

19.3 Discharging a capacitor

Name: _____ Class: _____ Date: _____

Total: 29 marks

KEY WORDS time constant 时间常数 • exponential decay 指数衰减 • resistor 电阻器
• resistance 电阻 • capacitor 电容器

Objective

Build the skills to answer exam questions on a **capacitor discharging** through a resistor.

You must be able to:

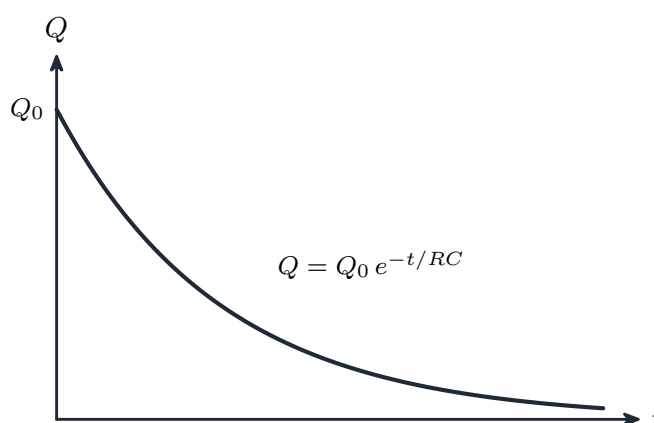
- use the **time constant** $\tau = RC$
- use $x = x_0 e^{-t/RC}$ for charge, p.d. or current
- interpret the discharge graphs

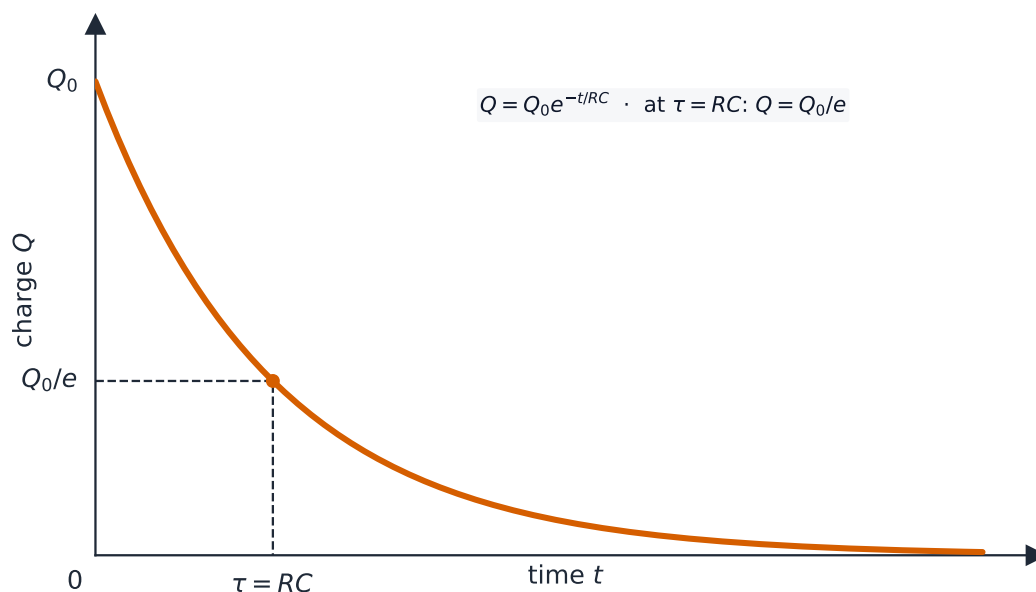
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Exponential decay

When a charged capacitor discharges through a resistor, the charge (and V , I) decays **exponentially**:





Exponential discharge of a capacitor

$$Q = Q_0 e^{-t/RC}, \quad V = V_0 e^{-t/RC}, \quad I = I_0 e^{-t/RC}.$$

The **time constant** $\tau = RC$ is the time for the charge to fall to $\frac{1}{e} \approx 37\%$ of its initial value.

Worked example. $470 \mu\text{F}$ through $2.0 \text{ k}\Omega$: $\tau = RC = (2000)(470 \times 10^{-6}) = 0.94 \text{ s}$.

■ Finding the time — take logs

To find the **time** for the charge (or V , or I) to fall to a given value, take the **natural log** of both sides of $V = V_0 e^{-t/RC}$:

$$\ln \frac{V}{V_0} = -\frac{t}{RC} \quad \Rightarrow \quad t = -RC \ln \frac{V}{V_0}.$$

Worked example. For the capacitor above (charged to 6.0 V , $\tau = 0.94 \text{ s}$), the time for the p.d. to fall to 2.0 V is $t = -0.94 \ln \frac{2.0}{6.0} = -0.94 \times (-1.10) = 1.0 \text{ s}$.

2 Practice

Now apply the methods above.

2.1 Write the formula for the **time constant**.

[1]

2.2 A $100\ \mu\text{F}$ capacitor discharges through a $10\ \text{k}\Omega$ resistor. Find the **time constant**. [2]

2.3 State what happens to the **charge** as a capacitor discharges. [1]

2.4 Write the discharge equation for **charge**. [1]

3 Exam-style questions

3.1 The time constant RC is the time for the charge to fall to: [1]

A	B
C	D

A zero

B half its initial value

C $1/e$ (about 37%) of its initial value

D $1/2e$ of its initial value

3.2 A $470\ \mu\text{F}$ capacitor charged to $6.0\ \text{V}$ discharges through a $2.0\ \text{k}\Omega$ resistor. Calculate (a) the time constant, (b) the p.d. after $1.0\ \text{s}$. [4]

3.3 Sketch a graph of **charge against time** for a discharging capacitor. [2]

3.4 State what happens to the time constant if the **resistance** is doubled. [1]

3.5 A $220\ \mu\text{F}$ capacitor charged to $9.0\ \text{V}$ discharges through a $15\ \text{k}\Omega$ resistor. Calculate the **time** for the p.d. to fall to $3.0\ \text{V}$. [4]

3.6 A capacitor C is charged and then discharged through a resistor R . A data logger records the p.d. V_R across the resistor and the p.d. V_C across the capacitor. At $t = 0$ the p.d. across the capacitor is 9.0 V, and the initial current is 1.8 mA. The p.d. across the capacitor falls to 3.3 V after 12 s.

(a) **State** the relationship between V_C and V_R during the discharge, and give a reason. [2]

(b) **Determine** the resistance R . [2]

(c) **Determine** the time constant τ of the circuit. [3]

(d) Hence **determine** the capacitance C , in μF . [2]

(e) **Explain**, in terms of the current in the circuit, why the decay of the charge on the capacitor is **exponential**. [3]
