A-LEVEL PHYSICS • EXERCISE SHEET

2.1.2 Velocity–time and acceleration–time graphs

Name: ______________ Class: __________ Date: ____________ **Total: 30 marks**

KEY WORDS velocity 速度 • acceleration 加速度 • displacement 位移 • change in velocity 速度变化 • gradient 斜率

Objective

Build the skills to answer exam questions on **velocity–time graphs** 速度 – 时间图像 and **acceleration–time graphs** 加速度 – 时间图像: an acceleration from a gradient, a displacement from an area, and one graph drawn from another. This sheet covers syllabus 2.1, learning outcomes 2, 3 and 5.

You must be able to:

- find an acceleration from the gradient of a velocity–time graph, including at one instant on a curve
- find a displacement from the area under a velocity–time graph, including area below the time axis
- tell distance from displacement on a velocity–time graph
- estimate the area under a curve by counting squares
- find a change in velocity from the area under an acceleration–time graph
- **sketch** one motion graph from another
- use **graphical methods** 图像法 to represent how speed, velocity and acceleration change with time

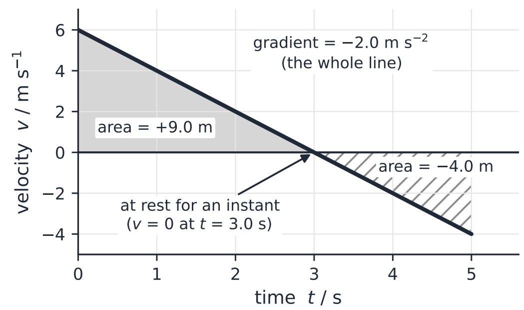
1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

■ Gradient and area of a velocity–time graph

On a velocity–time graph the **gradient** 斜率 is the **acceleration** 加速度, and the **area** 面积 between the line and the time axis is the **displacement** 位移. Area **below** the time axis is negative displacement: the object is moving backwards. The **distance** 距离 adds up the sizes of all the areas.

A trolley is pushed up a track at 6.0 m s^{-1}. It slows down, stops for an instant at $t = 3.0$ s, and rolls back.



- acceleration = gradient

$$a = \frac{\Delta v}{\Delta t} = \frac{-4.0 \text{ m s}^{-1} - 6.0 \text{ m s}^{-1}}{5.0 \text{ s}} = -2.0 \text{ m s}^{-2}$$

- the line is straight, so the acceleration is still -2.0 m s^{-2} at $t = 3.0$ s, when the velocity is zero
- area above the axis

$$\tfrac{1}{2} \times 3.0 \text{ s} \times 6.0 \text{ m s}^{-1} = +9.0 \text{ m}$$

- area below the axis

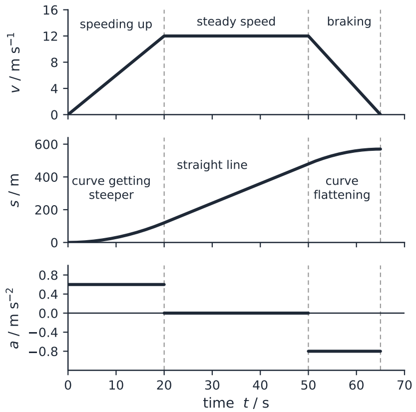
$$\tfrac{1}{2} \times 2.0 \text{ s} \times (-4.0 \text{ m s}^{-1}) = -4.0 \text{ m}$$

- displacement after 5.0 s: 9.0 m − 4.0 m = 5.0 m up the track; distance: 9.0 m + 4.0 m = 13 m

A negative gradient means the acceleration points in the negative direction. On an acceleration–time graph, the area under the line is the **change in velocity** 速度变化.

■ One journey, three graphs

A train starts from rest and speeds up uniformly to 12 m s^{-1} in 20 s. It keeps this speed for 30 s, then brakes uniformly to rest in 15 s.



- **acceleration–time**: each acceleration is a gradient of the velocity–time graph

$$a_1 = \frac{12\ \text{m s}^{-1}}{20\ \text{s}} = 0.60\ \text{m s}^{-2}, \qquad a_3 = \frac{-12\ \text{m s}^{-1}}{15\ \text{s}} = -0.80\ \text{m s}^{-2}$$

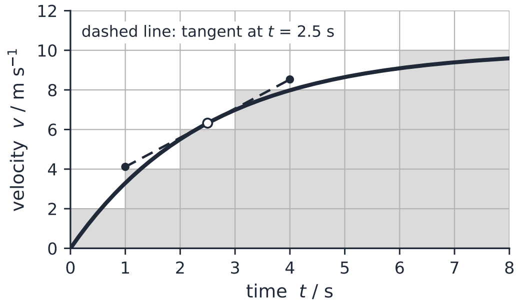
- while the speed is steady, $a = 0$
- **displacement–time**: its gradient is the velocity, so it gets steeper while the train speeds up, is a straight line while the speed is steady, and flattens to horizontal as the train stops
- total displacement = area under the velocity–time graph

$$s = \tfrac{1}{2}(20\ \text{s})(12\ \text{m s}^{-1}) + (30\ \text{s})(12\ \text{m s}^{-1}) + \tfrac{1}{2}(15\ \text{s})(12\ \text{m s}^{-1}) = 570\ \text{m}$$

When you **sketch** 画草图 a graph, draw and label both axes, and give each part the right shape: straight or curved, getting steeper or getting flatter.

■ A curve: a tangent and counting squares

When a velocity–time graph is a curve, the acceleration at one instant is the gradient of the **tangent** 切线 there, and the area is estimated by counting grid squares.



- the tangent at $t = 2.5$ s passes through $(1.0\ \text{s},\ 4.1\ \text{m s}^{-1})$ and $(4.0\ \text{s},\ 8.5\ \text{m s}^{-1})$

$$a = \frac{8.5\ \text{m s}^{-1} - 4.1\ \text{m s}^{-1}}{4.0\ \text{s} - 1.0\ \text{s}} = 1.5\ \text{m s}^{-2}$$

- one square is 1 s wide and 2 m s^{-1} tall, so it is worth

$$1\ \text{s} \times 2\ \text{m s}^{-1} = 2\ \text{m}$$

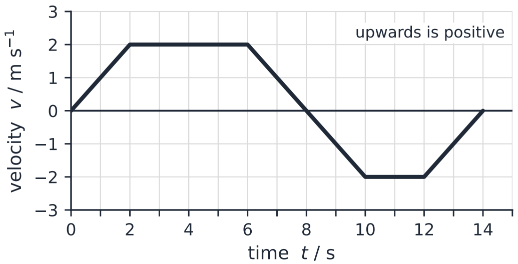
- count every square that is **at least half** under the curve: 28 squares
- displacement $\approx 28 \times 2\ \text{m} = 56\ \text{m}$

2 Practice

Now use the methods above.

2.1 State what is represented by (a) the gradient of a velocity–time graph; (b) the area under a velocity–time graph. [2]

2.2 The velocity–time graph shows a lift that goes up and then comes back down. Upwards is positive.



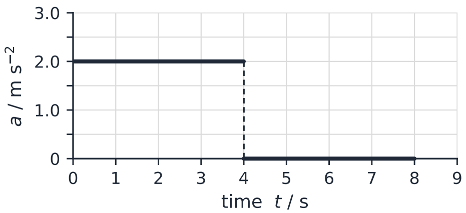
(a) Calculate the acceleration of the lift between $t = 0$ and $t = 2.0$ s. [1]

(b) State the times at which the lift is at rest. [1]

(c) Calculate the total distance travelled by the lift in the 14 s. *(Hint: for distance, every area counts as positive.)* [2]

(d) Determine the displacement of the lift from its starting point at $t = 14$ s. [1]

2.3 A car is moving at 5.0 m s^{-1} at time $t = 0$. The graph shows its acceleration.

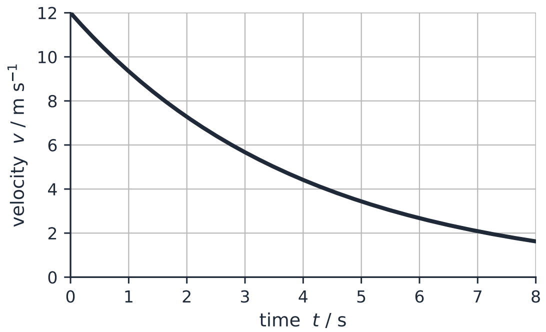


(a) Determine the change in velocity of the car between $t = 0$ and $t = 4.0$ s. [1]

(b) State the velocity of the car at $t = 6.0$ s. [1]

(c) **Sketch** the velocity–time graph of the car from $t = 0$ to $t = 8.0$ s. Label both axes and mark the values of velocity. [2]

2.4 A cyclist stops pedalling and slows down. The graph shows the velocity of the cyclist.

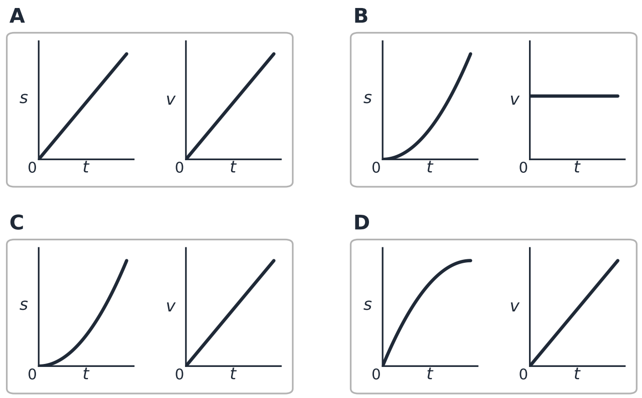


(a) By drawing a tangent, determine the acceleration of the cyclist at $t = 4.0$ s. [2]

(b) Estimate the distance travelled by the cyclist from $t = 0$ to $t = 8.0$ s by counting squares. [2]

3 Exam-style questions

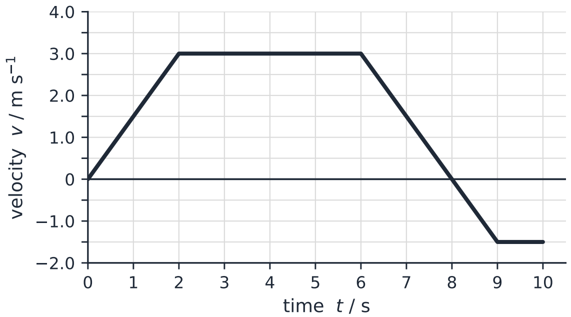
3.1 An object starts from rest and moves in a straight line. Which pair of graphs could show how its displacement s and its velocity v change with time t? [1]



3.2 The velocity–time graph of an object is a straight line from 8.0 m s^{-1} at $t = 0$ to -4.0 m s^{-1} at $t = 6.0$ s. What is the displacement of the object after 6.0 s? [1]

A 12 m
B 20 m
C 24 m
D 36 m

3.3 A remote-controlled toy car moves along a straight track. A **motion sensor** 运动传感器 records its velocity v. The graph shows how v varies with time t.



(a) **Define** acceleration. [1]

(b) **Compare** the acceleration of the car at $t = 1.0$ s with its acceleration at $t = 7.0$ s, in terms of magnitude and direction. [2]

(c) The car is at rest for an instant at $t = 8.0$ s. Determine the acceleration of the car at this instant. [2]

(d) Determine the greatest displacement of the car from its starting point, and the time at which it happens. [3]

(e) Determine the displacement of the car from its starting point at $t = 10.0$ s. [2]

(f) **Sketch** the variation with time of the displacement of the car from $t = 0$ to $t = 10.0$ s. Label both axes and mark the greatest displacement. [3]