Solutions

Each answer shows the steps, then the **result**.

2.1

- velocity is a **vector**, unit m s^{-1}
- acceleration is a **vector**, unit m s^{-2}

2.2

- known: $u = 24$ m s^{-1}, $v = 0$, $a = -3.0$ m s^{-2}; want s (no t)
- use $v^2 = u^2 + 2as$

$$0 = (24\ \text{m s}^{-1})^2 + 2(-3.0\ \text{m s}^{-2})s \Rightarrow s = \frac{576\ \text{m}^2\,\text{s}^{-2}}{6.0\ \text{m s}^{-2}} = \mathbf{96}\ \text{m}$$

2.3

- velocity is the **gradient** of a displacement–time graph

$$v = \frac{42\ \text{m} - 12\ \text{m}}{8.0\ \text{s} - 2.0\ \text{s}} = \frac{30\ \text{m}}{6.0\ \text{s}} = \mathbf{5.0}\ \text{m s}^{-1}$$

- the graph is a **straight** line, so v never changes; acceleration is the rate of change of velocity, so $a = 0$

2.4

- hold a steel ball with an electromagnet a measured height h above a trapdoor
- switching the magnet off starts an electronic timer and releases the ball; the ball opens the trapdoor and stops the timer, giving t
- use $h = \frac{1}{2}gt^2$, or plot h against t^2 and take $g = 2\times$ gradient
- reduce uncertainty: repeat each timing, and use several heights so a graph averages the readings

2.5

(a) acceleration = gradient of the v–t graph

$$a = \frac{8.0\ \text{m s}^{-1} - 0}{4.0\ \text{s}} = \mathbf{2.0}\ \text{m s}^{-2}$$

(b) displacement = area under the line; split into triangle + rectangle + triangle

- 0–4 s: $\frac{1}{2} \times 4.0\ \text{s} \times 8.0\ \text{m s}^{-1} = 16$ m
- 4–10 s: $6.0\ \text{s} \times 8.0\ \text{m s}^{-1} = 48$ m
- 10–12 s: $\frac{1}{2} \times 2.0\ \text{s} \times 8.0\ \text{m s}^{-1} = 8$ m
- total = 16 m + 48 m + 8 m = **72** m

3.1 A

- take up as positive: $u = 15$ m s^{-1}, $a = -9.81$ m s^{-2}, $t = 2.0$ s

$s = ut + \frac{1}{2}at^2 = (15\ \text{m s}^{-1})(2.0\ \text{s}) + \frac{1}{2}(-9.81\ \text{m s}^{-2})(2.0\ \text{s})^2 = 30\ \text{m} - 19.6\ \text{m} = 10.4\ \text{m}$

- closest to **10** m
- **B** forgot the $\frac{1}{2}at^2$ term; **C** is ut alone

3.2 B

- the area under an **acceleration–time** graph is the **change in velocity**
- **A** / **C** are the area under a **velocity–time** graph

3.3

(a) $u = 0$, $a = 0.50$ m s^{-2}, $t = 40$ s

$$v = u + at = 0 + (0.50 \text{ m s}^{-2})(40 \text{ s}) = \mathbf{20} \text{ m s}^{\mathbf{-1}}$$

(b) no v needed: $s = ut + \frac{1}{2}at^2$

$$s = \tfrac{1}{2}(0.50 \text{ m s}^{-2})(40 \text{ s})^2 = \mathbf{400} \text{ m}$$

(c) any one: the acceleration stays constant / the track is straight / no resistive forces

3.4

(a) vertical component $= u \sin\theta$

- $u_V = 20$ m s^{-1} $\times \sin 30° = 20 \times 0.50 = \mathbf{10}$ m s$^{\mathbf{-1}}$

(b) time to the top: $v_V = 0 = u_V - gt$

$$t_{\text{up}} = \frac{10 \text{ m s}^{-1}}{9.81 \text{ m s}^{-2}} = 1.02 \text{ s}$$

- total time in the air $= 2 \times 1.02$ s $= \mathbf{2.0}$ s

(c) $u_H = 20$ m s^{-1} $\times \cos 30° = 17.3$ m s^{-1}

- range $= u_H \times t = (17.3 \text{ m s}^{-1})(2.04 \text{ s}) = \mathbf{35}$ m

3.5

(a) same start time and positive direction

- car (constant v): $s = (18 \text{ m s}^{-1})t$
- motorcyclist (from rest): $s = \frac{1}{2}(3.0 \text{ m s}^{-2})t^2 = (1.5 \text{ m s}^{-2})t^2$
- they meet when the displacements are equal

$$(18 \text{ m s}^{-1})t = (1.5 \text{ m s}^{-2})t^2 \Rightarrow t = \frac{18 \text{ m s}^{-1}}{1.5 \text{ m s}^{-2}} = \mathbf{12} \text{ s}$$

(b) $s = (18 \text{ m s}^{-1})(12 \text{ s}) = \mathbf{216}$ m

(c) $v = at = (3.0 \text{ m s}^{-2})(12 \text{ s}) = \mathbf{36}$ m s$^{\mathbf{-1}}$

3.6

(a) acceleration is the **rate of change of velocity**

(b) a = gradient of the v–t graph

$$a = \frac{24 \text{ m s}^{-1} - 0}{8.0 \text{ s}} = \mathbf{3.0} \text{ m s}^{\mathbf{-2}}$$

(c) height = area under the line (a triangle)

$$s = \tfrac{1}{2} \times 8.0 \text{ s} \times 24 \text{ m s}^{-1} = \mathbf{96} \text{ m}$$

(d)

$$\Delta E_\text{P} = mgh = (1.8 \text{ kg})(9.81 \text{ m s}^{-2})(96 \text{ m}) = \mathbf{1.7 \times 10^3} \text{ J}$$

(e)

$$\Delta E_\text{K} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1.8 \text{ kg})(24 \text{ m s}^{-1})^2 = \mathbf{5.2 \times 10^2} \text{ J}$$

(f) useful energy gained = $\Delta E_\text{P} + \Delta E_\text{K}$

$$P = \frac{1.7 \times 10^3 \text{ J} + 5.2 \times 10^2 \text{ J}}{8.0 \text{ s}} = \mathbf{2.8 \times 10^2} \text{ W}$$

(g)

- axes drawn and labelled displacement / m (vertical) and time / s (horizontal)
- a curve starting at the origin
- getting **steeper** with time ($s \propto t^2$), reaching 96 m at 8.0 s

(h) **greater**

- the v–t graph is the same, so Δh and Δv are the same, but $\Delta E_\text{P} = mgh$ and $\Delta E_\text{K} = \frac{1}{2}mv^2$ both scale with **mass**, so more energy is gained in the same 8.0 s