

6.1.1 Deformation, the force-extension graph and measuring the Young modulus

Name: _____ Class: _____ Date: _____

Total: 31 marks

KEY WORDS young modulus 杨氏模量 • extension 伸长 • spring constant 弹簧常数
 • strain 应变 • stress 应力

Objective

Sheet 6.1 gave you Hooke's law and one Young-modulus substitution. This sheet is the rest of 6.1: the difference between **tensile** 拉伸 and **compressive** 压缩 **deformation** 形变, every named point on a force-extension graph and what each one means, springs joined in series and in parallel, why a **stress-strain** graph describes the **material** while force-extension describes the **specimen** 试样, and the Young modulus experiment done properly – including which measurement limits the accuracy. This sheet covers syllabus 6.1.

You must be able to:

- understand that deformation is caused by tensile or compressive forces
- use the terms **load** 负载, **extension** 伸长, compression and **limit of proportionality** 比例极限
- recall and use **Hooke's law** and the **spring constant** 弹簧常数 $k = F/x$
- define and use stress, strain and the **Young modulus** 杨氏模量
- describe an experiment to determine the Young modulus of a metal in the form of a wire

1 Worked examples

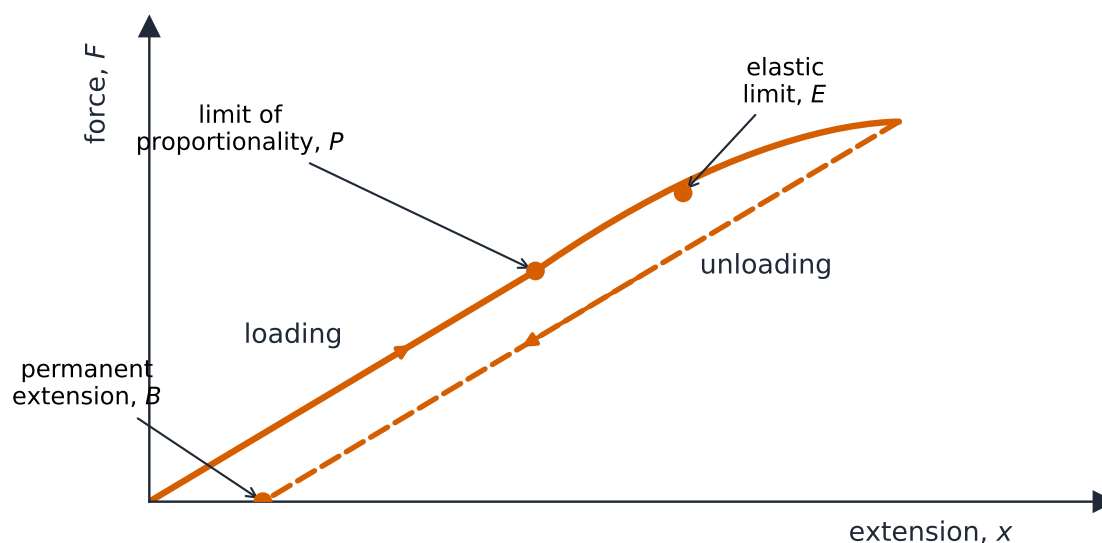
Study these first. Each one shows a **method** that a question later on this sheet needs.

■ Two kinds of deformation, and the words for them

- A **tensile** force **stretches** an object: the forces at the two ends pull **outwards**, and the object gets **longer**. The increase in length is the **extension**.
- A **compressive** force **squashes** it: the forces push **inwards**, and the object gets **shorter**. The decrease is the **compression**.
- Either way, the forces come in a **pair**. A single force acting on a free object accelerates it; it takes two opposing forces to deform it.
- The **load** is the force applied, usually the weight of the masses hung on the spec-

imen. Do not confuse it with the mass: $W = mg$.

■ Every named point on a force-extension graph

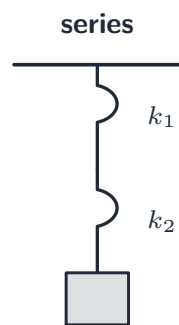


Hooke up to limit of proportionality · plastic beyond yield

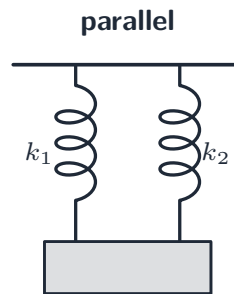
point	what it means
the straight region	extension is proportional to load; the specimen obeys Hooke's law
limit of proportionality	the point beyond which the graph is no longer straight ; $F \propto x$ fails here
elastic limit	the point beyond which the specimen does not return to its original length when the load is removed; it is just past the limit of proportionality
plastic region	the specimen is permanently deformed ; removing the load leaves an extension behind
breaking point	where it snaps

- The two limits are **different points** and the exam distinguishes them. Proportionality is about the **shape of the graph**; elasticity is about **what happens when you let go**.
- In the straight region the **gradient is the spring constant** k , so $k = \frac{F}{x}$. A **stiffer** specimen gives a **steeper** line.

■ Springs joined together



$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$



$$k = k_1 + k_2$$

Do not memorise formulae for these – work them out from the two questions that matter:

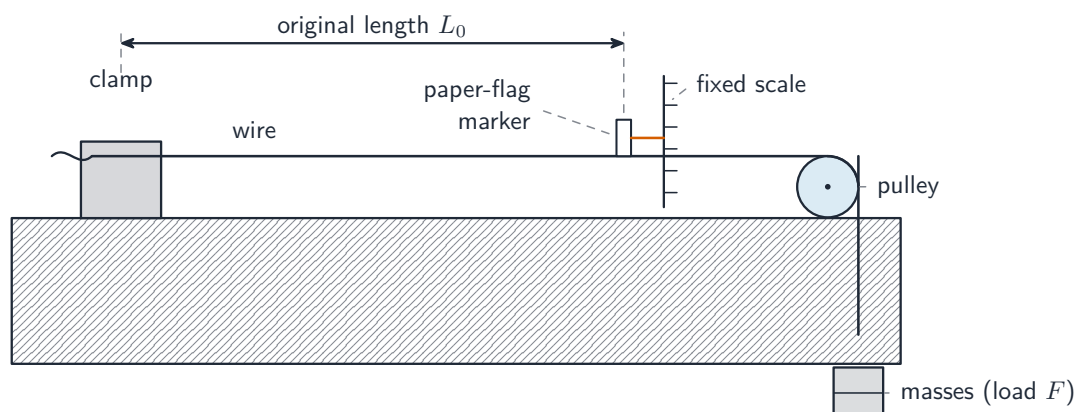
joined	what force does each spring carry?	what extension does the pair have?
in series (end to end)	the whole load F , each of them	each extends F/k , and the extensions add : total = $2F/k$, so the pair is less stiff
in parallel (side by side)	the load is shared , so $F/2$ each	each extends $F/2k$, and that is the extension of the pair: stiffer

■ Stress, strain, and why we bother

$$\text{stress } \sigma = \frac{F}{A} \quad (\text{Pa}) \quad \text{strain } \varepsilon = \frac{x}{L} \quad (\text{no unit}) \quad E = \frac{\sigma}{\varepsilon} \quad (\text{Pa})$$

- A **force-extension** graph describes **this specimen**: a thicker or shorter wire of the same metal gives a different line.
- Dividing the force by the **area** and the extension by the **original length** removes the specimen's dimensions, so a **stress-strain** graph describes the **material**. Every copper wire gives the same one.
- In the straight region the gradient of a stress-strain graph is the **Young modulus**, and it is a property of the material alone.
- Strain has **no unit**, so the Young modulus has the same unit as stress: **pascals**. Typical metals are around 10^{11} Pa; an answer of 10^9 usually means a unit was missed somewhere.

■ Measuring the Young modulus of a wire



$$E = \frac{\sigma}{\epsilon} = \frac{F/A}{x/L} = \frac{FL}{Ax}$$

so plotting **load against extension** gives a straight line of gradient $\frac{EA}{L}$, and

$$E = \text{gradient} \times \frac{L}{A}$$

What is measured, and with what:

quantity	instrument	note
original length L	metre rule	measured from the clamp to the marker; make it long so the extension is measurable
diameter d	micrometer , at several points and in different directions	$A = \pi d^2/4$ – the wire may not be perfectly round
extension x	marker against a millimetre scale , or a travelling microscope	take a reading before loading, as the zero
load F	known masses, $F = mg$	add and then remove them, checking the wire returns to its original length

- $\triangle$ **The diameter carries the largest percentage uncertainty**, because it is the smallest quantity measured – and it is **squared** in the area, so its percentage uncertainty is **doubled** in A and therefore in E . Measuring it carefully matters more than anything else in the experiment.
- Use a **long, thin** wire: a larger L and a smaller A both make the extension bigger and easier to measure.

2 Practice

Now use the methods above.

2.1 State what is meant by a **tensile** force and by a **compressive** force. [2]

2.2 Explain what is meant by the **limit of proportionality**, and state how it differs from the **elastic limit**. [3]

2.3 Two identical springs, each of spring constant 25 N m^{-1} , are joined end to end and a load of 3.0 N is hung from the pair. Calculate the total extension. [3]

2.4 State what the gradient of a **stress-strain** graph represents in its straight region. [1]

2.5 Explain why a stress-strain graph describes the **material** while a force-extension graph describes only that **specimen**. [2]

2.6 A wire of diameter 0.40 mm carries a load of 25 N . Calculate the stress in the wire. [3]

3 Exam-style questions

3.1 A wire obeys Hooke's law. Which quantity is given by the gradient of a graph of stress against strain? [1]

A	B
C	D

- A the spring constant of the wire
- B the Young modulus of the material
- C the stiffness of that particular wire
- D the breaking stress of the material

3.2 A spring obeys Hooke's law up to a load of 8.0 N, at which its extension is 0.16 m.

(a) Calculate the spring constant of the spring. [2]

(b) Two such springs are connected **in parallel** and the same 8.0 N load is hung from the pair. Calculate the extension. [2]

(c) A single spring is loaded beyond its elastic limit and the load is then removed. Describe what has happened to the spring. [2]

3.3 A student determined the Young modulus of a metal using a wire of original length 2.50 m and diameter 0.38 mm. A graph of load against extension was a straight line of gradient $9.0 \times 10^3 \text{ N m}^{-1}$ up to the limit of proportionality.

(a) Calculate the cross-sectional area of the wire. [2]

(b) Show that the Young modulus is given by $E = \text{gradient} \times \frac{L}{A}$. [2]

(c) Calculate the Young modulus of the metal. [2]

(d) State which measured quantity contributes the largest percentage uncertainty to the value of E , and explain why. [2]

(e) Suggest **two** changes that would improve the accuracy of the experiment. [2]