

4.1.1 Centre of gravity, moments at an angle, and couples

Name: _____ Class: _____ Date: _____

Total: 29 marks

KEY WORDS force 力 • moment 力矩 • weight 重力 • couple 力偶 • centre of gravity 重心

Objective

Sheet 4.1 gave you the definitions and one perpendicular-force calculation. This sheet is the part the exam actually sets: finding a **centre of gravity** 重心 and knowing why it matters, taking a **moment** 力矩 when the force is **not** perpendicular – by perpendicular distance or by resolving, both of which earn full marks – and the **couple** 力偶 and its **torque** 转矩 applied to a real object. This sheet covers syllabus 4.1.

You must be able to:

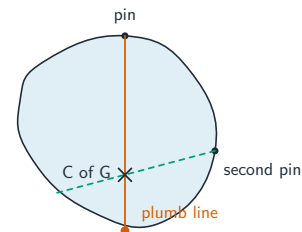
- understand that the **weight** of an object may be taken as acting at a single point, its centre of gravity
- define and apply the **moment of a force**
- understand that a couple is a pair of forces that produces **rotation only**
- define and apply the **torque of a couple**

1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

■ The centre of gravity, and how to find it

The **centre of gravity** is the single point at which the **whole weight** of an object may be taken to act. For a **uniform** object it is at the geometric centre – the middle of a uniform rod, the centre of a uniform disc.



hanging freely, the sheet settles with its centre of gravity vertically below the pin

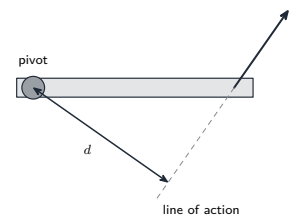
hang it from a second point: the two plumb lines cross at the centre of gravity

To find it for an irregular lamina:

- Hang the lamina from a pin through a hole near its edge, so it can swing freely.
 - Hang a **plumb line** from the same pin and draw the vertical line on the lamina.
 - Repeat from a **second** hole well away from the first.
 - The centre of gravity is where the two lines **cross**. A third hole checks the result.
- It works because the object hangs in equilibrium only when its centre of gravity is **directly below the point of suspension** – otherwise the weight has a moment about the pin and it swings.

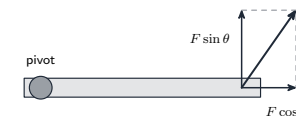
■ A moment when the force is not perpendicular

(a) perpendicular distance



$$d = L \sin \theta, \text{ so the moment is } F \times L \sin \theta$$

(b) resolve the force



only the perpendicular component turns it: $F \sin \theta \times L$

The two routes give the same answer, and either earns full marks. The component *along* the bar, $F \cos \theta$, has a line of action passing **through the pivot**, so its perpendicular distance is zero and it produces no turning effect at all.

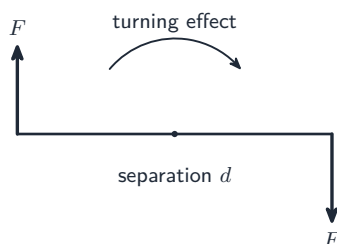
The **moment of a force** about a point is the **force** multiplied by the **perpendicular distance** from the point to the **line of action** of the force.

Every word of that definition is a marking point, and the phrase that is usually dropped is *line of action*. Two routes, both fully acceptable:

route	what to do
(a) perpendicular distance	extend the line of action of F , drop a perpendicular from the pivot onto it, and use $d = L \sin \theta$: moment $= FL \sin \theta$
(b) resolve	split F into $F \sin \theta$ perpendicular to the bar and $F \cos \theta$ along it; only the perpendicular part turns the bar: moment $= F \sin \theta \times L$

- The component **along** the bar has a line of action **through the pivot**, so its perpendicular distance is zero and it contributes **nothing**. That is the whole reason $\cos \theta$ never appears in the answer.
- A moment is measured in **newton metres, N m**. It is not a joule, although the units look the same: a moment is a force times a perpendicular distance, not a force times a distance moved.

■ A couple, and its torque



A **couple** is a pair of forces that satisfies **three** conditions, all of which are needed:

1. the forces are **equal in magnitude**;
2. they are **opposite in direction**;
3. their **lines of action are parallel but not the same line** – they are separated.

Because the forces are equal and opposite, the **resultant force is zero**: the object does **not** accelerate. It rotates, and only rotates. That is what makes a couple different from any other pair of forces.

The **torque of a couple** is one **force** multiplied by the **perpendicular distance between the lines of action** of the two forces.

$$\tau = F \times d$$

- Use **one** of the forces, not both, and not their sum. Doubling because “there are

two forces” is the standard error.

- d is the separation of the two lines, which for a wheel turned at opposite ends of a diameter is the **diameter**, not the radius.

2 Practice

Now use the methods above.

2.1 Describe how you would find the centre of gravity of an irregularly shaped flat sheet of card. [3]

2.2 A force of 40 N acts at the end of a spanner of length 0.25 m, at 60° to the spanner. Calculate its moment about the nut, using the **perpendicular distance**. [3]

2.3 Obtain the same answer by **resolving** the force, and state what this shows. [2]

2.4 Explain why the component of a force **along** a bar has no moment about a pivot at the end of that bar. [2]

2.5 Define the **torque of a couple**, and state its unit. [2]

3 Exam-style questions

3.1 A force F acts at a point a distance L from a pivot, at an angle θ to the line joining the pivot to that point. The moment of the force about the pivot is [1]

- | | |
|---|---|
| A | B |
| C | D |
- A** FL
B $FL \sin \theta$
C $FL \cos \theta$
D $FL \tan \theta$

3.2 A steering wheel of diameter 36 cm is turned by two forces of 15 N, applied tangentially at opposite ends of a diameter and in opposite directions.

(a) Explain why these two forces form a couple. [2]

(b) Calculate the torque of the couple. [2]

(c) State the resultant force on the wheel, and explain what this means for its motion. [2]

3.3 A uniform plank of weight 220 N and length 3.0 m rests on a single pivot placed 1.2 m from its left-hand end. A child of weight 300 N stands on the left-hand end.

(a) State where the weight of the plank acts, and explain why. [2]

(b) Calculate the moment of the child's weight about the pivot. [2]

(c) Calculate the moment of the plank's weight about the pivot. [2]

(d) State whether the plank turns, and in which direction, explaining your answer. [2]

(e) A second child stands on the right-hand end. Calculate the smallest weight this child can have if the plank is to stay horizontal. [2]