Solutions

Each answer shows the steps, then the **result**.

2.1

- horizontal: $6.0 \cos 30° = \mathbf{5.2}$ kg m $\mathrm{s^{-1}}$
- vertical: $6.0 \sin 30° = \mathbf{3.0}$ kg m $\mathrm{s^{-1}}$

2.2

- momentum is conserved along each axis, so the lump has 4.0 N s east and 3.0 N s north
- $p = \sqrt{4.0^2 + 3.0^2} = \mathbf{5.0}$ N s
- $\theta = \tan^{-1}(3.0/4.0) = \mathbf{37}°$ north of east

2.3

- the **total momentum** of a system of interacting bodies stays constant, provided **no resultant external force** acts on it
- momentum is a **vector**, so its components along two perpendicular directions are independent, and the condition holds along each of them separately

2.4

- $p = \sqrt{1.2^2 + 0.90^2} = 1.5$ N s
- $v = p/m = 1.5/0.50 = \mathbf{3.0}$ m $\mathrm{s^{-1}}$

2.5

- kinetic energy is a **scalar** and depends on the full speed, $\frac{1}{2}mv^2$; only vectors have components, so a component of the velocity gives an answer that is too small

2.6

- the total **kinetic energy** is conserved, as well as the momentum
- the **relative speed of approach** equals the **relative speed of separation**
- the relative-speed test is applied along the **line joining** the two bodies, so in two dimensions it is the components along that line that must satisfy it; testing the kinetic energy is the general method and is what question 3.2 uses

3.1

- along east: $(0.20)(5.0) = (0.20)(3.0)\cos 40° + (0.30)v_{Bx}$
- $1.00 = 0.460 + 0.30\,v_{Bx}$, so $v_{Bx} = 1.80$ m $\mathrm{s^{-1}}$
- along north, the total before is **zero**: $0 = (0.20)(3.0)\sin 40° + (0.30)v_{By}$
- $v_{By} = -1.29$ m $\mathrm{s^{-1}}$, that is 1.29 m $\mathrm{s^{-1}}$ south
- $v_B = \sqrt{1.80^2 + 1.29^2} = \mathbf{2.2}$ m $\mathrm{s^{-1}}$ at $\tan^{-1}(1.29/1.80) = \mathbf{36}°$ **south** of east
- a direction with no sense named ("36° to the east") does not earn the final mark: the question asks for a direction, and south is half of it

3.2

- before: $E_k = \frac{1}{2}(0.20)(5.0)^2 = 2.5$ J
- after: $\frac{1}{2}(0.20)(3.0)^2 + \frac{1}{2}(0.30)(2.2)^2 = 0.90 + 0.73 = 1.6$ J
- the kinetic energy has fallen by about 0.9 J, so the collision is **inelastic**
- both speeds are the **full** speeds, not components

3.3

- along east: $(2.0)(6.0) = (1.0)(5.0)\cos 53° + (1.0)v_x$
- $12.0 = 3.01 + v_x$, so $v_x = 9.0$ m s^{-1}
- along north: $0 = (1.0)(5.0)\sin 53° + (1.0)v_y$
- $v_y = -3.99$ m s^{-1}, that is 4.0 m s^{-1} south
- $v = \sqrt{9.0^2 + 4.0^2} = \mathbf{9.8}$ **m s^{-1}** at $\tan^{-1}(4.0/9.0) = \mathbf{24°}$ **south** of east
- an explosion conserves momentum exactly as a collision does, because no external resultant force acts during it

3.4

- momentum: with the masses equal, $m\vec{u} = m\vec{v_1} + m\vec{v_2}$, so $\vec{u} = \vec{v_1} + \vec{v_2}$
- elastic, so kinetic energy is conserved: $\frac{1}{2}mu^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$, giving $u^2 = v_1^2 + v_2^2$
- squaring the momentum equation: $u^2 = v_1^2 + 2\,\vec{v_1} \cdot \vec{v_2} + v_2^2$
- comparing the two gives $\vec{v_1} \cdot \vec{v_2} = 0$, so the velocities are **perpendicular**: the balls move off at 90° to each other
- the same result follows from a vector triangle whose sides satisfy $u^2 = v_1^2 + v_2^2$, which is Pythagoras, so the angle between v_1 and v_2 is a right angle

3.5

- momentum is conserved because there is **no resultant external force** on the system during the collision, and that is true whatever happens to the energy
- kinetic energy is not a conserved quantity: some of it is transferred to other forms – thermal energy, sound and the work done in deforming the objects
- the two are independent because momentum is a **vector** that only the external force can change, while kinetic energy is a **scalar** that internal forces can convert into other forms