Solutions

Each answer shows the steps, then the **result**.

2.1

- $F = BQv \sin\theta$, where θ is the angle between the velocity and the magnetic field
- the force is zero when $\theta = 0$, that is when the charge moves **along** the field

2.2

- the magnetic force is always at **right angles to the velocity**, so it changes the direction of motion but not its magnitude
- a force of constant magnitude always perpendicular to the velocity is a **centripetal** force, so the path is a circle
- since the force is perpendicular to the motion it does **no work**, so the kinetic energy and therefore the speed are constant

2.3

- the drifting charge carriers experience a magnetic force Bqv that pushes them towards **one face** of the slab
- charge builds up on that face, leaving the opposite face oppositely charged, so a **transverse electric field** appears across the slab
- the build-up stops when the **electric force on a carrier balances the magnetic force**, and the p.d. across the slab at that point is the Hall voltage

2.4

- $V_{\mathrm{H}} = \dfrac{BI}{ntq}$, so the Hall voltage is **inversely proportional to the number density** n
- a semiconductor has a far smaller n than a metal, so the voltage is large enough to be measured

2.5

- the **electric force and the magnetic force must be equal and opposite**, so $qE = Bqv$

$$v = \frac{E}{B}$$

3.1 D

- $V_{\mathrm{H}} = \dfrac{BI}{ntq}$, so a **small** n and a **small** t both make V_{H} larger
- **A** and **B** would give a tiny voltage, which is why a metal is useless as a Hall probe

3.2

(a)

- a carrier drifting at speed v feels a magnetic force Bqv pushing it to one face, so charge collects there and a transverse electric field E_{H} builds up

- equilibrium is reached when the electric force balances the magnetic force

$$qE_{\mathrm{H}} = Bqv \quad \Longrightarrow \quad E_{\mathrm{H}} = Bv, \qquad V_{\mathrm{H}} = E_{\mathrm{H}}d = Bvd$$

- eliminate v with $I = nAvq$ and $A = td$

$$v = \frac{I}{ntdq} \quad \Longrightarrow \quad V_{\mathrm{H}} = B\,\frac{I}{ntdq}\,d = \frac{BI}{ntq}$$

(b)

- substitute, with the thickness in metres

$$V_{\mathrm{H}} = \frac{0.25\ \mathrm{T} \times 0.015\ \mathrm{A}}{2.0 \times 10^{22}\ \mathrm{m}^{-3} \times 1.0 \times 10^{-4}\ \mathrm{m} \times 1.6 \times 10^{-19}\ \mathrm{C}} = \mathbf{1.2 \times 10^{-2}}\ \mathrm{V}$$

3.3

(a)

- the two forces balance, so $qE = Bqv$ and the charge cancels

$$v = \frac{E}{B} = \frac{3.0 \times 10^{4}\ \mathrm{V\ m^{-1}}}{0.12\ \mathrm{T}} = \mathbf{2.5 \times 10^{5}}\ \mathrm{m\ s^{-1}}$$

(b)

- the magnetic force Bqv **increases with speed**, while the electric force qE does not
- so for a faster ion $Bqv > qE$, the forces no longer balance, and the ion is deflected out of the beam and stopped by the slit

(c)

- the magnetic force provides the centripetal force

$$Bqv = \frac{mv^2}{r} \quad \Longrightarrow \quad m = \frac{rBq}{v}$$

$$m = \frac{0.086\ \mathrm{m} \times 0.35\ \mathrm{T} \times 1.6 \times 10^{-19}\ \mathrm{C}}{2.5 \times 10^{5}\ \mathrm{m\ s^{-1}}} = \mathbf{1.9 \times 10^{-26}}\ \mathrm{kg}$$

(d)

- the radius **doubles**
- $r = \dfrac{mv}{Bq}$, and with v, B and q unchanged the radius is proportional to the mass

(e)

- the magnetic force is always **perpendicular to the velocity**, so it has no component along the direction of motion
- work done is $Fs\cos\theta$ with $\theta = 90°$, so it is zero; the kinetic energy and the speed are unchanged and only the direction alters