3.3.1 Momentum in two dimensions, and testing a glancing collision for elasticity

Name: ______________ Class: __________ Date: ____________ **Total: 32 marks**

KEY WORDS momentum 动量 • elastic 弹性碰撞 • conservation of momentum 动量守恒 • collision 碰撞 • kinetic energy 动能

Objective

Build the skills to answer the **momentum 动量** questions that sheet 3.3 does not reach. That sheet conserves momentum along a line: collisions, explosions, elastic against inelastic, all in **one** dimension. The syllabus asks for interactions “in both one and **two** dimensions”, and the words component, resolve, angle and perpendicular appear nowhere in it. This sheet is the two-dimensional half.

You must be able to:

- apply the principle of conservation of momentum to interactions in **two dimensions**
- resolve a momentum into **perpendicular 垂直 components 分量**, and apply conservation along each axis on its own
- combine two perpendicular momenta into a magnitude and a direction
- decide whether a two-dimensional collision is **elastic 弹性碰撞**, using the **full** speed of each body

1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

One rule, applied twice

Momentum is a **vector**. So conservation of momentum is not one equation but **two**: the total momentum is conserved along each of two perpendicular directions, **independently**.

The method never changes:

1. **Choose the axes.** Take x along the motion of the incoming object and y across it. That makes the initial y-momentum zero, which is one equation almost solved before you start.
2. **Resolve every velocity.** A momentum p at angle θ to the x-axis has components $p\cos\theta$ along x and $p\sin\theta$ along y.

3. **Write the two equations**, "total before = total after", once for x and once for y.
4. **Recombine** if the question wants a speed and a direction: $p = \sqrt{p_x^2 + p_y^2}$ and $\theta = \tan^{-1}(p_y/p_x)$.

Worked example. *Ball A has momentum* 4.0 *N s due east and ball B has momentum* 3.0 *N s due north. They collide and stick together. Find the momentum of the combined object.*

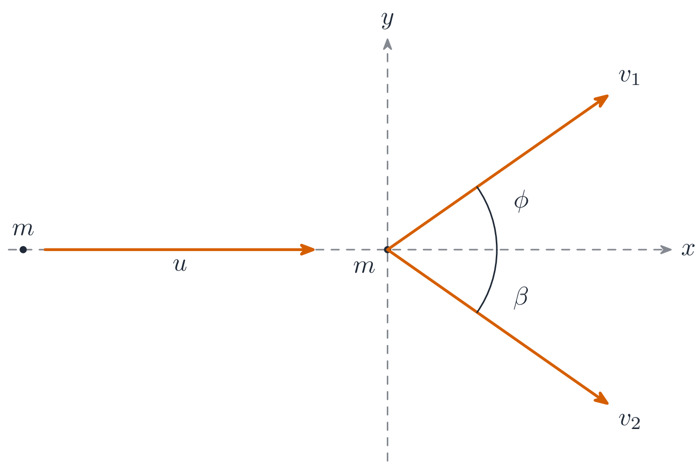
Nothing outside acts, so each axis keeps what it had: 4.0 N s east and 3.0 N s north.

$$p = \sqrt{4.0^2 + 3.0^2} = 5.0 \text{ N s}, \qquad \theta = \tan^{-1}\left(\frac{3.0}{4.0}\right) = 37°$$

so 5.0 N s at 37° north of east.

- A multiple-choice question showing two momentum arrows is asking exactly this: add them tip to tail.

A glancing collision



Worked example. *A* 0.20 *kg ball moving east at* 5.0 *m s*$^{-1}$ *strikes a stationary* 0.30 *kg ball. Afterwards the* 0.20 *kg ball moves at* 3.0 *m s*$^{-1}$ *at* 40° *north of east. Find the velocity of the* 0.30 *kg ball.*

Along x (east positive):

$$(0.20)(5.0) = (0.20)(3.0)\cos 40° + (0.30)v_{2x}$$

$$1.00 = 0.460 + 0.30\,v_{2x} \quad \Rightarrow \quad v_{2x} = 1.80 \text{ m s}^{-1}$$

Along y (north positive). Before the collision there is **no** y-momentum at all:

$$0 = (0.20)(3.0)\sin 40° + (0.30)v_{2y}$$

$$0 = 0.386 + 0.30\,v_{2y} \quad \Rightarrow \quad v_{2y} = -1.29 \text{ m s}^{-1}$$

The minus sign means south, which it must: one ball went north, so the other has to go south.

Recombine:

$$v_2 = \sqrt{1.80^2 + 1.29^2} = 2.2 \text{ m s}^{-1}, \qquad \theta = \tan^{-1}\left(\frac{1.29}{1.80}\right) = 36°$$

so 2.2 m s^{-1} at 36° **south** of east.

- Write the axis you are working on beside each equation. Almost every lost mark here is a sin where a cos belongs, and labelling the axis catches it.

■ Testing whether it was elastic

Momentum is conserved in **every** collision; kinetic energy is not. Test the energy separately, and use the **full speed** of each body.

For the collision above:

$$E_{k,\text{before}} = \tfrac{1}{2}(0.20)(5.0)^2 = 2.5 \text{ J}$$

$$E_{k,\text{after}} = \tfrac{1}{2}(0.20)(3.0)^2 + \tfrac{1}{2}(0.30)(2.2)^2 = 0.90 + 0.73 = 1.6 \text{ J}$$

About 0.87 J, or 35%, has gone, so the collision is **inelastic**.

- **Kinetic energy uses the whole speed, never a component.** $\frac{1}{2}mv_x^2$ is not a quantity in this subject. Momentum resolves; energy does not, because it is a scalar.

■ Equal masses, elastic, target at rest

A standard result worth recognising: when one object hits an equal mass at rest **elastically**, the two move off at 90° **to each other**.

Momentum gives $\vec{u} = \vec{v_1} + \vec{v_2}$, and kinetic energy gives $u^2 = v_1^2 + v_2^2$. Squaring the first,

$$u^2 = v_1^2 + 2\,\vec{v_1} \cdot \vec{v_2} + v_2^2$$

Comparing the two, $\vec{v_1} \cdot \vec{v_2} = 0$, so the two velocities are perpendicular.

2 Practice

Now use the methods above.

2.1 A momentum of 6.0 kg m s^{-1} acts at 30° above the horizontal.

Calculate its horizontal and vertical components. [2]

2.2 Two pieces of clay collide and stick together. Just before the collision one has momentum 4.0 N s due east and the other 3.0 N s due north.

Calculate the magnitude and direction of the momentum of the combined lump. [3]

2.3 State the **principle of conservation of momentum**, and state why it may be applied to each of two perpendicular directions separately. [2]

2.4 After a collision a 0.50 kg ball has momentum components 1.2 N s east and 0.90 N s north.

Calculate its speed. [2]

2.5 A student calculates the kinetic energy of a ball using only the horizontal component of its velocity. State why this is wrong. [1]

2.6 State the **two** things that are true of an **elastic** collision: one about energy, and one about relative speeds. [2]

3 Exam-style questions

3.1 On a frictionless horizontal surface, a ball **A** of mass 0.20 kg moving east at 5.0 m s^{-1} strikes a stationary ball **B** of mass 0.30 kg.

After the collision **A** moves at 3.0 m s^{-1} at 40° north of east.

Calculate the magnitude and direction of the velocity of **B**. [5]

3.2 Determine whether the collision in 3.1 is elastic. Show your working. [3]

3.3 An object of mass 2.0 kg moving east at 6.0 m s^{-1} explodes into two fragments, each of mass 1.0 kg.

One fragment moves at 5.0 m s^{-1} at 53° north of east.

Calculate the magnitude and direction of the velocity of the other fragment. [5]

3.4 A moving snooker ball strikes a second, identical ball that is at rest. The collision is elastic.

Show that after the collision the two balls move off at 90° to each other. [4]

3.5 In a two-dimensional collision the total kinetic energy after the collision is less than before, yet the total momentum is unchanged.

Explain how both statements can be true. [3]