Solutions

Each answer shows the steps, then the **result**.

2.1

- applying a p.d. across the crystal makes it **change shape**
- changing its shape makes it produce an **e.m.f.**

2.2

- $Z = \rho c$, the product of the medium's density and the speed of sound in it

2.3

- an alternating p.d. makes the crystal **vibrate**, emitting ultrasound
- a returning pulse makes it vibrate and produce an **e.m.f.**, which is detected

2.4

- $I = I_0 e^{-\mu x}$

3.1 B

- a large reflection when the two impedances are **very different**

3.2

- $I_\mathrm{R}/I_0 = (Z_2 - Z_1)^2/(Z_2 + Z_1)^2$

$$\frac{I_\mathrm{R}}{I_0} = \frac{(1.6 - 1.7)^2}{(1.6 + 1.7)^2} = \frac{(0.1)^2}{(3.3)^2} = \mathbf{9.2 \times 10^{-4}}$$

3.3

- without gel there is an **air gap**, whose acoustic impedance is very different from skin
- so almost all the ultrasound would be reflected; the gel removes the air gap so the ultrasound enters the body

3.4

- $I/I_0 = e^{-\mu x}$

$$\frac{I}{I_0} = e^{-(23\ \mathrm{m}^{-1})(0.050\ \mathrm{m})} = e^{-1.15} = \mathbf{0.32}$$

3.5

(a) the product of the **density** of the medium and the **speed of sound** in it, $Z = \rho c$

(b) the reflected wave's pressure variation makes the crystal **change shape**

- a piezoelectric crystal produces a **potential difference** across its faces when it is deformed, giving an electrical signal that varies with the received wave

(c) $I_\mathrm{R}/I_0 = (Z_2 - Z_1)^2/(Z_2 + Z_1)^2$

$$\frac{I_\mathrm{R}}{I_0} = \frac{(6.40 - 1.63)^2}{(6.40 + 1.63)^2} = \frac{22.8}{64.5} = \mathbf{0.35}$$

(d) with air

$$\frac{(1.63 \times 10^6 - 429)^2}{(1.63 \times 10^6 + 429)^2} \approx \mathbf{0.999}$$

- essentially **all** the ultrasound is reflected at a skin-air boundary; gel excludes the air layer and has an impedance close to that of skin, so the pulse actually enters the body

(e) the pulse travels **there and back**, a distance of 2×0.096 m $= 0.192$ m

$$c = \frac{0.192 \text{ m}}{1.2 \times 10^{-4} \text{ s}} = \mathbf{1.6} \times \mathbf{10^3} \text{ m s}^{-1}$$