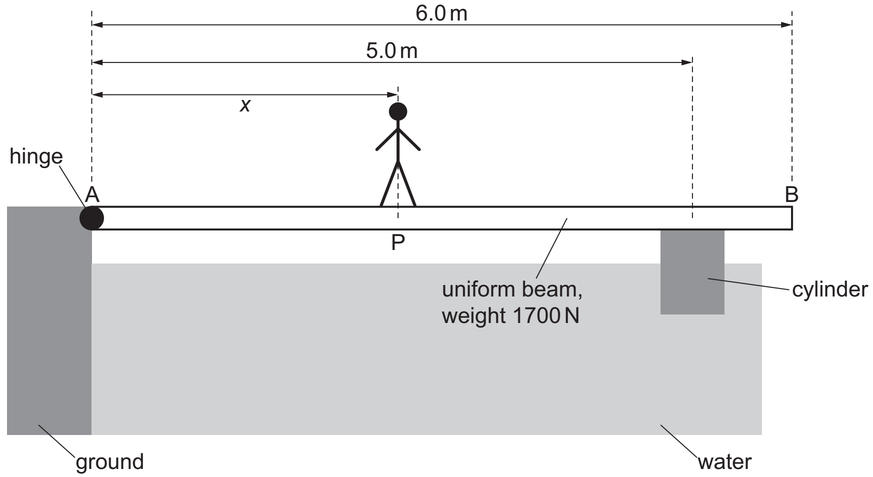


Fig. 2.1 (not to scale)

The beam has length 6.0 m and weight 1700 N. The beam is attached to solid ground with a hinge at end A.

The cylinder is floating vertically in the water. The top of the cylinder is attached at its centre to the beam at a horizontal distance of 5.0 m from end A. The cylinder applies a vertical force of 1300 N to the beam.

A person of weight 660 N stands on the beam at point P.

The beam AB is in equilibrium.

(i) By taking moments about end A, determine the distance x from A to P.

distance = ... m [3]

(ii) The bottom of the cylinder is submerged in the water to depth y as shown in Fig. 2.2. The beam is still attached to the cylinder but not shown.

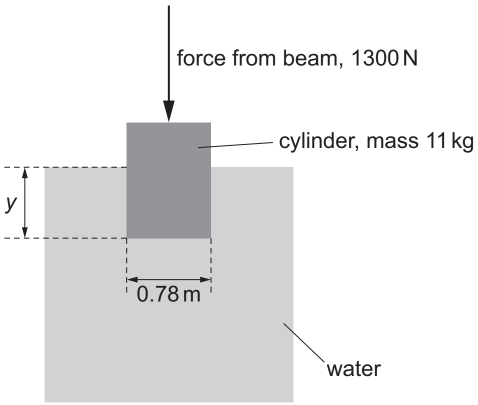


Fig. 2.2 (not to scale)

The cylinder has mass 11 kg and diameter 0.78 m. The beam exerts a vertical force of 1300 N on the cylinder. The cylinder is in equilibrium.

Show that the upthrust acting on the cylinder is 1400 N.

[1]

(iii) The water has density 990 kg m^{-3}.

Calculate the depth y.

y = ... m [2]

(iv) The person can stand anywhere between A and B.

On Fig. 2.3, sketch the variation of the depth of the bottom of the cylinder with the distance of the person from A, for distances between 0 and 6.0 m. Numerical values are not required.

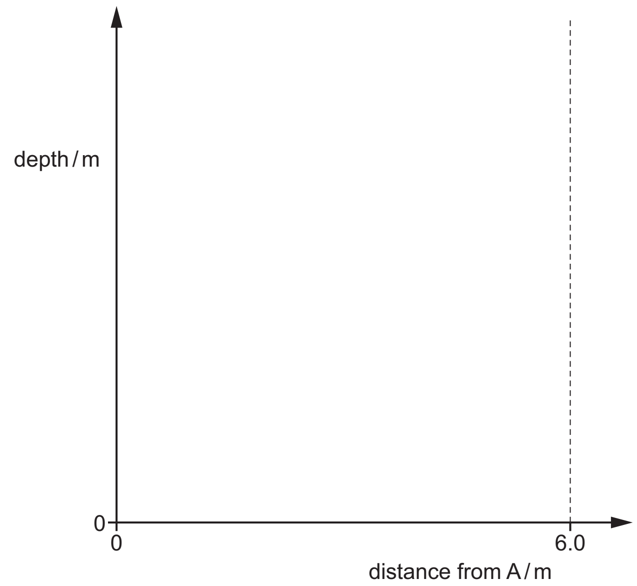


Fig. 2.3

[2]

[Total: 10]

A-LEVEL PHYSICS • EXERCISE SHEET

4.3 Density and pressure

Name: ____________ Class: ________ Date: __________ **Total: 31 marks**

KEY WORDS density 密度 • pressure 压强 • upthrust 浮力 • hydrostatic pressure 流体静压强 • atmospheric pressure 大气压强

Objective

Build the skills to answer exam questions on **density and pressure** — including hydrostatic pressure and upthrust.

You must be able to:

- use **density** $\rho = m/V$ and **pressure** $p = F/A$
- derive and use **hydrostatic pressure** $\Delta p = \rho g \Delta h$
- find **upthrust** with $F = \rho g V$ (Archimedes' principle)
- get a density from **measured** side lengths, and its **percentage uncertainty**
- decide whether an object **floats**, and how much of it sits below the surface

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Density and pressure

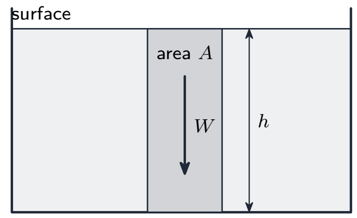
$$\rho = \frac{m}{V}\ (\text{kg m}^{-3}), \qquad p = \frac{F}{A}\ (\text{Pa} = \text{N m}^{-2}).$$

Water has $\rho = 1000$ kg m^{-3}.

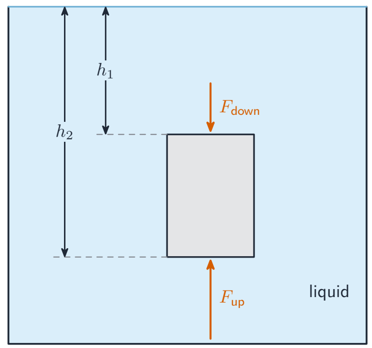
■ Hydrostatic pressure

A column of liquid (area A, depth h) has weight $W = \rho A h g$ pressing on its base, so the **extra** pressure is

$$\Delta p = \frac{W}{A} = \rho g h.$$



$\Delta p = \rho g h$



Upthrust arises from the pressure difference with depth

It depends only on **depth and density** — not on the container's shape. (For total pressure, add atmospheric pressure $\approx 1.0 \times 10^5$ Pa.)

■ Upthrust

A submerged object feels a larger pressure on its **bottom** than its top — this pressure difference gives an upward **upthrust**:

$$F = \rho g V,$$

where V is the volume of fluid displaced (Archimedes' principle).

■ Density from measurements, with its uncertainty

An exam density is rarely handed to you: it comes from a mass and three measured sides, each with an uncertainty.

Worked example. $m = (0.243 \pm 0.001)$ kg; $x = (5.41 \pm 0.01)$ cm, $y = (11.09 \pm 0.01)$ cm, $z = (1.62 \pm 0.01)$ cm.

$$V = xyz = 5.41 \times 11.09 \times 1.62 = 97.2\ \text{cm}^3 = 9.72 \times 10^{-5}\ \text{m}^3$$

$$\rho = \frac{m}{V} = \frac{0.243}{9.72 \times 10^{-5}} = 2500 \text{ kg m}^{-3}$$

For the uncertainty, add the **percentage** uncertainties of everything multiplied or divided:

$$\frac{0.001}{0.243} + \frac{0.01}{5.41} + \frac{0.01}{11.09} + \frac{0.01}{1.62} = 0.41 + 0.18 + 0.09 + 0.62 = 1.3\%$$

The **smallest** side carries the largest percentage — that is the measurement worth improving, and examiners ask exactly that.

■ **Floating**

An object floats when the upthrust can balance its weight **before** it is fully under:

$$\rho_{\text{object}} V_{\text{total}} g = \rho_{\text{fluid}} V_{\text{submerged}} g \quad \Rightarrow \quad \frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{\rho_{\text{object}}}{\rho_{\text{fluid}}}$$

So ice (920 kg m^{-3}) floats in water with 92% of its volume below the surface. If $\rho_{\text{object}} > \rho_{\text{fluid}}$ the fraction would exceed 1, which is the algebra telling you it sinks.

2 Practice

Now apply the methods above.

2.1 Define **density** and give its SI unit. [1]

2.2 A block has mass 2400 kg and volume 0.30 m^3. Find its **density**. [2]

2.3 A force of 200 N acts on an area of 0.50 m^2. Find the **pressure**. [2]

2.4 State what causes the **upthrust** on a submerged object. [1]

3 Exam-style questions

3.1 The extra pressure at the bottom of a liquid column depends on: [1]

- **A** the cross-sectional area only
- **B** the depth and the density
- **C** the shape of the container
- **D** the total volume of liquid

3.2 Calculate the extra pressure due to the water at the bottom of a pool 2.5 m deep. ($\rho = 1000$ kg m^{-3}, $g = 9.81$ m s^{-2}.) [2]

3.3 A block of volume 2.0×10^{-3} m^3 is fully submerged in water ($\rho = 1000$ kg m^{-3}). Calculate the **upthrust** on it. [2]

3.4 Derive the hydrostatic pressure equation $\Delta p = \rho g h$ from the definitions of **pressure** and **density**. [3]

3.5 A student measures a rectangular metal block to find the density of the metal. The results are

quantity	**measurement**
mass m	(0.512 ± 0.001) kg
length x	(8.02 ± 0.01) cm
width y	(4.15 ± 0.01) cm
height z	(2.05 ± 0.01) cm

(a) **Define** density. [1]

(b) **Determine** the density of the metal. [3]

(c) **Determine** the percentage uncertainty in the density. [3]

(d) **State**, with a reason, which single measurement the student should improve first. [2]

(e) The block is lowered into water ($\rho = 1000$ kg m^{-3}) until it is fully submerged. **Show that** the upthrust on it is about 0.67 N, and **explain** why the block still sinks. [3]

(f) A block of wood of density 650 kg m^{-3} floats in the same water. **Determine** the fraction of its volume that lies **below** the surface. [2]

(g) A swimming pool is 25 m long, 10 m wide and 2.0 m deep. **Estimate** the force that the water exerts on the bottom of the pool, and state one assumption you make. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- density is **mass per unit volume**, unit kg m^{-3}

2.2

- $\rho = m/V$

$$\rho = \frac{2400 \text{ kg}}{0.30 \text{ m}^3} = \mathbf{8000} \text{ kg m}^{-3}$$

2.3

- $p = F/A$

$$p = \frac{200 \text{ N}}{0.50 \text{ m}^2} = \mathbf{400} \text{ Pa}$$

2.4

- the **difference in pressure** between the bottom and top of the object (the bottom is deeper)

3.1 B

- $\Delta p = \rho g \Delta h$ depends only on **density** and **depth**
- **A** / **D** (volume, area) do not appear: a narrow tube and a wide tank of the same depth give the same pressure

3.2

- $\Delta p = \rho g h$

$$\Delta p = (1000 \text{ kg m}^{-3})(9.81 \text{ m s}^{-2})(2.5 \text{ m}) = \mathbf{2.5 \times 10^4} \text{ Pa}$$

3.3

- upthrust = weight of fluid displaced = $\rho g V$

$$F = (1000 \text{ kg m}^{-3})(9.81 \text{ m s}^{-2})(2.0 \times 10^{-3} \text{ m}^3) = \mathbf{19.6} \text{ N}$$

3.4

- mass of the column $m = \rho V = \rho A h$
- weight $W = mg = \rho A h g$ presses on area A

$$\Delta p = \frac{W}{A} = \rho g h$$

3.5

(a) density is **mass per unit volume**

(b) $V = 8.02 \text{ cm} \times 4.15 \text{ cm} \times 2.05 \text{ cm} = 68.2 \text{ cm}^3$

- convert: $V = 6.82 \times 10^{-5} \text{ m}^3$

$$\rho = \frac{0.512 \text{ kg}}{6.82 \times 10^{-5} \text{ m}^3} = \mathbf{7.5 \times 10^3} \text{ kg m}^{-3}$$

(c) percentage uncertainties

- mass: $0.001/0.512 = 0.20\%$; x: $0.01/8.02 = 0.12\%$; y: $0.01/4.15 = 0.24\%$; z: $0.01/2.05 = 0.49\%$
- total $= 0.20 + 0.12 + 0.24 + 0.49 = \mathbf{1.1}\%$

(d) the **height** z

- it is the smallest length, so the same ± 0.01 cm is the largest percentage of it

(e) upthrust $= \rho_{\text{water}} g V$

$$F = (1000 \text{ kg m}^{-3})(9.81 \text{ m s}^{-2})(6.82 \times 10^{-5} \text{ m}^3) = \mathbf{0.67} \text{ N}$$

- the block's weight is $mg = (0.512 \text{ kg})(9.81 \text{ m s}^{-2}) = 5.0$ N, much larger, so it **sinks**

(f) floating: upthrust = weight $\Rightarrow \rho_{\text{water}} V_{\text{sub}} = \rho_{\text{wood}} V_{\text{total}}$

$$\frac{V_{\text{sub}}}{V_{\text{total}}} = \frac{650}{1000} = 0.65 \Rightarrow \mathbf{65}\% \text{ below the surface}$$

(g)

$$\Delta p = \rho g h = (1000 \text{ kg m}^{-3})(9.81 \text{ m s}^{-2})(2.0 \text{ m}) = 1.96 \times 10^4 \text{ Pa}$$

- $A = 25 \text{ m} \times 10 \text{ m} = 250 \text{ m}^2$

$$F = pA = (1.96 \times 10^4 \text{ Pa})(250 \text{ m}^2) = \mathbf{4.9 \times 10^6} \text{ N}$$

- assumption: flat bottom (same depth everywhere); atmospheric pressure is **not** included

16 The formula for the upthrust on a block of wood partially submerged in water is shown.

$$F = \rho g V$$

What do the symbols ρ and V represent?

	ρ	V
A	density of water	volume of whole block
B	density of water	volume of block below surface of water
C	density of wood	volume of whole block
D	density of wood	volume of block below surface of water

A-Level Physics: **w25 12**

15 The mass and volume of an object are varied.

Which two changes, when made together, **must** increase the density of the object?

	mass	volume
A	decrease	decrease
B	decrease	increase
C	increase	decrease
D	increase	increase

16 On Earth, a solid object that is fully submerged in a liquid experiences an upthrust U_E.

On Mars, the same object, fully submerged in the same liquid, experiences an upthrust U_M.

The acceleration of free fall on Mars is $3.7\,m\,s^{-2}$.

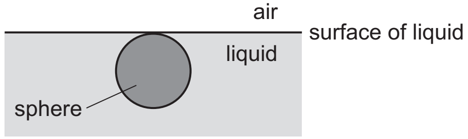
Assume that the liquid's density and the object's volume have the same values on Earth and Mars.

What is the ratio $\frac{U_M}{U_E}$?

A 0.38 **B** 1.0 **C** 2.7 **D** 3.6

12 The diagram shows a stationary sphere that is just fully submerged in a liquid. The radius of the sphere is r and the density of the liquid is ρ. The acceleration of free fall is g.

The air exerts pressure P_A on the surface of the liquid.



What is the pressure at the lowest point of the sphere?

A $P_A + \frac{4}{3}\pi r^3 \rho g$

B $P_A - \frac{4}{3}\pi r^3 \rho g$

C $2r\rho g + P_A$

D $2r\rho g - P_A$

2 **(a)** **(i)** Define pressure.

....

.... [1]

(ii) Explain how hydrostatic pressure results in an upthrust force acting on a solid object immersed in a liquid.

....

....

....

.... [2]

(b) A small steel ball of radius *r* and mass *m* falls vertically at terminal speed *v* through oil.

The viscous drag force *D* that acts on the ball is given by

$$D = 6\pi\eta r v$$

where η is a property of the oil called its viscosity.

(i) On Fig. 2.1, draw labelled arrows from the ball to show the directions of the **three** forces that act on the ball as it falls.

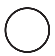


Fig. 2.1

[3]

(ii) Determine the SI base units of η.

base units .... [2]

(c) The oil in **(b)** has a density of $920\,kg\,m^{-3}$ and a viscosity of 4.7 in SI units.

The steel ball has a mass of $2.4 \times 10^{-3}\,kg$ and a radius of $4.2 \times 10^{-3}\,m$.

(i) Show that the upthrust force acting on the ball is $2.8 \times 10^{-3}\,N$.

[1]

(ii) Determine the terminal speed *v* of the ball.

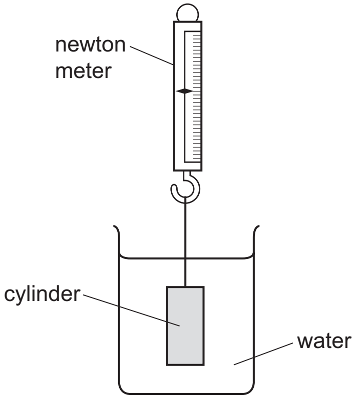
v = ... $m\,s^{-1}$ [3]

[Total: 12]

14 A uniform cylinder of weight 25.0 N is suspended from a newton meter.

The cylinder is fully submerged in water, as shown.

The reading on the newton meter is 10.0 N.



The water is replaced by a liquid with a density 10% greater than the density of water. The cylinder remains fully submerged.

What is the new reading on the newton meter?

A 8.5 N **B** 9.0 N **C** 11.0 N **D** 11.5 N

16 A submarine is at a depth of 130 m below the surface of the sea.

The pressure on the submarine due to the sea water is p.

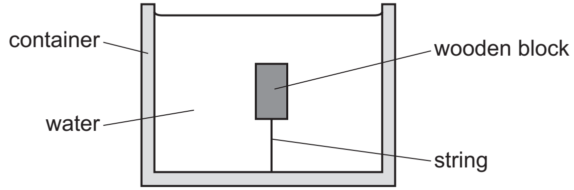
The submarine sinks to a depth of 260 m below the surface.

Assume that the density of sea water is constant.

What is the **difference** between the pressures on the submarine due to the sea water at the two depths?

A 0 **B** $0.5p$ **C** p **D** $2p$

14 A wooden block is held stationary in a container of water using a string that is attached to both the wooden block and the bottom of the container.



The wooden block has mass m and volume V. The water has density ρ. The acceleration of free fall is g.

What is the magnitude of the force acting on the block due to the tension in the string?

A ρgV **B** $mg + \rho gV$ **C** mg **D** $\rho gV - mg$

A-Level Physics: **s25 13**

13 What is the definition of density?

A mass of one cubic metre

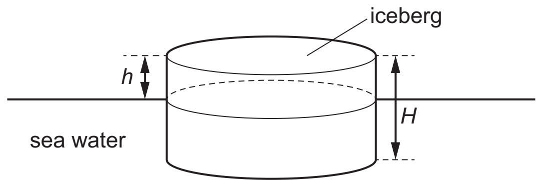
B mass of a unit volume

C mass per cubic metre

D mass per unit volume

A-Level Physics: **s25 14**

13 A cylindrical iceberg of height H floats in sea water. The top of the iceberg is at height h above the surface of the water.



The density of ice is ρ_i and the density of sea water is ρ_w.

What is the height h of the iceberg above the sea water?

A $\left(1-\dfrac{\rho_i}{\rho_w}\right)H$ **B** $\left(\dfrac{\rho_i}{\rho_w}-1\right)H$ **C** $\dfrac{\rho_w}{\rho_i}H$ **D** $\dfrac{\rho_i}{\rho_w}H$

10 An object is fully submerged in a liquid.

A student determines the ratio $\dfrac{\text{upthrust acting on the object}}{\text{weight of the object}}$.

Which single change would double the value of this ratio?

A Use a different liquid that has twice the density and the same volume as the original liquid.

B Use a different object that has half the volume and the same density as the original object.

C Use a different object that has twice the density and the same volume as the original object.

D Use a different object that has twice the volume and the same density as the original object.

A-Level Physics: **w24 21**

1 **(a)** Define density.

....

.... [1]

(b) Fig. 1.1 shows a cuboidal glass block.

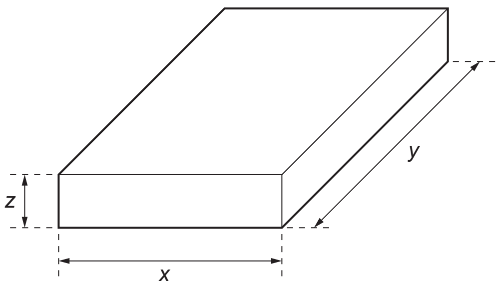


Fig. 1.1 (not to scale)

A student measures the mass m of the block and the side lengths x, y and z. The measurements are shown in Table 1.1.

Table 1.1

quantity	measurement
m	(0.243 ± 0.001) kg
x	(5.41 ± 0.01) cm
y	(11.09 ± 0.01) cm
z	(1.62 ± 0.01) cm

(i) Determine the density of the glass.

density = ... kg m^{-3} [2]

(ii) Calculate the percentage uncertainty in the density.

percentage uncertainty = ... % [3]

(c) The true value of the density of the glass is different from the answer in **(b)(i)** because of a systematic error in the measurements.

Suggest **one** possible cause of this systematic error.

....

....

.... [1]

[Total: 7]