Solutions

Each answer shows the steps, then the **result**.

2.1

- an **alternating p.d.** is applied across the crystal, which makes it change shape in step with the p.d. and so **vibrate**
- the vibration sends out an ultrasound wave; driving the crystal at its natural frequency gives the largest amplitude
- a returning wave **changes the crystal's shape**, and a crystal whose shape changes generates an **e.m.f.**, which is the detected signal

2.2

- the same crystal both transmits and receives, so it must be **silent** while it listens
- with a continuous wave the returning echo could not be separated from the outgoing wave, and no time of flight – and therefore no depth – could be measured

2.3

- the pulse travels to the boundary **and back**, so the distance travelled is $2d$

$$d = \frac{ct}{2} = \frac{1540\ \text{m s}^{-1} \times 52 \times 10^{-6}\ \text{s}}{2}$$

$$d = \mathbf{4.0 \times 10^{-2}}\ \text{m}\ (4.0\ \text{cm})$$

2.4

- the specific acoustic impedance of air is about 4×10^2 kg m^{-2} s^{-1}, thousands of times smaller than that of tissue
- the reflection coefficient $\dfrac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}$ is then almost exactly 1
- so almost all the intensity is **reflected at the air gap** and essentially none enters the body; the coupling gel excludes the air for this reason

3.1 B

- the reflection coefficient depends on $(Z_1 - Z_2)^2$, so it is large only when the two impedances are very **different**
- equal impedances give zero reflection; the absolute sizes do not matter on their own

3.2

(a)

- the **density** of the medium multiplied by the **speed of sound** in it, $Z = \rho c$

(b)

- substitute into the reflection coefficient

$$\frac{I_R}{I_0} = \frac{(6.40 - 1.63)^2}{(6.40 + 1.63)^2} = \frac{(4.77)^2}{(8.03)^2} = \mathbf{0.35}$$

- the powers of ten cancel, so they need not be written out

(c)

- substitute again, this time keeping the powers of ten

$$\frac{I_R}{I_0} = \frac{(1.63 \times 10^6 - 4.29 \times 10^2)^2}{(1.63 \times 10^6 + 4.29 \times 10^2)^2} = \mathbf{0.999}$$

- more than 99.9% of the intensity is reflected at a tissue-air boundary, so a scan is impossible through even a thin layer of air – hence the coupling gel

3.3

(a)

- use $I = I_0 e^{-\mu x}$ with x in metres

$$\frac{I}{I_0} = e^{-20 \text{ m}^{-1} \times 0.060 \text{ m}} = e^{-1.2}$$

$$\frac{I}{I_0} = \mathbf{0.30}$$

(b)

- the total path is **there and back**, 2×6.0 cm $= 12$ cm

$$\frac{I}{I_0} = e^{-20 \text{ m}^{-1} \times 0.120 \text{ m}} = e^{-2.4} = \mathbf{0.091}$$

(c)

- a higher frequency has a **shorter wavelength**, so smaller structures can be distinguished and the **resolution** is better
- but the attenuation coefficient **increases** with frequency, so the pulse is absorbed within a shorter distance and less of it returns from deep boundaries

(d)

- ultrasound is **non-ionising**, whereas X-rays are **ionising radiation**
- ionising radiation can damage or mutate the rapidly dividing cells of a developing fetus, so ultrasound is far safer for this use