Solutions

Each answer shows the steps, then the **result**.

2.1

- $\tau = RC$

2.2

- $\tau = RC$

$$\tau = (10 \times 10^3\ \Omega)(100 \times 10^{-6}\ \text{F}) = \mathbf{1.0}\ \text{s}$$

2.3

- it **decreases exponentially** towards zero

2.4

- $Q = Q_0 e^{-t/RC}$

3.1 C

- after one time constant, $Q = Q_0/e$ (about 37%)

3.2

(a) $\tau = RC$

$$\tau = (2.0 \times 10^3\ \Omega)(470 \times 10^{-6}\ \text{F}) = \mathbf{0.94}\ \text{s}$$

(b) $V = V_0 e^{-t/RC}$

$$V = (6.0\ \text{V})\, e^{-1.0/0.94} = (6.0\ \text{V})(0.345) = \mathbf{2.1}\ \text{V}$$

3.3

- a curve starting at the initial charge and **decaying exponentially** towards zero
- with the same shape (never quite reaching zero)

3.4

- it **doubles** ($\tau = RC$)

3.5

- $\tau = RC$

$$\tau = (15 \times 10^3\ \Omega)(220 \times 10^{-6}\ \text{F}) = 3.3\ \text{s}$$

- $t = -RC \ln(V/V_0)$

$$t = -3.3\ \text{s} \times \ln \frac{3.0\ \text{V}}{9.0\ \text{V}} = -3.3\ \text{s} \times (-1.10) = \mathbf{3.6}\ \text{s}$$

3.6

(a) they are **equal** at every instant, $V_C = V_R$

- the capacitor and resistor form a single loop with no other source, so the p.d. driving the current is the capacitor's own p.d.

(b) $R = V/I$

$$R = \frac{9.0\ \text{V}}{1.8 \times 10^{-3}\ \text{A}} = \mathbf{5.0 \times 10^3}\ \Omega$$

(c) $V = V_0 e^{-t/\tau}$, so $3.3/9.0 = e^{-12/\tau}$

$$\ln(0.367) = -\frac{12\ \text{s}}{\tau} \Rightarrow \tau = \mathbf{12}\ \text{s}$$

- the p.d. has fallen to $1/e$ of its start, which is what the time constant means

(d) $\tau = RC$, so $C = \tau/R$

$$C = \frac{12\ \text{s}}{5.0 \times 10^3\ \Omega} = 2.4 \times 10^{-3}\ \text{F} = \mathbf{2.4 \times 10^3}\ \mu\text{F}$$

(e) the current is $I = V_C/R$, and $V_C \propto Q$, so the current — the rate at which charge leaves — is **proportional to the charge still on the capacitor**

- a quantity whose rate of decrease is proportional to itself decays exponentially
- so as the charge falls the current falls with it, the discharge slows, and Q never quite reaches zero