Week 3

A-Level Physics

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

A-Level Physics Name: ____________ Score: ______

1. The rate of change of velocity with time is called ______.

   ____

2. When may the equations of motion be used? Select **all** that apply.

   ☐ when the acceleration is constant in magnitude

   ☐ when the acceleration is constant in direction

   ☐ for free fall with air resistance neglected

   ☐ for a body slowing under air resistance

   Tick every correct option.

3. A ball thrown horizontally and a ball dropped from the same height hit the ground at the same time (no air resistance).

   ☐ True ☐ False

4. Which quantity is the **rate of change of displacement**?

   ☐ speed

   ☐ velocity

   ☐ acceleration

   ☐ distance

5. A car starts at $u = 2\ \frac{\mathrm{m}}{\mathrm{s}}$ and accelerates at $a = 3\ \frac{\mathrm{m}}{\mathrm{s}^2}$ for $t = 4$ s. Find v.

   ________________ m/s

6. A projectile is launched at $20\ \frac{\mathrm{m}}{\mathrm{s}}$, 30° above the horizontal. What is the **vertical** part of the launch velocity?

   ________________ m/s

7. Deceleration is a completely separate quantity from acceleration.

   ☐ True ☐ False

8. Which equation is just the **gradient** of a velocity–time graph?

   ☐ $v = u + at$

   ☐ $s = ut + \frac{1}{2}at^2$

   ☐ $s = \frac{1}{2}(u + v)t$

- ☐ $v^2 = u^2 + 2as$

9. Select **all** the statements that are true for a projectile (ignore air).
   - ☐ The horizontal velocity stays constant
   - ☐ The vertical velocity is zero at the highest point
   - ☐ The whole velocity is zero at the highest point
   - ☐ The path is a parabola
   - ☐ A heavier projectile falls faster

   Tick every correct option.

10. On a displacement–time graph, a flat (horizontal) line means the object is at rest.

    ☐ True ☐ False

A-Level Physics

1. **acceleration**

   Acceleration is how fast the velocity changes — a vector, in $\frac{\mathrm{m}}{\mathrm{s}^2}$.

2. **when the acceleration is constant in magnitude; when the acceleration is constant in direction; for free fall with air resistance neglected**

   Air resistance makes the acceleration change, so only graph methods work there. A curved velocity-time graph is the visible warning.

3. **True**

   Yes — the vertical motion is the same free fall for both, and the horizontal speed does not change the fall time.

4. **velocity**

   Velocity is the rate of change of *displacement* (a vector). Speed is the rate of change of *distance* (a scalar).

5. **14 m/s**

   Use $v = u + at = 2 + 3 \times 4 = 14\ \frac{\mathrm{m}}{\mathrm{s}}$.

6. **10 m/s**

   Vertical part $= u \sin\theta = 20 \times \sin 30° = 20 \times 0.5 = 10\ \frac{\mathrm{m}}{\mathrm{s}}$.

7. **False**

   No — deceleration is just acceleration in the opposite direction to the motion (a negative acceleration).

8. $v = u + at$

   The gradient is $a = \frac{v-u}{t}$, which rearranges to $v = u + at$.

9. **The horizontal velocity stays constant; The vertical velocity is zero at the highest point; The path is a parabola**

   At the top only the *vertical* velocity is zero — the horizontal velocity $u\cos\theta$ carries on, so the ball keeps moving. Mass does not change free fall.

10. **True**

    A flat line has zero gradient, and the gradient is the velocity — so the velocity is zero (at rest).

2.1 Equations of motion

Name: ______________ Class: __________ Date: ____________ **Total: 49 marks**

KEY WORDS velocity 速度 • acceleration 加速度 • air resistance 空气阻力 • speed 速率 • distance 距离

Objective

Build the skills to answer exam questions on **uniformly accelerated motion 匀加速运动** — choosing and using the SUVAT equations, reading motion graphs, and solving free-fall and projectile problems.

You must be able to:

- find velocity from a displacement–time gradient, and acceleration + displacement from a velocity–time graph
- choose and use the four SUVAT equations for straight-line motion
- solve free-fall problems ($g = 9.81$ m s^{-2}, no air resistance)
- treat the horizontal and vertical motion of a projectile independently
- **describe** the experiment that measures the acceleration of free fall
- **sketch** a motion graph with both axes drawn and labelled

1 Worked examples

Study these first. Each one shows the **method 方法** for a question type used later — examiners give marks for showing the working, not only the final number.

■ Choosing a SUVAT equation

The five symbols are u (start velocity), v (final velocity), a (acceleration), s (displacement) and t (time). Each equation uses **four** of them and leaves one out:

equation	leaves out
$v = u + at$	s
$s = \frac{1}{2}(u + v)t$	a
$s = ut + \frac{1}{2}at^2$	v
$v^2 = u^2 + 2as$	t

Method: list what you know and what you want, then pick the row that leaves out the symbol you don't have.

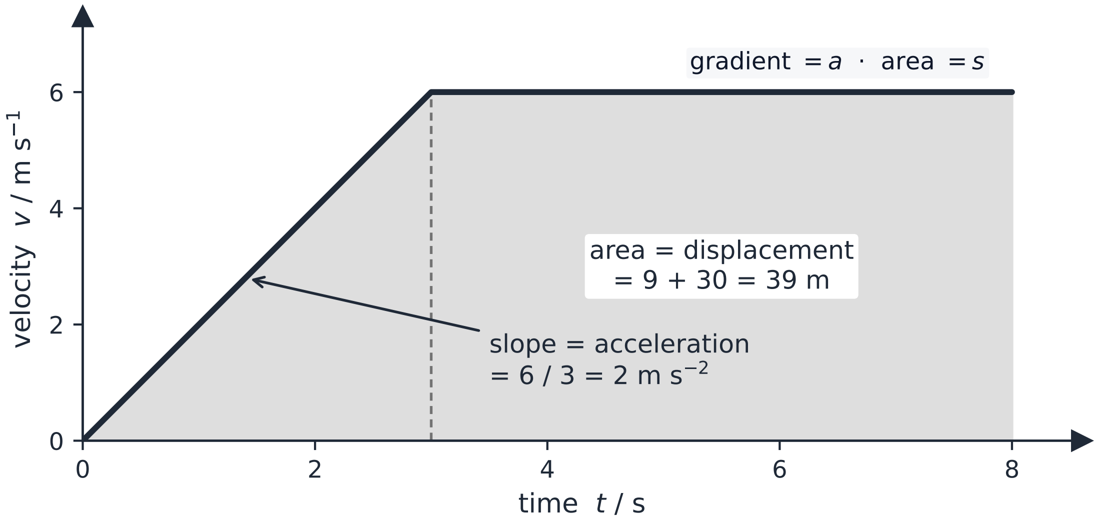
Example. A cyclist speeds up uniformly from 4.0 to 13 m s^{-1} in 6.0 s: so $u = 4.0$,

$v = 13$, $t = 6.0$; find a, then s.

- For a (no s): $v = u + at \Rightarrow a = \dfrac{v-u}{t} = \dfrac{13-4.0}{6.0} = 1.5\ \text{m s}^{-2}$.
- For s (no a): $s = \frac{1}{2}(u+v)t = \frac{1}{2}(4.0+13)(6.0) = 51$ m.

Reading a velocity–time graph

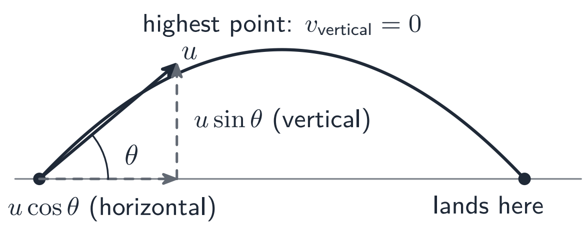
On a velocity–time graph the **gradient** 斜率 (steepness) is the acceleration, and the **area** under the line is the displacement.



(For an *acceleration*–time graph, the area under the line is instead the *change in velocity*.)

A projectile

Horizontal and vertical motion are **independent** — they share only the time t, so solve each as its own SUVAT problem. For a launch at angle θ, first **resolve** the speed: horizontal $u \cos\theta$ (stays constant) and vertical $u \sin\theta$ (changes).



Worked case — a ball thrown **horizontally** at 12 m s^{-1} from a cliff 25 m high ($g = 9.81$ m s^{-2}):

- Vertical (start 0): $25 = \frac{1}{2} g\, t^2 \Rightarrow t = \sqrt{2 \times 25/9.81} = 2.3$ s.
- Horizontal: $x = 12 \times 2.3 = 27$ m.

■ Measuring the acceleration of free fall

The syllabus asks you to **describe** this experiment, so learn it as five steps:

1. A steel ball is held by an **electromagnet** above a **trapdoor** switch, a measured height h above it.
2. Switching the magnet off starts an electronic **timer** and releases the ball at the same instant.
3. The ball opens the trapdoor on landing, which **stops** the timer, giving the fall time t.
4. The ball falls from rest, so $h = \frac{1}{2}gt^2$. Repeat for **several heights**, plot h against t^2, and the **gradient** is $\frac{1}{2}g$.
5. Measure h from the **bottom** of the ball to the trapdoor, and repeat each timing — the graph, not a single pair of readings, is what removes the random error.

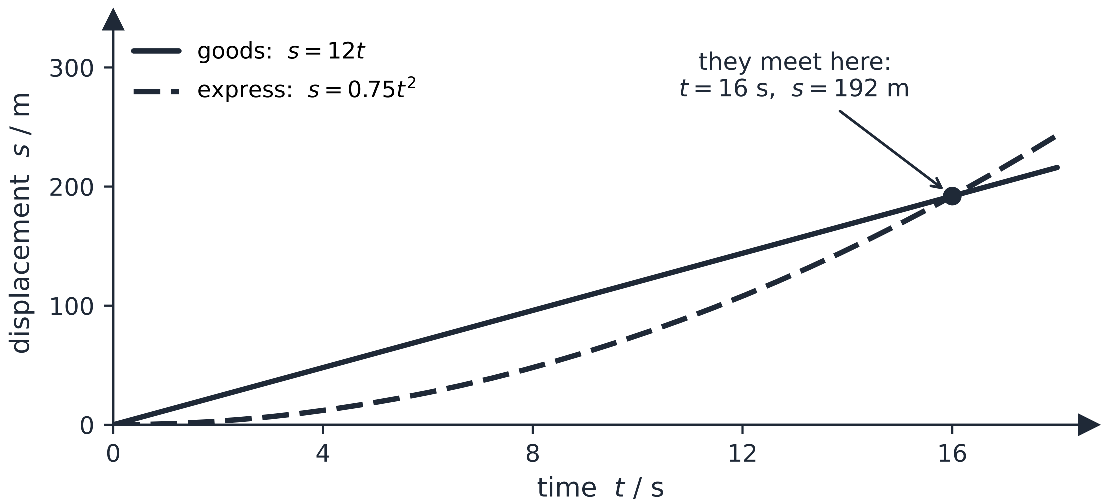
A steel ball is used because **air resistance** on it is negligible; the graph is a straight line **through the origin** only if there is no systematic timing delay.

■ Two objects meeting

Write a displacement equation for each object using the same start time and positive direction, then set the displacements equal.

A goods train moves at a constant 12 m s^{-1}. As it passes a signal, an express train starts from rest there with acceleration 1.5 m s^{-2}.

- Goods: $s = 12t$. Express: $s = \frac{1}{2}(1.5)t^2 = 0.75t^2$.
- Level when $12t = 0.75t^2 \Rightarrow t = \dfrac{12}{0.75} = 16$ s ($s = 12 \times 16 = 192$ m).



2 Practice

Now apply the methods above.

2.1 For each quantity, write **scalar** 标量 or **vector** 矢量, and state its SI unit: (a) velocity (b) acceleration. [2]

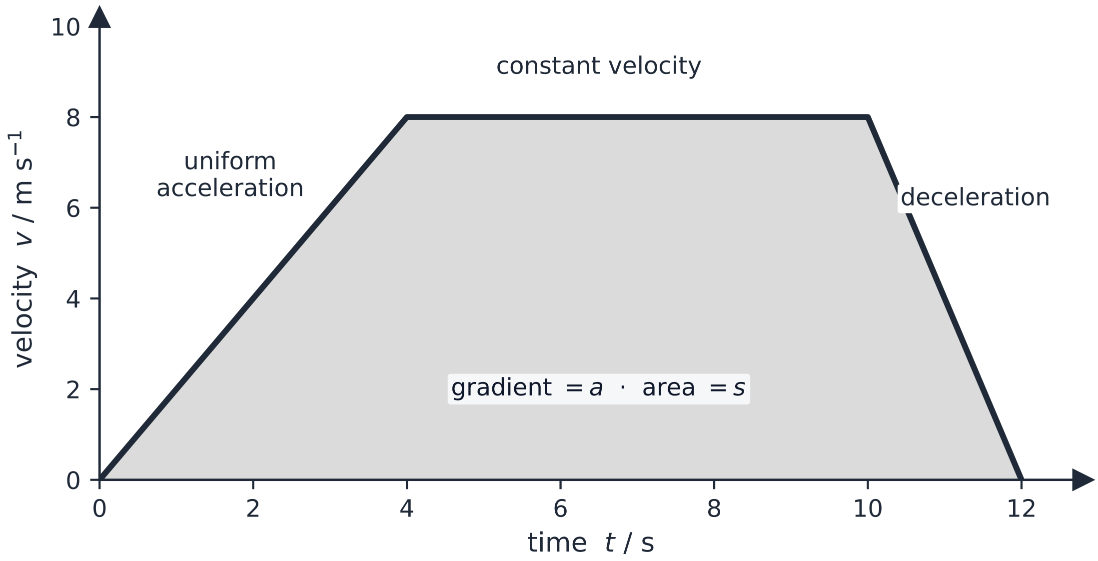
quantity	scalar or vector?	SI unit
velocity		
acceleration		

2.2 A car travelling at 24 m s^{-1} brakes uniformly at 3.0 m s^{-2} until it stops. Find the braking distance. *(Hint: which SUVAT equation links u, v, a and s?)* [3]

2.3 The displacement–time graph of a cyclist is a straight line passing through (2.0 s, 12 m) and (8.0 s, 42 m). **Determine** her velocity, and state how you know her acceleration is zero. [3]

2.4 Describe an experiment to determine the acceleration of free fall using a falling object. State the measurements you would take, how you would use them, and one way to reduce the uncertainty. [4]

2.5 The velocity–time graph shows a tram's journey.



Velocity–time graph for the tram's three-phase journey

(a) Find the acceleration during the first 4 s. [1]

(b) Find the total displacement of the tram. *(Hint: split the area into a triangle, a rectangle and a triangle.)* [3]

3 Exam-style questions

3.1 A stone is thrown vertically upwards at 15 m s^{-1}. Take $g = 9.81$ m s^{-2} and ignore air resistance. Its height above the launch point after 2.0 s is closest to: [1]

- **A** 10 m
- **B** 20 m
- **C** 30 m
- **D** 50 m

3.2 The area under an **acceleration–time graph** gives which quantity? [1]

- **A** displacement
- **B** change in velocity
- **C** distance
- **D** force

3.3 A train starts from rest and accelerates uniformly at 0.50 m s^{-2} for 40 s.

(a) Calculate the speed it reaches. [2]

(b) Calculate the distance travelled during these 40 s. [2]

(c) State one assumption you have made in your calculation. [1]

3.4 A ball is kicked from level ground at 20 m s^{-1}, at 30° above the horizontal. Take $g = 9.81$ m s^{-2} and ignore air resistance.

(a) Show that the vertical **component** 分量 of the initial velocity is about 10 m s^{-1}. [1]

(b) Calculate the time the ball is in the air before it lands. [2]

(c) Calculate the horizontal distance it travels (its range). [2]

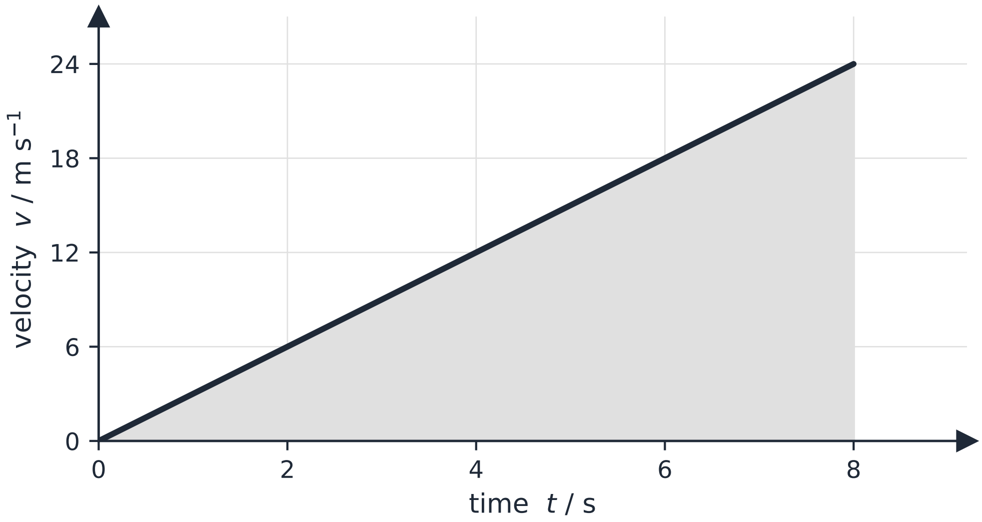
3.5 A car passes a stationary motorcyclist at a constant 18 m s^{-1}. At that instant the motorcyclist sets off from rest with a uniform acceleration of 3.0 m s^{-2}.

(a) Calculate the time the motorcyclist takes to catch up with the car. [3]

(b) Calculate how far each has travelled at that moment. [1]

(c) Calculate the motorcyclist's speed when she catches the car. [1]

3.6 A drone of mass 1.8 kg takes off from the ground and rises vertically. The graph shows its velocity for the first 8.0 s of the flight.



(a) **Define** acceleration. [1]

(b) **Determine** the acceleration of the drone. [2]

(c) **Show that** the drone rises 96 m in these 8.0 s. [2]

(d) Calculate the gain in **gravitational potential energy** of the drone. [2]

(e) Calculate the gain in **kinetic energy** of the drone. [2]

(f) **Determine** the average useful power output of the drone's motors during these 8.0 s. [2]

(g) In the space below, **sketch** the variation with time of the **displacement** of the drone over the same 8.0 s. **Draw** and **label** both axes. [3]

(h) A second drone of mass 2.7 kg takes off and follows exactly the same velocity–time graph. **State and explain** whether the average useful power output of its motors is less than, the same as, or greater than your answer to **(f)**. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- velocity is a **vector**, unit m s^{-1}
- acceleration is a **vector**, unit m s^{-2}

2.2

- known: $u = 24$ m s^{-1}, $v = 0$, $a = -3.0$ m s^{-2}; want s (no t)
- use $v^2 = u^2 + 2as$

$$0 = (24 \text{ m s}^{-1})^2 + 2(-3.0 \text{ m s}^{-2})s \Rightarrow s = \frac{576 \text{ m}^2\,\text{s}^{-2}}{6.0 \text{ m s}^{-2}} = \mathbf{96} \text{ m}$$

2.3

- velocity is the **gradient** of a displacement–time graph

$$v = \frac{42 \text{ m} - 12 \text{ m}}{8.0 \text{ s} - 2.0 \text{ s}} = \frac{30 \text{ m}}{6.0 \text{ s}} = \mathbf{5.0} \text{ m s}^{\mathbf{-1}}$$

- the graph is a **straight** line, so v never changes; acceleration is the rate of change of velocity, so $a = 0$

2.4

- hold a steel ball with an electromagnet a measured height h above a trapdoor
- switching the magnet off starts an electronic timer and releases the ball; the ball opens the trapdoor and stops the timer, giving t
- use $h = \frac{1}{2}gt^2$, or plot h against t^2 and take $g = 2\times$ gradient
- reduce uncertainty: repeat each timing, and use several heights so a graph averages the readings

2.5

(a) acceleration = gradient of the v–t graph

$$a = \frac{8.0 \text{ m s}^{-1} - 0}{4.0 \text{ s}} = \mathbf{2.0} \text{ m s}^{\mathbf{-2}}$$

(b) displacement = area under the line; split into triangle + rectangle + triangle

- 0–4 s: $\frac{1}{2} \times 4.0 \text{ s} \times 8.0 \text{ m s}^{-1} = 16$ m
- 4–10 s: $6.0 \text{ s} \times 8.0 \text{ m s}^{-1} = 48$ m
- 10–12 s: $\frac{1}{2} \times 2.0 \text{ s} \times 8.0 \text{ m s}^{-1} = 8$ m
- total $= 16 \text{ m} + 48 \text{ m} + 8 \text{ m} = \mathbf{72}$ m

3.1 A

- take up as positive: $u = 15$ m s^{-1}, $a = -9.81$ m s^{-2}, $t = 2.0$ s

$$s = ut + \tfrac{1}{2}at^2 = (15 \text{ m s}^{-1})(2.0 \text{ s}) + \tfrac{1}{2}(-9.81 \text{ m s}^{-2})(2.0 \text{ s})^2 = 30 \text{ m} - 19.6 \text{ m} = 10.4 \text{ m}$$

- closest to **10** m
- **B** forgot the $\frac{1}{2}at^2$ term; **C** is ut alone

3.2 B

- the area under an **acceleration–time** graph is the **change in velocity**
- **A** / **C** are the area under a **velocity–time** graph

3.3

(a) $u = 0$, $a = 0.50$ m s^{-2}, $t = 40$ s

$$v = u + at = 0 + (0.50 \text{ m s}^{-2})(40 \text{ s}) = \mathbf{20} \text{ m s}^{-1}$$

(b) no v needed: $s = ut + \frac{1}{2}at^2$

$$s = \tfrac{1}{2}(0.50 \text{ m s}^{-2})(40 \text{ s})^2 = \mathbf{400} \text{ m}$$

(c) any one: the acceleration stays constant / the track is straight / no resistive forces

3.4

(a) vertical component $= u \sin\theta$

- $u_V = 20$ m s^{-1} $\times \sin 30° = 20 \times 0.50 = \mathbf{10}$ m s^{-1}

(b) time to the top: $v_V = 0 = u_V - gt$

$$t_{\text{up}} = \frac{10 \text{ m s}^{-1}}{9.81 \text{ m s}^{-2}} = 1.02 \text{ s}$$

- total time in the air $= 2 \times 1.02$ s $= \mathbf{2.0}$ s

(c) $u_H = 20$ m s^{-1} $\times \cos 30° = 17.3$ m s^{-1}

- range $= u_H \times t = (17.3 \text{ m s}^{-1})(2.04 \text{ s}) = \mathbf{35}$ m

3.5

(a) same start time and positive direction

- car (constant v): $s = (18 \text{ m s}^{-1})t$
- motorcyclist (from rest): $s = \frac{1}{2}(3.0 \text{ m s}^{-2})t^2 = (1.5 \text{ m s}^{-2})t^2$
- they meet when the displacements are equal

$$(18 \text{ m s}^{-1})t = (1.5 \text{ m s}^{-2})t^2 \Rightarrow t = \frac{18 \text{ m s}^{-1}}{1.5 \text{ m s}^{-2}} = \mathbf{12} \text{ s}$$

(b) $s = (18 \text{ m s}^{-1})(12 \text{ s}) = \mathbf{216}$ m

(c) $v = at = (3.0 \text{ m s}^{-2})(12 \text{ s}) = \mathbf{36}$ m s^{-1}

3.6

(a) acceleration is the **rate of change of velocity**

(b) $a =$ gradient of the v–t graph

$$a = \frac{24 \text{ m s}^{-1} - 0}{8.0 \text{ s}} = \mathbf{3.0} \text{ m s}^{\mathbf{-2}}$$

(c) height = area under the line (a triangle)

$$s = \tfrac{1}{2} \times 8.0 \text{ s} \times 24 \text{ m s}^{-1} = \mathbf{96} \text{ m}$$

(d)

$$\Delta E_{\text{P}} = mgh = (1.8 \text{ kg})(9.81 \text{ m s}^{-2})(96 \text{ m}) = \mathbf{1.7 \times 10^3} \text{ J}$$

(e)

$$\Delta E_{\text{K}} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1.8 \text{ kg})(24 \text{ m s}^{-1})^2 = \mathbf{5.2 \times 10^2} \text{ J}$$

(f) useful energy gained $= \Delta E_{\text{P}} + \Delta E_{\text{K}}$

$$P = \frac{1.7 \times 10^3 \text{ J} + 5.2 \times 10^2 \text{ J}}{8.0 \text{ s}} = \mathbf{2.8 \times 10^2} \text{ W}$$

(g)

- axes drawn and labelled displacement / m (vertical) and time / s (horizontal)
- a curve starting at the origin
- getting **steeper** with time ($s \propto t^2$), reaching 96 m at 8.0 s

(h) **greater**

- the v–t graph is the same, so Δh and Δv are the same, but $\Delta E_{\text{P}} = mgh$ and $\Delta E_{\text{K}} = \frac{1}{2}mv^2$ both scale with **mass**, so more energy is gained in the same 8.0 s

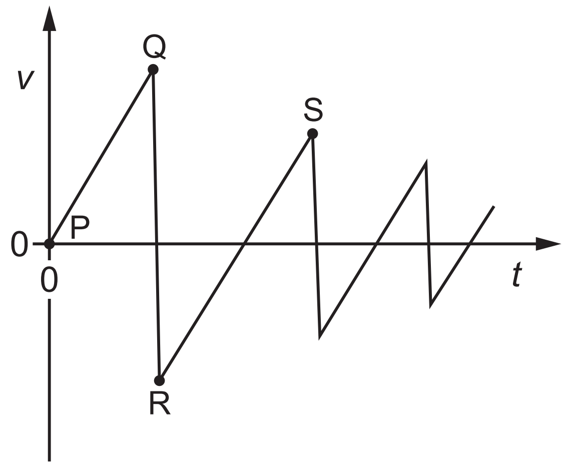
5 A goods train passes through a station at a constant speed of $10\,m\,s^{-1}$ at time $t = 0$. An express train is at rest at the station. The express train leaves the station with a uniform acceleration of $0.5\,m\,s^{-2}$ just as the goods train goes past. Both trains move in the same direction on straight, parallel tracks.

At which time t does the express train overtake the goods train?

A 6 s **B** 10 s **C** 20 s **D** 40 s

6 A ball is held above the ground and released. It falls to the ground and bounces several times.

The graph shows the variation with time t of the velocity v of the ball.



Four points on the graph are labelled P, Q, R and S.

Which statement is **not** correct?

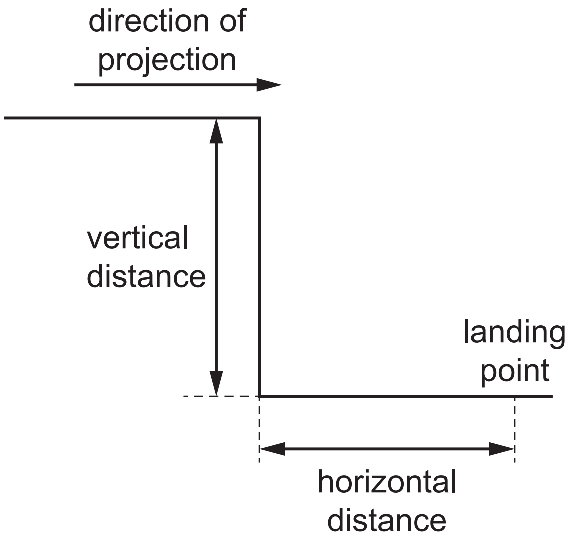
A The area under line PQ represents the initial height of the ball.

B The collisions of the ball with the ground are inelastic.

C The gradient of the line RS represents the acceleration due to free fall.

D The maximum upwards velocity of the ball is reached at point P.

7 An object is projected horizontally. The object falls a vertical distance *y* and travels a horizontal distance *x* before landing on the ground.



A second object is projected horizontally with the same initial velocity and falls a vertical distance 4*y* before landing on the ground.

Assume that air resistance is negligible.

Which horizontal distance does the second object travel?

A x **B** $2x$ **C** $4x$ **D** $16x$

A-Level Physics: **w25 12**

5 A car has an initial velocity *u*. The car then moves with constant acceleration *a* in a straight line through a displacement *d*. The car reaches a final velocity *v*.

Which expression gives the initial velocity *u* of the car?

A $v - ad$ **B** $\sqrt{v^2 - 2ad}$ **C** $v + ad$ **D** $v - \frac{1}{2}ad^2$

6 A bicycle brakes so that it undergoes uniform deceleration from a speed of $8\,\mathrm{m\,s^{-1}}$ to $6\,\mathrm{m\,s^{-1}}$ over a distance of 7 m.

The deceleration of the bicycle remains constant.

Which further distance will the bicycle travel before coming to rest?

A 7 m **B** 9 m **C** 16 m **D** 21 m

1 **(a)** Define acceleration.

....

.... [1]

(b) A rocket is launched vertically from the surface of the Earth.

Fig. 1.1 shows the variation of the velocity of the rocket with time for the first 20 s after its launch.

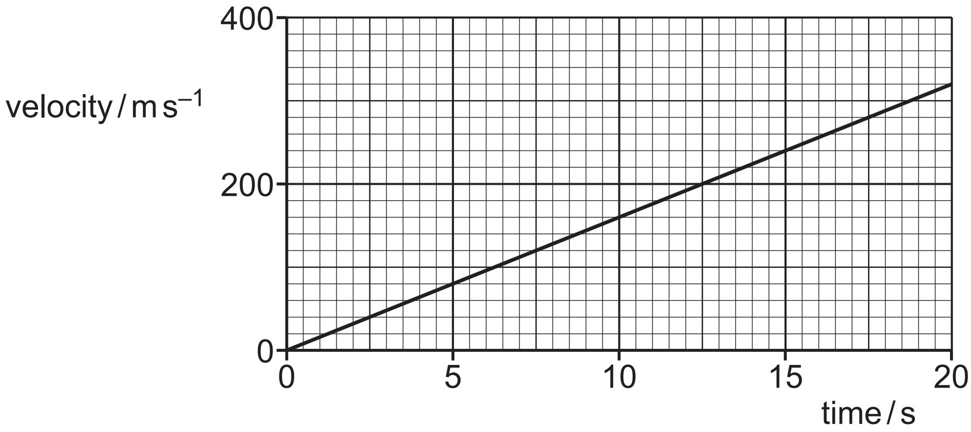


Fig. 1.1

(i) Determine the acceleration of the rocket.

acceleration = ... m s^{-2} [1]

(ii) Show that the height of the rocket above the surface of the Earth at a time of 20 s after launch is 3.2 km.

[2]

(c) The mass of the rocket in **(b)** is 2.9×10^6 kg. Assume that this mass remains constant.

For this rocket, from launch to its height at a time of 20 s after launch:

(i) calculate the gain in gravitational potential energy ΔE_P

ΔE_P = ... J [2]

(ii) calculate the gain in kinetic energy ΔE_K

ΔE_K = ... J [2]

(iii) determine the average power output of the rocket engines. Assume that resistive forces are negligible.

power = ... W [2]

[Total: 10]

1 A child kicks a ball so that it leaves horizontal ground with a velocity of $28\,\text{m}\,\text{s}^{-1}$ at an angle of 34° to the horizontal, as shown in Fig. 1.1.

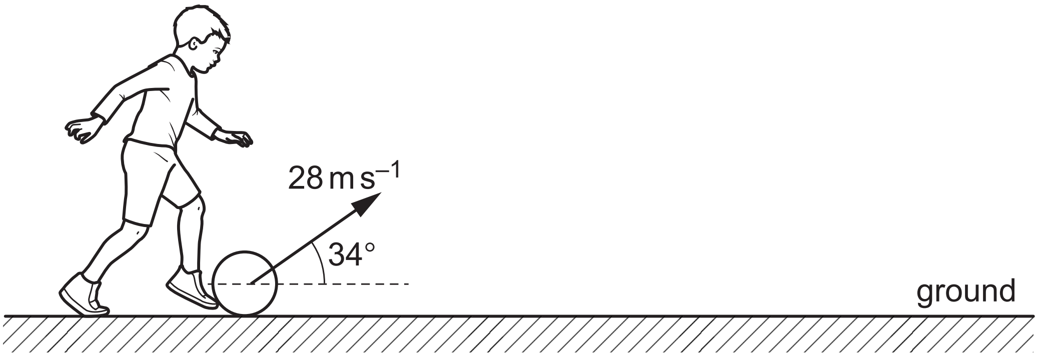


Fig. 1.1

Air resistance is negligible. The ball leaves the ground at time $t = 0$.

(a) (i) Calculate the horizontal component v_H and the vertical component v_V of the velocity of the ball immediately after it has left the ground.

v_H = ... $\text{m}\,\text{s}^{-1}$

v_V = ... $\text{m}\,\text{s}^{-1}$
[2]

(ii) Show that the ball reaches its maximum height at time $t = 1.6\,\text{s}$.

[1]

(iii) On Fig. 1.2, sketch the variation of v_H with time t between t = 0 and t = 3.2 s. Label your line H.

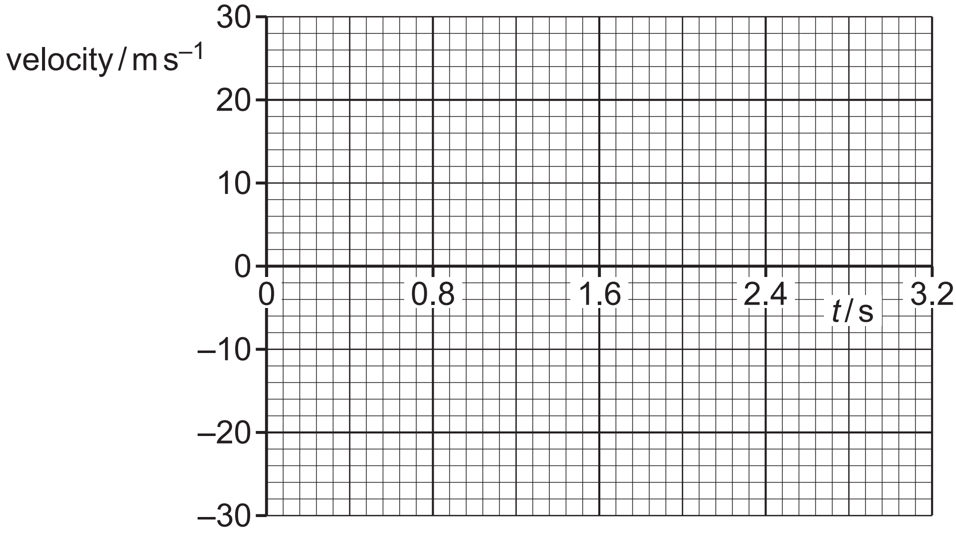


Fig. 1.2

[1]

(iv) On Fig. 1.2, sketch the variation of v_V with time t between t = 0 and t = 3.2 s. Assume that velocity in the upward direction is positive. Label your line V. [3]

(b) The total change in momentum of the ball between leaving the ground at t = 0 and landing on the ground at t = 3.2 s is 13 kg m s^{-1}.

(i) Define momentum.

....

.... [1]

(ii) Calculate the force that acts on the ball while it is in the air.

force = ... N [2]

(iii) Determine the mass of the ball.

mass = ... kg [1]

[Total: 11]

7 Which equation of uniformly accelerated motion can be derived using only the gradient of a velocity–time graph?

A $s = \frac{1}{2}(u + v)t$

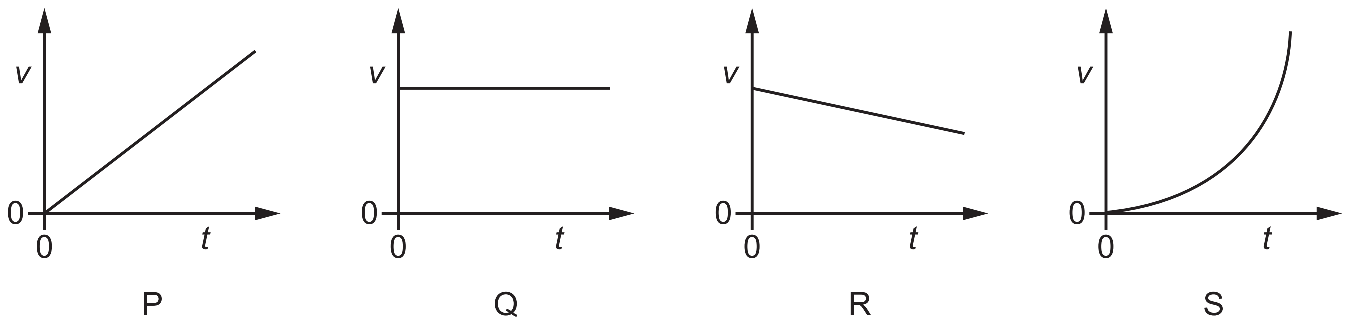
B $s = ut + \frac{1}{2}at^2$

C $v = u + at$

D $v^2 = u^2 + 2as$

8 An object is projected horizontally from a table at time t = 0. The object falls in a uniform gravitational field. Air resistance is negligible.

Graphs P, Q, R and S are velocity–time graphs.



Which graphs represent the horizontal and vertical components of the velocity of the object?

	horizontal	vertical
A	Q	P
B	Q	S
C	R	P
D	R	S

1 **(a)** Define acceleration.

....

.... [1]

(b) In an experiment, two objects A and B are released from the side of a building, as shown in Fig. 1.1.

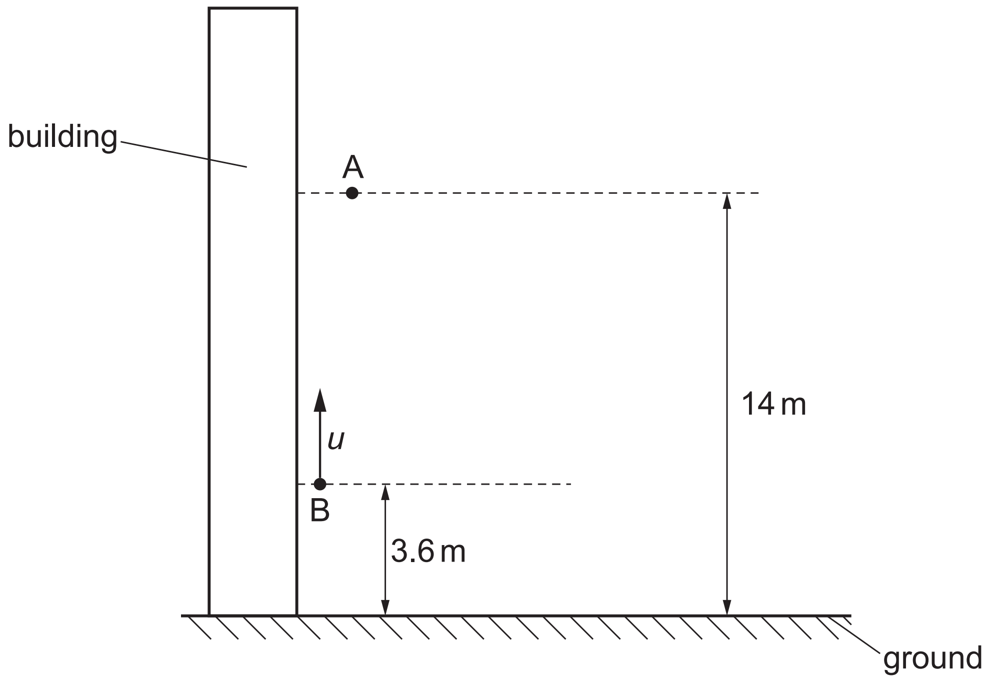


Fig. 1.1 (not to scale)

Object A is released from rest at a height of 14 m above the horizontal ground.
Object B is released with an initial upwards vertical velocity u at a height of 3.6 m above the ground.
Both objects take the same time to reach the ground and they do not collide with each other.
Air resistance is negligible.

(i) Calculate the time taken for object A to reach the ground.

time = ... s [2]

(ii) Use your answer in **(b)(i)** to calculate u.

u = ... $m s^{-1}$ [2]

(c) In a second experiment, object B is released from the same height and given the same initial speed as in **(b)** but at a release angle θ to the vertical, as shown in Fig. 1.2.

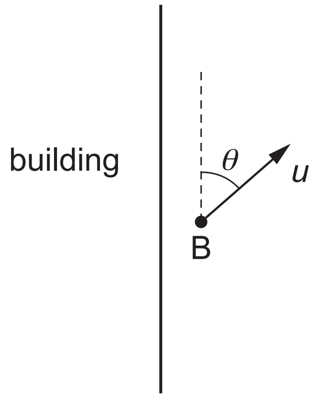


Fig. 1.2

(i) State and explain whether the time taken for object B to reach the ground is less than, the same as or greater than the time in **(b)(i)**.

....

....

.... [2]

(ii) By considering energy, state and explain the effect of the change in release angle on the speed at which B reaches the ground.

....

....

.... [2]

[Total: 9]

4 An aircraft, initially stationary on a runway, takes off with a speed of $85\,\text{km}\,\text{h}^{-1}$ in a distance of no more than 1.20 km.

What is the minimum constant acceleration necessary for the aircraft?

A $0.23\,\text{m}\,\text{s}^{-2}$ **B** $0.46\,\text{m}\,\text{s}^{-2}$ **C** $3.0\,\text{m}\,\text{s}^{-2}$ **D** $6.0\,\text{m}\,\text{s}^{-2}$

3 **(a)** A truck R of mass 9400 kg moves with constant acceleration in a straight line down a slope, as illustrated in Fig. 3.1.

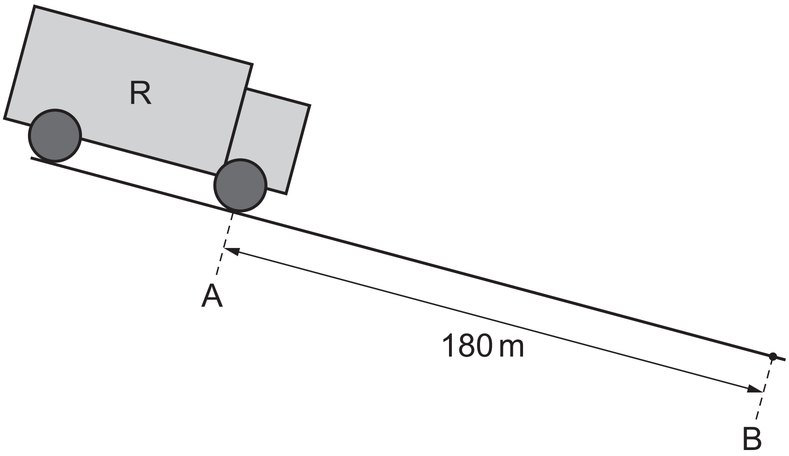


Fig. 3.1

At point A the speed of the truck is 13 m s^{-1} and at point B the speed of the truck is 22 m s^{-1}. A and B are a distance of 180 m apart.

(i) Calculate the acceleration of the truck between A and B.

acceleration = ... m s^{-2} [2]

(ii) Determine the gain in kinetic energy of the truck between A and B.

gain in kinetic energy = ... J [3]

(b) A short time after passing point B truck R moves in a straight line on horizontal ground. The driver of the truck applies the brakes. Fig. 3.2 shows the variation with time of the momentum of the truck.

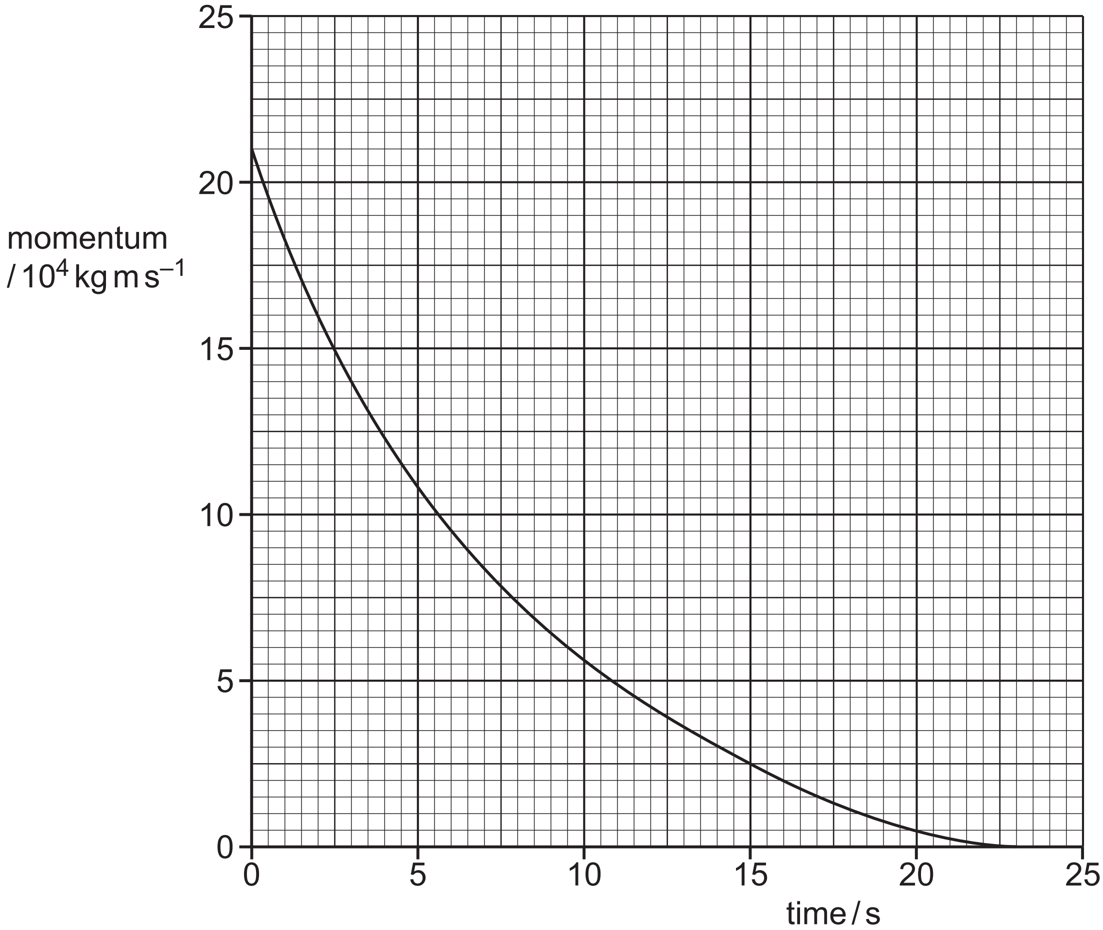


Fig. 3.2

(i) Define force.

....

.... [1]

(ii) Show that the average resultant force F acting on truck R between time t = 0 and t = 15 s is -1.2×10^4 N.

[1]

(iii) An identical truck S has the same initial momentum as truck R. Truck S experiences a constant force equal to the force *F* in **(b)(ii)**.

State and explain whether truck S will take more, less or the same amount of time to come to rest as truck R.

....

....

....

....

....

....

.... [3]

[Total: 10]

2.1.1 Describing motion and displacement–time graphs

Name: ______________ Class: __________ Date: ____________ **Total: 26 marks**

KEY WORDS velocity 速度 • acceleration 加速度 • speed 速率 • distance 距离 • displacement 位移

Objective

Build the skills to answer exam questions on **describing motion 描述运动**: the exact definitions, average speed and average velocity, and reading a velocity from a **displacement–time graph 位移 – 时间图像**. This sheet covers syllabus 2.1, learning outcomes 1, 2 and 4.

You must be able to:

- **define** distance 距离, displacement 位移, speed 速率, velocity 速度 and acceleration 加速度
- tell average speed from average velocity, and change km h^{-1} into m s^{-1}
- add two displacements that are at right angles
- find a velocity from the gradient of a displacement–time graph, using a tangent for a curve
- **describe** a motion from the shape of its displacement–time graph

1 Worked examples

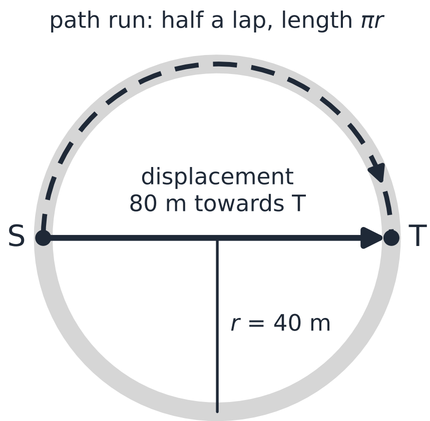
Study these first. Each one shows a **method 方法** that a question later on this sheet needs.

■ Definitions the examiner accepts

quantity	definition	scalar 标量 or vector 矢量
distance	the total length of the path travelled	scalar
displacement	the distance in a straight line from the start, in a stated direction	vector
speed	the rate of change of distance	scalar
velocity	the rate of change of displacement	vector
acceleration	the rate of change of velocity	vector

■ Distance or displacement?

An athlete runs half a lap of a circular track of radius 40 m, from S to T, in 25 s.



- distance: the path is half the **circumference** 周长

$$\pi r = \pi \times 40 \text{ m} = 126 \text{ m}$$

- displacement: the straight line from S to T is the **diameter** 直径, 80 m towards T
- **average speed** 平均速率 = total distance ÷ total time

$$\frac{126 \text{ m}}{25 \text{ s}} = 5.0 \text{ m s}^{-1}$$

- **average velocity** 平均速度 = displacement ÷ total time

$$\frac{80 \text{ m}}{25 \text{ s}} = 3.2 \text{ m s}^{-1} \text{ towards T}$$

After a **whole** lap the displacement is zero, so the average velocity is zero. The average speed is not.

■ Adding two displacements

A walker goes 600 m due west and then 800 m due south. The two displacements are at right angles, so they are the two short sides of a right-angled triangle (Topic 1).

- size of the displacement, by Pythagoras

$$\sqrt{(600 \text{ m})^2 + (800 \text{ m})^2} = 1000 \text{ m}$$

- direction, as an angle measured from west towards south

$$\tan\theta = \frac{800 \text{ m}}{600 \text{ m}} \Rightarrow \theta = 53^\circ$$

- the displacement is 1000 m at 53° south of west; the distance walked is 1400 m

Units. To change km h^{-1} into m s^{-1}, multiply by 1000 (km to m) and divide by 3600 (h to s):

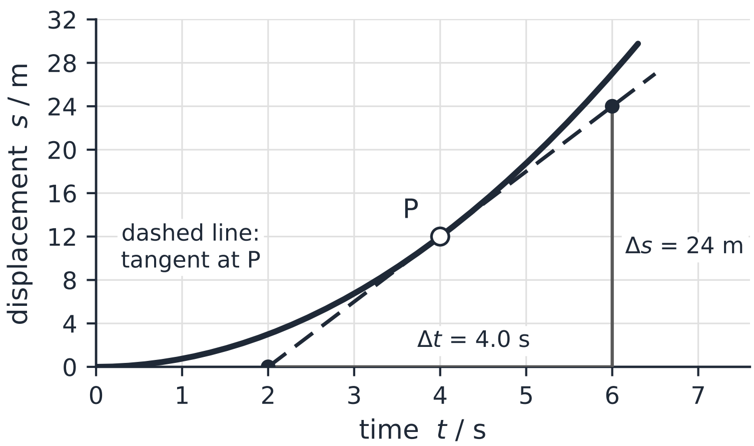
$$54 \text{ km h}^{-1} = \frac{54 \times 1000 \text{ m}}{3600 \text{ s}} = 15 \text{ m s}^{-1}$$

■ Velocity from a displacement–time graph

The **gradient** 斜率 of a displacement–time graph is the velocity. Use the shape to **describe** 描述 the motion:

- flat line: the object is at rest
- straight sloping line: constant velocity; a line sloping down means moving back towards the start (negative velocity)
- curve getting steeper: speeding up; curve getting flatter: slowing down

For a curve, draw a **tangent** 切线 at the time you need, make a **large** triangle on it, and find its gradient.



Example: the velocity of the car at P, where $t = 4.0$ s.

- the tangent passes through (2.0 s, 0) and (6.0 s, 24 m)

$$v = \frac{\Delta s}{\Delta t} = \frac{24 \text{ m} - 0}{6.0 \text{ s} - 2.0 \text{ s}} = 6.0 \text{ m s}^{-1}$$

- a small triangle would make your reading errors much larger
- this is the **instantaneous** 瞬时 velocity at P; the average velocity over a time is the total displacement divided by that time

2 Practice

Now use the methods above.

2.1 The examiner often starts a question with a definition.

(a) **Define** velocity. [1]

(b) State one way in which velocity is different from speed. [1]

2.2 A cyclist rides 3.0 km due east and then 5.0 km due north. The whole ride takes 25 minutes.

(a) State the total distance travelled. [1]

(b) **Determine** the size of the cyclist's displacement from the start. *(Hint: the two displacements are at right angles.)* [1]

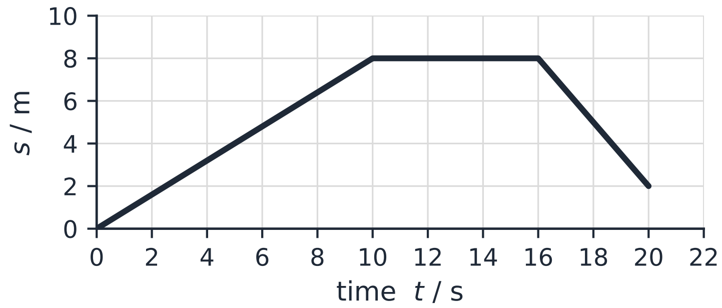
(c) Determine the direction of the displacement, as an angle measured from east towards north. [1]

(d) Calculate the average speed of the cyclist, in m s^{-1}. [2]

(e) Calculate the size of the average velocity of the cyclist, in m s^{-1}. [1]

2.3 Change the units. *(Hint: 1 km = 1000 m and 1 h = 3600 s.)* (a) 72 km h^{-1} into m s^{-1}; (b) 25 m s^{-1} into km h^{-1}. [2]

2.4 The displacement–time graph shows a model train on a straight track.

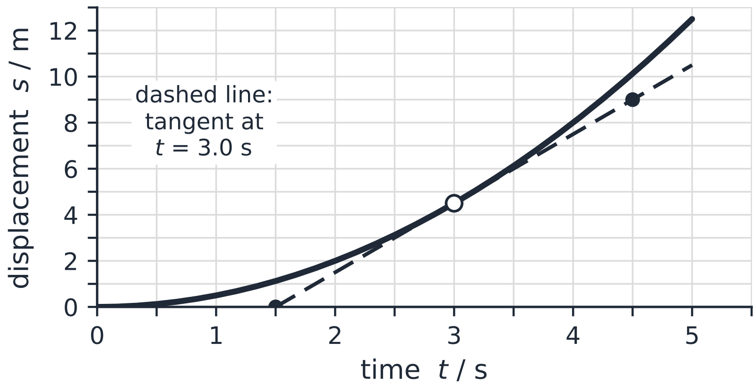


(a) Determine the velocity of the train between $t = 0$ and $t = 10$ s. [1]

(b) Describe the motion of the train between $t = 10$ s and $t = 16$ s. [1]

(c) Determine the velocity of the train between $t = 16$ s and $t = 20$ s. [1]

2.5 A **trolley** 小车 is released from rest at the top of a **ramp** 斜坡. The graph shows its displacement down the ramp. A tangent has been drawn at $t = 3.0$ s.



(a) Use the tangent to determine the velocity of the trolley at $t = 3.0$ s. [2]

(b) State how the graph shows that the trolley is speeding up. [1]

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3 Exam-style questions

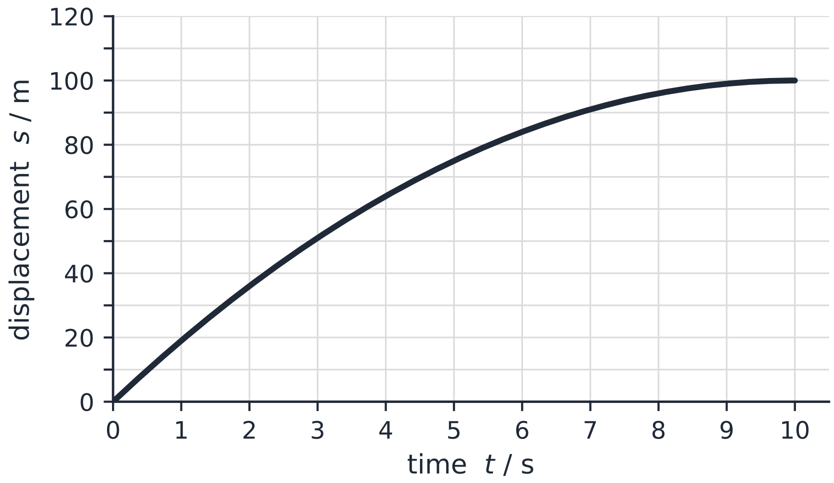
3.1 A runner completes 2.5 laps of a circular track of circumference 400 m in 300 s. What are her average speed and the size of her average velocity? [1]

- **A** speed 3.3 m s^{-1}, velocity 0
- **B** speed 3.3 m s^{-1}, velocity 0.42 m s^{-1}
- **C** speed 0.42 m s^{-1}, velocity 3.3 m s^{-1}
- **D** speed 3.3 m s^{-1}, velocity 3.3 m s^{-1}

3.2 A bus travels 12 km at an average speed of 40 km h^{-1}, then another 12 km at 60 km h^{-1}. What is its average speed for the whole journey? [1]

- **A** 13 m s^{-1}
- **B** 14 m s^{-1}
- **C** 48 m s^{-1}
- **D** 50 m s^{-1}

3.3 The graph shows the displacement s of a car along a straight road as the driver brakes to a stop.



(a) **Describe** the motion of the car from $t = 0$ to $t = 10$ s. [2]

(b) By drawing a tangent, determine the velocity of the car at $t = 4.0$ s. [3]

(c) Determine the average velocity of the car from $t = 0$ to $t = 10$ s. [2]

(d) Explain why your answers to (b) and (c) are different. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

(a) velocity is the **rate of change of displacement**

(b) velocity is a vector, so it has a direction; speed is a scalar, so it has none

2.2

(a) 3.0 km + 5.0 km = **8.0** km

(b) the displacements are at right angles, so use Pythagoras

$$\sqrt{(3.0\ \text{km})^2 + (5.0\ \text{km})^2} = \mathbf{5.8}\ \text{km}$$

(c) opposite side north, adjacent side east

$$\tan\theta = \frac{5.0\ \text{km}}{3.0\ \text{km}} \Rightarrow \theta = \mathbf{59}^\circ$$

north of east

(d) time = 25 × 60 s = 1500 s and distance = 8000 m

$$\text{average speed} = \frac{8000\ \text{m}}{1500\ \text{s}} = \mathbf{5.3}\ \text{m s}^{-1}$$

(e) use the displacement, not the distance

$$\text{average velocity} = \frac{5830\ \text{m}}{1500\ \text{s}} = \mathbf{3.9}\ \text{m s}^{-1}$$

2.3

(a) multiply by 1000 and divide by 3600

$$72\ \text{km h}^{-1} = \frac{72 \times 1000\ \text{m}}{3600\ \text{s}} = \mathbf{20}\ \text{m s}^{-1}$$

(b) in one hour the car travels 25 m × 3600 = 90 000 m = 90 km, so the speed is **90** km h^{-1}

2.4

(a) velocity = gradient

$$v = \frac{8.0\ \text{m} - 0}{10\ \text{s} - 0} = \mathbf{0.80}\ \text{m s}^{-1}$$

(b) the line is flat, so the displacement does not change: the train is **at rest**

(c) gradient of the line from 16 s to 20 s

$$v = \frac{2.0\ \text{m} - 8.0\ \text{m}}{20\ \text{s} - 16\ \text{s}} = \mathbf{-1.5}\ \text{m s}^{-1}$$

- the minus sign means the train moves back towards its start

2.5

(a) the tangent passes through $(1.5\ \text{s},\ 0)$ and $(4.5\ \text{s},\ 9.0\ \text{m})$

$$v = \frac{9.0\ \text{m} - 0}{4.5\ \text{s} - 1.5\ \text{s}} = \mathbf{3.0}\ \text{m s}^{-1}$$

(b) the curve gets **steeper**: its gradient, the velocity, increases with time

3.1 B

- distance $= 2.5 \times 400\ \text{m} = 1000\ \text{m}$

$$\text{average speed} = \frac{1000\ \text{m}}{300\ \text{s}} = 3.3\ \text{m s}^{-1}$$

- after 2.5 laps the runner is half a lap from the start, so the displacement is one diameter

$$d = \frac{400\ \text{m}}{\pi} = 127\ \text{m}, \qquad \text{average velocity} = \frac{127\ \text{m}}{300\ \text{s}} = \mathbf{0.42}\ \text{m s}^{-1}$$

- **A** is true only after whole laps; **C** swaps them; **D** uses the distance twice

3.2 A

- time for each part

$$t_1 = \frac{12\ \text{km}}{40\ \text{km h}^{-1}} = 0.30\ \text{h}, \qquad t_2 = \frac{12\ \text{km}}{60\ \text{km h}^{-1}} = 0.20\ \text{h}$$

- average speed = total distance ÷ total time

$$\frac{24\ \text{km}}{0.50\ \text{h}} = 48\ \text{km h}^{-1} = \frac{48 \times 1000\ \text{m}}{3600\ \text{s}} = \mathbf{13}\ \text{m s}^{-1}$$

- **B** averages the two speeds, but the bus spends longer at the lower speed; **C** and **D** forget to change the units

3.3

(a)

- the displacement increases, so the car moves forwards
- the gradient decreases to zero, so the car slows down and stops at $t = 10$ s

(b)

- tangent drawn at $t = 4.0$ s
- gradient from a large triangle, for example through $(0,\ 16\ \text{m})$ and $(8.0\ \text{s},\ 112\ \text{m})$

$$v = \frac{112\ \text{m} - 16\ \text{m}}{8.0\ \text{s} - 0} = \mathbf{12}\ \text{m s}^{-1}$$

(accept 11 to 13)

(c) average velocity = displacement ÷ time

$$\frac{100\ \text{m}}{10\ \text{s}} = \mathbf{10}\ \text{m s}^{-1}$$

(d) (b) is the velocity at one instant; (c) is the total displacement divided by the total time

A-LEVEL PHYSICS • EXERCISE SHEET

2.1.2 Velocity–time and acceleration–time graphs

Name: ______________ Class: __________ Date: ____________ **Total: 30 marks**

KEY WORDS velocity 速度 • acceleration 加速度 • displacement 位移 • change in velocity 速度变化 • gradient 斜率

Objective

Build the skills to answer exam questions on **velocity–time graphs 速度 – 时间图像** and **acceleration–time graphs 加速度 – 时间图像**: an acceleration from a gradient, a displacement from an area, and one graph drawn from another. This sheet covers syllabus 2.1, learning outcomes 2, 3 and 5.

You must be able to:

- find an acceleration from the gradient of a velocity–time graph, including at one instant on a curve
- find a displacement from the area under a velocity–time graph, including area below the time axis
- tell distance from displacement on a velocity–time graph
- estimate the area under a curve by counting squares
- find a change in velocity from the area under an acceleration–time graph
- **sketch** one motion graph from another

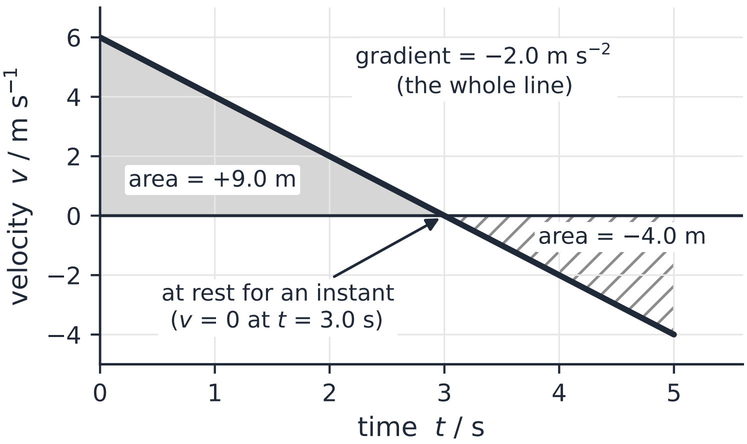
1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

■ Gradient and area of a velocity–time graph

On a velocity–time graph the **gradient 斜率** is the **acceleration 加速度**, and the **area 面积** between the line and the time axis is the **displacement 位移**. Area **below** the time axis is negative displacement: the object is moving backwards. The **distance 距离** adds up the sizes of all the areas.

A trolley is pushed up a track at 6.0 m s^{-1}. It slows down, stops for an instant at $t = 3.0$ s, and rolls back.



- acceleration = gradient

$$a = \frac{\Delta v}{\Delta t} = \frac{-4.0\ \text{m s}^{-1} - 6.0\ \text{m s}^{-1}}{5.0\ \text{s}} = -2.0\ \text{m s}^{-2}$$

- the line is straight, so the acceleration is still -2.0 m s^{-2} at $t = 3.0$ s, when the velocity is zero
- area above the axis

$$\tfrac{1}{2} \times 3.0\ \text{s} \times 6.0\ \text{m s}^{-1} = +9.0\ \text{m}$$

- area below the axis

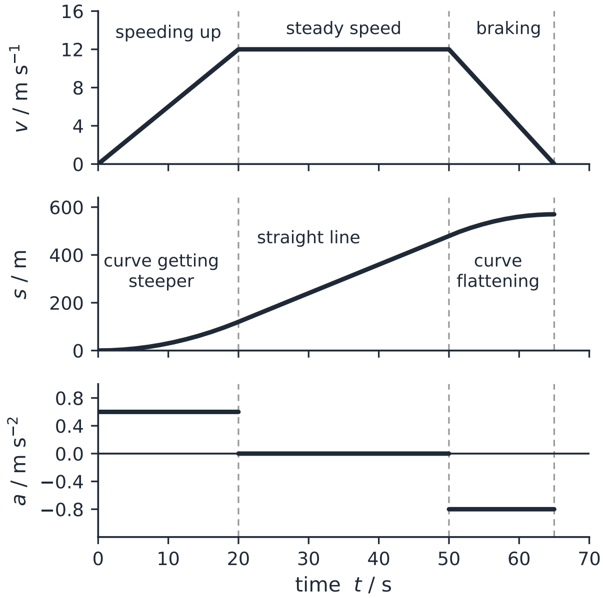
$$\tfrac{1}{2} \times 2.0\ \text{s} \times (-4.0\ \text{m s}^{-1}) = -4.0\ \text{m}$$

- displacement after 5.0 s: 9.0 m − 4.0 m = 5.0 m up the track; distance: 9.0 m + 4.0 m = 13 m

A negative gradient means the acceleration points in the negative direction. On an acceleration–time graph, the area under the line is the **change in velocity** 速度变化.

One journey, three graphs

A train starts from rest and speeds up uniformly to 12 m s^{-1} in 20 s. It keeps this speed for 30 s, then brakes uniformly to rest in 15 s.



- **acceleration–time**: each acceleration is a gradient of the velocity–time graph

$$a_1 = \frac{12\ \text{m s}^{-1}}{20\ \text{s}} = 0.60\ \text{m s}^{-2}, \qquad a_3 = \frac{-12\ \text{m s}^{-1}}{15\ \text{s}} = -0.80\ \text{m s}^{-2}$$

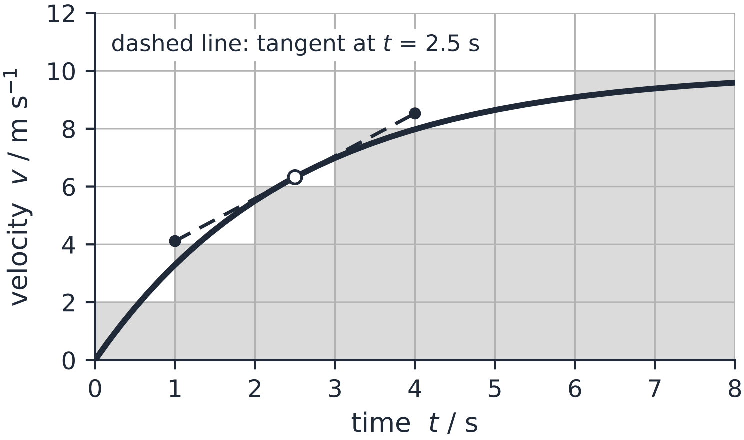
- while the speed is steady, $a = 0$
- **displacement–time**: its gradient is the velocity, so it gets steeper while the train speeds up, is a straight line while the speed is steady, and flattens to horizontal as the train stops
- total displacement = area under the velocity–time graph

$$s = \tfrac{1}{2}(20\ \text{s})(12\ \text{m s}^{-1}) + (30\ \text{s})(12\ \text{m s}^{-1}) + \tfrac{1}{2}(15\ \text{s})(12\ \text{m s}^{-1}) = 570\ \text{m}$$

When you **sketch** 画草图 a graph, draw and label both axes, and give each part the right shape: straight or curved, getting steeper or getting flatter.

■ A curve: a tangent and counting squares

When a velocity–time graph is a curve, the acceleration at one instant is the gradient of the **tangent** 切线 there, and the area is estimated by counting grid squares.



- the tangent at $t = 2.5$ s passes through (1.0 s, 4.1 m s^{-1}) and (4.0 s, 8.5 m s^{-1})

$$a = \frac{8.5\ \text{m s}^{-1} - 4.1\ \text{m s}^{-1}}{4.0\ \text{s} - 1.0\ \text{s}} = 1.5\ \text{m s}^{-2}$$

- one square is 1 s wide and 2 m s^{-1} tall, so it is worth

$$1\ \text{s} \times 2\ \text{m s}^{-1} = 2\ \text{m}$$

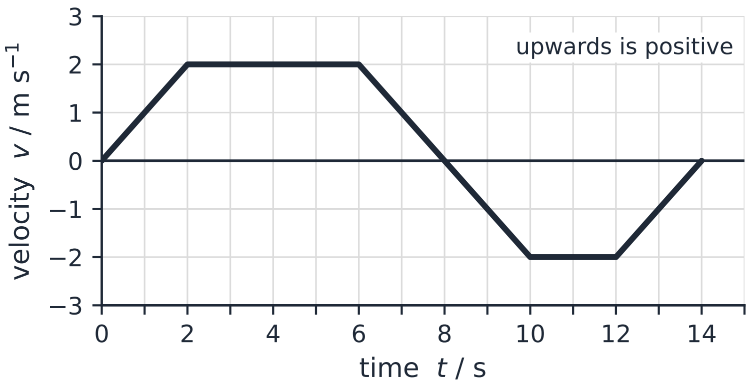
- count every square that is **at least half** under the curve: 28 squares
- displacement $\approx 28 \times 2$ m $= 56$ m

2 Practice

Now use the methods above.

2.1 State what is represented by (a) the gradient of a velocity–time graph; (b) the area under a velocity–time graph. [2]

2.2 The velocity–time graph shows a lift that goes up and then comes back down. Upwards is positive.



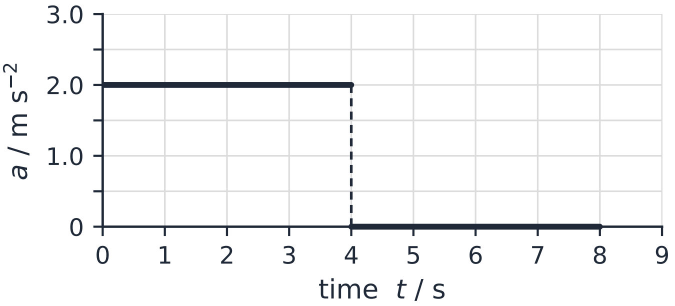
(a) Calculate the acceleration of the lift between $t = 0$ and $t = 2.0$ s. [1]

(b) State the times at which the lift is at rest. [1]

(c) Calculate the total distance travelled by the lift in the 14 s. *(Hint: for distance, every area counts as positive.)* [2]

(d) Determine the displacement of the lift from its starting point at $t = 14$ s. [1]

2.3 A car is moving at 5.0 m s^{-1} at time $t = 0$. The graph shows its acceleration.

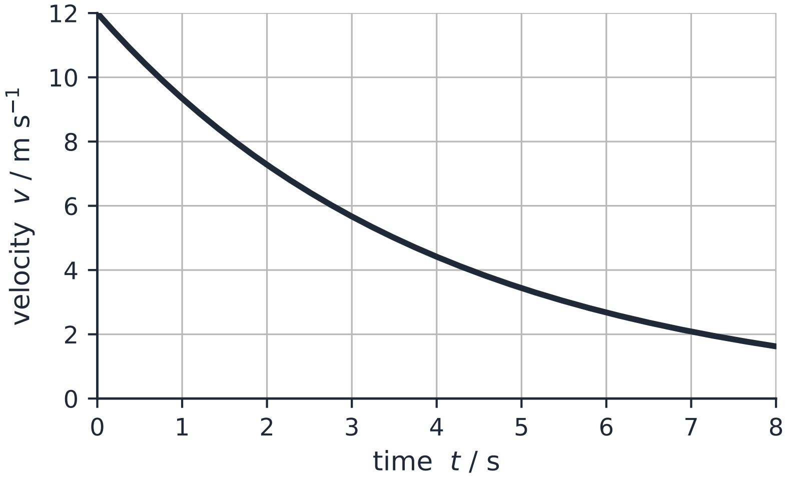


(a) Determine the change in velocity of the car between $t = 0$ and $t = 4.0$ s. [1]

(b) State the velocity of the car at $t = 6.0$ s. [1]

(c) **Sketch** the velocity–time graph of the car from $t = 0$ to $t = 8.0$ s. Label both axes and mark the values of velocity. [2]

2.4 A cyclist stops pedalling and slows down. The graph shows the velocity of the cyclist.

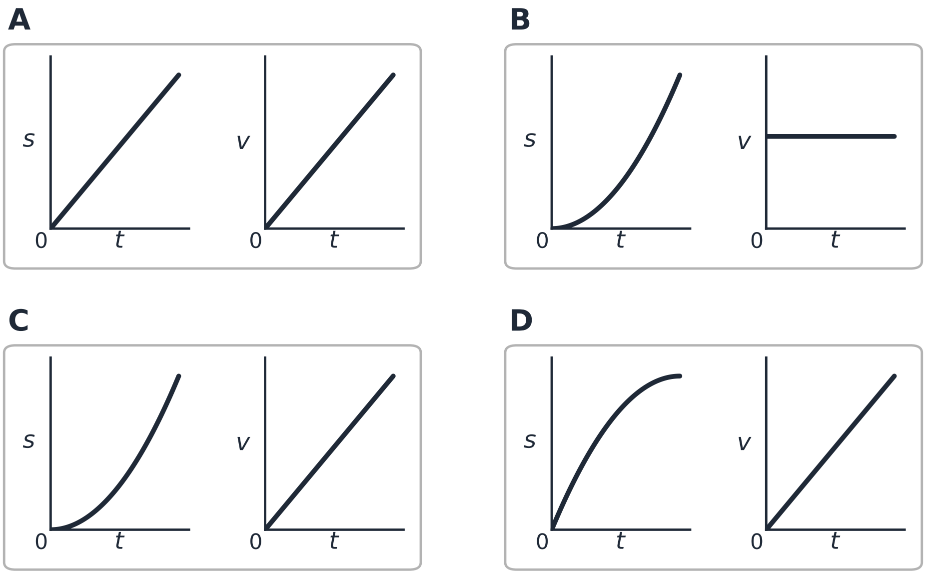


(a) By drawing a tangent, determine the acceleration of the cyclist at $t = 4.0$ s. [2]

(b) Estimate the distance travelled by the cyclist from $t = 0$ to $t = 8.0$ s by counting squares. [2]

3 Exam-style questions

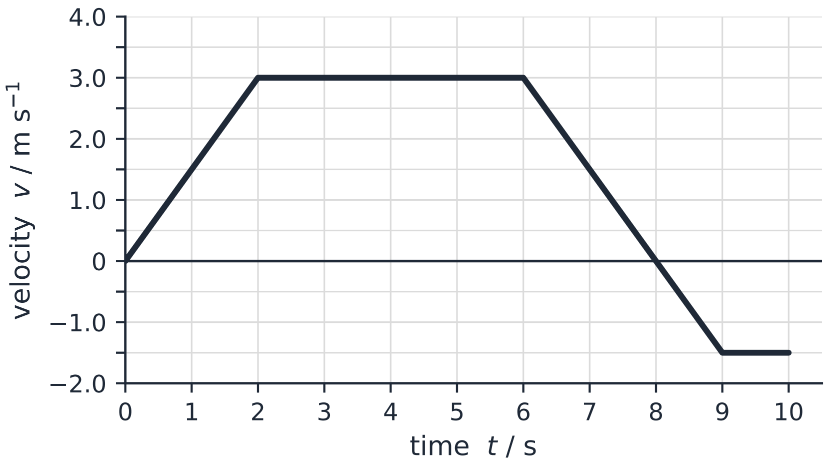
3.1 An object starts from rest and moves in a straight line. Which pair of graphs could show how its displacement s and its velocity v change with time t? [1]



3.2 The velocity–time graph of an object is a straight line from 8.0 m s^{-1} at $t = 0$ to -4.0 m s^{-1} at $t = 6.0$ s. What is the displacement of the object after 6.0 s? [1]

- **A** 12 m
- **B** 20 m
- **C** 24 m
- **D** 36 m

3.3 A remote-controlled toy car moves along a straight track. A **motion sensor** 运动传感器 records its velocity v. The graph shows how v varies with time t.



(a) **Define** acceleration. [1]

(b) **Compare** the acceleration of the car at $t = 1.0$ s with its acceleration at $t = 7.0$ s, in terms of magnitude and direction. [2]

(c) The car is at rest for an instant at $t = 8.0$ s. Determine the acceleration of the car at this instant. [2]

(d) Determine the greatest displacement of the car from its starting point, and the time at which it happens. [3]

(e) Determine the displacement of the car from its starting point at $t = 10.0$ s. [2]

(f) **Sketch** the variation with time of the displacement of the car from $t = 0$ to $t = 10.0$ s. Label both axes and mark the greatest displacement. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

(a) the gradient is the **acceleration**

(b) the area is the **displacement**

2.2

(a) gradient of the first line

$$a = \frac{2.0\ \text{m s}^{-1} - 0}{2.0\ \text{s}} = \mathbf{1.0}\ \text{m s}^{-2}$$

(b) $v = 0$ at $t = \mathbf{0}$, $\mathbf{8.0}$ s and $\mathbf{14}$ s

(c) going up, the area above the axis is a trapezium (or two triangles and a rectangle)

$$\tfrac{1}{2}(8.0\ \text{s} + 4.0\ \text{s})(2.0\ \text{m s}^{-1}) = 12\ \text{m}$$

- coming down, the area below the axis

$$\tfrac{1}{2}(6.0\ \text{s} + 2.0\ \text{s})(2.0\ \text{m s}^{-1}) = 8.0\ \text{m}$$

- total distance $= 12\ \text{m} + 8.0\ \text{m} = \mathbf{20}\ \text{m}$

(d) displacement $= 12\ \text{m} - 8.0\ \text{m} = \mathbf{4.0}$ m above the starting point

2.3

(a) change in velocity = area under the acceleration–time graph

$$\Delta v = 2.0\ \text{m s}^{-2} \times 4.0\ \text{s} = \mathbf{8.0}\ \text{m s}^{-1}$$

(b) $5.0\ \text{m s}^{-1} + 8.0\ \text{m s}^{-1} = 13\ \text{m s}^{-1}$ at 4.0 s; the acceleration is then zero, so at 6.0 s the velocity is still **13** m s^{-1}

(c)

- a straight line from $(0,\ 5.0\ \text{m s}^{-1})$ to $(4.0\ \text{s},\ 13\ \text{m s}^{-1})$
- then a horizontal line at 13 m s^{-1} to 8.0 s, with the axes labelled v / m s^{-1} and t / s

2.4

(a) tangent drawn at $t = 4.0$ s, for example through $(0,\ 8.8\ \text{m s}^{-1})$ and $(8.0\ \text{s},\ 0)$

$$a = \frac{0 - 8.8\ \text{m s}^{-1}}{8.0\ \text{s} - 0} = \mathbf{-1.1}\ \text{m s}^{-2}$$

(accept -1.0 to -1.2)

(b) about 21 squares are at least half under the curve, and each is worth $1\ \text{s} \times 2\ \text{m s}^{-1} = 2\ \text{m}$

$$\text{distance} \approx 21 \times 2\ \text{m} = \mathbf{42}\ \text{m}$$

(accept 40 m to 44 m)

3.1 C

- the curve in **C** gets steeper, so the velocity increases from zero, which matches a velocity line rising from the origin
- **A**: a straight displacement line means a constant velocity, not an increasing one
- **B**: a curve that gets steeper cannot have a constant velocity
- **D**: a curve that gets flatter means the velocity is decreasing

3.2 A

- the gradient is -2.0 m s^{-2}, so $v = 0$ at $t = 4.0$ s
- area above the axis and area below it

$$\tfrac{1}{2}(4.0\text{ s})(8.0\text{ m s}^{-1}) = 16\text{ m}, \qquad \tfrac{1}{2}(2.0\text{ s})(-4.0\text{ m s}^{-1}) = -4.0\text{ m}$$

- displacement $= 16$ m $- 4.0$ m $=$ **12** m
- **B** is the distance; **C** treats the whole graph as one triangle; **D** ignores the minus sign

3.3

(a) acceleration is the **rate of change of velocity**

(b) the gradients are

$$\frac{3.0\text{ m s}^{-1}}{2.0\text{ s}} = 1.5\text{ m s}^{-2} \quad \text{and} \quad \frac{-1.5\text{ m s}^{-1} - 3.0\text{ m s}^{-1}}{9.0\text{ s} - 6.0\text{ s}} = -1.5\text{ m s}^{-2}$$

- magnitude: **the same**
- direction: **opposite**

(c) the acceleration is the gradient of the straight line from 6.0 s to 9.0 s, which includes 8.0 s

$$a = \frac{-1.5\text{ m s}^{-1} - 3.0\text{ m s}^{-1}}{9.0\text{ s} - 6.0\text{ s}} = \mathbf{-1.5}\text{ m s}^{\mathbf{-2}}$$

- it is not zero: the velocity is zero for an instant, but it is still changing

(d) the car moves forwards until $v = 0$, so the greatest displacement is at $t =$ **8.0** s

- displacement $=$ area under the line from 0 to 8.0 s

$$\tfrac{1}{2}(2.0\text{ s})(3.0\text{ m s}^{-1}) + (4.0\text{ s})(3.0\text{ m s}^{-1}) + \tfrac{1}{2}(2.0\text{ s})(3.0\text{ m s}^{-1}) = \mathbf{18}\text{ m}$$

(e) area below the axis from 8.0 s to 10.0 s

$$\tfrac{1}{2}(1.0\text{ s})(-1.5\text{ m s}^{-1}) + (1.0\text{ s})(-1.5\text{ m s}^{-1}) = -2.25\text{ m}$$

- displacement $= 18$ m $- 2.25$ m $=$ **16** m

(f)

- axes labelled s / m and t / s; a curve getting steeper to 3.0 m at 2.0 s, then a straight line to 15 m at 6.0 s
- a curve getting flatter to a maximum of 18 m at 8.0 s, flat at the top
- the displacement then decreasing, to about 16 m at 10.0 s

2.1.3 The equations of uniformly accelerated motion

Name: ______ Class: ______ Date: ______ **Total: 37 marks**

KEY WORDS uniform acceleration 匀加速 • velocity 速度 • horizontal 水平 • acceleration 加速度 • speed 速率

Objective

Build the skills to answer exam questions with the **equations of uniformly accelerated motion** 匀加速运动方程: deriving them, choosing the right one, using signs, and solving problems with two stages or two objects. This sheet covers syllabus 2.1, learning outcomes 6 and 7 (motion in a straight line).

You must be able to:

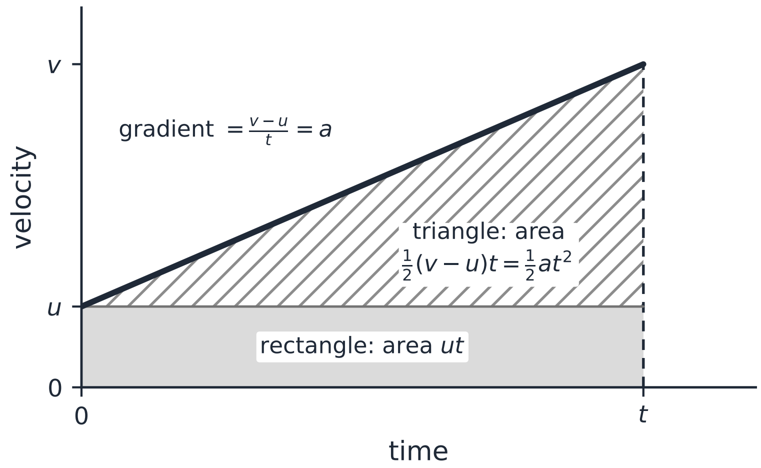
- **derive** 推导 the equations from the definition of acceleration and the area under a velocity–time graph
- state when the equations can be used
- choose the equation that links what you know to what you want
- choose a positive direction and give every vector a sign
- solve problems with two stages, or with two objects
- solve a **quadratic** 二次方程 with two answers, and decide which answers make sense

1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

■ Deriving the equations

An object has **uniform acceleration** 匀加速 a in a straight line. Its velocity changes from u to v in a time t, and its displacement is s.



- from the definition of acceleration (the gradient)

$$a = \frac{v-u}{t} \quad \Rightarrow \quad v = u + at$$

- displacement = area under the line, a **trapezium** 梯形

$$s = \tfrac{1}{2}(u+v)t$$

- put $v = u + at$ into the trapezium; this is the rectangle plus the triangle in the graph

$$s = \tfrac{1}{2}(u+u+at)t = ut + \tfrac{1}{2}at^2$$

- from the first equation $t = (v-u)/a$; put this into $s = \frac{1}{2}(u+v)t$

$$s = \frac{(v+u)(v-u)}{2a} \quad \Rightarrow \quad v^2 = u^2 + 2as$$

The equations work only when the acceleration is **constant** and the motion is in a **straight line**. $s = \frac{1}{2}at^2$ also needs the object to start from rest ($u = 0$).

■ Choosing an equation

Write down the five symbols s, u, v, a, t. Find the one you are not given and do not want: use the equation without it.

not given and not wanted	**use**
s	$v = u + at$
a	$s = \frac{1}{2}(u+v)t$
v	$s = ut + \frac{1}{2}at^2$
t	$v^2 = u^2 + 2as$

Example. A cyclist brakes uniformly from 9.0 m s^{-1} to rest in a distance of 12 m.

- $u = 9.0$ m s^{-1}, $v = 0$, $s = 12$ m; to find a, there is no t

$$a = \frac{v^2 - u^2}{2s} = \frac{0 - (9.0 \text{ m s}^{-1})^2}{2 \times 12 \text{ m}} = -3.4 \text{ m s}^{-2}$$

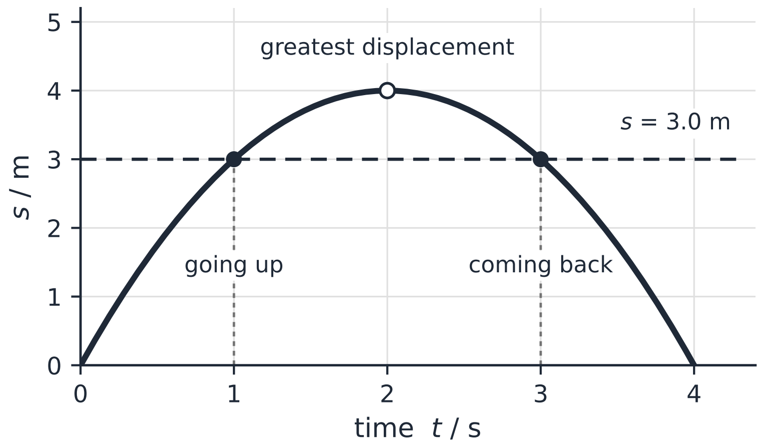
- to find t, there is no a

$$t = \frac{2s}{u+v} = \frac{2 \times 12\ \text{m}}{9.0\ \text{m s}^{-1}} = 2.7\ \text{s}$$

- the minus sign shows the acceleration is opposite to the velocity: a **deceleration** 减速度 of 3.4 m s^{-2}

■ Signs, and two answers

Choose one direction as positive and keep it for the whole problem. A ball rolls up a slope at 4.0 m s^{-1}. Its acceleration is 2.0 m s^{-2} down the slope, on the way up and on the way back down. Take **up the slope as positive**: $u = +4.0$ m s^{-1} and $a = -2.0$ m s^{-2}.



- when does it stop? Use $v = u + at$

$$0 = 4.0\ \text{m s}^{-1} + (-2.0\ \text{m s}^{-2})t \quad \Rightarrow \quad t = 2.0\ \text{s}$$

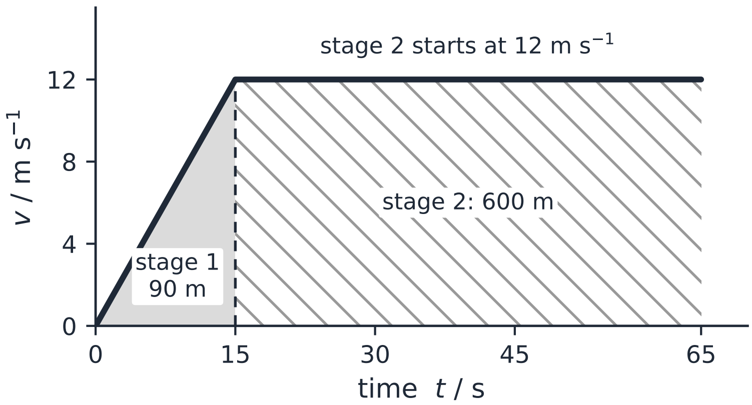
- when is it 3.0 m up the slope? Use $s = ut + \frac{1}{2}at^2$ (values in m and s)

$$3.0 = 4.0t - 1.0t^2 \quad \Rightarrow \quad t^2 - 4.0t + 3.0 = 0 \quad \Rightarrow \quad t = 1.0\ \text{s or } 3.0\ \text{s}$$

- both answers are real: at 1.0 s the ball is going up, and at 3.0 s it is coming back down

■ Two stages

A tram starts from rest with an acceleration of 0.80 m s^{-2} for 15 s. It then keeps a steady speed for 50 s.



- stage 1: the final velocity

$$v = u + at = 0 + (0.80\ \text{m s}^{-2})(15\ \text{s}) = 12\ \text{m s}^{-1}$$

- stage 1: the displacement

$$s_1 = \tfrac{1}{2}at^2 = \tfrac{1}{2}(0.80\ \text{m s}^{-2})(15\ \text{s})^2 = 90\ \text{m}$$

- the final velocity of stage 1 is the initial velocity of stage 2

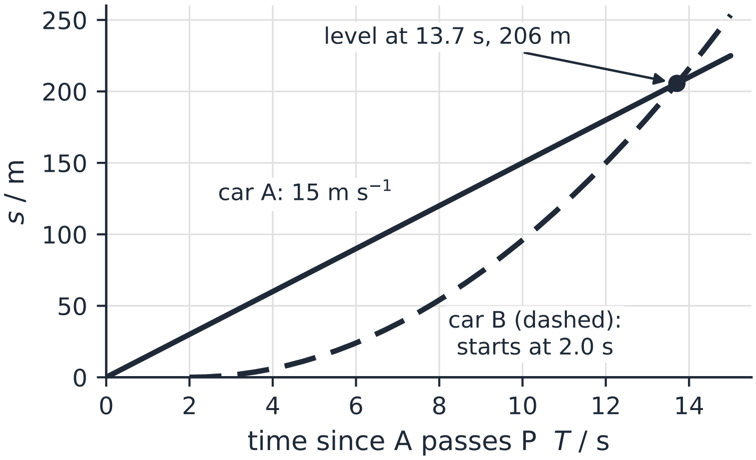
$$s_2 = (12\ \text{m s}^{-1})(50\ \text{s}) = 600\ \text{m}$$

- total displacement $= 90\ \text{m} + 600\ \text{m} = 690\ \text{m}$

A driver's **reaction time** 反应时间 is a stage too: the car keeps its speed, so the **thinking distance** 反应距离 is speed × reaction time. The **braking distance** 制动距离 comes from $v^2 = u^2 + 2as$, and the **stopping distance** 停车距离 is the sum of the two.

■ Two objects

Car A passes a point P at a constant 15 m s^{-1}. 2.0 s later, car B starts from rest at P with an acceleration of 3.0 m s^{-2}. Measure the time T from when A passes P.



- car A has moved for T and car B for $(T - 2.0)$, so (values in m and s)

$$s_\mathrm{A} = 15T, \qquad s_\mathrm{B} = \tfrac{1}{2}(3.0)(T - 2.0)^2 = 1.5(T - 2.0)^2$$

- they are level when the displacements are equal

$$15T = 1.5(T - 2.0)^2 \quad \Rightarrow \quad T^2 - 14T + 4.0 = 0 \quad \Rightarrow \quad T = 13.7 \text{ s or } 0.29 \text{ s}$$

- 0.29 s is before B starts, so it is not a meeting: B catches A at $T = 13.7$ s

$$s = (15 \text{ m s}^{-1})(13.7 \text{ s}) = 206 \text{ m}$$

2 Practice

Now use the methods above.

2.1 Show that $s = ut + \frac{1}{2}at^2$ follows from $v = u + at$ and $s = \frac{1}{2}(u + v)t$. [2]

2.2 The equations of uniformly accelerated motion can only be used under certain conditions.

(a) State two conditions that must both be true. [2]

(b) State the extra condition needed to use $s = \frac{1}{2}at^2$. [1]

2.3 Choose the right equation each time.

(a) $u = 2.0$ m s^{-1}, $a = 1.5$ m s^{-2}, $t = 4.0$ s. Find v. [1]

(b) $u = 0$, $v = 12$ m s^{-1}, $s = 18$ m. Find a. [1]

(c) $u = 5.0$ m s^{-1}, $v = 15$ m s^{-1}, $t = 4.0$ s. Find s. [1]

(d) $u = 3.0$ m s^{-1}, $a = 2.0$ m s^{-2}, $s = 10$ m. Find v. [1]

2.4 A car is travelling at 25 m s^{-1} when the driver sees a hazard. The driver's reaction time is 0.60 s. The car then brakes with a constant deceleration of 6.25 m s^{-2}.

(a) Calculate the thinking distance. [1]

(b) Calculate the braking distance. [2]

(c) State the stopping distance. [1]

2.5 A puck slides up a smooth icy ramp at 6.0 m s^{-1}. Its acceleration is 1.5 m s^{-2} down the ramp, on the way up and on the way back down. Take up the ramp as positive.

(a) Calculate the time taken for the puck to stop. [1]

(b) Calculate the greatest distance the puck travels up the ramp. [2]

(c) Calculate the two times at which the puck is 10 m up the ramp. *(Hint: you will need to solve a quadratic.)* [3]

3 Exam-style questions

3.1 A car moving at 5.0 m s^{-1} accelerates uniformly. In the next 4.0 s it travels 44 m. What is its acceleration? [1]

- **A** 1.5 m s^{-2}
- **B** 3.0 m s^{-2}
- **C** 5.5 m s^{-2}
- **D** 6.0 m s^{-2}

3.2 A lift starts from rest and accelerates uniformly at 1.2 m s^{-2} for 2.5 s. It then moves at a constant velocity for 6.0 s, and finally slows uniformly to rest in 2.0 s. How far does the lift travel? [1]

- **A** 18 m
- **B** 25 m
- **C** 32 m
- **D** 33 m

3.3 A delivery van travels along a straight road at a constant velocity of 16 m s^{-1}. It passes a police car that is at rest. 3.0 s later, the police car sets off after the van. It accelerates uniformly at 2.5 m s^{-2} until it reaches 30 m s^{-1}, and then keeps this velocity.

(a) Calculate the time the police car takes to reach 30 m s^{-1}. [1]

(b) Calculate the distance the police car travels while it is accelerating. [2]

(c) **Show that** the police car has not caught the van when it reaches 30 m s^{-1}. [2]

(d) Determine the time, measured from when the van passes the police car, at which the police car catches the van. [3]

(e) On one set of axes, **sketch** the velocity–time graphs of the van and of the police car up to the moment the police car catches the van. Label each line. [2]

3.4 The equation $v^2 = u^2 + 2as$ links velocity and displacement.

(a) Starting from $v = u + at$ and $s = \frac{1}{2}(u + v)t$, **derive** $v^2 = u^2 + 2as$. [3]

(b) A skateboarder accelerates uniformly from rest to 9.0 m s^{-1} over a distance of 18 m. Calculate her acceleration. [2]

(c) Explain why this equation could not be used if her acceleration decreased as she sped up. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- substitute $v = u + at$ into $s = \frac{1}{2}(u + v)t$

$$s = \tfrac{1}{2}(u + u + at)t = \tfrac{1}{2}(2u + at)t$$

- multiply out the bracket: $s = ut + \frac{1}{2}at^2$

2.2

(a) the acceleration is constant; the motion is in a straight line

(b) the object starts from rest, so $u = 0$

2.3

(a) no s, so $v = u + at = 2.0\ \text{m s}^{-1} + (1.5\ \text{m s}^{-2})(4.0\ \text{s}) = \mathbf{8.0}\ \text{m s}^{-1}$

(b) no t, so use $v^2 = u^2 + 2as$

$$a = \frac{v^2 - u^2}{2s} = \frac{(12\ \text{m s}^{-1})^2}{2 \times 18\ \text{m}} = \mathbf{4.0}\ \text{m s}^{-2}$$

(c) no a, so use $s = \frac{1}{2}(u + v)t$

$$s = \tfrac{1}{2}(5.0\ \text{m s}^{-1} + 15\ \text{m s}^{-1})(4.0\ \text{s}) = \mathbf{40}\ \text{m}$$

(d) no t, so use $v^2 = u^2 + 2as$

$$v^2 = (3.0\ \text{m s}^{-1})^2 + 2(2.0\ \text{m s}^{-2})(10\ \text{m}) = 49\ \text{m}^2\,\text{s}^{-2} \Rightarrow v = \mathbf{7.0}\ \text{m s}^{-1}$$

2.4

(a) the speed is constant during the reaction time: $(25\ \text{m s}^{-1})(0.60\ \text{s}) = \mathbf{15}\ \text{m}$

(b) $u = 25\ \text{m s}^{-1}$, $v = 0$, $a = -6.25\ \text{m s}^{-2}$; use $v^2 = u^2 + 2as$

$$s = \frac{0 - (25\ \text{m s}^{-1})^2}{2(-6.25\ \text{m s}^{-2})} = \mathbf{50}\ \text{m}$$

(c) $15\ \text{m} + 50\ \text{m} = \mathbf{65}\ \text{m}$

2.5

(a) at the top $v = 0$

$$t = \frac{v - u}{a} = \frac{0 - 6.0\ \text{m s}^{-1}}{-1.5\ \text{m s}^{-2}} = \mathbf{4.0}\ \text{s}$$

(b) at the top $v = 0$; use $v^2 = u^2 + 2as$

$$s = \frac{0 - (6.0\ \text{m s}^{-1})^2}{2(-1.5\ \text{m s}^{-2})} = \mathbf{12}\ \text{m}$$

(c) $s = ut + \frac{1}{2}at^2$ with $s = 10$, $u = 6.0$, $a = -1.5$ (values in m and s)

$$10 = 6.0t - 0.75t^2 \quad \Rightarrow \quad 0.75t^2 - 6.0t + 10 = 0$$

- use the quadratic formula

$$t = \frac{6.0 \pm \sqrt{36 - 30}}{1.5}$$

- $t = \mathbf{2.4}$ s (going up) and $t = \mathbf{5.6}$ s (coming back down)

3.1 B

- $u = 5.0$ m s^{-1}, $t = 4.0$ s, $s = 44$ m; there is no v, so use $s = ut + \frac{1}{2}at^2$

$$44 \text{ m} = (5.0 \text{ m s}^{-1})(4.0 \text{ s}) + \tfrac{1}{2}a(4.0 \text{ s})^2 \Rightarrow a = \frac{24 \text{ m}}{8.0 \text{ s}^2} = \mathbf{3.0} \text{ m s}^{\mathbf{-2}}$$

- **A** leaves out the $\frac{1}{2}$; **C** leaves out ut; **D** subtracts u from the average speed but does not divide by the time

3.2 B

- top speed: $(1.2 \text{ m s}^{-2})(2.5 \text{ s}) = 3.0 \text{ m s}^{-1}$
- distance = area under the velocity–time graph

$$\tfrac{1}{2}(2.5 \text{ s})(3.0 \text{ m s}^{-1}) + (6.0 \text{ s})(3.0 \text{ m s}^{-1}) + \tfrac{1}{2}(2.0 \text{ s})(3.0 \text{ m s}^{-1}) = \mathbf{25} \text{ m}$$

- **A** is the steady stage only; **C** uses the top speed while the lift speeds up and slows down; **D** uses the top speed for all 11 s

3.3

(a)

$$t = \frac{v - u}{a} = \frac{30 \text{ m s}^{-1} - 0}{2.5 \text{ m s}^{-2}} = \mathbf{12} \text{ s}$$

(b) no a needed once t is known

$$s = \tfrac{1}{2}(u + v)t = \tfrac{1}{2}(0 + 30 \text{ m s}^{-1})(12 \text{ s}) = \mathbf{180} \text{ m}$$

(c) the van has been moving for $3.0 \text{ s} + 12 \text{ s} = 15$ s

$$s_{\text{van}} = (16 \text{ m s}^{-1})(15 \text{ s}) = 240 \text{ m}$$

- 240 m is more than 180 m, so the van is still ahead

(d) the gap at 15 s is $240 \text{ m} - 180 \text{ m} = 60$ m

- the police car closes the gap at $30 \text{ m s}^{-1} - 16 \text{ m s}^{-1} = 14 \text{ m s}^{-1}$

$$\frac{60 \text{ m}}{14 \text{ m s}^{-1}} = 4.3 \text{ s} \quad \Rightarrow \quad t = 15 \text{ s} + 4.3 \text{ s} = \mathbf{19} \text{ s}$$

(e)

- van: a horizontal line at 16 m s^{-1} from $t = 0$

- police car: zero until 3.0 s, a straight line up to 30 m s^{-1} at 15 s, then horizontal to about 19 s

3.4

(a)

- from $v = u + at$, $t = (v - u)/a$
- substitute into $s = \frac{1}{2}(u + v)t$

$$s = \frac{(u + v)(v - u)}{2a} = \frac{v^2 - u^2}{2a}$$

- so $2as = v^2 - u^2$, which gives $v^2 = u^2 + 2as$

(b) $u = 0$

$$a = \frac{v^2}{2s} = \frac{(9.0 \text{ m s}^{-1})^2}{2 \times 18 \text{ m}} = \mathbf{2.3} \text{ m s}^{\mathbf{-2}}$$

(c) the equation is true only when the acceleration is constant