

20.3.1 The Hall effect, the Hall probe and velocity selection

Name: _____ Class: _____ Date: _____

Total: 30 marks

KEY WORDS hall voltage 霍尔电压 • electric field 电场 • hall probe 霍尔探头
 • magnetic flux density 磁通量密度 • velocity 速度

Objective

Sheet 20.3 covered $F = BQv \sin \theta$ and the circular path. This sheet is the rest of 20.3, and it is the part with the **derivations**: where the **Hall voltage** 霍尔电压 comes from and how $V_H = \frac{BI}{ntq}$ is obtained, how a **Hall probe** 霍尔探头 measures a **magnetic flux density** 磁通量密度, and how crossed electric and magnetic fields **select one speed** out of a beam. This sheet covers syllabus 20.3.

You must be able to:

- determine the direction of the force on a charge moving in a magnetic field, and recall and use $F = BQv \sin \theta$
- understand the origin of the Hall voltage, and **derive** and use $V_H = \frac{BI}{ntq}$, where t is the thickness
- understand the use of a Hall probe to measure magnetic flux density
- describe the motion of a charged particle in a uniform magnetic field perpendicular to its motion
- explain how electric and magnetic fields are used in **velocity selection** 速度选择

1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

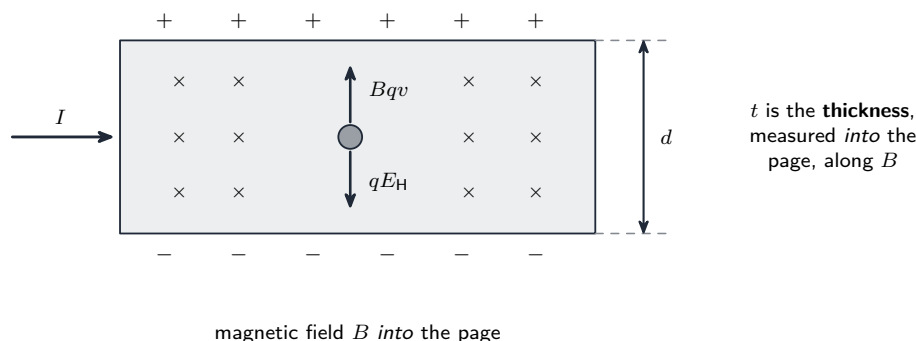
■ What you already have, in two lines

- $F = BQv \sin \theta$, with θ measured between the **velocity and the field**. The force is **zero** when the charge moves **along** the field ($\theta = 0$), and **maximum** when it moves at right angles.
- Perpendicular to a uniform field, the force is always at right angles to the velocity, so it is a **centripetal** force and the path is a **circle**:

$$BQv = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{BQ}$$

- The speed never changes, because a force at right angles to the motion does **no work**.

■ Where the Hall voltage comes from, and the derivation



The magnetic force pushes the carriers to one face until the electric field they build pushes back just as hard: $qE_H = Bqv$, so $E_H = Bv$ and $V_H = E_H d = Bvd$.
 Substituting the drift speed from $I = nAvq$ with $A = td$ gives $V_H = \frac{BI}{ntq}$.

The derivation is four steps, and each one is a marking point:

1. A carrier drifting with speed v through the field feels a magnetic force Bqv , which pushes it to **one face** of the slab.
2. Charge builds up on that face, leaving the opposite face oppositely charged, so a **transverse electric field** E_H appears **across** the slab.
3. Charge stops building up when the **electric force balances the magnetic force**:

$$qE_H = Bqv \implies E_H = Bv$$

and since the field is uniform across the width d , the **Hall voltage** is

$$V_H = E_H d = Bvd$$

4. Eliminate the drift speed using $I = nAvq$ with $A = td$:

$$v = \frac{I}{ntdq} \implies V_H = B \left(\frac{I}{ntdq} \right) d = \frac{BI}{ntq}$$

- n is the **number density** of charge carriers and t is the **thickness measured along the field**, not the width.
- Notice the d **cancels**: the Hall voltage does not depend on the width of the slab.

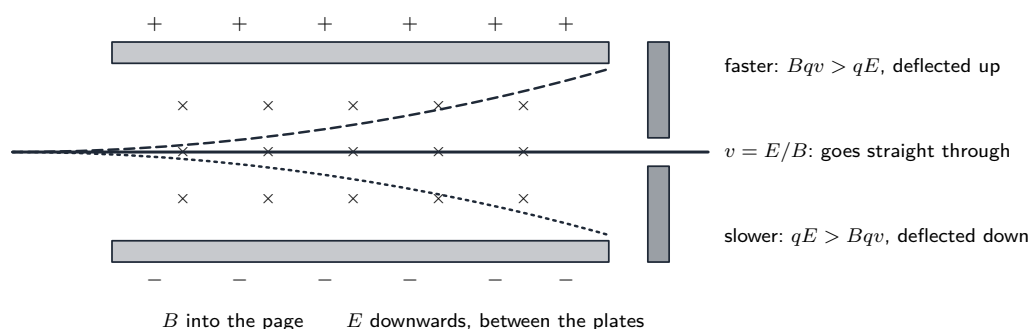
■ Why a Hall probe is made of semiconductor

$$V_H = \frac{BI}{ntq}$$

- With I , t and q fixed, $V_H \propto B$: measure the voltage and you have measured the **flux density**. That is the whole instrument.

- A **semiconductor** 半导体 has a number density n perhaps 10^7 times smaller than a metal's, and $V_H \propto \frac{1}{n}$, so the voltage is large enough to measure. A metal probe would give microvolts.
- Making the slab **thin** helps for the same reason.
- In use: the probe must be held with its **face perpendicular to the field**, because it responds to the component of B through it, and the current must be kept **constant**.

■ Velocity selection



The electric force qE and the magnetic force Bqv act in *opposite* directions, and they balance only when $qE = Bqv$, that is when $v = \frac{E}{B}$. Every other speed is deflected and stopped by the slit, whatever the particle's charge or mass.

Put an electric field and a magnetic field at right angles to each other and to the beam, so that the two forces on a charge are in **opposite** directions:

$$qE = Bqv \implies v = \frac{E}{B}$$

- Only a charge with **exactly** this speed passes through undeflected and reaches the slit.
- **Faster:** the magnetic force Bqv grows with v while the electric force does not, so $Bqv > qE$ and the charge is deflected towards the magnetic force.
- **Slower:** $qE > Bqv$, and it is deflected the other way.
- The charge q **cancels**, so the selected speed does not depend on the charge or the mass – the same selector works on any ion.

2 Practice

Now use the methods above.

2.1 Write the equation for the force on a charge moving at an angle to a magnetic field, and state the condition for the force to be zero. [2]

2.2 Explain why a charged particle entering a uniform magnetic field at right angles to it follows a circular path at constant speed. [3]

2.3 Explain how a Hall voltage arises across a slab carrying a current in a magnetic field. [3]

2.4 Explain why a Hall probe is made from a semiconductor rather than a metal. [2]

2.5 State the condition for a charged particle to pass undeflected through a velocity selector, and give the expression for the selected speed. [2]

3 Exam-style questions

3.1 A Hall probe is to give the largest possible Hall voltage for a given flux density. The slab should have [1]

A	B
C	D

- A** a large number density of charge carriers and a large thickness.
- B** a large number density of charge carriers and a small thickness.
- C** a small number density of charge carriers and a large thickness.
- D** a small number density of charge carriers and a small thickness.

3.2 A rectangular slab of width d and thickness t carries a current I . A uniform magnetic field of flux density B acts perpendicular to the face of the slab, and the charge carriers each have charge q and number density n .

(a) Derive the expression $V_H = \frac{BI}{ntq}$ for the Hall voltage across the slab. [4]

(b) A Hall probe uses a slab of thickness 0.10 mm in which $n = 2.0 \times 10^{22} \text{ m}^{-3}$ and $q = 1.6 \times 10^{-19} \text{ C}$. The current through it is 15 mA. Calculate the Hall voltage when the probe is placed in a field of flux density 0.25 T. [2]

3.3 Positive ions, each of charge $+1.6 \times 10^{-19}$ C, pass through a velocity selector in which the electric field strength is 3.0×10^4 V m⁻¹ and the magnetic flux density is 0.12 T. They then enter a second region of uniform magnetic field of flux density 0.35 T, perpendicular to their motion.

- (a) Calculate the speed of the ions that pass through the selector undeflected. [2]

- (b) Explain why ions travelling faster than this do not pass through. [2]

- (c) In the second region an ion follows a circular path of radius 0.086 m. Calculate its mass. [3]

- (d) State and explain the effect on the radius of the path of using an ion of the same charge and speed but **twice** the mass. [2]

- (e) Explain why the magnetic field in the second region does no work on the ion. [2]