

17.1 Simple harmonic oscillations

Name: _____ Class: _____ Date: _____

Total: 34 marks

KEY WORDS simple harmonic motion 简谐运动 • acceleration 加速度 • displacement 位移
• period 周期 • frequency 频率

Objective

Build the skills to answer exam questions on **simple harmonic motion** — the defining equation, the solutions, and the graphs.

You must be able to:

- use the SHM condition $a = -\omega^2 x$ and $x = x_0 \sin \omega t$
- use $v = \pm \omega \sqrt{x_0^2 - x^2}$ and $v_{\max} = \omega x_0$
- interpret the displacement, velocity and acceleration graphs

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

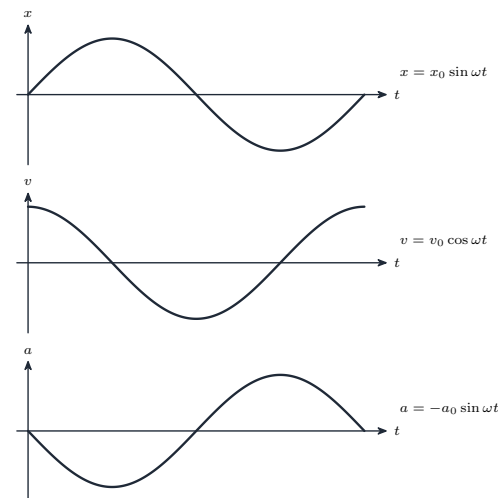
■ What SHM is

A particle moves with **SHM** when its acceleration is **proportional to displacement** and **directed back** to equilibrium:

$$a = -\omega^2 x, \quad x = x_0 \sin \omega t, \quad T = \frac{2\pi}{\omega}.$$

■ Velocity and the graphs

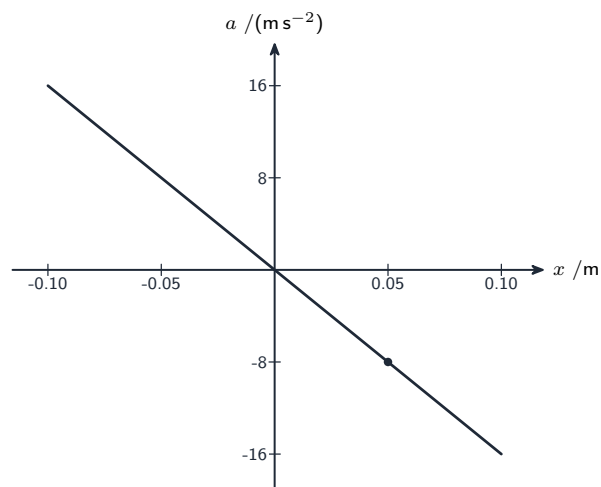
$$v = v_0 \cos \omega t, \quad v = \pm \omega \sqrt{x_0^2 - x^2}, \quad v_{\max} = \omega x_0, \quad a_{\max} = \omega^2 x_0.$$



Displacement and acceleration are **antiphase**; velocity leads displacement by a **quarter cycle**.

■ The acceleration–displacement graph

Because $a = -\omega^2 x$, a graph of a against x is a **straight line through the origin** with a **negative gradient** of magnitude ω^2 :



This is the **test for SHM**: $a \propto x$ (straight line through the origin) and **opposite in direction** (negative gradient). Read the gradient to find ω : gradient $= -\omega^2$, so $\omega = \sqrt{|\text{gradient}|}$.

Worked example. The line above passes through $(0.05 \text{ m}, -8 \text{ m s}^{-2})$, so gradient $= -8/0.05 = -160 \text{ s}^{-2} = -\omega^2$; thus $\omega = \sqrt{160} = 12.6 \text{ rad s}^{-1}$ and $T = 2\pi/\omega = 0.50 \text{ s}$.

2 Practice

Now apply the methods above.

2.1 State the **two** conditions for simple harmonic motion. [2]

2.2 Define the **amplitude** of an oscillation. [1]

2.3 A particle has $\omega = 5.0 \text{ rad s}^{-1}$ and amplitude 0.040 m . Find its **maximum speed**. [2]

2.4 State the **phase relationship** between displacement and acceleration in SHM. [1]

3 Exam-style questions

3.1 In simple harmonic motion, the acceleration is: [1]

- | | |
|---|---|
| A | B |
| C | D |
- A** proportional to displacement and in the same direction
B proportional to displacement and in the opposite direction
C constant
D always zero

3.2 A mass on a spring oscillates with period 0.50 s and amplitude 0.030 m . Find (a) the angular frequency, (b) the maximum speed, (c) the maximum acceleration. [4]

3.3 A particle in SHM has amplitude 0.10 m and $\omega = 8.0 \text{ rad s}^{-1}$. Find its **speed** when $x = 0.060 \text{ m}$. [3]

3.4 Sketch the **displacement–time** and **acceleration–time** graphs for SHM, and state their phase relationship. [3]

3.5 An object oscillates. A graph of its acceleration a against displacement x is a straight line through the origin passing through $(0.05 \text{ m}, -8.0 \text{ m s}^{-2})$.

(a) Explain how this graph shows the motion is **simple harmonic**. [2]

(b) Use the **gradient** to find the angular frequency ω and the period T . [3]

3.6 A small ball hangs on a spring and is set oscillating vertically. A motion sensor records its displacement, and the graph obtained is a smooth curve whose successive maximum displacements are equal, with a maximum displacement of x_0 and 8 complete oscillations in 6.4 s.

(a) A second graph plots the ball's **acceleration** against its **displacement** and gives a **straight line through the origin with a negative gradient**. Explain how this shows that the motion is simple harmonic. [2]

(b) **Determine** the period T and the angular frequency ω of the oscillations. [3]

(c) The magnitude of the gradient of that acceleration-displacement line is 61 s^{-2} . **Show that** this is consistent with your value of ω . [2]

(d) **State** what is meant by **damping**. [2]

(e) The ball is now made to oscillate in a beaker of oil, and is released from the same displacement x_0 . On the axes below, **sketch** a possible variation of its displacement with time from $t = 0$ to $t = 2T$. [3]

