Solutions

Each answer shows the steps, then the **result**.

2.1

- energy is continually transferred between **kinetic** and **potential** forms
- with no damping the **total** stays constant

2.2

- KE is maximum at the **equilibrium** ($x = 0$)
- PE is maximum at the **extremes** ($x = \pm x_0$)

2.3

- $E = \frac{1}{2}m\omega^2 x_0^2$

3.1 B

- $E \propto x_0^2$, so energy is proportional to the **amplitude squared**

3.2

- $E = \frac{1}{2}m\omega^2 x_0^2$

$$E = \tfrac{1}{2}(0.20\text{ kg})(10\text{ rad s}^{-1})^2(0.050\text{ m})^2 = \mathbf{0.025}\text{ J}$$

3.3

- KE: a **downward** parabola, maximum at $x = 0$, zero at $\pm x_0$
- PE: an **upward** parabola, zero at $x = 0$, maximum at $\pm x_0$; their sum is constant

3.4

- it becomes **four times** larger ($E \propto x_0^2$)

3.5

(a) the KE varies at **twice** the oscillation frequency (the object passes the centre twice per cycle)

- so the oscillation period is **twice** the KE-graph period: $T = 2 \times 0.40\text{ s} = \mathbf{0.80}\text{ s}$

(b) $\omega = 2\pi/T$

$$\omega = \frac{2\pi}{0.80\text{ s}} = \mathbf{7.9}\text{ rad s}^{\mathbf{-1}}$$

3.6

(a) the kinetic energy is zero at **both** ends of the swing and maximum at the centre twice per cycle, so the energy curve repeats **twice** per oscillation

- the period of the mass is therefore $2 \times 0.40\text{ s} = 0.80\text{ s}$

(b) $\omega = 2\pi/T$

$$\omega = \frac{2\pi}{0.80\text{ s}} = \mathbf{7.9}\text{ rad s}^{\mathbf{-1}}$$

(c) the total energy equals the maximum kinetic energy, $E = \frac{1}{2}m\omega^2 x_0^2$

$$E = \tfrac{1}{2}(0.25\ \text{kg})(7.85\ \text{rad s}^{-1})^2(0.040\ \text{m})^2 = \mathbf{1.2 \times 10^{-2}}\ \text{J}$$

(d) $v = \omega\sqrt{x_0^2 - x^2}$

$$v = (7.85\ \text{rad s}^{-1})\sqrt{(0.040\ \text{m})^2 - (0.020\ \text{m})^2} = (7.85\ \text{rad s}^{-1})(0.0346\ \text{m}) = \mathbf{0.27}\ \text{m s}^{\mathbf{-1}}$$

(e) light damping barely changes the period, so the time between maxima stays about 0.40 s

- the maxima get **lower** with time
- because the amplitude decays and the total energy is proportional to the **square** of the amplitude, so the energy falls faster than the amplitude does